On Energy-Delay Tradeoff in Uncoordinated MAC

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***Abstract*—Polyanskiy [1] proposed a framework for the MAC problem with a large number of users, where users employ a common codebook in the finite blocklength regime. In this work, we extend [1] to the case when the number of active users is random and there is also a delay constraint. We first define a random-access channel and derive the general converse bound. Our bound captures the basic tradeoff between the required energy and the delay constraint. Then we propose an achievable bound for block transmission. In this case, all packets are transmitted in the second half of the block to avoid interference. We then study treating interference as noise (TIN) with both single user and multiple users. Last, we derive an achievable bound for the packet splitting model, which allows users to split each packet into two parts with different blocklengths. Our numerical results indicate that, when the delay is large, TIN is effective; on the other hand, packet splitting outperforms as the delay decreases.**

I. Introduction

With the rapid development of Internet of Things, wireless communications such as video streaming, smart home and smart cities involve a massive number of devices with delay constraints. The relationship between delay and energy in wireless communications has attracted many research efforts. Having a delay constraint can certainly guarantee the QoS of end users. However, the desire of energy conservation is the opposite of delay-sensitive communications. This calls for a good understanding of the tradeoff between energy and delay.

On one hand, the fundamental limit on the energy efficiency achievable for the MAC under a finite-blocklength (FBL) constraint was derived in [1]. The framework in [1] has recently been extended to the quasi-static fading channel [2], multiple-antenna channel [3], [4], and a setup with common alarm messages [5]. In the achievability bound of [1] and in most of its extensions, the number of active users is assumed to be fixed and known to the receiver. However, in practice packets are often generated randomly and consequently the number of active users at any time instant is random. An achievability bound for the Gaussian MAC with unknown number of active users was presented in [6]. The authors considered both mis-detection and false alarms, which can serve as a benchmark for unsourced multiple access with random user activities. In our work, we loosen this constraint and consider the case in which the number of active users is a random variable but known to the receiver.

On the other hand, energy consumption of communications is becoming an increasing focus in next-generation communication systems. With the help of FBL theory, it is possible to precisely analyze what is the actual energy needed to meet a given delay constraint. The energy-delay tradeoff for communication over fading channels has been studied in [7]–[9]. An adaptive blocklength framework to minimize the important quantity of end-to-end delay for the single-user case was proposed in [10]. In [11], the authors employed variable-length coding to jointly encode packets to obtain an optimal reliability-latency tradeoff under variable-length FBL coding. The authors of [12] studied the energy-delay tradeoff between power control and link adaption based on a Markov queuing model. The authors of [13] studied the delay of rate adaptation systems with imperfect channel state information (CSI), where FBL coding was used to reduce the transmission delay. However, there is still no research work on the theoretical performance limit for this problem.

To summarize, a new theoretical framework aiming to address the tradeoff between delay constraints and energy consumption in the FBL regime with random packet generation is still missing. This paper consists of three parts. In part one, we first extend Polyanskiy's bound [1] to the case where the number of active users is random yet known to the receiver. We derive a general converse bound with the tradeoff between the required energy and the delay constraint. We then assume that packets are transmitted in the last half of blocks to avoid interference. We develop an achievable bound for the Gaussian MAC under this assumption. In part two, we allow packets to be transmitted at any time interval. In this case we consider the achievability results for treating interference as noise (TIN) with both single-user and joint decoding schemes. Our results show that TIN performs well when the delay is large. In part three, we develop a packet splitting model, which allows users to split one packet into two different blocklengths. Achievability bound shows that packet splitting outperforms when the delay is small.

The remainder of the paper is organized as follows. In Section II, we present our system model. We propose a general converse random-coding bounds for the Gaussian MAC with a delay constraint in Section III. The achievable bound for block transmission is developed in Section IV. In Section V, we consider the continuous user arrival case and study achievability bounds for the TIN model. In Section VI, we evaluate the packet splitting model. We present numerical results and discussions in Section VII. Section VIII concludes the paper.

Work supported in part by NSF ECCS grants 1923751 and 1923803.

II. SYSTEM MODEL

Consider a MAC channel with K_{total} users subject to a strict delay constraint. At each point in time tT_p, $t \in \mathbb{Z}$ a user generates a packet with probability P_p. T_p is the packet timing, which we will later set to 1 without loss of generality. This has to be decoded at time $(t+d)T_p$ ($d \in \mathbb{Z}_+$). An (M, d, ϵ) code consists of the following

- An encoder $f : [M] \to \mathcal{X}^d$.
- A decoder $g : \mathcal{Y}^d \to 2^M$; the decoder at time t outputs the list of messages that were transmitted at time $t-d$.

