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To cite this article: A Pitna *et al* 2023 *J. Phys.: Conf. Ser.* **2544** 012009

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Transmission of magnetic island modes across interplanetary shocks: comparison of theory and observations

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Abstract. Interplanetary shock waves are observed frequently in turbulent solar wind. They naturally enhance the temperature/entropy of the plasma through which they propagate. Moreover, many studies have shown that they also act as an amplifier of the fluctuations incident on the shock front. Solar wind turbulent fluctuations can be well described as the superposition of quasi-2D and slab components, the former being energetically dominant. In this paper, we address the interaction of fast forward shocks observed by the Wind spacecraft at 1 AU and quasi-2D turbulent fluctuations in the framework of the Zank et al. (2021) transmission model and we compare model predictions with observations. Our statistical study includes 378 shocks with varying upstream conditions and Mach numbers. We estimate the average ratio of the downstream observed and theoretically predicted power spectra within the inertial range of turbulence. We find that the distributions of this ratio for the whole set and for the subset of shocks that met the assumptions of the model, are remarkably close. We argue that a large statistical spread of the distributions of this ratio is governed by the inherent variation of the upstream conditions. Our findings suggest that the model predicts the downstream fluctuations with a good accuracy and that it may be adopted for a wider class of shocks than it was originally meant for.

1. Introduction

One of the outstanding open question of heliospheric physics that has recently attracted attention is the interaction of magnetohydrodynamic (MHD) turbulence and interplanetary (IP) shocks. The difficulty in solving this question lies in the complexity of the interaction, i.e., the character of the incident fluctuations, upstream bulk plasma parameters, the type of an IP shock, its obliquity, Mach number, compression ratio, shock evolution, etc., since all of these factors influence the resulting downstream state of the plasma. Thus, any dedicated study can only address a part of the questions.

Knowledge of how the upstream fluctuations are transmitted through the IP shock front provide insight into particle acceleration mechanisms, since the inhomogeneities in the upstream and downstream plasma play a crucial role in such processes [1]. The coupling of the solar wind plasma and Earth's magnetosphere is dependent on the level of the magnetic field fluctuations [2],



thus, the enhanced downstream fluctuations may intensify this coupling, but a full understanding of the underlying physical mechanisms is missing.

It has been known for decades that IP shocks increase the levels of the incident/upstream fluctuations downstream [see e.g., review [3] and the references therein]. Most studies analyzed magnetic field fluctuations ([4, 5, 6, 7, 8, 9]). Recent investigations focus on the transmission of density [10], solar wind bulk velocity [9] and entropy fluctuations [11]. The transmission of Alfvén waves was studied from a theoretical perspective by McKenzie and Westphal [12]. Adhikari et al. (2016) [7] studied the transport of incompressible turbulent fluctuations across quasi-parallel and quasi-perpendicular IP shocks using an incompressible MHD formulation of a turbulent transport model [13]. They reported that the turbulent energy increases across the shock and that the correlation length is approximately conserved across the shock.

Zank et al. (2021) [11] (hereafter Zank21) introduced a novel framework in which the transmission of MHD turbulent fluctuations can be straightforwardly investigated. The formulation was motivated by the observations that (1) the solar wind turbulent fluctuations can be successfully described as a superposition of an energetically dominant 2D and a minority slab component [14, 15] and (2) the majority of fast forward (FF) IP shocks are quasi-perpendicular [16], i.e., the angle between the shock normal, $\mathbf{n}$, and upstream magnetic field, $\mathbf{B}_u$, θ_{Bn} , is larger than 45° . The model describes the linear interaction of incident vortical, magnetic island, entropy and acoustic normal modes with planar quasi-perpendicular shock. It assumes (1) a high upstream plasma beta, $\beta_p^u \gtrsim 1$, (the ratio of the plasma and magnetic pressure) and (2) strong magnetic field in a sense that the magnetic fluctuations, δB , are of the same order as the average B , $\delta B/B \approx 1$, (3) that the incident plasma velocity vector lies in the plane perpendicular to the average upstream magnetic field, and (4) wave vectors of the fluctuations are constrained in the same plane. Finally, Zank21 derived a set of linearized higher-order jump conditions for the velocity, density, pressure and magnetic field fluctuations. The jump equations for the magnetic field fluctuations read as

$$[U_x \delta B_y] = 0; \quad (1)$$

$$[\delta B_x] = 0, \quad (2)$$

where square brackets denote the difference of the quantity from upstream to downstream, U_x is the normal component of upstream velocity in the shock frame, δB_x and δB_y are the normal and perpendicular components of the magnetic field fluctuation vector $\delta \mathbf{B}$. These equations are fully decoupled from the gas dynamic counterparts and thus can be solved separately.

