Three-Dimensional Fully Coupled Thermo-Hydro-Mechanical Model for Thaw Consolidation of Permafrost

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Abstract: A fully coupled three-dimensional (3D) thermo-hydro-mechanical (THM) model is developed for simulating the complex multiphysics process of permafrost thaw. The 3D formulation allows the analysis of thaw consolidation problems with complex geometry and boundary conditions. The thermal, hydraulic, and mechanical fields are coupled in this model. Governing equations are derived based on the laws of conservation of each field: conservation of energy for the thermal field, conservation of mass for the hydraulic field, and conservation of momentum for the mechanical field. Physical processes such as heat conduction, phase change, thermal convection, fluid flow due to pore water pressure, elevation, thermal gradients, and force equilibrium based on effective stress theory are considered in this model. The model is then applied to simulate the thaw consolidation of permafrost. The simulation results show that excess pore water pressure is generated in the soil during thawing. The soil then experiences a time-dependent settlement following the dissipation of excess pore water pressure. The results prove that the THM model adequately captures the thaw consolidation process of permafrost.

1. Introduction

Degradation of permafrost, especially thaw consolidation, has caused severe damage to civil infrastructures, such as roads, buildings, pipelines, and powerlines (Nelson et al. 2001; Larsen et al. 2008; Melvin et al. 2016; Hjort et al. 2018; Liew et al. 2022b). It is therefore important to investigate the effects of permafrost degradation on civil infrastructure. One approach is the development of numerical models for predicting the long-term effects of thaw consolidation on foundations under various climate scenarios. Thaw consolidation is a complex multi-physics process that involves heat transfer, moisture transport, and stress-strain equilibrium. A model framework that is fully coupled with these three different physics is commonly known as the thermo-hydro-mechanical (THM) model.

Several complex thaw consolidation models were developed in recent years. Yao et al. (2012) formulated a three-dimensional large strain thaw consolidation theory using the Cauchy strain rate tensor and Jaumann stress rate. In this model, the thawing process was governed by thermal conduction and was calculated using the heat transfer equation. To couple the thermal and hydromechanical processes, Yao et al. (2012) adopted a two-step calculation with thermal calculation applied for the entire soil domain while the hydromechanical calculation is suppressed. The hydromechanical calculation was only then applied to the post-thawed domain after the thermal calculation converged. A smaller time step is also selected for the hydromechanical calculation. In the model by Dumais and Konrad (2018), a one-dimensional large strain consolidation theory is coupled with a heat transfer equation to simulate the thaw settlement of frozen soil. The large strain functionality was achieved by formulating empirically derived compressibility and hydraulic conductivity as functions of void ratio. One of the major benefits of a large-strain thaw consolidation model is its capability to simulate excessive settlement in permafrost. This phenomenon is quite typical for thawing permafrost that is imposed by relatively high overburden stress or ice-rich permafrost undergoing degradation. On the other hand, a three-dimensional model is well suited for thaw consolidation problems with complex boundary conditions such as a moving heat source and interactions between permafrost and a complex foundation system. Currently, there is no three-dimensional thaw consolidation model in which the three physical fields are fully coupled.

This paper focuses on the formulation of a three-dimensional fully coupled THM model for simulating the thaw consolidation of degrading permafrost. The physics of the thermal field considered in this model include heat conduction, phase change, and thermal convection. For the hydraulic field, fluid flow due to pore water pressure, elevation, and the thermal gradient is considered. In the mechanical field, the stress-strain relationship is formulated based on linear elastostatics and the effective stress principle. Since all three physical fields are fully coupled, the primary variables of this model are calculated at each time step simultaneously. This THM model with three-dimensional formulations can be applied to solving thaw consolidation problems with complex boundary conditions.

2. Theory

Thaw consolidation can be simulated by coupling three physical fields: thermal, hydraulic, and mechanical fields. The three physical fields are governed by their respective laws of conservation: conservation of momentum for the mechanical field, conservation of energy for the thermal field, and conservation of mass for the hydraulic field. The primary variables of the mechanical field are the displacement vector (i.e., u, v, and w for a three-dimensional model); the primary variable of the thermal field is temperature, T; the primary variable of the hydraulic field is pore water pressure, p. The partial differential equations of the fully coupled three-dimensional thaw consolidation model are derived and presented in the following subsections.