Following [1] we only care about obtaining a list of messages, not from which transmitters they are transmitted. Additionally, all users employ the same codebook. Correspondingly there are two type of errors: first is $E'_j \triangleq \{M_j(t-d) \notin g(y(t))\}$, the probability that a transmitted message is not in the list of decoded messages. Second is the possibility that two transmitters transmitted the same message at the same time, which results in a collision since they use the same codebook; this type of error depends on exactly what type of transmission scheme is used, but as argued in [1] the probability of this type of error is small. We call the total error probability E_j. We now have the condition that the average (per-user) probability of error (PUPE) satisfies

$$\frac{1}{K_{total}} \mathbb{E}_{K_a}\left[\sum_{j=1}^{K_a} \mathbb{P}(E_j)\right] \leq \epsilon. \tag{1}$$

In general, packets will cause interference to each other. Consider the situation at time t. Assume that messages transmitted before time $t-d$ have been decoded correctly, except a small fraction. The contribution from these messages can therefore be subtracted from the received signal, and will not cause interference. The decoder has to decode messages transmitted at time $t-d$. Messages transmitted at time $t-d < t' \leq t$ will then cause interference. We consider two extreme cases of this situation. In the first *block transmission* scheme, interference is completely avoided. The relationship between n and d is $n = L \cdot d$, where L is the bandwidth. In the TIN and packet splitting scheme, no attempt is made to manage interference. Any user can transmit a packet at any time, which leads to $n = 2 \cdot L \cdot d$.

III. GENERAL CONVERSE BOUND

In order to find a converse, we consider a simplified system model. All users think the system is subject to a delay constraint d so that they have to finish transmission after time d. However, the receiver is allowed to wait till time T to decode all packets transmitted prior to $T-d$. We let $T \to \infty$ to avoid endpoint affects. This gives a converse, as the receiver is still allowed to decode a packet immediately, resulting in

Theorem 1. *Every $(M, n, K_{total}, P_p, \epsilon, P)$ random-access code must satisfy*

$$\frac{n}{2}\mathbb{E}[\log 2\pi(1+PB)] - \frac{1}{2}\log(2\pi) \geq MH\left(\frac{\mathbb{E}[K_a]}{M}\right)$$
$$-\mathbb{E}[K_a](\epsilon \log \frac{Me}{\epsilon\mathbb{E}[K_a]} + H(\epsilon)) \tag{2}$$

Here: $H(x) = x\log\frac{1}{x} + (1-x)\log\frac{1}{1-x}$, and $B \sim \mathcal{B}(d * K_{total}, P_p)$, $K_a \sim \mathcal{B}(K_{total}, P_p)$

Proof. The proof is based on [14]. At each point in time t an active user transmits one of M messages. Let $U \in \{0,1\}^{MT}$, where $U_{t,i} = 1$ if message number i is transmitted at time t. Similarly $\hat{U}_{t,i} = 1$ if the decoder outputs message i for time t. We let $d(\cdot,\cdot)$ denote Hamming distance. Let $K_T = \sum_{t=1}^{T} K_a(t)$. We can then write the error criterion (1) as

$$\lim_{T\to\infty} \frac{1}{T}\mathbb{E}[d(U,\hat{U})] \leq \lim_{T\to\infty} \frac{2}{T}\mathbb{E}[K_T]\epsilon = 2\mathbb{E}[K_a]\epsilon \tag{3}$$

where $U \sim \text{Uniform}\left[\binom{TM}{K_T}\right]$. We now have the Markov chain

$$U \to X^T \to Y^T \to \hat{U} \tag{4}$$

where at each point in time the received signal is

$$Y_i = \sum_{t=i-d+1}^{i} X_{t,1} + X_{t,2} + \cdots + X_{t,K_a(t)} + Z \tag{5}$$

here each $X_{t,i}$ and Z are actually vectors of length L, with each component of $X_{t,i}$ subject to a power constraint P. This is a channel with state $K_a(t)$ and memory d. We can bound the capacity as

$$I(X^T; Y^T | K_a^T) = h(Y^T|K_a^T) - h(Y^T|X^T, K_a^T) \tag{6}$$
$$= h(Y^T|K_a^T) - \sum_{i=1}^{T} h(Y_i|X^i) \tag{7}$$
$$\leq \sum_{i=1}^{T} h(Y_i|K_a^T) - h(Y_i|X^i) \tag{8}$$
$$= Th(Y_i|K_a^T) - Th(Y_i|X^i). \tag{9}$$

It is clear that $h(Y_i|K_a^T)$ is maximized if the components of $X_{t,i}$ are iid Gaussian with power P. Then

$$h(Y_i|K_a^T) = \frac{1}{2}E[\log 2\pi e(1+PB)] \tag{10}$$

where the expectation is over $B \sim \mathcal{B}(d * K_{total}, P_p)$.