Since the wave vector of the magnetic island mode is perpendicular to the average $\mathbf{B}$, this mode is non-propagating and it characterizes magnetic island/flux rope phenomenon [17, 18]. Adopting the NI turbulence approximation, magnetic field fluctuations in the upstream solar wind plasma can be viewed as a nonlinearly interacting set of the energetically dominant magnetic island modes. Solving eqs. (1) and (2) for incident magnetic island modes with upstream wave number k_1 and amplitude δB_1 yields the following solutions for the transmitted wave number k_2 and amplitude δB_2 ,

$$\tan \theta_2 = R^{-1} \tan \theta_1; \quad (3)$$

$$k_2 = R \frac{\cos \theta_1}{\cos \theta_2} k_1; \quad (4)$$

$$\delta B_2 = R \frac{\cos \theta_1}{\cos \theta_2} \delta B_1, \quad (5)$$

where θ_1 and θ_2 denote the upstream and downstream angles between the corresponding wave vector and shock normal, respectively, and R is the shock compression ratio.

In this paper, we analyze *in situ* measurements of a large set (378) of IP shocks observed by the Wind spacecraft with the aim of testing the theoretical basis for the transmission of MHD fluctuations introduced by Zank21. We analyze one hour intervals in both upstream and downstream regions. By employing a continuous wavelet transform (CWT), we estimate the trace power spectral densities (PSDs) of the magnetic field fluctuations. Based on the upstream PSDs of turbulent fluctuations, we estimate the theoretically predicted downstream PSDs and compare them with the observed downstream PSDs. We compute the average ratio of the observed and theoretically predicted PSDs within the inertial range for each case and we show that for a subset of IP shocks with high plasma beta, $\beta_p^u > 2$ and large obliquities, $\theta_{Bn} > 70^\circ$, the median value of this ratio is ~ 0.73 . We argue that the spread of the distribution of this ratio is given mainly by the variable upstream conditions and not due to the uncertainty of the model. Furthermore, we suggest that the Zank21 model can be useful also for a wider range of IP shocks than it was originally derived for.

2. Data analysis

Our statistical study is based on the IP shock observations made by the Wind spacecraft. We utilized the Heliospheric Shock Database generated at the University of Helsinki (<http://ipshocks.fi/database>) and we analyzed data from the Solar Wind Experiment (SWE) and Magnetic Field Investigation (MFI) instruments on board Wind [19, 20]. Our set includes IP shocks detected between December 1994 and May 2017. We excluded IP shocks that contain a large number of data gaps in both upstream and downstream one hour intervals (with less than 90% data coverage for MFI and 70% for SWE). The final data set includes 378 fast forward (FF) IP shocks. In Figure 1, we show the scatter plot of the angle, θ_{Bn} vs. upstream plasma beta, β_p^u for the whole statistical set. Additionally, we define a subset (52 cases) of highly perpendicular IP shocks with large upstream beta, i.e., $\theta_{Bn} > 70^\circ$ and $\beta_p^u > 2$, marked by the green diamonds. For the shocks in this subset, we calculate the level of the normalized magnetic field fluctuations within one hour, $\delta B/B_0$, in the upstream and downstream regions. Their mutual relation is shown in 2.

To determine the magnetic field power spectra in each one hour interval, we employ the widely used CWT [21, 22]. We estimate the trace power spectrum, P_B , and we denote the upstream and downstream PSDs as $P_B^u = P_B^u(k)$ and $P_B^d = P_B^d(k)$, respectively, where k denotes the wave number. We employed the Taylor hypothesis, i.e., we estimate k from the spacecraft frame frequencies f as $k = 2\pi f/v_{sw}$, where v_{sw} denotes the average solar wind speed within the interval.