2.1 Mechanical Field

The law of conservation of momentum states that the momentum of a system remains constant if there is no external force acting on the system. The governing equation for the mechanical field can be stated as follows.

$$\sigma_{ij,j} + b_i = 0 \tag{1}$$

where $\sigma_{ij,j}$ is the differentiation of the total stress tensor σ_{ij} with respect to x_j, and b_i is the body force. Given the effective stress theory, total stress is the sum of effective stress σ'_{ij} and pore water pressure p. The equation is expressed as

$$\sigma_{ij} = \sigma'_{ij} + \delta_{ij} p \tag{2}$$

where δ_{ij} is the Kronecker delta. Following the generalized Hooke's law, the effective stress tensor is defined as

$$\sigma'_{ij} = D_{ijkl} \varepsilon_{kl}^{\sigma'} \tag{3}$$

where D_{ijkl} is the elasticity matrix, and $\varepsilon_{kl}^{\sigma'}$ is the strain tensor due to effective stress. Assuming an isotropic soil,

$$D_{ijkl} = \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) + \lambda\left(\delta_{ij}\delta_{kl}\right) \tag{4}$$

where λ and μ are Lamé's first and second parameters, respectively. Their relationships to bulk modulus K and shear modulus G are given as follows.

$$\lambda = K - \frac{2}{3}G \tag{5}$$

$$\mu = G \tag{6}$$

Given that elastic moduli of frozen soil are higher than the moduli of thawed soil (Liew et al. 2022a), K can be expressed as

$$K = \begin{cases} K_{\mathrm{f}}, & \text{if } T < T_{\mathrm{zero}} \\ K_{\mathrm{u}}, & \text{if } T \geq T_{\mathrm{zero}} \end{cases} \tag{7}$$

where K_{f} is the frozen bulk modulus, and K_{u} is the unfrozen bulk modulus. T_{zero} can be taken as 273.15 K if assuming no freezing point depression. Similarly, G can be expressed in terms of the frozen shear modulus G_{f} and unfrozen shear modulus G_{u}.

$$G = \begin{cases} G_{\mathrm{f}}, & \text{if } T < T_{\mathrm{zero}} \\ G_{\mathrm{u}}, & \text{if } T \geq T_{\mathrm{zero}} \end{cases} \tag{8}$$

Considering that changes in soil temperature can lead to thermal expansion, the total strain tensor ε_{kl} is defined as the sum of the strain tensor induced by effective stress and the strain tensor induced by temperature change ε_{kl}^{T}.

$$\varepsilon_{kl} = \varepsilon_{kl}^{\sigma'} + \varepsilon_{kl}^{T} \tag{9}$$

And ε_{kl}^{T} can be expressed as

$$\varepsilon_{kl}^{T} = \alpha_T \delta_{kl} dT \tag{10}$$

where α_T is the coefficient of thermal expansion of soil, δ_{kl} is the Kronecker delta, and dT is the temperature change. Assuming no thermal expansion below the freezing point of water, the following function can be adopted for α_T and dT.

$$\alpha_T = \begin{cases} \alpha_T, & \text{if } T > 273.15 \text{ K} \\ 0, & \text{if } T \leq 273.15 \text{ K} \end{cases} \tag{11}$$

$$dT = T - T_{\mathrm{ref1}} \tag{12}$$

where T_{ref1} is the reference temperature at which the soil starts to expand and can be taken as 273.15 K. Substituting Eq. 2, 3, 9, and 10 into Eq. 1, the governing equation of the mechanical field coupled with the effects of the hydraulic and thermal fields can be written as

$$\left[D_{ijkl}\left(\varepsilon_{kl} - \alpha_T \delta_{kl} dT\right) + \delta_{ij} p\right]_{,j} + b_i = 0 \tag{13}$$

where b_i in the x_3 direction can be expressed as

$$b_3 = \rho g \tag{14}$$

where ρ is the overall density of the soil and can be calculated as the weighted average of the density of each soil constituent.

$$\rho = \theta_{\mathrm{W}}\rho_{\mathrm{W}} + \theta_{\mathrm{I}}\rho_{\mathrm{I}} + \theta_{\mathrm{S}}\rho_{\mathrm{S}} \tag{15}$$

where θ is the volumetric content; ρ is density; the subscripts, W, I, and S, represent water, ice, and soil grains, respectively.

