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Integrating dial-a-ride with transportation network companies for cost efficiency: A Maryland case study

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ABSTRACT

Dial-a-ride (DAR) is a shared-ride service that provides mobility to transportation-disadvantaged individuals who are unable to use public transit. While most DAR studies focus on optimizing operations, our research explores the feasibility and benefits of outsourcing outlier trips to transportation network companies (TNCs) to minimize the combined service delivery cost. To achieve this goal, we formulate a multi-vehicle DAR problem (DARP) with trip outsourcing to TNCs, which can be solved optimally for small scale instances. To solve larger instances, we propose a two-stage solution framework to improve DAR routes from commercially available software. Firstly, we develop an integer programming model to re-optimize individual routes with trip outsourcing. Secondly, we design a multi-vehicle heuristic that considers reinserting trips initially designated for outsourcing back into the DAR fleet, as well as reinsertion and exchange of remaining trips among routes. We apply the approach to a medium-sized DAR operator in Maryland and achieve cost reductions of 7%-13% depending on the TNC volume discount negotiated by the DAR company.

1. Introduction

The Americans with Disabilities Act (ADA) of 1990 requires transit agencies across the United States to provide door-to-door transportation services for individuals who cannot use conventional transit. These ride-sharing services, commonly known as dial-a-ride (DAR) or ADA paratransit, are typically provided through subcontractors (Rodman and High, 2018). However, although DAR customers are required to pay a fare similar to conventional transit, the actual cost of a DAR trip can be 10 times higher due to low vehicle occupancy, specialized vehicles, and increased labor per passenger (Gonzales et al., 2019). Consequently, DAR services are heavily subsidized, and service providers are under pressure to reduce their operating costs. To address this challenge, algorithms have been developed to efficiently route and schedule DAR vehicles, which is known as the DAR problem (or DARP) in the literature (Cordeau and Laporte, 2003a, 2007).

In addition to developing efficient vehicle routing algorithms, researchers and practitioners have explored innovative solutions, strategies, and partnerships to further reduce service costs. One such strategy involves integrating DAR with transportation network companies (TNCs), which has been adopted by some transit agencies such as the Massachusetts Bay Transportation Authority's collaboration with Uber in the RIDE Flex program and The Big Blue Bus (BBB) partnership with Lyft in the Mobility on Demand

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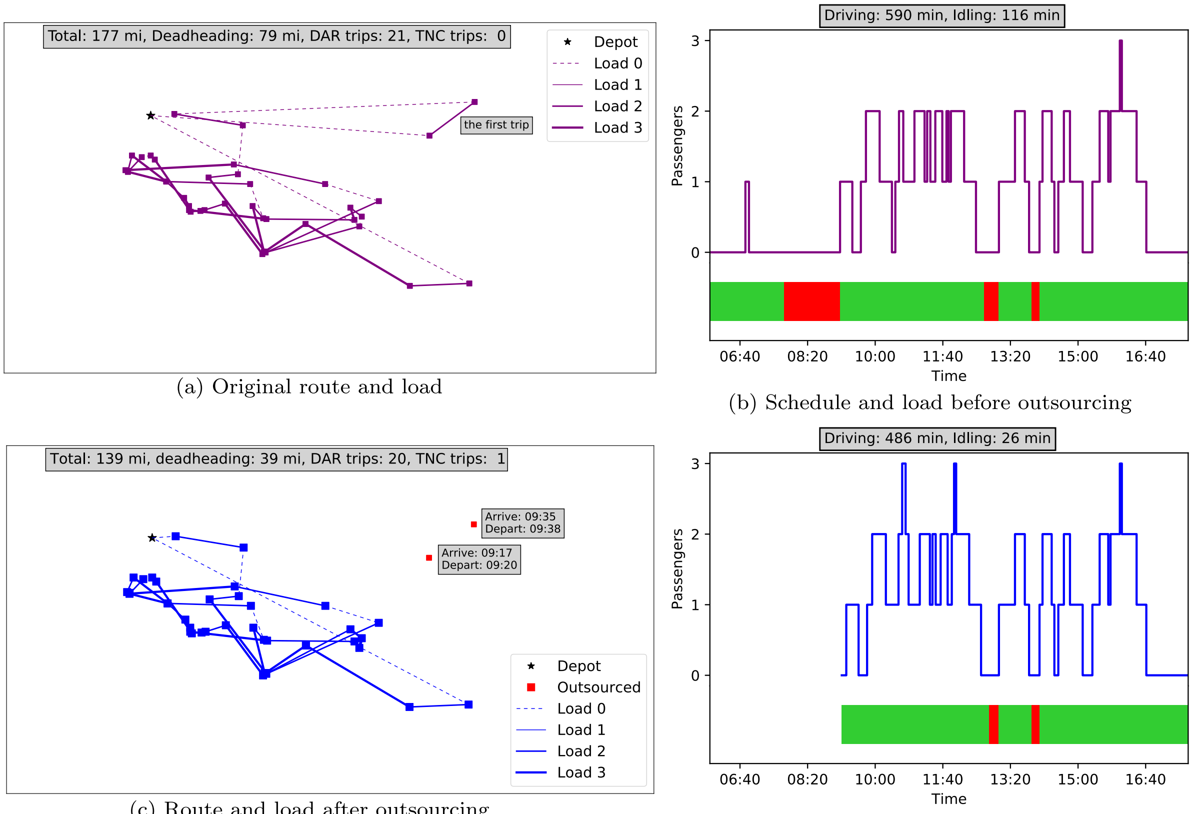


(a) Original route and load

(b) Schedule and load before outsourcing

(c) Route and load after outsourcing

(d) Schedule and load after outsourcing

Fig. 1. Example of a route and benefits of outsourcing a trip that represents a spatio-temporal outlier.

Everyday (MODE) program in Santa Monica, California. A review of relevant partnerships in the U.S. is available in Section 2.2, highlighting the significance of considering modal integration in the context of the DARP.

The benefits of integrating DAR with TNCs depend on the spatio-temporal distribution of demand, which consists of thousands of diverse requests every day. A trip request includes the pickup address, drop-off location, and the desired pickup or drop-off time. Requests that have similar spatio-temporal characteristics facilitate ridesharing and significantly reduce service costs. However, requests that are incompatible with the rest of the demand may result in excessive costs due to extended operating hours in the case of temporal outliers and significant deadheading in the case of spatial outliers. This raises the question of whether outsourcing some of the outlier trips to TNCs would be more cost-effective for the DAR service provider.

Our main hypothesis is that the cost of DAR services can be substantially reduced by identifying and outsourcing trips that are spatio-temporal outliers from the rest of the demand. To illustrate this phenomenon, we consider the example route shown in Fig. 1. From Fig. 1(a), we can see that outsourcing the first trip could be economical because its origin and destination are far away from the depot and other demand locations. Additionally, the user-imposed service time windows for this trip produce extensive idle time in the vehicle schedule, as shown in Fig. 1(b), making it a suitable candidate for outsourcing. By outsourcing this trip, we can reduce routing distance and duration, as illustrated in Figs. 1(c) and 1(d), respectively.

This illustrative example suggests that integrating DAR with TNC could be a viable approach to reduce the overall cost of the transportation service. However, this promising modal integration has not yet been systematically studied in the literature, particularly in the context of using real-world data to inform the decision-making of transit agencies.

The lack of systematic evaluation of the benefits of integrating DAR with TNC could be attributed to methodological and computational barriers. Even the DARP without any modal integration is notoriously difficult, and the largest instance solved optimally only includes 8 routes and 96 trips (Ropke et al., 2007; Rist and Forbes, 2021). Consequently, various heuristics are used to tackle real-world DARP instances with thousands of trips, producing quality solutions that are near-optimal at best. DAR operators have access to these heuristics through software suites like Trapeze[1] and Mobile Resource Management System (MRMS),[2]

[1] http://www.trapezegroup.com/
[2] http://itcurves.net/

which allow them to route thousands of trips into hundreds of routes in a short time. However, these heuristics do not consider the integration of DAR with other services or transportation modes. To address this issue, we propose a two-stage approach to improve DAR routes from commercially available software. First, we develop methods to re-optimize routes one at a time while also allowing for the outsourcing of trips to TNC. Second, we propose a multi-vehicle heuristic that considers the reinsertion of trips initially designated for outsourcing back to the DAR fleet, as well as the reinsertion and exchange of remaining DAR trips among the routes to further reduce operational costs.

The main contributions of the paper are described as follows:

1. We consider the DARP with trip outsourcing to TNC as a means to reduce the combined service delivery cost of both the DAR and TNC systems. In contrast to related studies that have mainly treated taxis as a backup option for when the DAR system capacity is exceeded or DAR vehicles are delayed, we explore the integration of DAR with TNC for cost reduction. Additionally, we consider the cost of DAR vehicle idle time, which is often ignored in previous studies, but is an important factor that needs to be accounted for when optimizing the tradeoff between DAR and TNC costs.
2. We propose a two-stage heuristic approach to improve DAR routes generated by commercially available software by identifying and outsourcing outlier trips to TNC. Our approach is evaluated on small-scale instances that can be solved optimally using mathematical programming methods. Through comparisons with optimal solutions, we demonstrate that our two-stage heuristic approach quickly returns near-optimal solutions. Our performance evaluation demonstrates that our approach is effective and efficient for real-world instances.
3. We demonstrate the application of our proposed two-stage approach to a dataset from a mid-size DAR operator in Maryland. We compare the results to the original MRMS routes and find considerable cost reductions, with total savings ranging from 7% to 13%, depending on the volume discount negotiated by the DAR operator with a TNC. Since these savings are estimated based on real-world DAR and TNC data, our results provide critical insights for transportation authorities that are considering the integration of these types of services.

The paper is organized as follows. Section 2 reviews the relevant literature and highlights our contributions. Section 3 presents a two-stage algorithmic framework for integrating DAR with TNC. Section 4 provides a description of the data used in this study. Section 5 applies the developed model to DAR operations in Maryland and explores the resulting economic benefits. Finally, Section 6 draws conclusions and suggests future extensions of this work.

2. Literature review

We begin by reviewing different variants of DARP and their solution approaches, highlighting a significant gap between academic literature and industry practice. Next, we focus on the integration of DARP and TNC, and review the current state of practice. After examining studies that consider the integration of DAR with taxis and conventional transit, we highlight our contributions to the literature.

2.1. State-of-the-art in DARP modeling and solution methods

The DARP deals with designing routes and schedules for demand-responsive vehicles to serve passengers requesting pickups and drop-offs between origins and destinations (Cordeau and Laporte, 2003a). Typically, a solution is sought to minimize the cost of vehicle routes, subject to a set of constraints that ensure operational requirements and service quality (Heilporn et al., 2011). With the increasing demand for demand-responsive vehicles, additional efforts are required to make their operations more economical while maintaining high service quality (Rahimi et al., 2018). Therefore, a large number of studies are devoted to optimizing routes and schedules in demand-responsive vehicle operations to improve cost efficiency. As the DARP problem generalizes the NP-hard Vehicle Routing Problem with Pickup and Delivery (Cordeau, 2006) and often deals with a large number of requests, various (meta)heuristics have been developed to address the problem in a reasonable amount of time (Reinhardt et al., 2013). Interested readers are referred to Cordeau and Laporte (2007), Molenbruch et al. (2017b), and Ho et al. (2018) for comprehensive reviews of various solution approaches to DARP. Some of the recent solution approaches are discussed briefly in the remainder of this subsection.

Cordeau and Laporte (2003b) developed a tabu search heuristic based on passengers' origin–destination specifications, time windows, and a fleet of vehicles with a common depot. They proposed a procedure to verify the maximum ride time constraints, which is still widely used today. Mathematical programming-based heuristics such as branch-and-cut (BC) were proposed by Cordeau (2006) and Ropke et al. (2007), while a branch-cut-and-price algorithm was put forward by Gschwind and Irnich (2015) and Qu and Bard (2015). Several studies have utilized insertion heuristics Diana and Dessouky, 2004; Luo and Schonfeld, 2007; Marković et al., 2015; Vallée et al., 2020 to solve DARP. Parragh et al. (2010) used a variable neighborhood search (VNS) procedure for DARP, which was further extended in their later study (Parragh, 2011) to consider heterogeneous users and vehicles. A multi-depot DARP was addressed via a Local Neighborhood Search (LNS) algorithm by Molenbruch et al. (2017a).

Hybrid algorithms have also been gaining popularity for solving different variants of the DARP. Zidi et al. (2012) utilized a Simulated Annealing algorithm combined with other heuristics to solve a multi-objective DARP, while Braekers et al. (2014) implemented BC and deterministic annealing algorithms for dealing with multi-depot DARP. Masmoudi et al. (2016) proposed three heuristic solution methods for addressing multi-depot heterogeneous DARP based on Adaptive Large Neighborhood Search (ALNS) and Hybrid Bees Algorithm. Masmoudi et al. (2017) later proposed a genetic algorithm combined with a local search

Table 1
Innovative pilot programs for paratransit riders.