In calculations of the theoretical spectra, we used the same methodology as Zank21. We assumed that the underlying 2D upstream power spectrum is isotropic. We can then calculate the source terms for the upstream magnetic field fluctuations and we can use eqs. (3) to (5) to determine the downstream angle of a wave vector with respect to the shock normal, downstream wave number and magnetic field fluctuation amplitude. We estimate the power spectrum of the downstream fluctuations in k - θ_2 space and we integrate over θ_2 in order to obtain the downstream theoretical omni-directional spectrum, $P_B^{d,Th}$.

3. Comparison of experimental and theoretical PSDs

In Figure 3, we present an example of experimental upstream/downstream power spectra and theoretical downstream power spectra for an IP shock observed by the Wind spacecraft on May 7, 2007 at 07:03 UT. We see that the agreement between P_B^d and $P_B^{d,Th}$ in the inertial range is very good. We quantify the similarity between the observed and theoretically predicted spectrum by calculating the average ratio between these spectra in a wave number range $k \in \langle 10^{-4}, 10^{-3} \rangle \text{ km}^{-1}$, denoted as $R_{OT} = \langle P_B^d/P_B^{d,Th} \rangle$. We choose this range because it

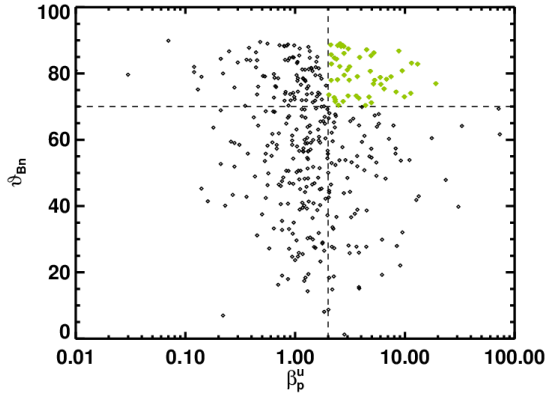


Figure 1. Diamonds show the variation of angle θ_{Bn} between the shock normal and upstream magnetic field as a function of upstream proton plasma β_p^u for the whole set of 378 IP shocks. Green diamonds identify the sub-sample with $\beta_p^u > 2$ and $\theta_{Bn} > 70^\circ$.

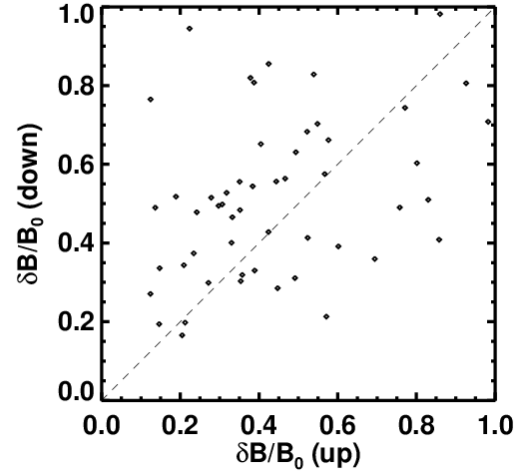


Figure 2. Scatter plot of downstream normalized magnetic field fluctuations versus the corresponding upstream values for the subset.

avoids PSD values that either (1) exhibit large errors due to statistical under-sampling for low k and (2) are sufficiently above the characteristic kinetic scales (high k).

We estimate R_{OT} for each case in the subset of quasi-perpendicular high beta IP shocks. Figure 4a shows the distribution of R_{OT} . The median value of the distribution is $0.73^{+0.05}_{-0.09}$, signifying that the theoretical PSDs exhibit slightly higher values than the corresponding observed PSDs. On the other hand, the distribution itself is highly non-gaussian with a large variance. In order to quantify the spread of the distribution, we employ an estimator that is less sensitive to outliers, a so-called central confidence interval factor (CIF) [23], which is defined as the ratio of the upper and lower values of the central confidence interval (CI). We choose a CI that contains 68.3% of all values. The calculation yields $CIF(R_{OT}) = 6.7^{+2.8}_{-0.9}$. Roughly, this number can be viewed as an 'uncertainty factor' between the observed and modeled quantity. One can say that the model differs from the observation by a factor of $CIF(R_{OT})/2$, on average. Three main factors that govern the value of CIF are (1) the model uncertainty, (2) uncertainty in the levels of upstream fluctuations and (3) uncertainty in compression ratio. We address these aspects in the following paragraphs.