In this model, a fully saturated soil is assumed. As such, the frozen soil consists of soil grains, ice, and water, while the thawed soil consists of only soil grains and water. The amount of each of these soil constituents can be calculated using their volumetric ratios. Here, θ_{W} is the ratio of the volume of water to the total volume of soil. Similarly, θ_{I} is the ratio of the volume of ice to the total volume of soil. Assuming that the soil is fully saturated, the sum of the volumetric ice content and the volumetric water content is equivalent to the soil porosity n and is expressed as

$$n = \theta_{\mathrm{W}} + \theta_{\mathrm{I}} \tag{16}$$

Correspondingly, θ_{S} is the ratio of the volume of soil grains to the total volume of soil.

$$\theta_{\mathrm{S}} = 1 - n \tag{17}$$

Substituting Eq. 16 and 17 into Eq. 15, the overall density of soil can be re-expressed as

$$\rho = \theta_{\mathrm{W}}\rho_{\mathrm{W}} + \left(n - \theta_{\mathrm{W}}\right)\rho_{\mathrm{I}} + \left(1 - n\right)\rho_{\mathrm{S}} \tag{18}$$

Following the infinitesimal strain theory, the strain tensor in Eq. 13 can be expressed as

$$\varepsilon_{ij} = u_{(i,j)} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{19}$$

where u_i is the displacement vector.

2.2 Hydraulic Field

The law of conservation of mass states that the difference between the mass moving into a control volume and the mass leaving is equivalent to the mass change in the control volume. The governing equation of the hydraulic field can therefore be stated as

$$v_{i,i}\rho_{\mathrm{W}} dV + \dot{m}_{\mathrm{W}} + \dot{m}_{\mathrm{I}} = 0 \tag{20}$$

where v_i is the Darcy's velocity; ρ_{W} is the density of water; dV is the infinitesimal change of control volume; $\dot{m}_{\mathrm{W}}$ and $\dot{m}_{\mathrm{I}}$ are the rates of change of water mass and ice mass in the soil system, respectively. According to Darcy's law,

$$v_i = -k_{ij}^{H} h_{,j} \tag{21}$$

where k_{ij}^{H} is the hydraulic conductivity tensor, and $h_{,j}$ is the hydraulic gradient. Darcy's velocity v_i can be expressed in terms of the pore water pressure p_{total} as

$$v_i = -\frac{k_{ij}^{H}}{\rho_{\mathrm{W}} g} p_{\mathrm{total},j} \tag{22}$$

where ρ_{W} is the density of water, and g is the acceleration due to gravity.

Fluid flow in degrading permafrost is governed by pore water movement due to (1) pore water pressure, (2) elevation difference, and (3) temperature gradient (Thomas et al. 2009). The water movement induced by temperature gradient is derived using the Clapeyron equation following works by Thomas et al. (2009). The Clapeyron equation states that

$$\frac{p_{\mathrm{W}}^{\mathrm{T}}}{\rho_{\mathrm{W}}} - \frac{p_{\mathrm{I}}^{\mathrm{T}}}{\rho_{\mathrm{I}}} = L \ln \frac{T}{T_{\mathrm{ref2}}} \tag{23}$$

where $p_{\mathrm{W}}^{\mathrm{T}}$ and $p_{\mathrm{I}}^{\mathrm{T}}$ are the water and ice pressure induced by temperature gradient, respectively; L is the latent heat of fusion; T_{ref2} is the reference temperature, which is set as 273.15 K. As presented in Eq. 23, the temperature gradient is responsible for the pore water movement, i.e., pore water moves from the active layer towards the permafrost table. Re-arranging and simplifying Eq. 23, cryogenic suction P_{T} can be expressed as the difference between $p_{\mathrm{W}}^{\mathrm{T}}$ and $p_{\mathrm{I}}^{\mathrm{T}}$.

$$P_{\mathrm{T}} \approx \rho_{\mathrm{I}} L \frac{T - T_{\mathrm{ref2}}}{T_{\mathrm{ref2}}} \tag{24}$$

The total pore water pressure p_{total} is therefore the sum of those three components (Thomas et al. 2009).

