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Loaded Microstrip Network Transmision-Line Metamaterials on an Irregular Grid

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***Abstract*—A planar loaded microstrip network metamaterial on an irregular grid for modeling 2-D electromagnetic environment in TE polarization is presented. The technique is applied to a Luneburg lens example toward printed circuit implementation and verification.**

I. Introduction

Transmission-line (TL) metamaterials can model 2-D electromagnetic environment using a planar network of loaded TL segments [1], [2]. They have been investigated to model devices involving simple, anisotropic [3], and bianisotropic [4] materials and devices. Most TL metamaterials have been modeled and fabricated on a periodic grid, allowing for efficient and accurate design using unit-cell analysis. However, a periodic regular grid places limitations on modeling curved boundaries. Furthermore, the planar surface dictated by a periodic grid is fundamentally incompatible with a doubly-curved surface profile.

In this paper, a printed TL metamaterial design using impedance-loaded microstrip (MS) network on an irregular or unstructured grid is presented. TL metamaterials on an irregular grid adds flexibility in modeling such as curved device or domain boundaries. Expanding on our previous work [5], we design a planar MS network with lumped series and shunt loads to move toward fabrication using standard printed-circuit technologies.

II. A Polygon Loaded Microstrip Network Cell

The TL metamaterial is a union of cells having irregular polygonal cell boundary, where a single cell comprises a network of microstrip segments loaded with lumped elements. A typical cell is shown in Fig. 1(a), which occupies an area S in the xy-plane. The node is placed at the origin and the size and shape of the cell is set by the irregular grid generated using an unstructured mesher. The n-th branch comprises an MS segment of width w_n and length l_n in series with a lumped reactive element. The total branch impedance is Z_n. The shunt admittance from the node to the ground is Y. It is desired that each series reactive impedance be realized by adjusting the MS width w_n. An additional lumped inductor or capacitor element may be needed.

III. Realization of Effective Medium Parameters

Since free space cannot be modeled using an MS network, we define a reference medium for defining the relative material parameters as an *unloaded* MS network on a *periodic hexagonal* grid with a cell size (i.e., the node-to-node distance) of p_0. We choose $p_0 = \lambda/8$, where λ is the wavelength of the reference medium. The equilateral hexagonal cell having an area S_0 for the reference MS medium is shown in Fig. 1(b), where the immitance values are Z_0 and Y_0.

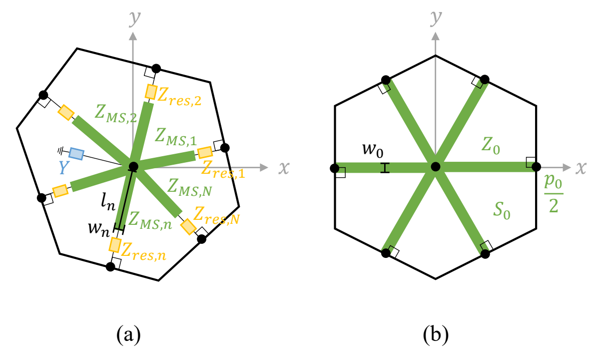

Fig. 1. Loaded MS network cells. (a) A general N-sided polygon cell. (b) A regular hexagonal cell with MS width w_0. Series impedance for each branch is Z_0. The shunt admittance for the cell is Y_0.

For per-unit-length reactance and susceptance of the MS comprising the network, we have $X'_{\text{MS}} = k_{\text{MS}} Z_{\text{MS}}$ and $B'_{\text{MS}} = k_{\text{MS}}/Z_{\text{MS}}$, respectively, where k_{MS} and Z_{MS} are the propagation constant and the characteristic impedance of the MS, respectively. The immittance values for the reference cell in Fig. 1(b) are

$$Z_0 = jX_0 = jX'_{\text{MS}}\left(\frac{p_0}{2}\right), \ Y_0 = jB_0 = jB'_{\text{MS}}(3p_0). \quad (1)$$

The series and shunt immittance values for modeling a 2-D environment in TE polarization with a relative permittivity ϵ_{zz} and a tensor permeability $\boldsymbol{\mu} = [\mu_{xx}\ \mu_{xy}; \mu_{yx}\ \mu_{yy}]$ in the xy-plane are known in terms of the geometrical details of the cell [5]. Let these relative medium parameters be defined relative to the reference MS medium. The shunt load Y is expressed as

$$Y = \epsilon_{zz} Y_0 \left(\frac{S}{S_0}\right). \quad (2)$$

The N number of series impedances Z_n are found by solving five linear equations in N unknowns [5]. They are found to be a function of S/S_0, p_0, B_0, and $\boldsymbol{\mu}$.