We now consider constraint (3). We need to find $\min \mathbb{E}[I(U;\hat{U})]$ subject to the constraint. Different from [14], we have the expectation over K_T. However, from [14] we still have

$$H(U|\hat{U}) \leq \epsilon\mathbb{E}[K_T]\log\frac{eMT}{\epsilon E\mathbb{E}} + \mathbb{E}[K_T]H(\epsilon) \tag{11}$$

and therefore the rate corresponding to ϵ is bounded by

$$R(\epsilon) \geq \frac{1}{T}\mathbb{E}\left[\log\binom{MT}{K_T}\right]$$
$$-\left(\mathbb{E}[K_a]\log\frac{eM}{\epsilon\mathbb{E}[K_a]} + \mathbb{E}[K_a]H(\epsilon)\right). \tag{12}$$

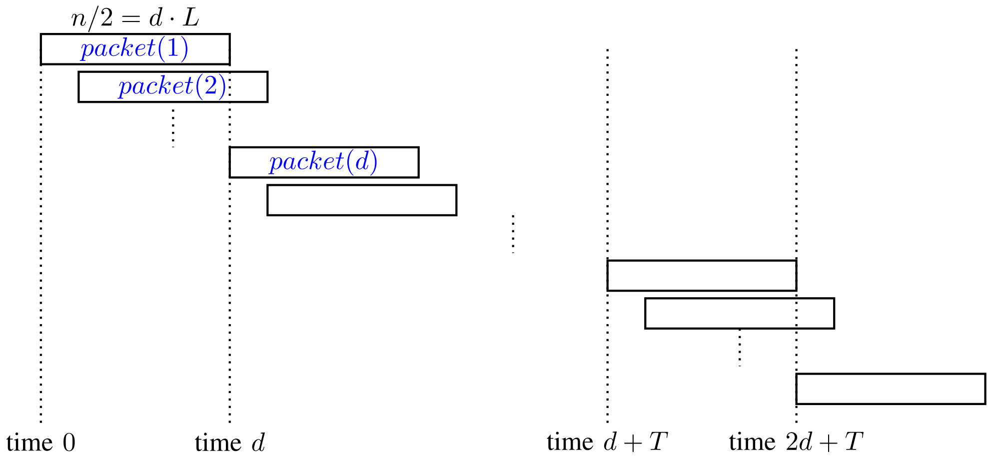

Fig. 1: General Converse

According to [15, Section 13.2],

$$\log \binom{MT}{K_T} \geq MTH\left(\frac{K_T}{MT}\right) + \frac{1}{2}\log\frac{MT}{8K_T(MT-K_T)} \tag{13}$$

By the LLN, $\frac{K_T}{T} \xrightarrow{P} \mathbb{E}[K_a]$, and since the argument is uniformly integrable, we also have

$$\lim_{T\to\infty}\frac{1}{T}\mathbb{E}\left[\log\binom{MT}{K_T}\right] \geq MTH\left(\frac{\mathbb{E}[K_a]}{M}\right). \tag{14}$$

□

IV. Block Transmission

In block transmission, transmitters send packet only during the second half of intervals and are then jointly decoded so as to completely avoid interference (see Fig.2). All packets that were generated between t_0 and $t_0 + Ld/2$ are transmitted at time $t_0 + Ld/2$, and decoded at time $t_0 + Ld$. Hence, the expectation of the number of active users is $K_{total} \cdot P_p \cdot d/2$. The energy critically depends on the two parameters K_{total} and d, and the aim is to find the energy per bit required for transmission.

A. An Achievability Bound

We first give a random-coding achievability bound.

Theorem 2. *(Random-coding bound with random K_a): Fix $P' < P$, there exists an $(M, n, d, K_{total}, P_p, \epsilon)$ random-access code satisfying the power constraint P and*

$$\epsilon \leq \frac{2}{dK_{total}}\sum_{K=0}^{K_T}\mathbb{P}(K_T = K)\left(\sum_{t=1}^{K}\frac{t}{K}p(t) + p_0\right), \tag{15}$$

where

$$p_0 = \frac{2}{dM}\binom{K_T}{2} + K_T\mathbb{P}\left[\frac{1}{n}\sum_{j=1}^{n}Z_j^2 > \frac{P}{P'}\right], \tag{16}$$

$$p(t) = e^{-nE(t)}, \tag{17}$$

$$E(t) = \max_{\rho,\rho_1\in[0,1]} -\rho\rho_1 tR_1 - \rho_1 R_2 + E_0(\rho,\rho_1), \tag{18}$$

$$E_0(\rho,\rho_1) = \rho_1 a + \ln(1-\rho_1 P_2 b), \tag{19}$$

with $a = \rho\ln(1+P't\lambda) + \ln(1+P't\mu)$*,* $b = \rho\lambda - \frac{\mu}{1+P't\mu}$*,* $\mu = \frac{\rho\lambda}{1+P't\lambda}$*,* $\lambda = \frac{P't-1+\sqrt{D}}{4(1+\rho\rho_1)P't}$*,* $D = (P't-1)^2 + 4P't\frac{1+\rho\rho_1}{1+\rho}$*,* $R_1 = \frac{1}{n}\log dM/2 - \frac{1}{nt}\log(t!)$*, and* $R_2 = \frac{1}{n}\log\binom{K_T}{t}$*.*