Addressing the model accuracy, we inspect the dependence of upstream β_p^u , θ_{Bn} and $\delta B/B_0$ as a function of R_{OT} , in Figures 4b, 4c and 4d, respectively. The expectation from the Zank21 theory would be that for high β_p^u , high θ_{Bn} and/or large $\delta B/B_0$, the values of R_{OT} should cluster around 1 and for lower β_p^u , θ_{Bn} and $\delta B/B_0$, one would expect a deviation from unity. However, we see no clearly distinguishable trend in these figures. Due to the absence of the anticipated deviations in R_{OT} , we analyze all cases (378) in the same manner.

We estimate R_{OT} for the whole statistical sample and we plot its distribution in Figure 5a. We see that the parameters of the distribution, $median(R_{OT}) = 0.63^{+0.06}_{-0.07}$ and $CIF(R_{OT}) = 8.0^{+0.9}_{-0.7}$, are remarkably close to those of the sub-sample. It is reasonable to expect that the median value of the whole sample should differ from the median value of the sub-sample which satisfies the assumptions of the theory, and moreover, the CIF should exhibit larger values for the whole

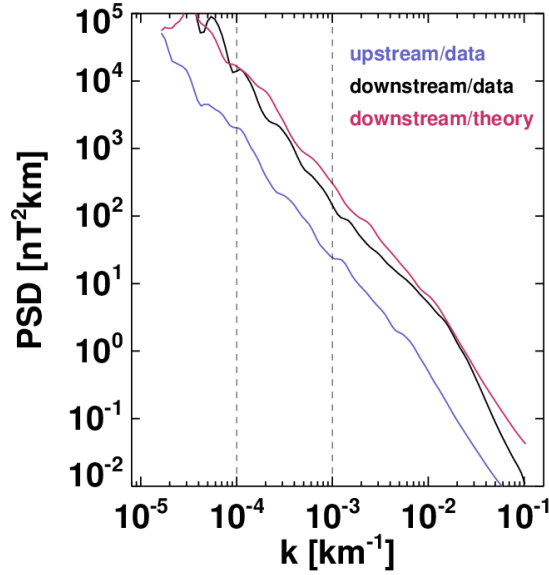


Figure 3. Magnetic field power spectra upstream (blue) and downstream (black) of IP shock front. The red curve shows the theoretically predicted power spectrum. Dashed vertical lines mark the wave number range within which the average of the ratio between the observed and theoretically predicted downstream spectra are computed. For this case, $R_{OT} = 0.65$.

statistical set. We observe this trend, but the uncertainties¹ prohibit a firm conclusion about its significance.

The dependencies of R_{OT} on β_p^u and θ_{Bn} for the whole set show no correlations between these quantities, as Figures 5b and 5d demonstrate. On the other hand, Figure 5d shows $\delta B/B_0$ as a function of R_{OT} and we see that for $\delta B/B_0 \lesssim 0.2$, R_{OT} tends to be higher than 1, which suggests that the theory is not applicable for very low levels of upstream fluctuations. This is not surprising, since it is one of the assumptions of the theory. However, we have now obtained an empirical value for a lower acceptable limit of $\delta B/B_0$.

We also investigate the uncertainty in P_B^u . The "uncertainties" in the levels of upstream fluctuations can be understood in the following sense: the original levels of fluctuations that were transmitted through the shock front and which give rise to the downstream observed fluctuations (P_B^d) are not the same as the fluctuations actually measured in the upstream region later (P_B^u). Let us denote the PSD of these fluctuations as $P_B^{u,ori}$. The key question that has to be addressed is: what is the distribution of the ratio $\langle P_B^{u,ori}/P_B^u \rangle$? It is obviously impossible to estimate this ratio from single point measurements for any particular IP shock. However, an excellent proxy for this distribution is statistics of the ratio of PSDs of two consecutive 1-hour intervals in the pristine solar wind.