The per-unit-length reactance of the MS plays a key role in designing a 2-D MS circuit network. A range of realizable

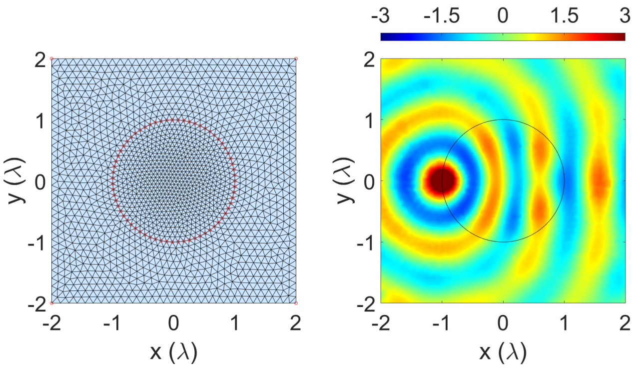

Fig. 2. A 2λ-diameter Luneburg lens. (a) An unstructured mesh over $4\lambda \times 4\lambda$ with fixed points during mesh generation indicated as red circles. (b) Voltage distribution in mV for a 1 mA current source at $(-\lambda, 0)$.

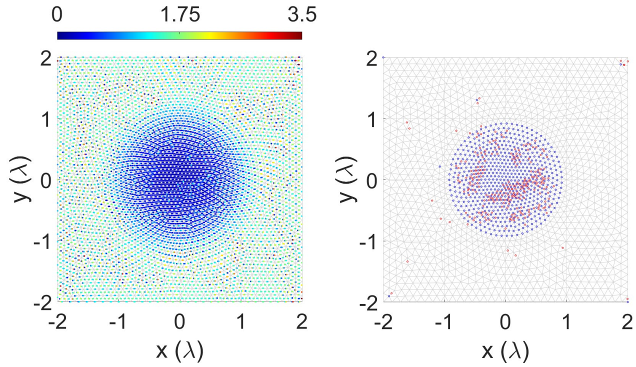

Fig. 3. Design details for the Luneburg lens. (a) The MS width distribution in mm. (b) Locations of the additional L, C elements (series elements as red dots and shunt elements as blue dots).

X'_{MS} is calculated based on the available microstrip widths at a fabrication facility. Since there is a limited range of X'_{MS} available, we split the series reactance X_n in $Z_n = jX_n$ into MS-contributed and residual values as

$$X_n = X_{\text{MS},n} + X_{\text{res},n}. \tag{3}$$

If the MS reactance $X_{\text{MS},n}$ with a width w_n within the practical range cannot realize X_n by itself, a lumped reactive component is introduced to account for $X_{\text{res},n}$. It should be remembered that commercially available L, C components come in discrete element values.

IV. Design Example

The device chosen to model using the loaded MS network is the Luneburg lens of an $a = 2\lambda$ diameter, which is an isotropic, continuous medium device with a relative permitivity of

$$\epsilon_{zz} = 2 - \left(\frac{\rho}{a}\right)^2 \tag{4}$$

inside the lens boundary at $\rho = a$ and the reference medium outside the lens. The device is modeled over a $4\lambda \times 4\lambda$ grid, as shown in Fig. 2(a). There are 1,957 nodes and 4,981 branches in the grid.

Manufacturer requirements and component availability constrain the design for fabrication. In this design, an FR-4 substrate of thickness $d = 1.0$ mm, dielectric constant $\epsilon_r = 4.5$, and loss tangent $\tan\delta = 0.02$ is selected. To meet the maximum board size requirement, the reference medium wavelength of $\lambda = 90$ mm is chosen. Copper traces has a 0.035 mm thickness and the widths are bounded to $[0.127, 3.5]$ mm. The design frequency is determined to be $f = 1.29$ GHz.

The width distribution for the MS branches is shown in Fig. 3(a). The reference cell has $w_0 = 1.5$ mm, and the majority of MS branches have widths close to w_0. As a result, not many lumped loads are required, as shown in Fig. 3(b). It is found that 256 lumped inductors and 615 lumped capacitors are needed.

The loaded MS network metamaterial has been modeled and analyzed using Pathwave ADS. A current source injects 1 mA at $(x', y') = (-\lambda, 0)$, and the resulting voltage distribution is plotted in Fig. 2(b). The cylindrical wave is transformed to have planar wavefronts by the lens in the $+x$-direction. The lumped loads have been selected from commercially available L, C components, with the inductance in the range [1.3,5.2] nH and [68,91] nH for series and shunt components, respectively. Similarly, the capacitance values are in the range of [160,2200] pF and [0.15,1] pF for series and shunt components, respectively.

V. Conclusion

A 2-D MS-network TL metamaterial design has been presented together with a Luneburg lens example. The circuit network parameters are found from the required device parameters and the irregular-polygon cell geometry. The required immitance values are translated into MS geometries and additional lumped loads. Lumped-distributed circuit simulation validates the design.

Acknowledgment

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