Agency	Program	Location	Partner (s)	Launch
BBB	Mobility on Demand Everyday	Santa Monica, CA	Lyft	2018
rabbittransit	Paratransit	Central Pennsylvania	Lyft, Uber	2017
NYC Transit	Access-a-Ride E-hail	New York City, NY	Uber, Lyft	2017
Omnitrans	Uber RIDE	San Bernardino, CA	Lyft	2016
PSTA	P4-MOD	Pinellas County, FL	Uber	2016

for solving heterogeneous DARP. More recently, Gschwind and Drexl (2019) implemented an adaptive LNS combined with a set covering approach, while Malheiros et al. (2021) utilized a combination of iterative local search with a set partitioning approach for addressing heterogeneous DARP. In the context of shared autonomous vehicles with flow-dependent travel times, Liang et al. (2020) applied a customized Lagrangian relaxation algorithm within a rolling horizon framework to solve DARP for shared autonomous vehicles. Recent studies Dong et al., 2022; Azadeh et al., 2022 incorporated users' preferences in the DARP framework via logit models, where user requests accept/reject decisions are made using profit maximization as the optimization objective.

In a conventional DAR system in the United States, the fare collected from riders must be comparable to the cost of a similar fixed-route transit trip. However, the cost of providing the DAR trip can be ten times larger (Gonzales et al., 2019), which means that DAR operations in the U.S. are never profitable and heavily subsidized. Therefore, a common optimization objective is to minimize the total operating cost while requiring that each rider is served e.g., Vodopivec et al., 2015. Conversely, a few studies e.g., Parragh et al., 2015; Jafari et al., 2016; Tafreshian et al., 2021; Zhao et al., 2022 have considered maximizing the profit by rejecting trips that are not profitable. This is tackled in a mathematical program through a modified objective that maximizes the profit and by adding additional constraints to accept/reject trips based on a binary decision variable. Given the differences in the optimization objective and assumption, it is likely that a heuristic designed for a for-profit operation may not work well for non-profit operations.

The above review demonstrates that researchers in the fields of transportation and operations research have developed various DAR formulations and proposed different types of solution algorithms, which have significantly advanced the DAR practice. However, a gap remains between the academic community and practitioners. For example, the trade-off between vehicle idling and vehicle miles has rarely been explored. Seeking a solution that only minimizes vehicle miles may yield excessive vehicle idle time (as shown in Appendix A) and thus a significant waste of drivers' time (Xiang et al., 2006). In practice, vehicle idling is a key issue that must be addressed, as demonstrated in this paper by considering the operations of Challenger Transportation, a DAR operator serving the Washington, D.C. metro area. In addition, due to the negative impact of vehicle idling on the environment (e.g., air pollution), an eco-routing strategy for paratransit has been introduced by researchers (Li et al., 2022) and many municipalities have enacted regulations to mitigate vehicle idling (USDOE, 2022). Second, the potential of improving the DAR routes and schedules through collaborative strategies, such as the integration of DAR and TNC, is not explored in these studies. The practical aspects of such integration will be discussed next.

2.2. Survey of DAR-TNC integration in practice

The Federal Transit Administration (FTA) has been a strong supporter of innovative partnerships involving public transit, as evidenced by its Mobility on Demand (MOD) Sandbox Program announced in 2016 (FTA, 2022). One such partnership supported by the program involved collaboration between the Pinellas Suncoast Transit Authority (PSTA) in Florida and Lyft, with the objective of improving the cost-effectiveness and quality of PSTA's paratransit services. An independent evaluation report (Martin et al., 2022) confirmed substantial benefits of the innovative service partnership, such as increased ridership, improved satisfaction, and reduced idle time.

In another notable example, the Massachusetts Bay Transportation Authority (MBTA) offers two choices for its paratransit customers: RIDE and RIDE Flex. The latter represents a collaboration between MBTA and two ride-hailing companies, Uber and Lyft, with the objective of improving the flexibility and cost-effectiveness of the MBTA's paratransit services (MBTA, 2022). The two programs are operated independently, and the services provided also differ. For instance, drivers in the RIDE program will assist customers with embarking and disembarking vehicles, while Lyft or Uber drivers are not required to provide such assistance. Eligible customers pay differently for the service. For example, customers pay a flat fare of $3 per trip while the MBTA covers the rest of the cost up to $40. One clear advantage of Lyft or Uber is that advanced trip bookings are no longer needed due to the high responsiveness of these TNC services. When customers book with The RIDE Access Center (TRAC), their trips may be transferred to Lyft or Uber unless they request to opt-out of the RIDE Flex program.

Table 1 provides additional examples of pilot programs for paratransit riders involving partnerships between transit agencies and TNCs. The launch dates indicate that such partnerships are still in their infancy, and a series of challenges still need to be addressed. For example, it may be difficult to require all TNC partners to accept cash payments and make additional accommodations for customers with special needs. Moreover, transit agencies may require a call center for customers without smartphones, while TNCs may not be willing to offer such a service. For further discussion about the partnerships between public transit agencies and TNCs, readers may refer to Terra et al. (2019) and Shurna and Schwieterman (2020).

2.3. Integration of DAR with taxis

While the integration of DAR and TNC has not been systematically examined in the literature, some relevant studies have explored the complementary role of taxis in serving paratransit demand as a recourse or overflow option when a paratransit vehicle is delayed or capacity is exceeded. These studies are discussed below.

A few studies have investigated the integration of taxis with DAR in the context of using taxis as a backup option to serve trips that would have been delayed due to traffic conditions or other disruptions in vehicle schedules (Cremers et al., 2009; Heilporn et al., 2011; Vodopivec et al., 2015). For example, Heilporn et al. (2011) studied a single-vehicle DARP with stochastic passenger delays, where a passenger missing a DAR trip is served with alternative taxi services. They considered the expected cost of a delay from the absent passenger in the objective function of the single-vehicle DARP, which was solved using an integer L-shaped algorithm. In another study, Vodopivec et al. (2015) integrated DAR with taxis based on decision policies of the probability of delay and expected cost of recourse. They conducted extensive numerical experiments with varying parameters to evaluate taxi recourse options under different decision policies. Finally, in the context of a day-ahead paratransit planning problem, Cremers et al. (2009) considered the integration of taxis with DAR in their two-stage integer recourse model. The first stage considers all early requests, and the second stage considers the late requests that can be outsourced to taxis. Clustering and assignment heuristics were utilized in the two stages to deal with the preparation of routes and assigning DAR vehicles and taxis to the routes.

Other studies have also explored the integration of taxis with DAR to serve trips that could not be accommodated by the existing fleet due to capacity constraints. For example, Toth and Vigo (1996) considered transportation of people with disabilities and used taxi services to supplement the DAR fleet and serve isolated demand. However, their solution approach relied on heuristics only. Rahimi et al. (2018) employed continuum approximation models to identify the level of paratransit demand where taxis could be integrated with paratransit to reduce operating costs. Although their method can define service areas covered efficiently by paratransit and areas where taxis could be used to supplement the service, it uses an aggregate analytical model for the outsourcing decision to taxis, rather than an integer program that can handle the problem at a disaggregate level. More recently, Schenekemberg et al. (2022) proposed using an in-house fleet or a common carrier to jointly cover DAR travel requests and developed a branch-and-cut algorithm to solve the resulting mixed integer program. They also proposed a hybrid genetic algorithm and Q-learning metaheuristic to tackle large instances of the problem. However, similar to other DAR studies, they only considered the routing cost for the DAR vehicles and did not explicitly model vehicle idling costs. This is an important aspect to consider when optimizing the trade-off between DAR and TNC costs since removing idle time cost from the objective results in loss of savings, as discussed in Section 5.3.

2.4. Integration of DAR with public transit

Several studies have investigated the integration of DAR and public transit. At the operational level, Aldaihani and Dessouky (2003) explored a hybrid system consisting of DAR and transit, where they minimized two metrics: the DAR vehicle driving distance and passenger travel time. They used a construction-then-improvement procedure to generate initial solutions for tabu search, which was then used for further optimization. Häll et al. (2009) introduced the concept of integrated DARP, where a part of a DAR trip can be covered by public transit. They developed a vehicle-arc formulation and solved it for four rider requests. In a follow-up study, Posada et al. (2017) improved the modeling realism of transfers between DAR and transit by explicitly considering waiting time at the transfer location. However, in their numerical experiments, they found it computationally challenging to solve instances consisting of more than six requests within two hours. Therefore, the operational integration of DAR and transit remains challenging, and only small instances (with fewer than six requests) can be solved.

A few related studies considered integration of DAR and public transit at the planning level. In a transit network design study considering demand uncertainty, An and Lo (2014) determined the network structure and service frequencies for fixed-route transit in the first stage and optimized the deployment of flexible services, such as DAR, in the second stage. They demonstrated their proposed method on a ten-node network. Chu et al. (2022) tackled a related problem with a two-stage stochastic program involving a single transit line. It was assumed that DAR was used only when public transit was unavailable. In situations where neither DAR nor transit was available, a penalty was incurred. In contrast with the studies that considered integration with public transit at the operational level, the aforementioned planning studies considered DAR as a supplementary element in the design.

2.5. Summary

Despite the extensive research on DARP in the past few decades, a major research gap remains unaddressed, which is the cost trade-off between DAR and TNC during the routing and scheduling stage. To fill this gap, we propose a two-stage framework to integrate TNC into DAR operations and explore the potential savings from outsourcing spatio-temporal outlier trips. In the first stage, we formulate an integer program for a single-route DAR problem with trip outsourcing to re-optimize individual routes. In the second stage, we develop a multi-vehicle improvement procedure that considers reinsertion of some trips initially designated for outsourcing into the DAR fleet, while also reinserting and exchanging the remaining trips among the routes.

3. Methodology

The proposed research methodology is structured as follows. In the first part, we introduce a formulation of the multi-vehicle DARP with trip outsourcing, which can be solved optimally for small-scale instances. For larger instances, we propose a two-stage framework. The first stage optimizes individual routes by solving a single-route DARP with trip outsourcing. This problem can be solved optimally within a reasonable amount of time, and we also propose an effective heuristic to speed up computations. In the second stage, we present a multi-vehicle heuristic that considers reinserting trips initially designated for outsourcing back into the DAR fleet. The multi-vehicle heuristic also considers the reinsertion and exchange of remaining DAR trips among the routes to further reduce operating costs.

3.1. Multi-vehicle DARP with TNC

Graph. Using the notation from Ropke et al. (2007), we let n denote the number of passengers that were initially assigned to the route. The multi-vehicle DARP with outsourcing to TNC is defined on a complete directed graph $G = (N, A)$, where $N = P \cup D \cup \{0, 2n+1\}$, $P = \{1, \dots, n\}$, and $D = \{n+1, \dots, 2n\}$. Subsets P and D contain pick-up and drop-off nodes, respectively, while nodes 0 and $2n+1$ represent the depot. Associated with each user i are thus an origin node i and a destination node $n+i$.

Parameters. Let K denote the set of DAR vehicles. Each DAR vehicle $k \in K$ has a capacity of $Q^{\max}$ passengers, and the total duration of its route cannot exceed a known time limit $T^{\max}$. Each node $i \in N$ is associated with a load q_i and a non-negative service duration d_i such that, for $i = 1, \dots, n$, we have $q_0 = q_{2n+1} = 0$, $q_i = -q_{n+i}$ and $d_0 = d_{2n+1} = 0$. A time window $[e_i, l_i]$ is associated with node $i \in N$, where e_i and l_i represent the earliest and latest time, respectively, at which service may begin at node i. With each arc $(i, j) \in A$, there is a routing cost c_{ij} and a travel time t_{ij}. Let f and α denote the fixed cost for using the DAR vehicle and the variable cost of vehicle idling, respectively. We denote by ξ_i the cost of outsourcing trip i to TNC. Let r_i denote the maximum ride time coefficient for trip i, such that $r_i \cdot t_{i,n+i}$ represents the maximum ride time for user i, which prevents excessive ride time that could otherwise occur due to detours.

Variables. For each arc $(i, j) \in A$, let x^k_{ij} be a binary variable that equals 1 if the DAR vehicle k travels from node i to node j directly and 0 otherwise. For each node $i \in N$, let z^k_{min} and z^k_{max} be the time at which the vehicle k begins and ends service, and T^k is the routing duration for vehicle k. Let Q^k_i be the load of the vehicle k after visiting node i. For each user i, let L^k_i be the ride time of user i on the vehicle k. To enable outsourcing of trips, let y_i be a binary variable equal to 1 if customer i is outsourced to TNC and 0 otherwise. Note that if a rider is served by TNC, the maximum ride time will not be exceeded as TNC in this study offers non-shared services. Furthermore, for trips that cannot be outsourced for any reason, we let the corresponding $y_i = 0$.