Proof. We generate the $Md/2$ codewords $c_i \cdots c_{Md/2} \sim \mathcal{N}(0, P')$ with power $P' < P$, each user before transmitting c_i makes sure that $||c_i||^2 \leq nP$. The decoder receives

$$Y = c_1 + c_2 + \cdots + c_{K_T} + Z. \tag{20}$$

Now we define the sum-codewords as $c(S) = \sum_{i\in S} c_i$. Then the maximum likelihood decoder is

$$\hat{S} = \arg\min_S ||c(S) - Y||. \tag{21}$$

The error rate is defined as per-user (PUPE) as in (1). We assume t of K_T messages are mis-decoded, which is the same event as when we let $S_0 \subset [K_T]$ of messages be replaced with $S_0' \subset \{K_T+1, \cdots, dM/2\}$ and $|S_0| = |S_0'| = t$. The received symbol becomes

$$Y = c([K_T]\backslash S_0) + c(S_0) + Z. \tag{22}$$

We can define an error event as $F(S_0, S_0') = \{||c(S_0) - c(S_0') + Z|| < ||Z||\}$. Then,

$$\mathbb{P}[t\text{-misdecoded}] \leq \mathbb{P}\Big[\bigcup_{S_0\in\binom{K_T}{t}}\ \bigcup_{S_0'\in\binom{dM/2-K_T}{t}} F(S_0, S_0')\Big]. \tag{23}$$

Next, given $c(S_0)$ and Z, it holds for every $\lambda > -\frac{1}{2tP'}$ that

$$\mathbb{P}[F(S_0,S_0')|c(S_0),Z] \leq e^{\lambda||Z||^2}\mathbb{E}[e^{-\lambda||c(S_0)-c(S_0')+Z||^2}]$$
$$= (1+2\lambda tP')^{-n}\exp\left(\lambda||Z||^2 - \frac{-\lambda||c(S_0)+Z||^2}{1+2\lambda tP'}\right), \tag{24}$$

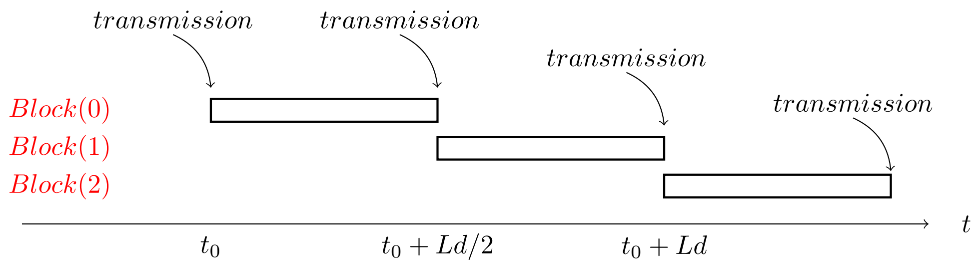

Fig. 2: The Block Transmission

where the inequality in (24) follows from the Chernoff bound and by computing the expectation using

$$\mathbb{E}[e^{-\lambda||\sqrt{a}Z+u||_2^2}] = (1+2\lambda a)^{-n/2} \exp(-\frac{\lambda||u||_2^2}{1+2\lambda a}) \quad (25)$$

for $\forall \lambda > -\frac{1}{2a}, Z \sim \mathcal{N}(0,1)$.

Then, we apply Gallager's ρ-trick for any $\rho \in [0,1]$ to obtain

$$\mathbb{P}[F(S_0)|c(S_0), Z] \leq \binom{dM/2-K_T}{t}^{\rho} (1+2\lambda t P')^{-n\rho/2} \cdot \exp(\lambda\rho(||Z||^2 - \frac{||c(S_0)+Z||^2}{1+2\lambda t P'})). \quad (26)$$

Taking the expectation over $c(S_0)$ using (25) and Gallager's ρ-trick for any $\rho_1 \in [0,1]$, we have

$$\mathbb{P}[\cup F(S_0)] \leq \binom{K_T}{t}^{\rho_1} \binom{dM/2-K_T}{t}^{\rho\rho_1} e^{-n\rho_1 a}(1-\rho_1 b)^{-n} = p(t). \quad (27)$$

□

V. TIN

The disadvantage of block transmission is that packet length is limited to $d/2$. In the second transmission method, each user transmits a packet as soon as it becomes available and the packet length therefore can be the full length d. As mentioned above, messages transmitted at time $t-d < t' \leq t$ will then cause interference; we treat this interference as noise (TIN).