$$p_{\mathrm{total}} = p + \rho_{\mathrm{W}} g h_{\mathrm{elev}} + \rho_{\mathrm{I}} L \frac{T - T_{\mathrm{ref2}}}{T_{\mathrm{ref2}}} \tag{25}$$

where h_{elev} is the elevation, which is the distance between the free water surface and datum. The datum can be set at any depths where the soil temperature is seasonally invariant. For the simulation of in situ permafrost degradation, the midpoint of permafrost layer can be taken as the datum. For the simulation of a laboratory test, the bottom surface of the soil column can be taken as the datum. Substituting Eq. 25 into Eq. 22, v_i can be re-expressed as

$$v_i = -\frac{k_{ij}^{\mathrm{H}}}{\rho_W g} \left(p + \rho_{\mathrm{W}} g h_{\mathrm{elev}} + \rho_{\mathrm{I}} L \frac{T - T_{\mathrm{ref2}}}{T_{\mathrm{ref2}}} \right)_{,j} \tag{26}$$

The second and third terms in Eq. 20 can be respectively expressed in terms of volumetric ratios as

$$m_{\mathrm{W}} = \theta_{\mathrm{W}} \rho_{\mathrm{W}} dV \tag{27}$$

and

$$m_{\mathrm{I}} = \theta_{\mathrm{I}} \rho_{\mathrm{I}} dV \tag{28}$$

In Eq. 27 and 28, since dV (i.e., the total infinitesimal volume of the frozen soil) is not a constant, it can be redefined as

$$dV = (1 + e)\, dV_{\mathrm{S}} \tag{29}$$

where e is the void ratio of the soil, and dV_{S} is the volume of the soil grains. Differentiating m_{W} and m_{I} with respect to time yields

$$\dot{m}_{\mathrm{W}} = \rho_{\mathrm{W}} dV_{\mathrm{S}} (\theta_{\mathrm{W}} \dot{e} + (1+e) \dot{\theta}_{\mathrm{W}}) \tag{30}$$

and

$$\dot{m}_{\mathrm{I}} = \rho_{\mathrm{I}} dV_{\mathrm{S}} (\theta_{\mathrm{I}} \dot{e} + (1+e) \dot{\theta}_{\mathrm{I}}) \tag{31}$$

Substituting Eq. 26, 30, and 31 into the original governing equation of the hydraulic field (Eq. 20), the final governing equation of the hydraulic field becomes

$$\begin{aligned} &(\rho_{\mathrm{W}} \theta_{\mathrm{W}} - \rho_{\mathrm{I}} \theta_{\mathrm{W}} + \rho_{\mathrm{I}}) \dot{\varepsilon}_{ii} + (\rho_{\mathrm{W}} - \rho_{\mathrm{I}}) \theta_{,T}^{\mathrm{W}} \dot{T} \\ &-\frac{1}{g} \left(k_{ij}^{\mathrm{H}} p_{,j} \right)_{,i} - \rho_{\mathrm{W}} \left(k_{ij}^{\mathrm{H}} h_{\mathrm{elev},j} \right)_{,i} - \frac{1}{g} \rho_{\mathrm{I}} L \left(k_{ij}^{\mathrm{H}} \frac{(T - T_{\mathrm{ref2}})_{,j}}{T_{\mathrm{ref2}}} \right)_{,i} = 0 \end{aligned} \tag{32}$$

where $\dot{\varepsilon}_{ii}$ is the volumetric strain rate; $\theta_{,T}^{\mathrm{W}}$ is the differentiation of θ_{W} with respect to temperature;

$\dot{T}$ is the rate of change of temperature.