Formulation. With the notation described above and also summarized in Table 2, we can state the multi-vehicle DARP with outsourcing to TNC as an integer program:

$$\min \quad \underbrace{f \sum_{k\in K} \sum_{j\in P} x^k_{0,j}}_{\text{fixed cost}} + \underbrace{\sum_{k\in K} \sum_{i\in N} \sum_{j\in N} c_{ij} x^k_{ij}}_{\text{routing cost}} + \underbrace{\sum_{k\in K} (T^k - \sum_{i\in N} \sum_{j\in N} t_{ij} x^k_{ij})\alpha}_{\text{idle time cost}} + \underbrace{\sum_{i\in P} \xi_i\, y_i}_{\text{outsourcing cost}} \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in K} \sum_{j\in N} x^k_{ij} = 1 - y_i, \quad \forall i \in P \tag{2}$$

$$\sum_{j\in N} x^k_{ij} - \sum_{j\in N} x^k_{n+i,j} = 0, \quad \forall i \in P,\ k \in K \tag{3}$$

$$\sum_{j\in N} x^k_{0,j} \le 1, \quad \forall k \in K \tag{4}$$

$$\sum_{j\in N} x^k_{ji} - \sum_{j\in N} x^k_{ij} = 0, \quad \forall i \in P \cup D,\ k \in K \tag{5}$$

$$\sum_{i\in N} x^k_{i,2n+1} \le 1, \quad \forall k \in K \tag{6}$$

$$B^k_j \geqslant B^k_i + d_i(1 - y_i) + t_{ij} - M_i(1 - x^k_{ij}), \quad \forall i \in N,\ j \in N,\ k \in K \tag{7}$$

$$e_i - M_i \cdot y_i \le B^k_i \le l_i + M_i \cdot y_i, \quad \forall i \in N,\ k \in K \tag{8}$$

$$y_i = y_{i+n}, \quad \forall i \in P \tag{9}$$

$$Q^k_j \geqslant Q^k_i + q_j - W_i(1 - x^k_{ij}), \quad \forall i \in N,\ j \in N,\ k \in K \tag{10}$$

$$Q^k_i \le Q^{\max}, \quad \forall i \in N,\ k \in K \tag{11}$$

$$L^k_i = B^k_{n+i} - (B^k_i + d_i(1 - y_i)), \quad \forall i \in P,\ k \in K \tag{12}$$

$$t_{i,i+n} \le L^k_i \le r_i \cdot t_{i,i+n} \quad \forall i \in P,\ k \in K \tag{13}$$

$$z^k_{max} - z^k_{min} \le T^{\max} \quad \forall k \in K \tag{14}$$

$$T^k \le C \sum_{i\in P} x^k_{0,i} \quad \forall k \in K \tag{15}$$

$$T^k \le z^k_{max} - z^k_{min} \quad \forall k \in K \tag{16}$$

Table 2
Nomenclature.

Sets and indices			
i	Node/user index	P	Set of pick-up nodes
D	Set of drop-off nodes	N	Set of nodes
A	Set of arcs (i,j)	K	Set of DAR vehicles
Parameters			
t_{ij}	Travel time of arc (i,j)	d_i	Service duration at node i
e_i	Earliest time of service at node i	l_i	Latest time of service at node i
r_i	Maximum ride time coefficient	q_i	Load at visiting node
$Q^{\max}$	Passenger capacity of DAR vehicle	$T^{\max}$	Maximum route duration
f	Fixed cost of DAR vehicle	c_{ij}	Routing cost of arc (i,j)
α	Variable cost of vehicle idling	ξ_i	Cost of outsourcing to TNC
M_i	Large integer	W_i	Large integer
C	Large integer	n	Number of passengers
Decision variables			
x_{ij}^k	Routing status for arc (i,j)	y_i	Ridesourcing status
Q_i^k	Load after visiting node i	L_i^k	Ride time for user i
z_{min}^k	Starting time of service for vehicle k	z_{max}^k	Ending time of service for vehicle k
T^k	Routing duration for vehicle k		

$$T^k \geq z_{max}^k - z_{min}^k - C\sum_{i\in P} x_{0,i}^k \quad \forall k \in K \tag{17}$$

$$x_{ij}^k, y_i \in \{0,1\};\ B_i^k, L_i^k, T^k, z_{max}^k, z_{min}^k \in \mathbb{R}^+;\ Q_i^k \in \mathbb{Z}^+, \quad \forall i \in N,\ j \in N,\ k \in K. \tag{18}$$

The objective function (1) minimizes fixed and variable vehicle routing and trip outsourcing costs. Constraints (2)–(3) ensure that each request is served exactly once or is outsourced to a TNC. Constraints (4)–(6) guarantee that, unless all the trips are outsourced, the route starts and ends at the depot. Constraints (7) are used to compute service times at each node. For those trips that are not outsourced, constraints (8) ensure that the service begins within the designated time window. Eq. (9) ensure that if a pickup of a trip is outsourced, then the drop-off of that trip is also outsourced. Consistency of the load variable is ensured by constraints (10), which is bounded by constraints (11). Eq. (12) define the ride time of each user, which is bounded by constraints (13). Inequality (14) bounds the duration of the route. Constraints (15)–(17) are used to ensure that routing duration is considered only when vehicles are dispatched from the depot. Finally, (18) indicate which variables are binary, continuous, or integer. It is noteworthy to mention that the values of M_i and C are specified using $M_i = \max\{0, l_i\}$ and $C = T^{max}$, and W_i is specified using $W_i = \min\{Q^{max}, Q^{max} + q_i\}$ following Cordeau (2006).

Next, we explain how the proposed model differs from the classical DARP. As stated earlier, the integer program (1)–(18) builds upon the DARP formulation from Ropke et al. (2007). Specifically, we introduce the trip outsourcing variable y_i, and ensure with (2)–(3) that the DAR vehicle does not visit the pickup and drop-off locations of outsourced trips. By using '≤' instead of '=' in (4) and (6), we allow the DAR vehicle not to be dispatched if all the trips are outsourced. Also, modifications to (8) ensure the removal of time window constraints for the outsourced trips, as they will not be visited by the DAR vehicle. Eq. (9) ensure equal treatment of pickups and drop-offs with respect to lifting the time windows. Modifications in Eqs. (7) and (12) ensure that service duration for a trip is considered only when the trip is not outsourced. Lastly, we introduce three new terms in the objective function (1), where the first two enable comprehensive computation of DAR costs and balance them with the TNC outsourcing cost modeled in the last term. It is worth noting that the first term is commonly considered in studies related to fleet sizing (Golden et al., 1984). The third term represents the cost of idle time, which is the time a DAR vehicle spends waiting for the passenger at a scheduled stop. Neglecting this cost can result in inefficient routes with significant idle time (Xiang et al., 2006). To accurately calculate the routing duration for idle time cost, constraints (15)–(17) are used to ensure that routing duration is considered only when vehicles are dispatched from the depot. The inclusion of idle time cost is the primary differentiating factor of our model from the one recently proposed by Schenekemberg et al. (2022). Therefore, we will evaluate the importance of idle time in the case study.

Since the integer program presented above cannot be solved optimally for real-world instances, we propose a two-stage solution approach to handle larger problem sizes. The proposed framework is initialized based on the classical DARP solution without outsourcing, which can be generated using any DARP heuristic. In our study, we have utilized MRMS to generate the initial routes from the trip requests. In the first stage of our framework, we propose a single-route DAR problem with trip outsourcing to TNC to re-optimize each route from MRMS individually. Next, we use a multi-vehicle heuristic that considers the reinsertion into the DAR fleet of trips previously designated for outsourcing, as well as reinsertion and exchange of the remaining DAR trips among the routes. The proposed framework's workflow is shown in Fig. 2.

3.2. First stage: Single-route DARP with TNC

In the first stage of our approach, we re-optimize each route generated by software systems like MRMS, Trapeze, or other DAR scheduling software suites while considering the option to outsource trips to TNC. To accomplish this, we simplify the multi-vehicle

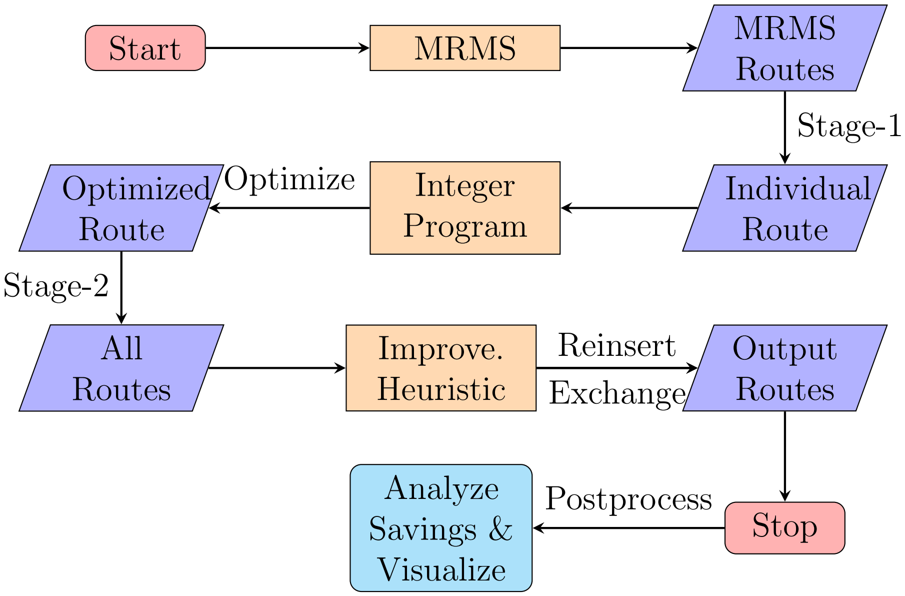


Fig. 2. Proposed two-stage framework.

DARP with TNC model (1)–(18) by dropping the index k, and solve the resulting single-vehicle DARP with TNC model (19)–(36) optimally using an integer programming solver.

$$\min \quad \underbrace{f\sum_{j\in P} x_{0,j}}_{\text{fixed cost}} + \underbrace{\sum_{i\in N}\sum_{j\in N} c_{ij}x_{ij}}_{\text{routing cost}} + \underbrace{(T - \sum_{i\in N}\sum_{j\in N} t_{ij}x_{ij})\alpha}_{\text{idle time cost}} + \underbrace{\sum_{i\in P} \xi_i\, y_i}_{\text{outsourcing cost}} \tag{19}$$

$$\text{s.t.} \quad \sum_{j\in N} x_{ij} = 1 - y_i, \quad \forall i \in P \tag{20}$$

$$\sum_{j\in N} x_{ij} - \sum_{j\in N} x_{n+i,j} = 0, \quad \forall i \in P \tag{21}$$

$$\sum_{j\in N} x_{0,j} \le 1 \tag{22}$$

$$\sum_{j\in N} x_{ji} - \sum_{j\in N} x_{ij} = 0, \quad \forall i \in P \cup D \tag{23}$$

$$\sum_{i\in N} x_{i,2n+1} \le 1 \tag{24}$$

$$B_j \geqslant B_i + d_i(1-y_i) + t_{ij} - M_i(1-x_{ij}), \quad \forall i \in N,\ j \in N \tag{25}$$

$$e_i - M_i \cdot y_i \le B_i \le l_i + M_i \cdot y_i, \quad \forall i \in N \tag{26}$$

$$y_i = y_{i+n}, \quad \forall i \in P \tag{27}$$

$$Q_j \geqslant Q_i + q_j - W_i(1-x_{ij}), \quad \forall i \in N,\ j \in N \tag{28}$$