A. Single-user Decoding

In single-user decoding, each packet is decoded separately while treating all other packets as interference. We include this as a benchmark.

Theorem 3. *(Single User Decoding TIN Model) Consider a single user decoding TIN model with random K_a, the error of probability ϵ for a given power P is bounded by*

$$\epsilon \leq \frac{2}{\sqrt{n}}(\frac{\log 2}{\sqrt{2\pi}} + 2B_1) + Q(B_2) + B_1, \quad (28)$$

where

$$B_1 = \frac{\mathbb{E}[\frac{15K_a^3P^3}{8(1+K_aP)^3}] - \frac{3}{8}\mathbb{E}[\frac{K_aP}{1+K_aP}]\mathbb{E}[\frac{K_a^2P^2+4}{(1+K_aP)^2}] + \mathbb{E}^3[\frac{K_aP}{1+K_aP}]}{8\sqrt{n/2}(\mathbb{E}[\frac{K_a^2P^2+4}{(1+K_aP)^2}] - \mathbb{E}^2[\frac{K_aP}{1+K_aP}])^{3/2}},$$

$$B_2 = \frac{\sqrt{n/2}\mathbb{E}[\frac{K_aP}{1+K_aP}] - \log\frac{dM-1}{2}}{\sqrt{n/2}(\mathbb{E}[\frac{K_a^2P^2+4}{(1+K_aP)^2}] - \mathbb{E}^2[\frac{K_aP}{1+K_aP}])^{1/2}},$$

Proof. Consider a single user was transmitted at time t, the received signal can be equivalently expressed as

$$Y_i = X_i + Z + Z_I^i, \quad (29)$$

where $Z \sim \mathcal{N}(0,1)$, $Z_I \sim \mathcal{N}(0, PK_I^i)$ with $K_I^i = \sum_{j=1}^{i} K_a^j$. $K_a^j \sim \mathcal{B}(K_{total}, P_p)$ denotes the number of active users in the j^{th} packet.

We consider the average error probability of the TIN model from the dependence testing bound [16]

$$\epsilon \leq \mathbb{P}[i(X;Y) \leq \log\frac{dM-1}{2}] + \frac{dM-1}{2}\mathbb{P}[i(X;Y) > \log\frac{dM-1}{2}], \quad (30)$$

The information density can be calculated as

$$i(x^n;y^n) = \log\frac{(2\pi(1+K_IP))^{-\frac{n}{2}}\exp(-\frac{||y^n-x^n||^2}{2(1+K_IP)})}{(2\pi(1+K_IP+P))^{-\frac{n}{2}}\exp(-\frac{||y^n||^2}{2(1+K_IP+P)})} = \sum_{j=1}^{n} W_{Tj}, \quad (31)$$

First, in order to use the central limit theorem, we need to show that the information density satisfies the Lindenberg-Feller condition as follows

$$\lim_{n\to\infty}\frac{\sum_{j=1}^n \mathbb{E}|W_{Tj} - \mathbb{E}(W_{Tj})|^4}{(\sum_{j=1}^n \mathrm{Var}(W_{Tj}))^2} = 0, \quad (32)$$

where the calculation is based on $\mathbb{E}[g(X)] \leq 1$, with $g(X) = \frac{1}{X^n}, n = 2,3,\cdots$ and $X \sim \mathcal{B}(K_{total}, P_p)$.

To upper bound the probability of error, we derive the Berry-Esseen ratio as follows

$$B_1 = \frac{T}{\mathrm{Var}^{3/2}} = \frac{\sum_{j=1}^n \mathbb{E}[|W_{Tj} - \mathbb{E}[W_{Tj}]|^3]}{(\sum_{j=1}^n \mathrm{Var}[W_{Tj}])^{\frac{3}{2}}}. \quad (33)$$

By the Berry-Esseen theorem [16], the first part of the dependence testing bound (30) can be bounded as

$$\mathbb{P}[\sum_{j=1}^n W_{Tj} \leq \log\frac{dM-1}{2}] \leq Q(B_2) + B_1. \quad (34)$$