Unfrozen water content exists in permafrost even when the temperature is below the freezing point of water (Liew et al. 2022a). The unfrozen water content of a soil can be expressed as a function of temperature following the empirical equation by Anderson and Tice (1972). However, the THM model uses volumetric unfrozen water content θ_{W}, while Anderson and Tice (1972) used gravimetric water content. So, the soil-dependent parameters α and β need to be fitted using volumetric data points instead. Then, θ_{W} is modified to include n as follows.

$$\frac{\theta_{\mathrm{W}}}{n} = \begin{cases} \frac{\alpha}{n}\left(273.15 - T\right)^{\beta}, & T < T_{\mathrm{zero}} \\ 1, & T \geq T_{\mathrm{zero}} \end{cases} \tag{33}$$

If an isotropic fluid flow is assumed,

$$k_{ij}^{\mathrm{H}} = \delta_{ij} k_{\mathrm{H}} \tag{34}$$

where δ_{ij} is the Kronecker delta, and k_{H} is the hydraulic conductivity. Since the hydraulic conductivity of soil varies with temperature (Liew et al. 2022a), the following function is adopted for k_{H}.

$$k_{\mathrm{H}} = \begin{cases} k_{\mathrm{H\_f}}, & \text{if } T < T_{\mathrm{zero}} \\ k_{\mathrm{H\_u}}, & \text{if } T \geq T_{\mathrm{zero}} \end{cases} \tag{35}$$

2.3 Thermal Field

The law of conservation of energy states that the difference between the heat energy flowing into a control volume and the heat energy flowing out from the control volume is equivalent to the energy change. The governing equation can be stated as

$$\dot{E} + q_{i,i} dV = 0 \tag{36}$$

where $\dot{E}$ is rate of change of internal heat energy, q_i is heat flux, and dV is the infinitesimal control volume. The change in internal heat energy E can be expressed in terms of the phase change and temperature change as follows.

$$E = mc(T - T_{\mathrm{ref3}}) + m_{\mathrm{W}} L \tag{37}$$

where m is the total soil mass, c is the overall specific heat capacity, T is temperature (i.e., the primary variable of the thermal field), m_{W} is the mass of moisture, and L is the latent heat of fusion. T_{ref3} is equivalent to the initial temperature of the soil domain and can be set by users depending on the problems to be solved. Re-expressing Eq. 37 in terms of dV,

$$E = \rho c(T - T_{\mathrm{ref3}}) dV + \rho_{\mathrm{W}} L dV_{\mathrm{W}} \tag{38}$$

where ρ is the overall density of the soil, and ρ_{W} is the density of water.

The overall specific heat capacity c of the soil domain can be defined as a weighted function of the specific heat capacities of the soil constituents.

$$\rho c = \theta_{\mathrm{W}} \rho_{\mathrm{W}} c_{\mathrm{W}} + \theta_{\mathrm{I}} \rho_{\mathrm{I}} c_{\mathrm{I}} + \theta_{\mathrm{S}} \rho_{\mathrm{S}} c_{\mathrm{S}} \tag{39}$$

Variations of the specific heat capacity and density of each soil constituents with temperature are assumed to be negligible. Substituting Eq. 16, 17, and 39 into Eq. 38, the equation becomes

$$E=\left(\rho_{\mathrm{W}}c_{\mathrm{W}}-\rho_{\mathrm{I}}c_{\mathrm{I}}\right)dV_{\mathrm{S}}\theta_{\mathrm{W}}\left(1+e\right)\left(T-T_{\mathrm{ref3}}\right)+\left(\rho_{\mathrm{I}}c_{\mathrm{I}}\right)dV_{\mathrm{S}}e\left(T-T_{\mathrm{ref3}}\right)$$
$$+\rho_{\mathrm{S}}c_{\mathrm{S}}dV_{\mathrm{S}}\left(T-T_{\mathrm{ref3}}\right)-\rho_{\mathrm{I}}dV_{\mathrm{S}}L\left(n-\theta_{\mathrm{W}}+en-e\theta_{\mathrm{W}}\right) \tag{40}$$