$$Q_i \le Q^{\max}, \quad \forall i \in N \tag{29}$$

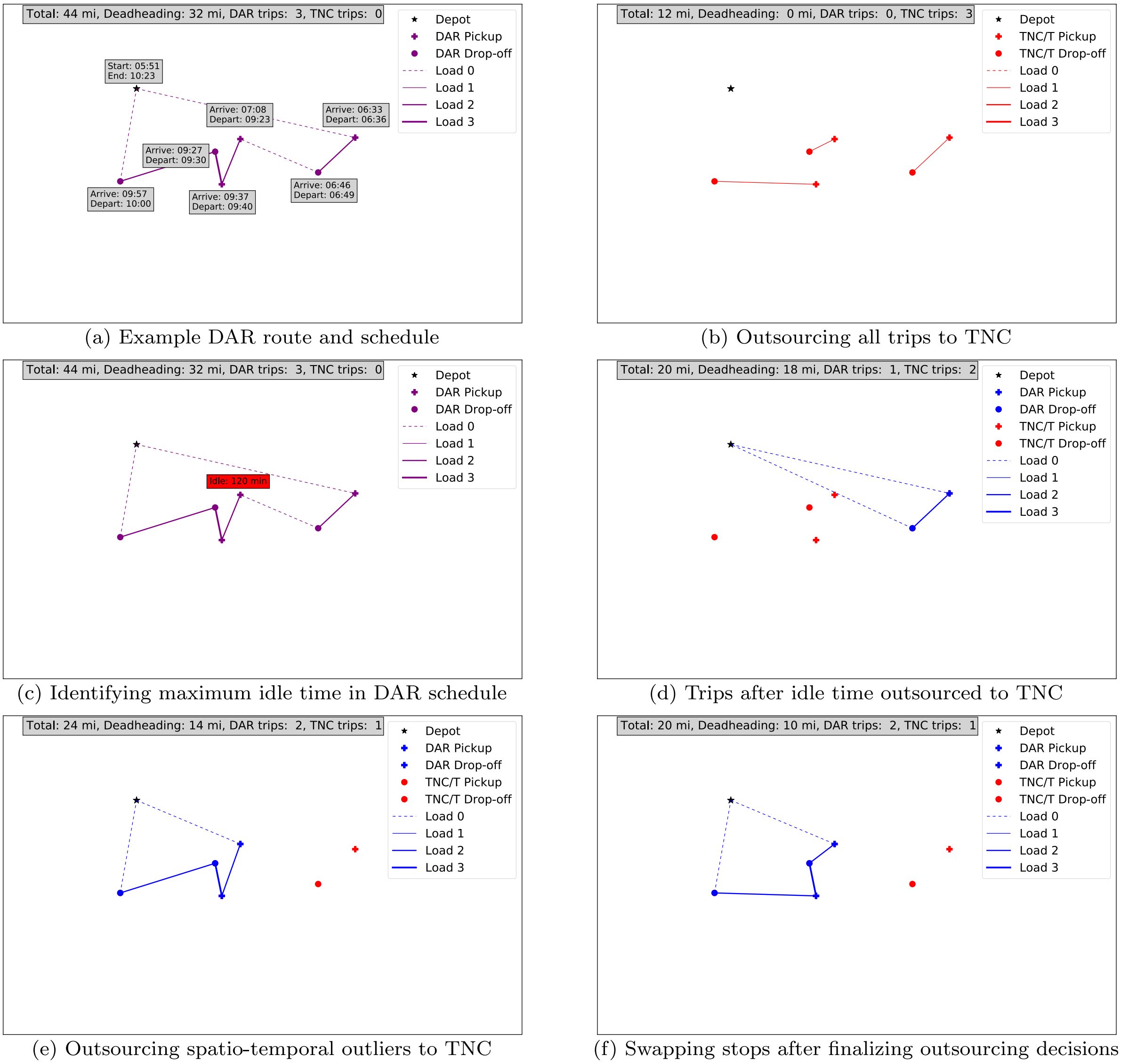
$$L_i = B_{n+i} - (B_i + d_i(1-y_i)), \quad \forall i \in P \tag{30}$$

$$t_{i,i+n} \le L_i \le r_i \cdot t_{i,i+n} \quad \forall i \in P \tag{31}$$

$$z_{max} - z_{min} \le T^{\max} \tag{32}$$

$$T \le C \sum_{i\in P} x_{0,i} \tag{33}$$

$$T \le z_{max} - z_{min} \tag{34}$$



(a) Example DAR route and schedule
(b) Outsourcing all trips to TNC
(c) Identifying maximum idle time in DAR schedule
(d) Trips after idle time outsourced to TNC
(e) Outsourcing spatio-temporal outliers to TNC
(f) Swapping stops after finalizing outsourcing decisions

Fig. 3. Illustration of steps in the custom heuristic.

$$T \geq z_{max} - z_{min} - C \sum_{i \in P} x_{0,i} \tag{35}$$

$$x_{ij}, y_i \in \{0,1\};\; B_i, L_i, T, z_{max}, z_{min} \in \mathbb{R}^+;\; Q_i \in \mathbb{Z}^+, \quad \forall i \in N,\; j \in N. \tag{36}$$

3.2.1. Custom heuristic

A typical DAR route includes about 10–20 trips, which translates to 20–40 pickups and deliveries. For such problem instances, the single-route DARP with TNC can be optimally solved within a few minutes using commercial integer programming solvers such as CPLEX and Gurobi. However, not all DAR operators, whether public or private, have access to commercial solvers as they require licenses. Thus, practical solutions are necessary that can be used with existing scheduling software suites such as Trapeze. To facilitate the implementation of the proposed framework and enable quick exploration of potential savings, we have developed a custom heuristic based on the optimal solutions to (19)–(36) obtained by an integer programming solver. These solutions have revealed the following insights:

1. When a route includes "significant" idle time, the optimal solution to (19)–(36) often outsources all the trips to be served either before or after the idle time. This strategy avoids vehicle idling and the associated driver cost.
2. When the DAR vehicle is underused outside of peak periods, the optimal solution is often to outsource all the trips and dissolve the route altogether. This approach reduces costs by avoiding the fixed cost of a route.

3. If the origin or destination of a trip is distant from the depot or other demand points, the optimal solution is often to outsource that distant trip. This approach reduces costs by minimizing deadheading and the associated cost.
4. After outsourcing trips, some routes can be further improved by only altering the sequence of stops. This adjustment can result in additional savings.

Based on these insights, we propose a heuristic (Algorithm 1) to find a feasible solution to (19)–(36). To illustrate the heuristic steps, we prepare a hypothetical route (Fig. 3(a)). The steps of the heuristic are as follows:

- First, it checks whether outsourcing all the trips and dissolving the route would be more economical than serving it with DAR (Fig. 3(b)). Let $\psi\big(G(N,A)\big)$ represent the cost of DAR trips in a route without allowing any outsourcing, i.e., the sum of fixed cost, routing cost, and idle time cost. If the cost of outsourcing a trip is ξ_i, the cost of outsourcing all the trips in the route is $\sum_{i\in P}\xi_i$. Therefore, based on condition $\sum_{i\in P}\xi_i < \psi\big(G(N,A)\big)$, we can decide whether to outsource every trip i in the route.
- Afterward, it identifies the maximum idle time (Fig. 3(c)) in the schedule and checks whether outsourcing all trips before or after the maximum idle time of a route can provide a more economical route by removing the large idle time from the schedule (Fig. 3(d)). Let $j^* = \operatorname{argmax}_j B_j - B_{j-1}$ represents the node after the maximum idle time in the schedule, where idle time between two consecutive nodes $j-1$ and j in a route is $B_j - B_{j-1}$. By comparing the cost of outsourcing all trips before and after j^* against the cost of DAR routes without outsourcing, it can be decided whether to outsource all trips before or after j^*.
- Next, it checks whether a route contains spatio-temporal outliers by removing each trip at a time and evaluating the before-after route cost (Fig. 3(e)). Let the updated graph after outsourcing trip i is $G(N',A')$. If the cost of the route after outsourcing the removed trip is less than the cost of the route without removing the trip, i.e., $C\big(G(N',A')\big) \le \psi\big(G(N,A)\big)$, the removed trip is identified as a spatio-temporal outlier.
- After deciding on the cheapest outsourcing strategy (e.g., taking route in Fig. 3(e) as the cheapest option after outsourcing), it checks whether the routes can be further improved by swapping stops (Fig. 3(f)) using Algorithm 2. The algorithm removes each trip pick-up i and drop-off $i+n$ and tries to find the minimum cost indices r_i^* and r_{i+n}^* for reinsertion.
- If none of the aforementioned steps reduce the cost of a route, then the route is kept intact (Fig. 3(a)). It is noteworthy to mention that the route schedule is updated and the feasibility of the resulting route is checked after each of the outsourcing steps, which helps to minimize the routing duration. Readers can refer to Luo and Schonfeld (2007) for detailed procedures on updating route schedules and feasibility checks.

3.3. Second stage: Multi-vehicle improvement heuristic

In the second stage, we consider the multi-vehicle extension of the approach outlined in Section 3.2 by using additional improvement procedures using Algorithm 3, which are conducted in two steps as described below:

- First, we consider the reinsertion into existing routes of the trips that the single-route model previously designated for outsourcing. A trip designated for outsourcing is only reinserted if the cost of the trip after reinsertion is lower than the outsourcing cost. The algorithm removes each outsourced trip pick-up i and drop-off $i+n$ and tries to find the minimum cost indices k_i^* and k_{i+n}^* for reinsertion. If the cost after the minimum cost reinsertion is $\psi\big(G(N'\cup N_{k_i}\cup N_{k_{i+n}},A)\big)$, by comparing the reinsertion cost $\psi\big(G(N'\cup N_{k_i}\cup N_{k_{i+n}},A)\big)-\psi\big(G(N,A)\big)$ with the outsourcing cost of the trip $\xi_i y_i$, the algorithm determines whether to reinsert the trip or outsource it permanently.
- Once outsourcing decisions are finalized, we conduct further improvement procedure by considering reinsertion and exchange of remaining DAR trips among the routes. For exchange of the DAR trips, each possible pair of trips $\{i,i+n\}$ and $\{j,j+n\}$ to be exchanged are checked to ensure that there are no service time conflicts, i.e., $e_i \le l_j \wedge e_j \le l_i$, and trips to be exchanged are not same, i.e., $\{i,i+n\} \ne \{j,j+n\}$. Afterwards, the algorithm looks for minimum cost indices (r_i^*, r_{i+n}^*) and (r_j^*, r_{j+n}^*) for exchange. For reinsertion, the algorithm removes each DAR trip $i, i+n$ and tries to find the minimum cost indices k_i^* and k_{i+n}^* for reinsertion after ensuring that the reinsertion indices are not depots ($k_i \ne 0 \wedge k_{i+n} \ne 0$) and pickup index is before drop-off index ($k_i \le k_{i+n}$).

4. Data

This section provides a description of our dataset. We sourced the data from Challenger Transportation,[3] a DAR company that is one of the five subcontractors of the Washington Metropolitan Area Transit Authority (WMATA). The company has been providing door-to-door paratransit services in Maryland since 2000.

4.1. Trip data

The demand dataset comprises 1,057 trips on a typical day, selected by Challenger Transportation. Fig. 4(a) presents the spatio-temporal distribution of demand, which shows that most trips occur away from the depot, between 9 am and 5 pm. The temporal

[3] http://www.challengertrans.com/

Algorithm 1. A custom heuristic to find a feasible solution to (1)–(18)

Input: $\psi\big(G(N,A)\big) = f \sum_{j\in P} x_{0,j} + \sum_{i\in N}\sum_{j\in N} c_{ij}x_{ij} + (B_{2n+1} - B_0 - \sum_{i\in N}\sum_{j\in N} t_{ij}x_{ij})\alpha;$

$C\big(G(N,A)\big) = f \sum_{j\in P} x_{0,j} + \sum_{i\in N}\sum_{j\in N} c_{ij}x_{ij} + (B_{2n+1} - B_0 - \sum_{i\in N}\sum_{j\in N} t_{ij}x_{ij})\alpha + \sum_{i\in P} \xi_i\, y_i;$

Sets, indices, and parameters from Table 2.

Output: A feasible solution to (1)–(18).

procedure Outsourcing DAR trips
  for any route **do**
    ▷ Check if outsourcing all the trips would be cheaper than providing the DAR service
    if $\sum_{i\in P} \xi_i < \psi\big(G(N,A)\big)$ **then**
      for $i \in P$ **do**
        $y_i \leftarrow 1;$
      end for
      for $(i,j) \in A$ **do**
        $x_{i,j} \leftarrow 0;$
      end for
    end if
    ▷ Find the longest idle time and try outsourcing all the trips before or after
    $j^* \leftarrow \text{argmax}_j\ B_j - B_{j-1};$
    if $\sum_{i\in P: i\le j^*} \xi_i < \sum_{i\in P: i>j^*} \xi_i < \psi\big(G(N,A)\big)$ **then**
      for $i \in N : 0 < i \le j^*$ **do**
        $y_i \leftarrow 1;$
        $x_{i,i+1} \leftarrow 0;$
      end for
    end if
    Check feasibility and update schedule;
    if $\psi\big(G(N,A)\big) > \sum_{i\in P: i\le j^*} \xi_i > \sum_{i\in P: i>j^*} \xi_i$ **then**
      for $i \in N : j^* \le i < 2n+1$ **do**
        $y_i \leftarrow 1;$
        $x_{i,i+1} \leftarrow 0;$
      end for
    end if
    Check feasibility and update schedule;
    ▷
    If outsourcing a trip reduces the cost, proceed with its removal
    for $i \in P$ **do**
      $N' \leftarrow N \backslash \{i, i+n\}$
      if $C\big(G(N',A')\big) \le \psi\big(G(N,A)\big)$ **then**
        $y_i \leftarrow 1;$
        $x_{i,i+1} \leftarrow 0;$
      end if
    end for
    Select min. cost route and check rearrangement of stops using Algorithm 2;
  end for
end procedure

distribution of trips exhibits a common bi-modal pattern, with peaks at around 9 am and 2 pm. Silver Spring is the biggest generator and attractor of trips, followed by Hyattsville and Rockville, as shown in Fig. 4(b). The summary statistics and distribution of trip duration and distance are displayed in Figs. 4(c) and 4(d), respectively. The boxplot and histogram of trip duration and distance indicate that the trips are relatively short, with a median direct-ride time of 12 minutes and a distance of 8 miles.

4.2. Baseline routes

The 1,057 DAR trips were assigned to 75 routes using MRMS, a software suite for managing DAR services from trip request management to dispatch and operations. Currently, over a dozen DAR operators across the U.S. use this software suite. At the core of the system is an algorithm for efficient routing and scheduling of vehicles, subject to various operational and level-of-service constraints. This algorithm was developed using insertion and improvement heuristics Luo and Schonfeld, 2007; Marković et al., 2015; Vallée et al., 2020. Algorithm 4 outlines the routing procedure implemented in MRMS, and the schedules of the routes produced by MRMS are displayed in Fig. 5.