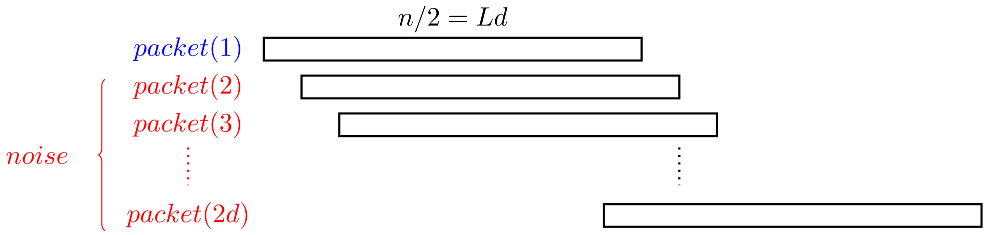

Fig. 3: The TIN Model

The second part can be expressed as

$$\frac{dM-1}{2}\mathbb{P}[\sum_{j=1}^{n} W_{Tj} > \log\frac{dM-1}{2}] \leq \frac{2}{\sqrt{n}}(\frac{\log 2}{\sqrt{2\pi}} + 2B_1). \tag{35}$$

Finally, by substituting (34) and (35) into (30), we obtain the final result. □

B. Joint Decoding

In joint decoding, messages transmitted at the same time are jointly decoded; only those transmitted later are treated as interference.

Theorem 4. *(Joint Decoding TIN Model) Fix $P' < P$, there exists an $(M, n, d, K_{total}, P_p, \epsilon)$ random-access code with K_a-random number of active users satisfying the power constraint P and*

$$\epsilon \leq \frac{1}{dK_{total}}\sum_{K=0}^{K_T}\mathbb{P}(K_T = K)(\sum_{t=1}^{K}\frac{t}{K}A(t)c(t)^{-\frac{1}{2}} + p_0), \tag{36}$$

where

$$p_0 = \frac{\binom{K_T}{2}}{dM} + K_T\mathbb{P}[\frac{1}{n}\sum_{j=1}^{n} Z_j^2 > \frac{P}{P'}], \tag{37}$$

$$A(t) = \max_{\rho,\rho_1\in[0,1]}\binom{K_T}{t}^{\rho_1}\binom{dM-K_T}{t}^{\rho\rho_1} (1+2\lambda tP')^{-\rho\rho_1\frac{n}{2}}(1+2tP'\mu)^{-\rho_1\frac{n}{2}}, \tag{38}$$

$$c(t) = \max_{\rho,\rho_1\in[0,1]}(1-2\gamma)^n - 2\gamma nK_{total}P_p(1-2\gamma)^{n-1} + 4\gamma^2\frac{n(n-1)}{2}K_{total}P_p(1-P_p)(1-2\gamma)^{n-2}, \tag{39}$$

$\mu = \frac{\rho\lambda}{1+2tP'\lambda}$, *and* $\gamma = \lambda\rho\rho_1 - \frac{\mu\rho_1}{1+2tP'\mu}$.

Proof. Consider that $c_1, \cdots, c_{K_T}$ were transmitted at time t, the received signal can be equivalently expressed as

$$Y_i = \sum^{K_T} X + Z + Z_I^i, \tag{40}$$

where $Z \sim \mathcal{N}(0,1)$ and $Z_I^i \sim \mathcal{N}(0, PK_I^i)$ follows from (29).

We can similarly rewrite the probability of error event like in Section IV as

$$\begin{aligned}&\mathbb{P}[F(S_0, S_0')|c(S_0), Z, Z_I]\\ &\leq e^{\lambda||Z+Z_I||_2^2}\mathbb{E}[e^{-\lambda||c(S_0)-c(S_0')+Z+Z_I||_2^2}]\\ &= e^{\lambda||Z+Z_I||^2}(1+2\lambda tP')^{-\frac{n}{2}}\exp{(-\frac{\lambda||c(S_0)+Z+Z_I||_2^2}{1+2\lambda tP'})}.\end{aligned}$$

Next, we use Gallager's ρ-trick for any $\rho \in [0,1]$ to get

$$\begin{aligned}&\mathbb{P}[F(S_0)|c(S_0), Z, Z_I]\\ &\leq \binom{dM-K_T}{t}^{\rho}(1+2\lambda tP')^{-\rho\frac{n}{2}}\\ &\cdot\exp{(\lambda\rho(||Z+Z_I||_2^2 - \frac{||c(S_0)+Z+Z_I||_2^2}{1+2\lambda tP'}))}.\end{aligned} \tag{41}$$