Differentiating Eq. 40,

$$\dot{E}=\left(\rho_{\mathrm{W}}c_{\mathrm{W}}-\rho_{\mathrm{I}}c_{\mathrm{I}}\right)dV_{\mathrm{S}}\left(1+e\right)\left(T-T_{\mathrm{ref3}}\right)\dot{\theta}_{\mathrm{W}}+\left(\rho_{\mathrm{W}}c_{\mathrm{W}}-\rho_{\mathrm{I}}c_{\mathrm{I}}\right)dV_{\mathrm{S}}\theta_{\mathrm{W}}\left(T-T_{\mathrm{ref3}}\right)\dot{e}$$
$$+\left(\rho_{\mathrm{W}}c_{\mathrm{W}}-\rho_{\mathrm{I}}c_{\mathrm{I}}\right)dV_{\mathrm{S}}\theta_{\mathrm{W}}\left(1+e\right)\dot{T}+\left(\rho_{\mathrm{I}}c_{\mathrm{I}}\right)dV_{\mathrm{S}}\dot{e}\left(T-T_{\mathrm{ref3}}\right)$$
$$+\left(\rho_{\mathrm{I}}c_{\mathrm{I}}\right)dV_{\mathrm{S}}e\dot{T}+\rho_{\mathrm{S}}c_{\mathrm{S}}dV_{\mathrm{S}}\dot{T}-\rho_{\mathrm{I}}dV_{\mathrm{S}}L\left[\left(1-\theta_{\mathrm{W}}\right)\dot{e}-\left(1+e\right)\dot{\theta}_{\mathrm{W}}\right] \tag{41}$$

The second term of Eq. 36 can be expressed in terms of heat conduction (i.e., the first term on the right-hand side of the equation) and convection (i.e., the second term on the right-hand side of the equation) as follows.

$$q_{i,i}dV=-\left(k_{ij}^{\mathrm{T}}T_{,j}\right)_{,i}dV+\left(c_{\mathrm{W}}\rho_{\mathrm{W}}v_i\left(T-T_{\mathrm{ref3}}\right)\right)_{,i}dV \tag{42}$$

where k_{ij}^{T} is the thermal conductivity tensor of soil, and v_i is the seepage velocity. Assuming an isotropic soil, k_{ij}^{T} is defined as

$$k_{ij}^{\mathrm{T}}=\delta_{ij}k_{\mathrm{T}} \tag{43}$$

where k_{T} can be expressed as the volume-weighted average of the thermal conductivity of water, ice, and soil grains as in Eq. 44.

$$k_{\mathrm{T}}=\theta_{\mathrm{W}}k_{\mathrm{W}}+\left(n-\theta_{\mathrm{W}}\right)k_{\mathrm{I}}+\left(1-n\right)k_{\mathrm{S}} \tag{44}$$

Finally, the governing equation of the thermal field coupled with the effects of mechanical and hydraulic fields can be re-expressed as Eq. 45.

$$\left[\left(\rho_{\mathrm{W}}c_{\mathrm{W}}-\rho_{\mathrm{I}}c_{\mathrm{I}}\right)\left(T-T_{\mathrm{ref3}}\right)\theta_{,T}^{\mathrm{W}}+\rho_{\mathrm{I}}L\theta_{,T}^{\mathrm{W}}+\left(\rho_{\mathrm{W}}c_{\mathrm{W}}-\rho_{\mathrm{I}}c_{\mathrm{I}}\right)\theta_{\mathrm{W}}+\left(\rho_{\mathrm{I}}c_{\mathrm{I}}\right)\frac{e}{1+e}+\rho_{\mathrm{S}}c_{\mathrm{S}}\frac{1}{1+e}\right]\dot{T}$$
$$+\left[\left(\rho_{\mathrm{W}}c_{\mathrm{W}}\right)\theta_{\mathrm{W}}\left(T-T_{\mathrm{ref3}}\right)+\rho_{\mathrm{I}}c_{\mathrm{I}}\left(1-\theta_{\mathrm{W}}\right)\left(T-T_{\mathrm{ref3}}\right)-\rho_{\mathrm{I}}L\left(1-\theta_{\mathrm{W}}\right)\right]\dot{\varepsilon}_{ii}$$
$$-\left(k_{ij}^{\mathrm{T}}T_{,j}\right)_{,i}-\frac{c_{\mathrm{W}}}{g}\left(T-T_{\mathrm{ref3}}\right)\left(k_{ij}^{\mathrm{H}}p_{,j}\right)_{,i}-c_{\mathrm{W}}\rho_{\mathrm{W}}\left(T-T_{\mathrm{ref3}}\right)\left(k_{ij}^{\mathrm{H}}h_{\mathrm{elev},j}\right)_{,i}$$
$$-\frac{c_{\mathrm{W}}\rho_{\mathrm{I}}L}{gT_{\mathrm{ref2}}}\left(T-T_{\mathrm{ref3}}\right)\left(k_{ij}^{\mathrm{H}}\left(T-T_{\mathrm{ref2}}\right)_{,j}\right)_{,i}=0 \tag{45}$$

The void ratio in Eq. 45 is calculated as

$$e=e_0+\left(1+e_0\right)\left(\varepsilon_{ii}\right) \tag{46}$$

where e_0 is the initial void ratio, and ε_{ii} is the volumetric strain.