Algorithm 2. Outline of the stop swapping procedure

Input:
$\psi\big(G(N,A)\big) = f \sum_{j\in P} x_{0,j} + \sum_{i\in N}\sum_{j\in N} c_{ij}x_{ij} + (B_{2n+1} - B_0 - \sum_{i\in N}\sum_{j\in N} t_{ij}x_{ij})\alpha$;
Sets and parameters from Table 2;
Output: Improved $G(N,A)$ after rearrangement of stops.

procedure Rearranging stops in a route
  for $\{i, i+n\} \in N$ **do**
  ▷ Find best sequence of stops
    $N' \leftarrow N \backslash \{i, i+n\}$;
    for r_i in indices **do**
      for r_{i+n} in indices **do**
      ▷ Ensure indices are not depots and pick-up index is before drop-off index
        if $r_i \neq 0 \wedge r_{i+n} \neq 0 \wedge r_i \leq r_{i+n}$ **then**
          $N_{r_i} \leftarrow i$;
          $N_{r_{i+n}} \leftarrow i+n$;
          $(r_i^*, r_{i+n}^*) \leftarrow \text{argmin}_{(r_i, r_{i+n})}\, \psi\big(G(N' \cup N_{r_i} \cup N_{r_{i+n}}, A)\big)$;
          $N \leftarrow N' \cup N^*_{r_i} \cup N^*_{r_{i+n}}$;
          Check feasibility and update schedule;
        end if
      end for
    end for
  end for
end procedure

Table 3
Parameter values for DAR.

Parameter	Value	Parameter	Value
Capacity of DAR vehicle ($Q^{\max}$)	7 passengers	Maximum route duration ($T^{\max}$)	660 min
Maximum ride time co-efficient (r_i)	2	Service duration at stop (d_i)	3 min
Fixed Cost of DAR vehicle (f)	\$15/route	Idle time cost (α)	\$0.4/min

4.3. Parameter values and unit costs

The capacity of a DAR vehicle is assumed to be $Q^{\max} = 7$ passengers. The maximum route duration is $T^{\max} = 660$ min, and the maximum ride time coefficient is $r_i = 2$. The service duration at each stop is set to $d_i = 3$ min. The estimated fixed cost of DAR operation is $f = \$15$ per route, which includes vehicle preparation and cleaning costs. The company's variable cost is composed of a unit distance cost of \$1.3/veh-mi and a unit time cost of \$0.3/veh-min, resulting in a combined cost of \$0.8/veh-min, assuming an average speed of 25 miles per hour. The routing cost is computed as $c_{ij} = 0.8 \cdot t_{ij}$. The company incurs a variable cost of $\alpha =$ \$0.4/veh-min when the vehicle remains idle during the service time.

In contrast, the unit cost of outsourcing to TNC is based on the breakdown of Uber unit cost.[4] This cost includes a unit distance cost of \$2.8/veh-mi and a unit time cost of \$0.35/veh-min, which can be combined as a unit cost of \$1.55/veh-min, assuming an average speed of 25 mi/h. Additionally, there is a fixed cost of \$10.55, which comprises the base fare and booking fee. The TNC costs are computed as $\xi_i = 10.55 + 1.55 \cdot t_{i,i+n}$, where $t_{i,i+n}$ is the travel time for trip i. The parameter values and unit costs are summarized in Table 3.

5. Results

The numerical results are presented as follows. In Sections 5.1–5.4, we compare the costs of MRMS routes with and without TNC integration. We present the results for individually re-optimized MRMS routes with TNC integration obtained by solving (19)–(36) both optimally and with the custom heuristic (Algorithm 1). In Section 5.5, we present the results from the multi-vehicle improvement procedure (Algorithm 3), which is applied to the routes resulting from the single-route optimization approach. In Section 5.6, we evaluate the performance of the proposed two-stage framework on small-scale instances that can be solved optimally using mathematical programming methods. Finally, we conduct a sensitivity analysis of the overall approach in Section 5.7.

[4] http://taxis-fare.com/uber-fare-finder.php

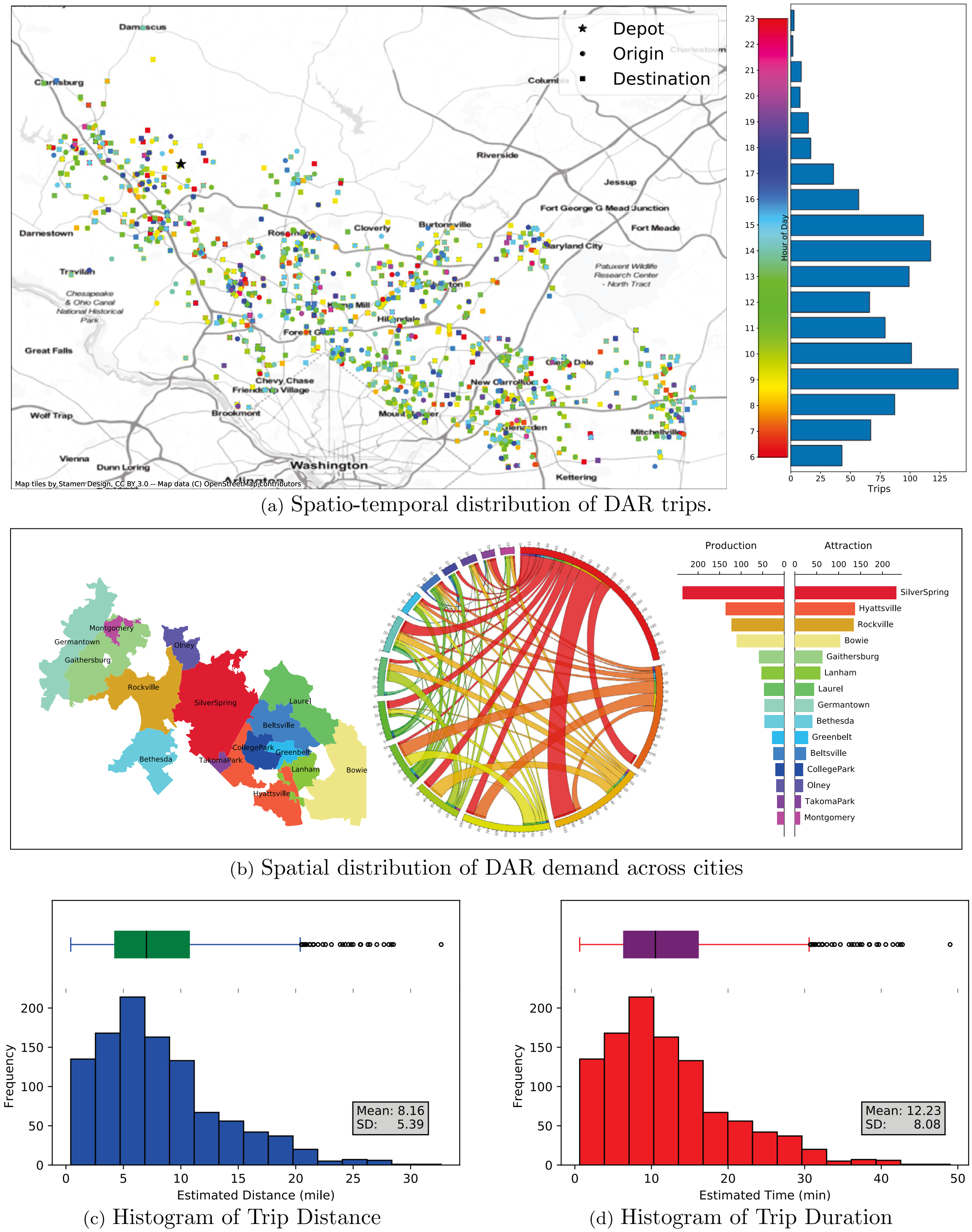
Algorithm 3. Outline of the improvement procedure

Input:
$\psi\big(G(N,A)\big) = f\sum_{j\in P} x_{0,j} + \sum_{i\in N}\sum_{j\in N} c_{ij}x_{ij} + (B_{2n+1} - B_0 - \sum_{i\in N}\sum_{j\in N} t_{ij}x_{ij})\alpha$;
Sets and parameters from Table 2;
Output: Improved $G(N,A)$ after exchange and reinsertion.

1: **procedure** Reinsertion and exchange of trips among routes
2: **for** $\{i, i+n\} \in N$ **do**
▷ Check reinsertion of outsourced trips
3: **if** $y_i = 1$ **then**
4: Conduct steps 24-28;
5: **if** $\psi\big(G(N' \cup N_{k_i} \cup N_{k_{i+n}}, A) - \psi\big(G(N,A) < \xi_i y_i$ **then**
6: $N \leftarrow N' \cup N^*_{k_i} \cup N^*_{k_{i+n}}$;
7: Check feasibility and update schedule;
8: **end if**
9: **end if**
10: **for** $\{j, j+n\} \in A$ **do**
▷ Check trips to be exchanged are not outsourced
11: **if** $x_{i,i+n} \neq 1 \vee x_{j,j+n} \neq 1$ **then**
12: continue;
13: **end if**
▷ Exchange trip indices r provided no service time conflicts between trips
14: **if** $\{i,i+n\} \neq \{j,j+n\} \wedge e_i \leq l_j \wedge e_j \leq l_i$ **then**
15: $N' \leftarrow N \backslash \{i, i+n, j, j+n\}$;
16: $N_{r_i}, N_{r_j} \leftarrow j, i$;
17: $N_{r_{i+n}}, N_{r_{j+n}} \leftarrow j+n, i+n$;
18: $(r^*_i, r^*_{i+n}) \leftarrow \text{argmin}_{(r_i, r_{i+n})}\, \psi\big(G(N' \cup N_{r_i} \cup N_{r_{i+n}}, A)\big)$;
19: $(r^*_j, r^*_{j+n}) \leftarrow \text{argmin}_{(r_j, r_{j+n})}\, \psi\big(G(N' \cup N_{r_j} \cup N_{r_{j+n}}, A)\big)$;
20: $N \leftarrow N' \cup N^*_{r_i} \cup N^*_{r_{i+n}} \cup N^*_{r_j} \cup N^*_{r_{j+n}}$;
21: Check feasibility and update schedule;
22: **end if**
23: **end for**
▷ Find best reinsertion for a trip
24: $N' \leftarrow N \backslash \{i, i+n\}$;
▷ Ensure new indices k are not depots and pick-up is before drop-off
25: **if** $k_i \neq 0 \wedge k_{i+n} \neq 0 \wedge k_i \leq k_{i+n}$ **then**
26: $N_{k_i} \leftarrow i$;
27: $N_{k_{i+n}} \leftarrow i+n$;
28: $(k^*_i, k^*_{i+n}) \leftarrow \text{argmin}_{(k_i, k_{i+n})}\, \psi\big(G(N' \cup N_{k_i} \cup N_{k_{i+n}}, A)\big)$;
29: $N \leftarrow N' \cup N^*_{k_i} \cup N^*_{k_{i+n}}$;
30: Check feasibility and update schedule;
31: **end if**
32: **end for**
33: **end procedure**

Algorithm 4. Outline of the route generation procedure implemented in MRMS

Initialize: Sort requests according to desired pick-up time and introduce the first vehicle.
Step 1: Consider the next unassigned request.
Step 2: For each introduced vehicle: Generate all feasible insertions of the request into the schedule and compute the change in the objective function.
Step 3: If there exists a feasible insertion of the request, then the insertion with the minimum increase in the objective function is selected and the request is inserted while updating the schedule. If a feasible insertion does not exist, a new vehicle is introduced and the request is assigned to it.
Step 4: If there are unassigned requests, then go to step 1.



(a) Spatio-temporal distribution of DAR trips.

(b) Spatial distribution of DAR demand across cities

(c) Histogram of Trip Distance

(d) Histogram of Trip Duration

Fig. 4. Spatio-temporal distribution of demand and trip statistics.

5.1. Route-by-route optimization: High-level summary

The re-optimization of the MRMS routes is performed by solving (19)–(36) using GAMS (General Algebraic Modeling System) with CPLEX as the integer programming solver. Each route is analyzed individually to identify and designate the outlier trips for

Fig. 5. Schedules of MRMS routes. Green and red colors indicate spans of driving and idle periods. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

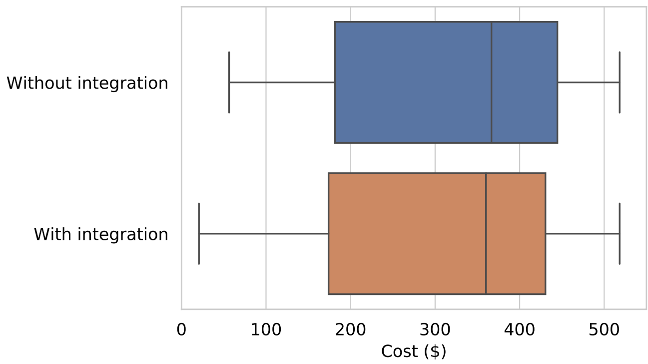


Fig. 6. Route cost distribution.