Since R_{OT} is defined as the average in the specific k -range, we estimate the relevant

¹ In the estimation of the confidence intervals for median and CIF, we employ a bootstrap re-sampling method [24].

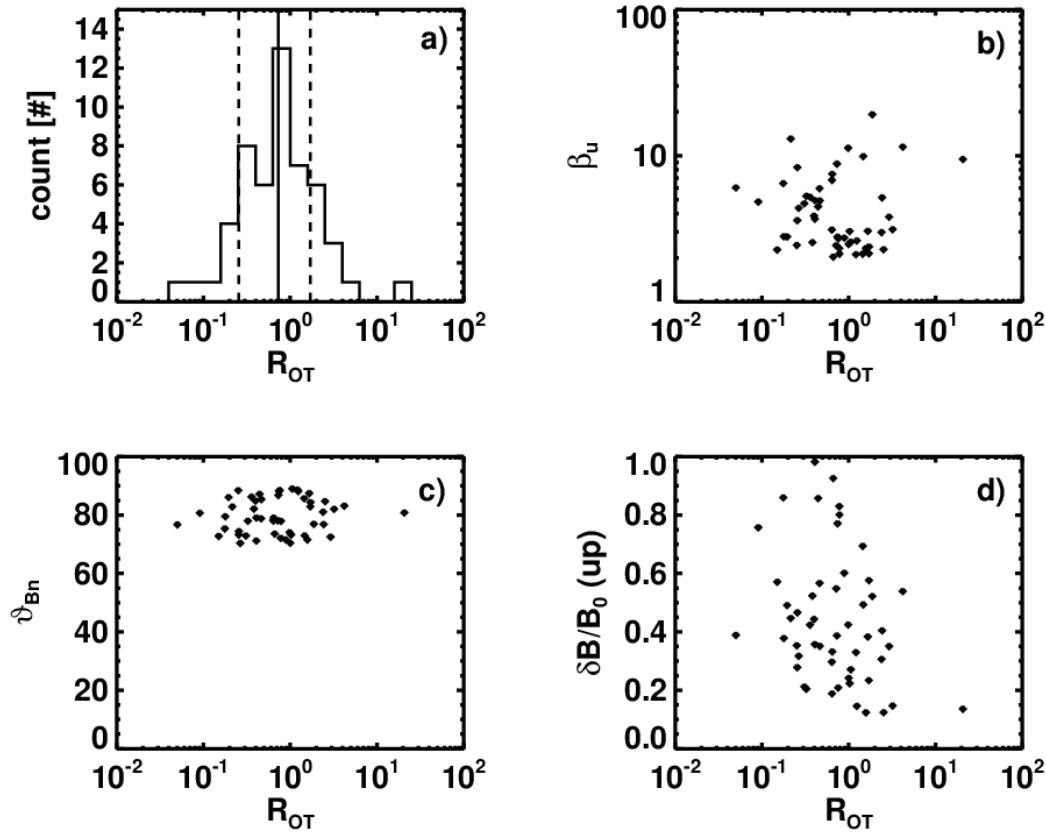


Figure 4. (a) Distribution of R_{OT} for the high proton plasma beta quasi- $\perp$ subset. Dashed vertical lines identify the central confident interval containing 68.3 % of data. Scatter plots on panels (b), (c) and (d) show the variation of β_p^u , θ_{Bn} and upstream $\delta B/B_0$ versus R_{OT} , respectively.

distribution as it follows: (1) We select the year of 2015 and estimate PSDs within all one-hour non-overlapping intervals. After applying the same selection criteria for the intervals (excluding intervals with large portions of data gaps), we obtain 6477 cases (out of 8760 possible), (2) We calculate the average ratio of PSDs, R_{PSDB} , between all pairs of two consecutive intervals. The resulting distribution of R_{PSDB} is shown in Figure 6. Its median value is essentially one and its $CIF = 3.73^{+0.05}_{-0.09}$, which is less than $CIF(R_{OT})$ for the subset, as might be expected. On the basis of this CIF value, we assert that roughly 50 % of the width of the distribution of R_{OT} (in terms of CIF) is given by the variability of the upstream plasma.