Applying (25) again to calculate the expectation of $c(S_0)$, we get

$$\begin{aligned}&\mathbb{P}[F(S_0)|Z, Z_I]\\ &\leq \binom{dM-K_T}{t}^{\rho}(1+2\lambda tP')^{-\rho\frac{n}{2}}(1+2tP'\mu)^{-\frac{n}{2}}\\ &\cdot\exp{(\lambda\rho||Z+Z_I||_2^2(1-\frac{1}{(1+2\lambda tP')(1+2tP'\mu)}))},\end{aligned} \tag{42}$$

where $\mu = \frac{\rho\lambda}{1+2tP'\lambda}$. Employing Gallager's ρ-trick for any $\rho_1 \in [0,1]$ and (25) again, we have

$$\mathbb{P}[\bigcup_{S_0} F(S_0)] \leq A(t)\mathbb{E}[\prod_{k=1}^{n}(1-2\gamma-2\gamma K_I^k)^{-\frac{1}{2}}], \tag{43}$$

where $\gamma = \lambda\rho\rho_1 - \frac{\mu\rho_1}{1+2tP'\mu}$. Finally, we calculate the expectation and upper bound it to have the result. □

VI. Packet Splitting

The advantage of TIN is that the packet length can be longer than $d/2$, but noise quickly deteriorates the performance as the spectral efficiency increases (see Fig. 5 later). We therefore consider a transmission scheme that can be seen as a compromise between TIN and block transmission. The transmission system we consider is shown in Fig. 4. An active user separately transmits its message in two parts. The first part of the message is encoded to a variable length packet of length d_m. The second part of the message is transmitted with the blocklength $d/2$. The number of information bits needs to be allocated differently in each half. Based on d_m, to make it simple the number of information bits is evenly

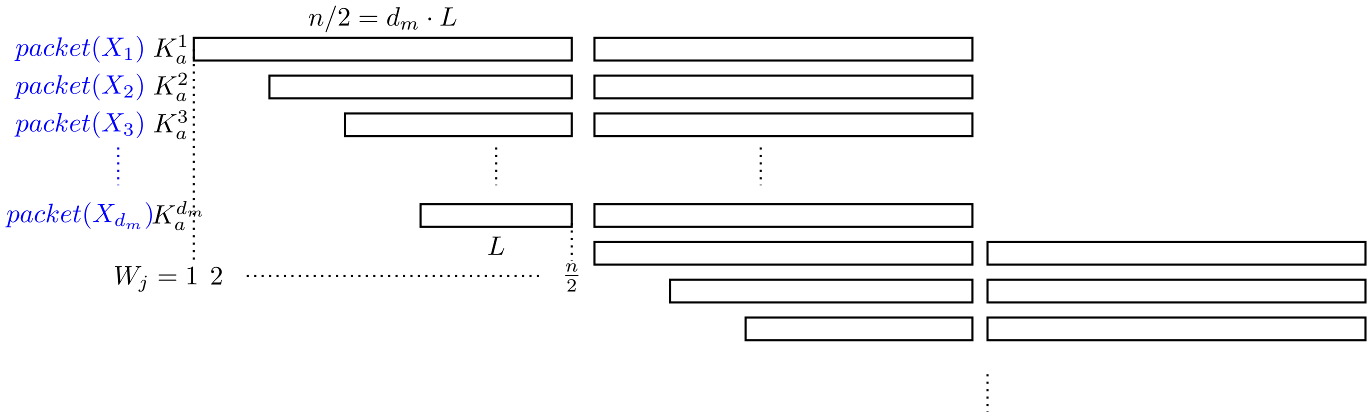

Fig. 4: The Packet Splitting Model

divided, i.e. the first half transmits $k \cdot \frac{d_m}{d}$ bits. This is not necessarily the best solution. However, compared with block transmission, a packet can be transmitted over longer than $d/2$, while the receiver can jointly decode all packets without any interference.

Theorem 5. *(Packet Splitting Model) The error of probability ϵ for a given power P is bounded by*

$$\epsilon \leq G(t_1) + \frac{d_m M - 1}{2} G(t_2) \quad (t_1 < 0, t_2 > 0) \tag{44}$$

Proof. The information density can be calculated as

$$i(x^n; y^n) = \sum_{i=1}^{n/2} \frac{1}{2} \log \sigma_{Y_i}^2 + \frac{1}{2} \log e \cdot (\frac{y_i^2}{\sigma_{Y_i}^2} - Z_i^2) \tag{45}$$

where $Z_i \sim \mathcal{N}(0, 1)$,

$$\sigma_{Y_i}^2 = \begin{cases} 1 + (K_a^1)^2 P & i = 1 \sim L \\ 1 + (K_a^1 + K_a^2)^2 P & i = L \sim 2L \\ \vdots & \\ 1 + (\sum_{i=1}^{d_m} K_a^i)^2 P & i = (d_m - 1)L \sim d_m L \end{cases} \tag{46}$$

and

$$y_i = \begin{cases} K_a^1 \sqrt{P} + Z & j = 1 \sim L \\ (K_a^1 + K_a^2)\sqrt{P} + Z & j = L \sim 2L \\ \vdots & \\ \sum_{i=1}^{d_m} K_a^i \sqrt{P} + Z & j = (d_m - 1)L \sim d_m L \end{cases} \tag{47}$$