Table 4
Types of trip outsourcing with savings.

Type	Routes		Savings	
Partial outsourcing	20	(27%)	$556	(43%)
Full outsourcing	8	(11%)	$599	(46%)
No outsourcing	47	(62%)	$135	(11%)
Total	75		$1,290	

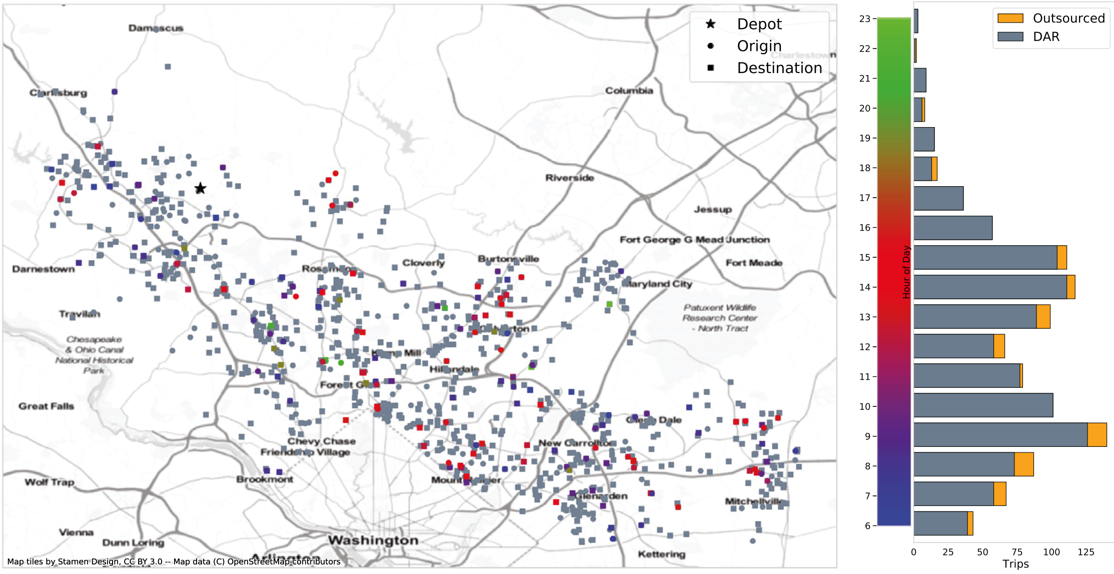
Table 5
Summary of routes before and after outsourcing.

	Trips	Routes	Fixed cost ($)		Veh-min		Veh-miles		idle time	Deadheading
			DAR	TNC	DAR	TNC	DAR	TNC	(min)	(miles)
Before	1,057	75	1,125	–	22,231	–	8,550	–	13,433	3,406
After	976	67	1,005	855	19,562	912	7,524	391	10,175	2,770
Change	8%	11%	11%		12%		12%		24%	19%
Net			$120	−$855	$801	−$319	$1,335	−$1,095	$1,303	

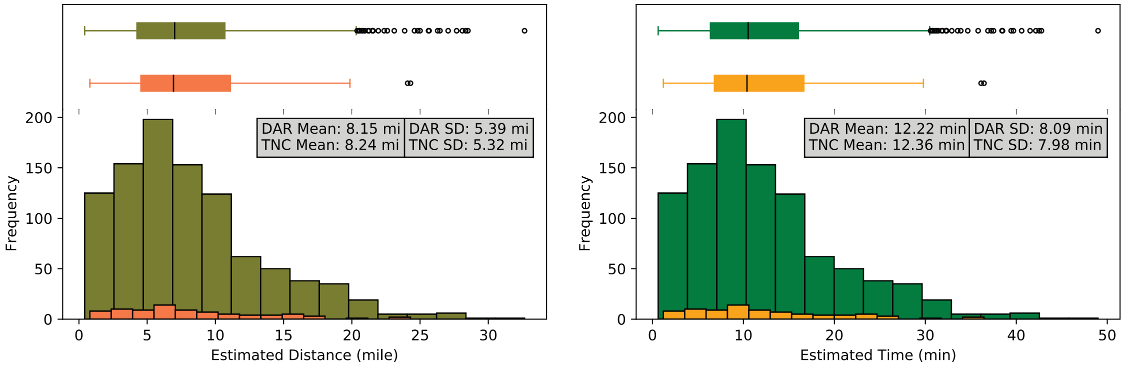
outsourcing, in order to minimize the total route cost. On average, the optimization for each route takes approximately 1.6 minutes using an Intel Core i3-7020U processor (2.30 GHz) and 8 GB RAM.

The comparison between the routes without TNC integration and the re-optimized routes with TNC integration is presented in Table 5, and can be summarized as follows:

- Although the outsourcing cost accounts for 10% of the total cost ($2,269), outsourcing trips leads to a 15% reduction in operating costs ($3,559), resulting in a net savings of 5% ($1,290) (as shown in Table 4). Approximately 28% of the routes saw improvements through either partial or full outsourcing, which contributed to 89% of the total savings. It is noteworthy that the 11% savings from the routes without outsourcing were achieved through changes in the route sequence, such as swapping trips in routes that were initially suboptimal.
- The distribution of costs for the routes without TNC integration and the re-optimized routes with TNC integration are presented in Fig. 6. The average cost for MRMS routes is estimated as $324, while the average cost for re-optimized routes is found to be $307. Statistical tests conducted to determine the difference in means and medians revealed that the costs of the re-optimized routes with TNC integration are significantly lower ($p < 0.01$) than those of MRMS routes.
- The origins and destinations of the outsourced trips, as shown in Fig. 7(a) (left), appear to be uniformly distributed across the service area. While outsourced trips are on average 4%–5% longer than trips served by DAR (Fig. 7(b)), statistical tests indicate no significant difference in means and medians ($p > 0.05$). Around 80% of the outsourced trips occurred during peak hours (Fig. 7(a) (right)), enabling cost-effective removal of eight routes (routes 1, 8, 10, 18, 38, 44, 64, and 75 in Fig. 5) that were underused in off-peak periods. However, a few underused routes (routes 17, 20, 33, 34) were not removed due to their lower DAR cost compared to the outsourcing cost for the trips.
- A significant proportion of the observed cost savings can be attributed to a 12% reduction in vehicular minutes and miles, as indicated by Table 5. Notably, since only 8% of the trips were outsourced, the greater reduction in vehicular minutes and miles suggests that only "difficult" trips (i.e., spatio-temporal outliers) were outsourced. Another significant contributor to the



(a) Spatio-temporal distribution of trip origins and destinations for both DAR and TNC



(b) Histogram of distance and duration for DAR and TNC trips

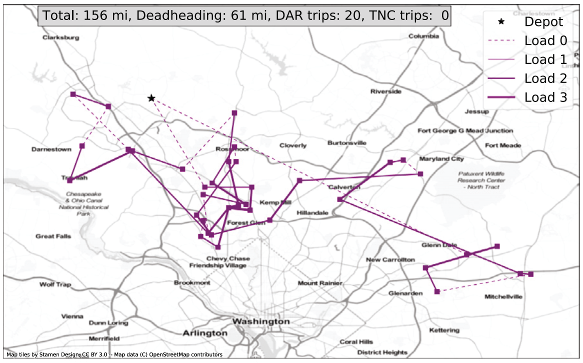
Fig. 7. Distribution of DAR and TNC trips.

observed savings was a 24% reduction in vehicle idle time, which amounted to $1303 (as shown in Table 5). This will be further explained through the analysis of individual routes.

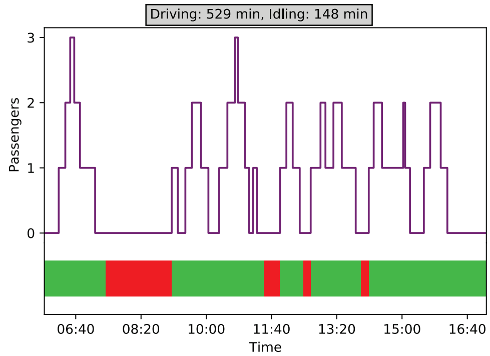
5.2. Route-by-route optimization: Detailed analysis

To provide additional insights into the results, we have included visualizations of several MRMS routes without TNC integration and re-optimized routes with TNC integration in Figs. 8–13, which display the schedule and load distribution of the dispatched vehicle. It is clear from the figures that the re-optimized routes have been simplified by outsourcing the outlier trips. The visual comparison indicates that the savings were achieved in the following ways:

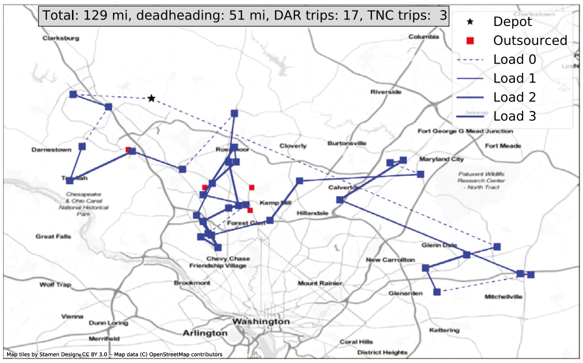
- In the case of routes with "significant" idle times, the trips before or after the idle time were all outsourced, effectively avoiding the idle times and the corresponding driver cost. This strategy is illustrated in examples of these routes in Figs. 8–9.
- Some of the routes contained pickup or drop-off far away from the depot or other demand locations. Outsourcing those trips significantly reduced both driving time and idle time. Examples of such routes are shown in Figs. 10–11.
- Some routes were improved using a combination of strategies. For example, consider the two routes in Figs. 12–13, where outsourcing occurred both in the beginning and end, thereby squeezing the overall route duration in addition to idle time. In Fig. 13, outsourcing allowed for greater loads in addition to avoiding idle time.
- Some of the suboptimal MRMS routes were improved without outsourcing by only altering the sequence of stops. This resulted in shorter route duration, as evident from Figs. 14–15.



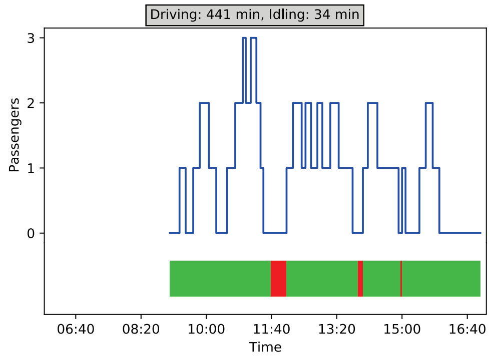
(a) Original route and load



(b) Original schedule and load

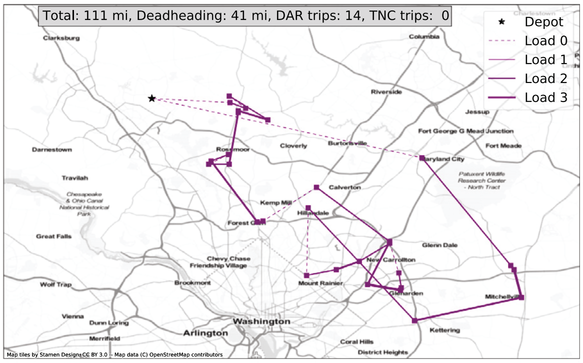


(c) Route and load after outsourcing

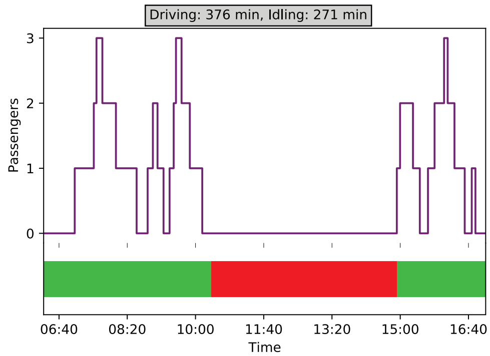


(d) Schedule and load after outsourcing

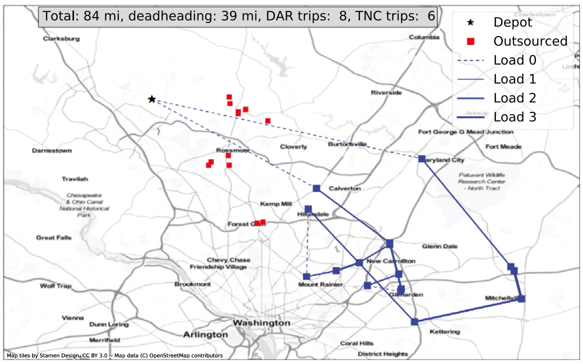
Fig. 8. Outsourcing avoids 1.5 h idle time in the schedule and saves $19 for route 13.