4. Discussion and conclusions

In this paper, we investigated the transmission of MHD scale fluctuations using the theoretical approach introduced by Zank21. Zank21 showed that for three particular cases observed by Wind, Ulysses and Voyager 2 at 1, 5 and 84 AU, respectively, the theoretically predicted power spectra match the observed counterparts very well. Here, we extended their study and we apply their methodology to a larger set of IP shocks observed by Wind at L1.

Perhaps the most surprising result is that the distributions of the ratios of observed and theoretically predicted PSD for the whole statistical set and the sub-sample (high beta, quasi-

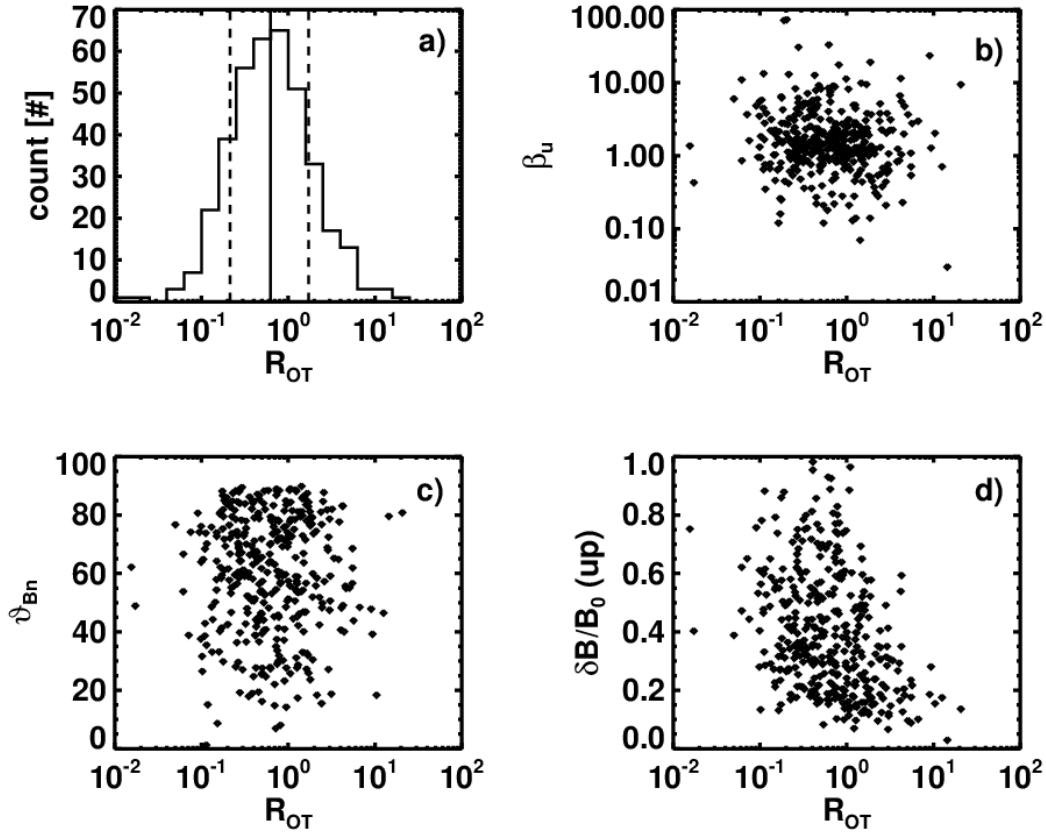


Figure 5. Distribution of R_{OT} for the whole statistical sample and the corresponding scatter plots. The format is the same as in Figure 4.