To evaluate the distribution of $i(X; Y)$, a number of other inequalities must be used. Applying $1 + x \leq e^x$, we have

$$i(X; Y) \leq \sum_{i=1}^{n/2} \frac{1}{2} (\sum_{j=1}^{i} K_a^j)^2 P - \frac{1}{2} \log e \cdot [\sum_{i=1}^{n/2} Z_i^2 - \sum_{i=1}^{n/2} 2 Z_i (\sum_{j=1}^{i} K_a^j) \sqrt{P} - \frac{n}{2}]. \tag{48}$$

Now, using the Chernoff bound, we can obtain a bound of the left tail, for $t_1 < 0$

$$\mathbb{P}[i(X; Y) \leq \log \frac{d_m M - 1}{2}] \leq M(t_1) (\frac{d_m M - 1}{2})^{-t_1} = G(t_1) \tag{49}$$

and on the right tail, for $t_2 > 0$

$$\mathbb{P}[i(X; Y) > \log \frac{d_m M - 1}{2}] \leq M(t_2) (\frac{d_m M - 1}{2})^{-t_2} = G(t_2). \tag{50}$$

The moment-generating function $M(t)$ can be calculated as

$$M(t) = (1 + \frac{1}{2} t P K_{total} P_p (1 - P_p) (\sum_{d=1}^{d_m} d!)) \cdot (e^{\frac{tn}{4}}) \cdot (e^{-\frac{t}{2}} (1 - 2t)^{-\frac{n}{4}}). \tag{51}$$

Plugging (49) and (50) into the dependence testing bound (30), we have the final result. □

VII. Numerical Results

In this section we plot some numerical results for the bounds developed above. In Fig. 5 we compare various strategies in the following setting: $K_a \sim \mathcal{B}(K_{total}, P_p)$ with $K_{total} = 3000$ and $P_p = 0.01$. Each active users is sending $k = 100$ bits and the target per-user probability of error is 0.1. The blocklength n varies of delay d as $n = Ld$ in block transmission, $n = 2Ld$ for the TIN and $n = (d_m + d/2)L$ with packet splitting. Specifically, we set bandwidth L to 3000 in our simulations. We use the average energy per bit $\frac{E_b}{N_0} = \frac{nP}{2k}$ as the performance measure.

The general converse gives the lower bound for all different schemes. We observe that for large delay, TIN outperforms block transmission, but for smaller delay the energy per bit rapidly increases. This is not surprising as the interference then becomes much larger than the noise. It is seen that the energy increases very rapidly when the delay is small and packet splitting is significantly better than block transmission. In principle, packet splitting should always be better than block transmission. In fact, if the first half transmit zero bits, the scheme is the same as block transmission. However, the reason it is worse when the delay is large is because the way we split the packets is suboptimal. We leave the optimal way to split information bits to future work.

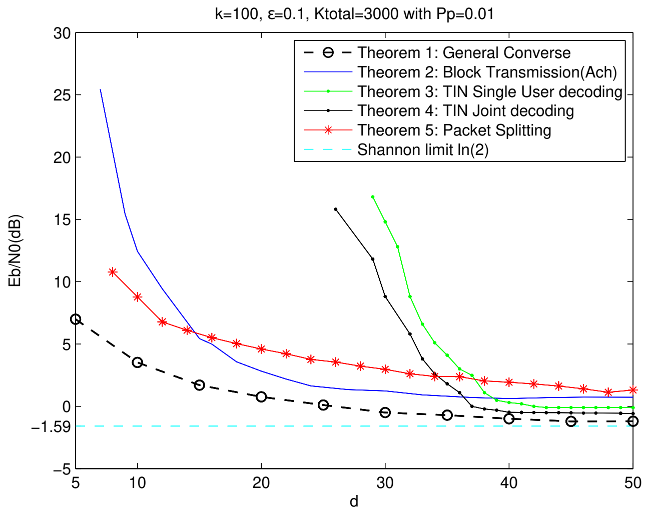

Fig. 5: The tradeoff between the energy per bit and the delay constraint for the general converse bound and different achievabletransmission schemes.

VIII. Conclusions

We investigated the tradeoff between the delay constraint and energy consumption in the FBL regime when the number of active users is random. We derived a general random-coding converse bound for uncoordinated MAC. Our bound can serve as a benchmark to assess the performance of practical schemes. We also developed achievable bounds for three different schemes: block transmission, TIN, and packet splitting. Numerical results show that for uncoordinated MAC, the TIN model outperforms when the delay is large whereas the packet splitting model is better when the delay is relatively small.

It is seen from Fig. 5 that there is still a large gap between the general converse and the achievable performance of packet splitting. The challenge is to devise transmission schemes that can close this gap. This is what we will consider in future works.

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