3. Simulation of Thaw Consolidation of Degrading Permafrost

The THM model is applied to study the thaw consolidation of a three-dimensional soil column. The height of the rectangular soil column is 100 mm, and its cross-section is 10 mm by 10 mm. The bottom boundary of the soil column is fixed, and its side boundaries are laterally confined. All boundaries of the soil column are impermeable, except for the top boundary, which is a free-drainage boundary. The soil column has an initial temperature of 272.15 K. Then, an overburden stress of 50 kPa was applied on the top boundary and the temperature of the top boundary was increased to 293.15 K for 18 hours. The soil is silty clay with a dry unit weight of 17.8 kN/m^3 and a total moisture content of 19%. Other model input parameters are presented in Table 1.

Table 1. Model input parameters.

Parameters	Value	Unit
Shear modulus of frozen soil, G_f	1.4×10^6	Pa
Bulk modulus of frozen soil, K_f	3.0×10^6	Pa
Shear modulus of unfrozen soil, G_u	2.0×10^5	Pa
Bulk modulus of unfrozen soil, K_u	4.3×10^5	Pa
Coefficient of thermal expansion, α_T	0	1/K
Density of water, ρ_W	1000	kg/m^3
Density of ice, ρ_I	918	kg/m^3
Density of soil grain, ρ_S	2650	kg/m^3
Gravimetric specific heat capacity of water, c_w	4184	J/kg/K
Gravimetric specific heat capacity of ice, c_I	2100	J/kg/K
Gravimetric specific heat capacity of soil grain, c_S	800	J/kg/K
Thermal conductivity of water, k_W	0.613	J/m/s/K
Thermal conductivity of ice, k_I	2.31	J/m/s/K
Thermal conductivity of soil grain, k_S	1.10	J/m/s/K
Hydraulic conductivity of unfrozen soil, k_{H_u}	1×10^{-8}	m/s
Hydraulic conductivity of frozen soil, k_{H_f}	1×10^{-12}	m/s
Acceleration due to gravity, g	9.81	m/s^2
Latent heat of fusion, L	333500	J/kg
Initial void ratio, e_0	0.23	Unitless
Reference temperature 1, T_{ref1}	273.15	K
Reference temperature 2, T_{ref2}	273.15	K
Reference temperature 3, $T_{ref3} = T_{initial}$	272.15	K
Parameter 1 for calculating water content, α	0.08	Unitless
Parameter 2 for calculating water content, β	−0.5	Unitless

Figure 1 shows how the three primary variables (i.e., temperature, pore water pressure, and displacement) vary with depth over time at the centerline of the soil column. The top boundary of the soil column is at the height of 0.1 m; the bottom boundary is at 0 m. As shown in Figure 1a, the temperatures of the top and bottom boundaries are fixed at 293.15 K and 272.15 K, respectively. At 0.20 hour, the sharp reduction of temperature at height of 0.088 m is due to the phase change of ice to water. Since heat energy is used for phase change, the soil temperature remains relatively the same at the initial temperature (i.e., 272.15 K) from the depth of 0 m to 0.088 m. Similarly, at 0.50 hour, the sharp reduction of temperature occurs at 0.062 m indicating that the soil at this depth is experiencing phase change. As time increases, the soil temperature gradually increases, creating a thermal equilibrium between the top and bottom boundaries.

As temperature and consequently the unfrozen water content increase, the overburden stress imposed on the top surface of the soil column is transferred from the ice matrix to pore water in the soil. As a result, as shown in Figure 1b, pore water pressure has the highest value near the beginning of the simulation and gradually dissipates with time. Since unfrozen water exists in permafrost, the pore water pressure of the soil column below the melting point of ice is not zero but follows the hydrostatic pressure profile of the soil. Meanwhile, the pore water pressure at the top surface (i.e., the height of 0.1 m) is zero at all time given that water can freely drain through the top surface. The consolidation process of degrading permafrost is illustrated in Figures 1b and 1c. As pore water pressure dissipates, the soil column experiences settlement. The settlement of the soil column is indicated by the increase in the magnitude of the vertical displacement at the top boundary over time as presented in Figure 1c. The vertical displacement at the bottom boundary is zero at all times since this boundary is fixed.