$\perp$ IP shocks) are remarkably similar. When we realize that 2D/magnetic island fluctuations dominate the turbulent energy budget in the solar wind, and majority of IP shocks exhibit quasi-perpendicular geometry, it can be then anticipated that we observe only small differences between the distributions. The distribution of R_{PSDB} demonstrates that the largest factor that influences the distribution of R_{OT} is the variable level of upstream fluctuations over the scales of interest. Moreover, the compression ratio plays a pivotal role in the model and any uncertainty in its estimation translates into uncertainty of the estimated R_{OT} . A quantitative assessment of the interplay between model and data uncertainties is not straightforward. However, a comparison of the CIFs of R_{PSDB} and R_{OT} suggests that the uncertainty is rather small.

The most obvious way to correct for the variability of the upstream solar wind conditions is to utilize multi-point spacecraft observations of the same IP shock. This can be achieved by either (1) radial alignment of sufficiently separated spacecraft (see, e.g., [25, 26]) or (2) by exploiting a standing bow shock (BS) (e.g., the terrestrial BS), where concurrent upstream and downstream measurements are commonplace (see, e.g., [27]). In both cases, the information about the incident fluctuations will be available and the source terms that enter the calculation (eqs. (3) to (5)) will be known with greater accuracy.

Numerical simulations of IP shocks propagating into magnetized turbulent plasma can provide a direct test of any turbulent transmission model. Indeed, Nakanotani et al. (2022) [28] ran a 2D hybrid kinetic simulation of a moderately strong shock (sonic Mach number of 2.4) propagating

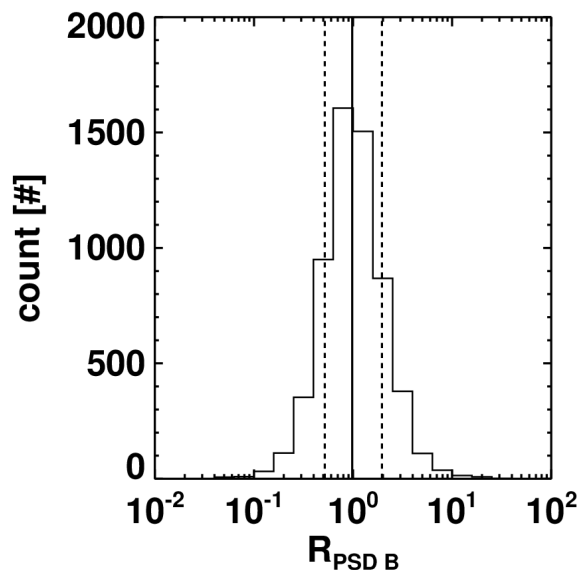


Figure 6. Distribution of the average ratio R_{PSDB} of power spectra estimated within two consecutive one hour long intervals of the solar wind (during year 2015). Dashed vertical lines identify the central confidence interval CI containing 68.3% of data, whereas the solid vertical line identifies the median of the distribution.

into a turbulent plasma initiated by magnetic reconnection of alternating force-free magnetic field. They observed an enhancement of the levels of magnetic field fluctuations across the shock front that was consistent with the Zank21 model. Interestingly, the ratio R_{OT} for the simulation data is lower than unity, which is consistent with our findings.

In conclusion, we showed that the transmission of the magnetic field fluctuations across IP shocks can be successfully modeled by the Zank21 model. We suggest that the model can predict the downstream levels of fluctuations even in scenarios where the assumptions of the theory are not well met. We show that the variability of the solar wind within the inertial range of turbulence effectively prohibits accurate determination of the downstream levels with a good accuracy. We have found that any particular prediction of downstream PSDs suffers an error of a factor ~ 3 . We plan to extend our study where we will address the transmission of velocity and density fluctuations across IP shocks and terrestrial bow shock, employing the methodology described above.

Acknowledgments

This work is supported by the Czech Grant Agency under contract (23-06401S). This paper uses data from the Heliospheric Shock Database, generated and maintained at the University of Helsinki. G.P.Z., M.N., L-L.Z, and L.A. acknowledge the partial support of a NASA Parker Solar Probe contract SV4-84017, an NSF EPSCoR RII-Track-1 Cooperative Agreement OIA - 2148653, a NASA IMAP subaward under NASA contract 80GSFC19C0027, and a NASA award 80NSSC20K1783.

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