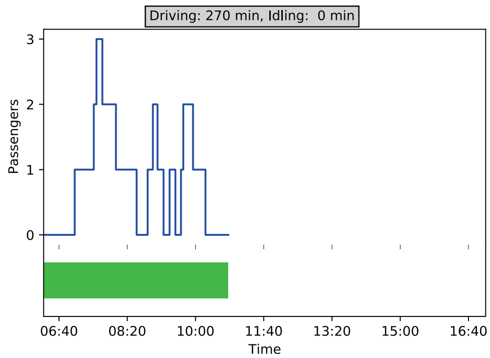
(a) Original route and load



(b) Original schedule and load

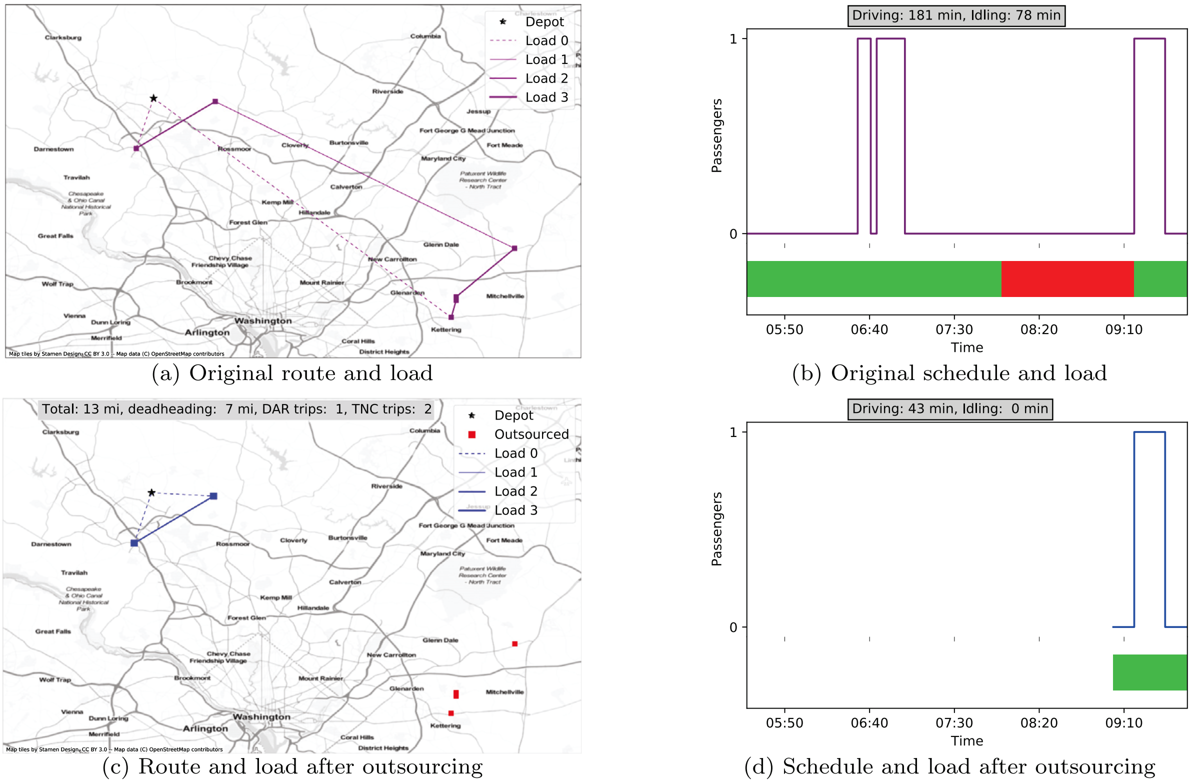


(c) Route and load after outsourcing



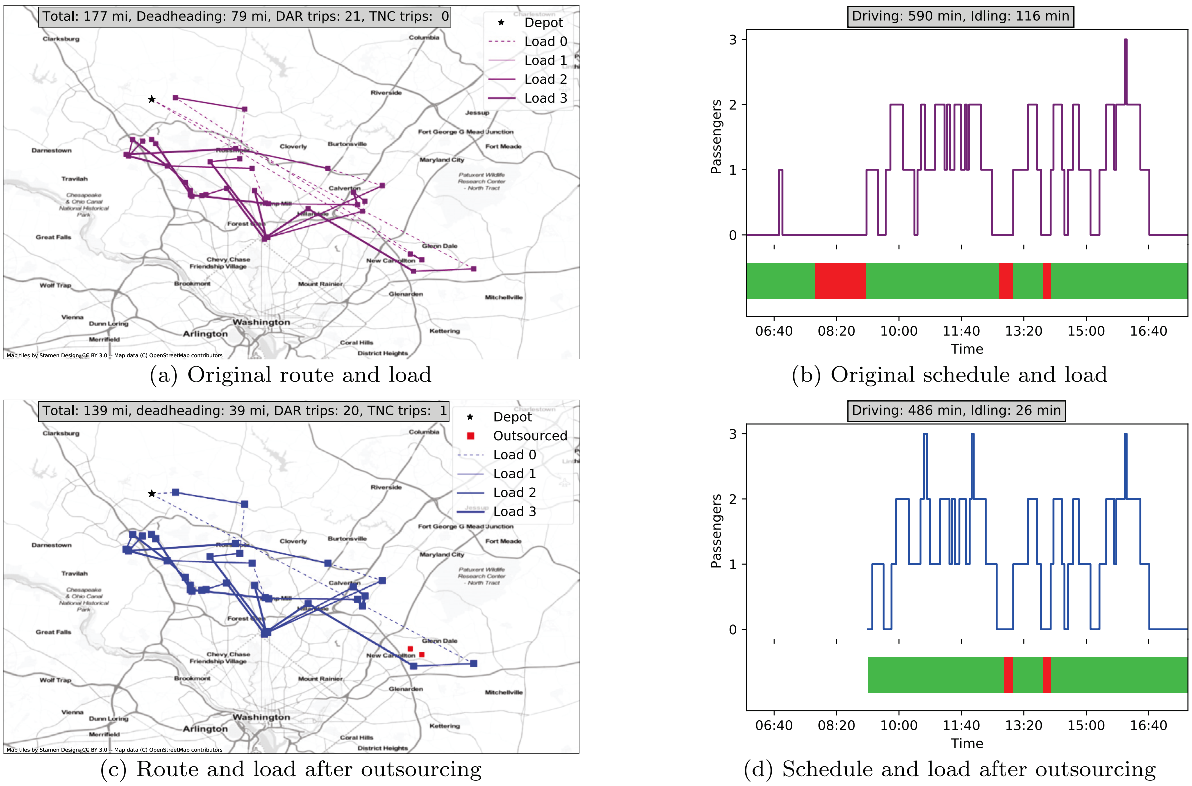
(d) Schedule and load after outsourcing

Fig. 9. Outsourcing avoids 4.5 h idle time in the schedule and saves $32 for route 4.



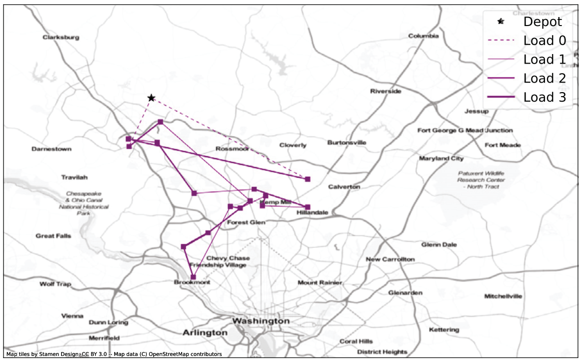
(a) Original route and load
(b) Original schedule and load
(c) Route and load after outsourcing
(d) Schedule and load after outsourcing

Fig. 10. Outsourcing avoids 1.3 h idle time to produce $100 saving for route 64.

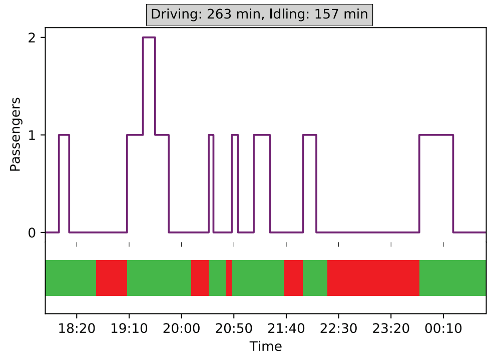


(a) Original route and load
(b) Original schedule and load
(c) Route and load after outsourcing
(d) Schedule and load after outsourcing

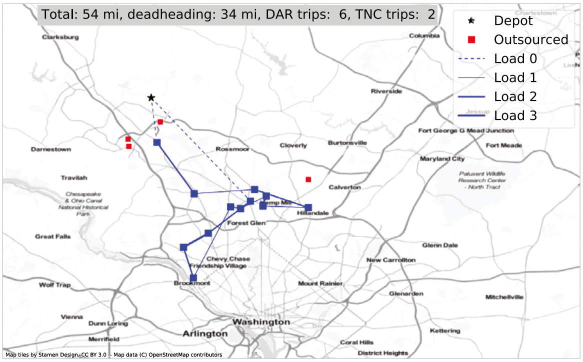
Fig. 11. Outsourcing avoids 1-hour idle time in schedule and saves $103 for route 30.



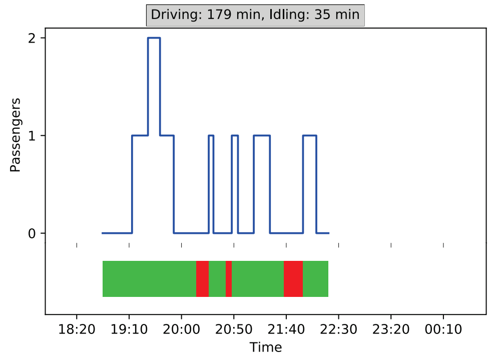
(a) Original route and load



(b) Original schedule and load

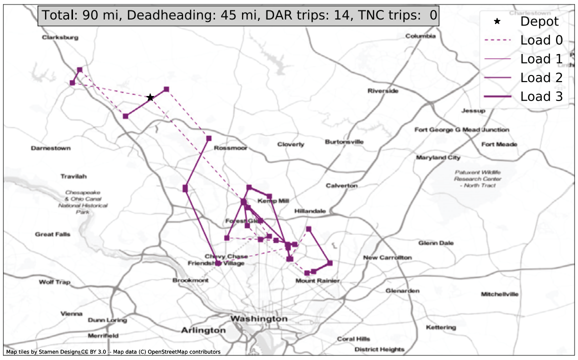


(c) Route and load after outsourcing

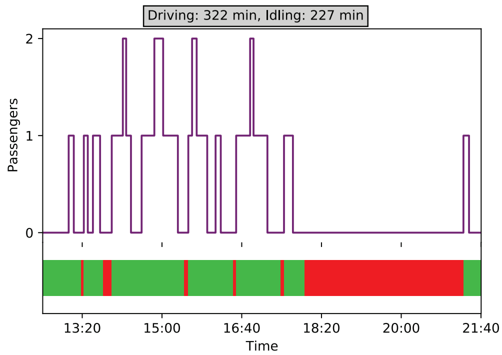


(d) Schedule and load after outsourcing

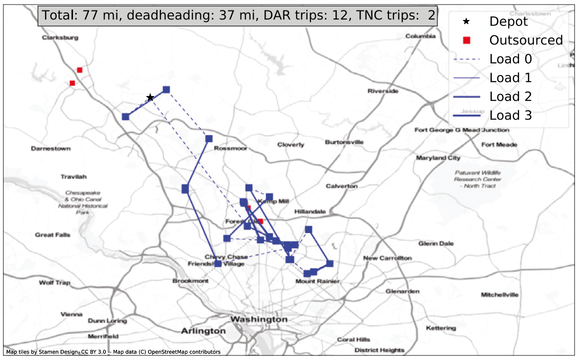
Fig. 12. Outsourcing avoids zig-zagging and idle time to produce $10 saving for route 44.



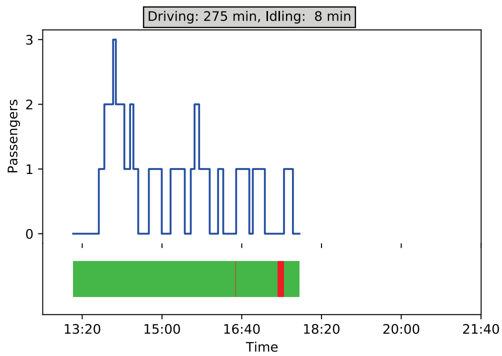
(a) Original route and load



(b) Original schedule and load

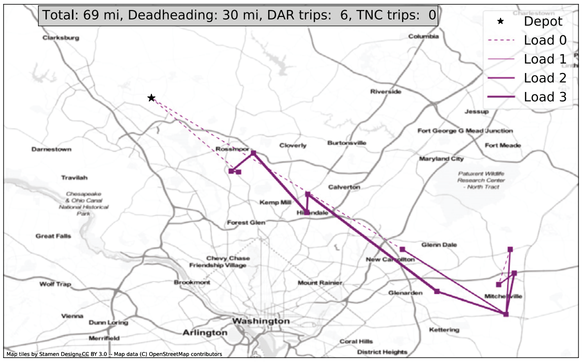


(c) Route and load after outsourcing

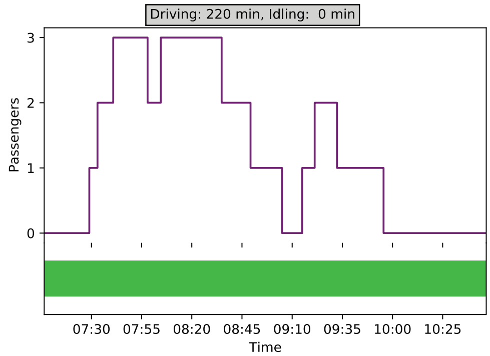


(d) Schedule and load after outsourcing

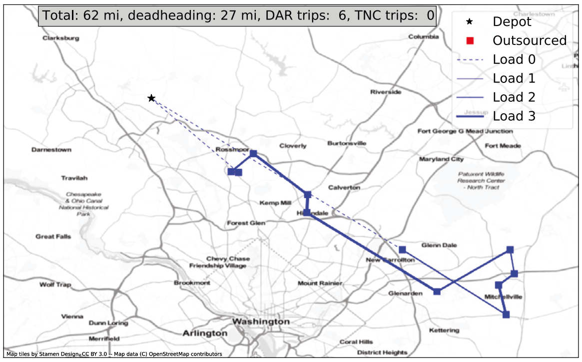
Fig. 13. Outsourcing avoids 3.5 h idle time and generates $88 saving for route 39.