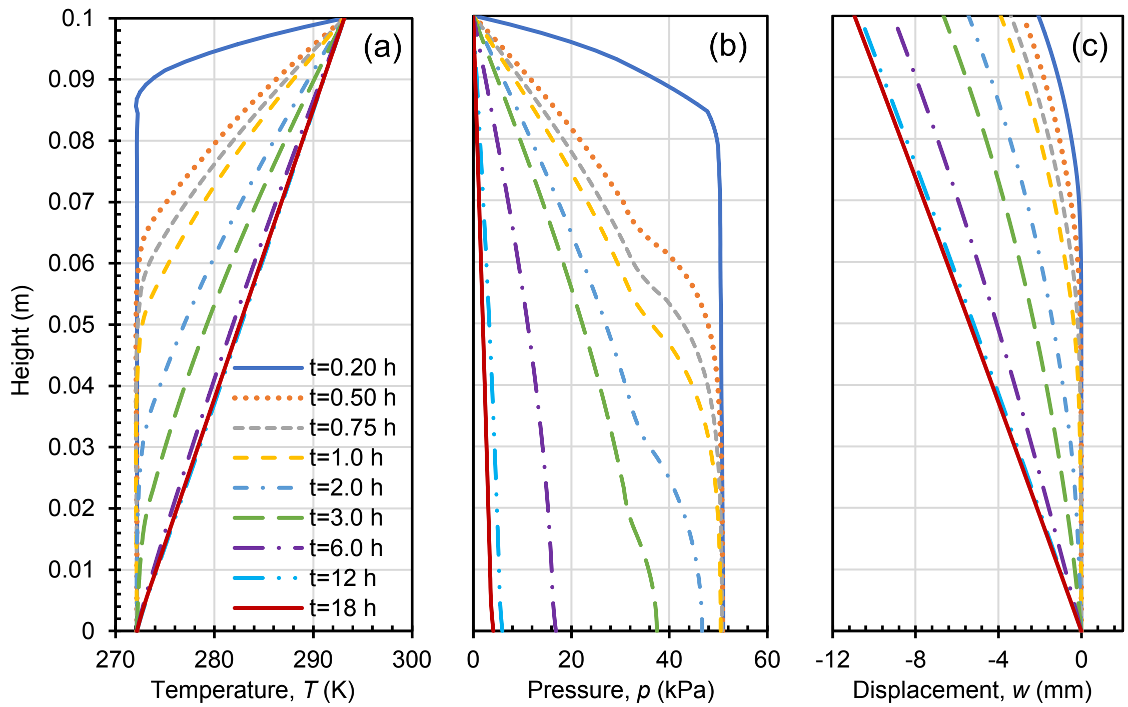

Figure 1. Model simulation profiles of (a) temperature, (b) pore water pressure, and (c) vertical displacement over time at the centerline of the soil column.

4. Conclusions

This paper presents a fully coupled three-dimensional THM model for simulating the thaw consolidation of permafrost. The model is implemented using the finite element method. The following physical processes are considered: heat conduction, thermal convection, phase change, pore water pressure generation and dissipation, volumetric expansion due to phase change and temperature change, and deformation due to drainage and dissipation of pore water pressure. By defining different values for the compressibility and hydraulic conductivity of thawed and frozen regions, the proposed model can simulate the hydromechanical behaviors of thawing permafrost. However, soil behaviors at the ice-water interface cannot be captured under current formulations. Since the THM model is fully coupled, the thaw penetration depth of the degrading permafrost is calculated by solving the three governing equations; the ice melting temperature effectively indicates the boundary between the thawed and frozen layers. The model simulations show that as heat transfers into the soil system, the soil temperature increases, and ice changes into water, generating excess pore water pressure. The soil column settles as pore water dissipates with time. The results also show that the pore water pressure follows the profile of hydrostatic pressure when the soil temperature is below the freezing point of water since unfrozen water exists in permafrost even below this temperature.

5. Acknowledgements

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6. References

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