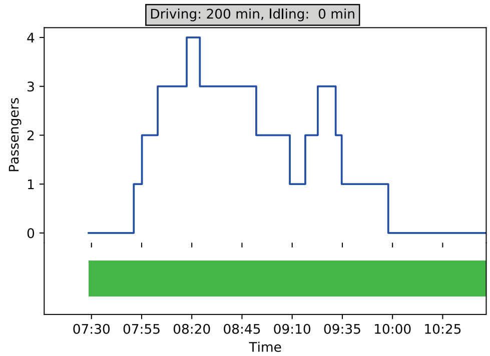
(a) Original route and load



(b) Original schedule and load

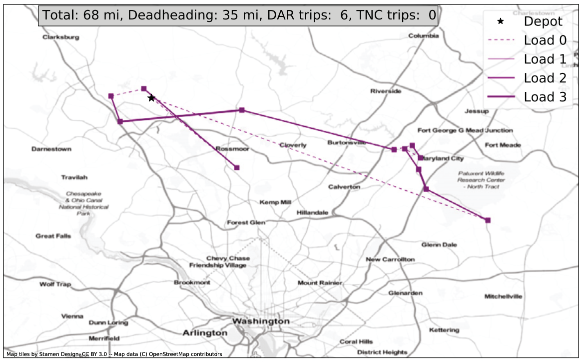


(c) Route and load after outsourcing

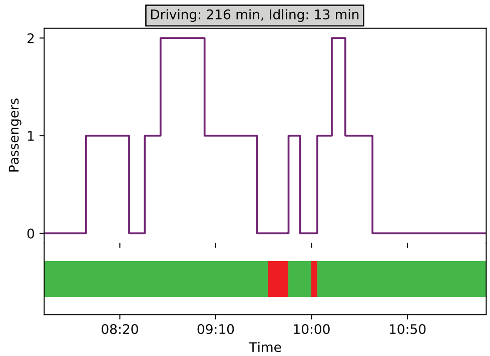


(d) Schedule and load after outsourcing

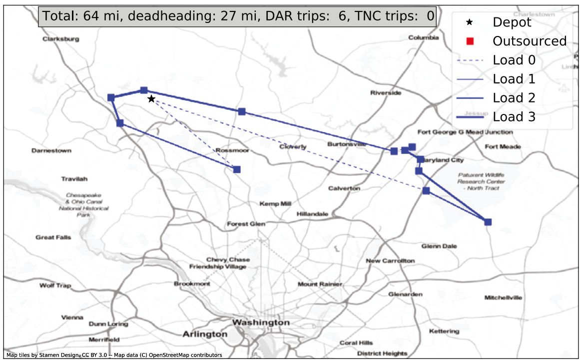
Fig. 14. Changes in trip sequence reduces driving time and generates $16 savings for route 58.



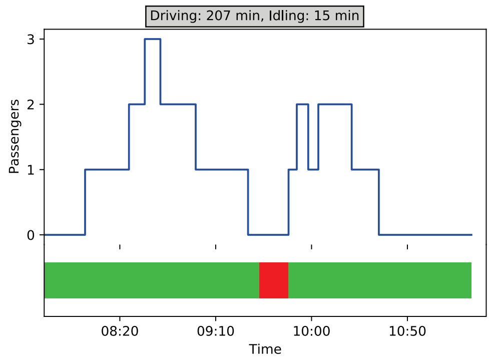
(a) Original route and load



(b) Original Schedule and load

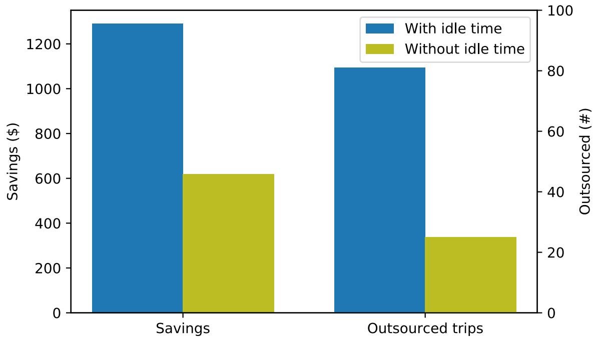


(c) Route and load after outsourcing

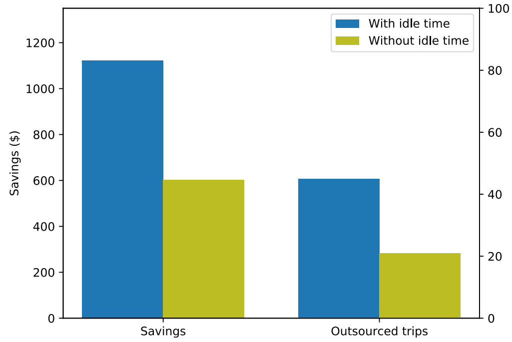


(d) Schedule and load after outsourcing

Fig. 15. Changes in trip sequence reduces total routing time and generates $6 savings for route 67.



(a) Savings and outsourced trips for math program



(b) Savings and outsourced trips for heuristics

Fig. 16. Effect of idle time.

Table 6
Runtime comparison between custom heuristic and GAMS.

Type	Runtime Mean	Runtime Std.
Exact solution	96.3 s	128.2 s
Custom heuristic	29.1 s	51.3 s

Table 7
Types of trip outsourcing with savings.

Outsourcing type	Routes		Savings	
Before/after idle time	2	(3%)	$144	(13%)
Spatio-temporal outliers	7	(9%)	$364	(32%)
Route eliminated	6	(8%)	$339	(30%)
No outsourcing	60	(80%)	$275	(25%)
Total	75		$1,122	

Additional examples of the savings mechanisms discussed above are provided in Appendix B, where route-level analyses for other routes are presented.

5.3. Route-by-route optimization: Effect of idle time

Previous research on DAR has often overlooked the impact of idle time, which is a significant issue when integrating TNC with DAR. To emphasize its importance, we re-optimized the routes by excluding the idle time cost from the objective function. While this should result in fewer outsourced trips since it makes DAR more cost-efficient than it actually is, it is crucial to evaluate decisions based on the comprehensive objective function. Our results show that not accounting for idle time reduces the number of outsourced trips and leads to a 48% loss in savings, as demonstrated in Fig. 16. This highlights the need to consider idle time when optimizing DAR routes with TNC integration.

5.4. Route-by-route optimization: Performance of the custom heuristic

The proposed heuristic was implemented in MATLAB and applied to the MRMS routes, with an average computation time of approximately 30 seconds per route using an Intel Core i5-10210U processor (2.11 GHz) and 16 GB RAM. Table 6 compares the runtime of the custom heuristic and the integer program, demonstrating that the heuristic is three times faster than the exact solution approach. This computational advantage would only increase with larger instances.

The breakdown and summary of the savings achieved using the proposed heuristic are presented in Table 7 and Fig. 17. The application of the proposed heuristic results in savings of $1,122, which corresponds to a roughly 4.6% reduction in operational cost. This indicates that the heuristic is able to recover about 87% of the savings achieved with the integer programming approach. While there is a compromise in savings, it is noteworthy that the heuristic is much easier to implement in DAR operations as it does not require mathematical programming. Additionally, the computation time for the heuristic is significantly faster than the exact solution approach, as shown in Table 6, making it a practical option for large-scale instances.

5.5. Multi-vehicle improvement procedure

The savings achieved by applying the multi-vehicle improvement procedure are reported in Table 8. By reinserting into DAR vehicles the trips that were initially designated for outsourcing, the number of outsourced trips reduces, leading to a significant

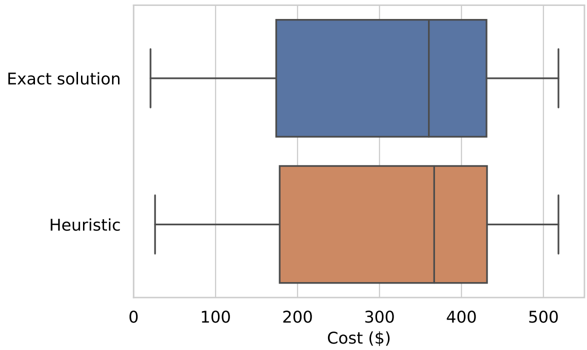
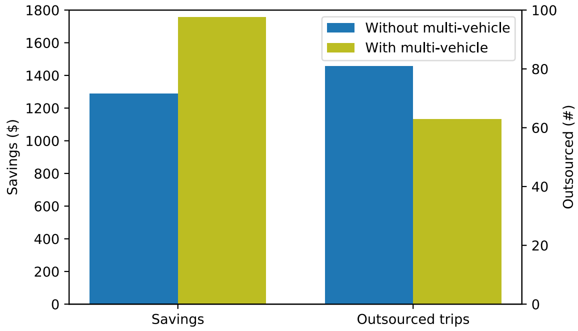
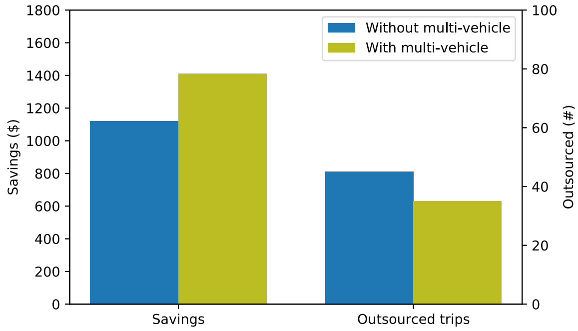


Fig. 17. Route cost distribution.



(a) Savings and outsourced trips for math program



(b) Savings and outsourced trips for heuristic

Fig. 18. Effect of multi-vehicle improvement procedure.

Table 8
Additional savings from improvement procedure.

Improvement type	Exact solution routes		Custom heuristic routes	
Reinsertion of outsourced trips	$367	(21%)	$128	(9%)
Exchange and reinsertion of DAR trips	$100	(6%)	$162	(11%)
Total increase in savings	$467	(27%)	$290	(21%)

increase in savings, as shown in Fig. 18. The reduction in operational cost from the integer program and heuristic increased from 5.3% to 7.2% and from 4.6% to 5.8%, respectively. To ensure the consistency of the estimated savings, we applied the proposed algorithmic framework to another day of the company's operations, and the results are summarized in Appendix C

5.6. Performance evaluation of the proposed two-stage framework

To evaluate the quality of solutions proposed by the two-stage framework, we prepared 10 small-scale instances with 6–15 requests and solved (1)–(18) in GAMS. Eight of these instances can be solved optimally within 2 hours. The mean gap between the optimal solution and our solution from the proposed two-stage framework is 3.9%. The results are summarized in Table 9.

5.7. Sensitivity analysis for the overall framework

In the previous analysis, we used unit costs for TNC that apply to the general public. While these unit costs represent a reasonable baseline, they may be relatively high compared to what a DAR company or a transit agency could potentially negotiate with Uber and other TNCs. Clearly, lowering the unit costs of TNCs would result in additional savings for the DAR operator. To quantify these additional savings, we re-did the previous analysis based on discounted TNC costs, which include 10%–70% discounts on the original costs considered in this study. The results are presented in Fig. 19, which shows intuitive trends. Specifically, it is

Table 9
Comparison between multi-vehicle DARP with TNC and proposed two-stage framework.

Instance #	$\|P\|$	$\|K\|$	Runtime (s)	Multi-veh. cost ($)	Two-stage cost ($)	Gap (%)
1	6	2	2.6	141.1	143.5	1.7
2	7	2	2.9	199.4	199.9	0.2
3	8	2	6.2	216.5	225.9	4.4
4	9	2	22.6	255.1	267.4	4.8
5	10	3	8.0	280.1	289.3	3.3
6	11	3	47.8	306.2	326.1	6.5
7	12	3	837.2	290.1	310.7	7.1
8	13	3	1,742.6	377.9	390.3	3.3
9	14	3	7,200.0	unsolvable	414.3	–
10	15	3	7,200.0	unsolvable	416.6	–

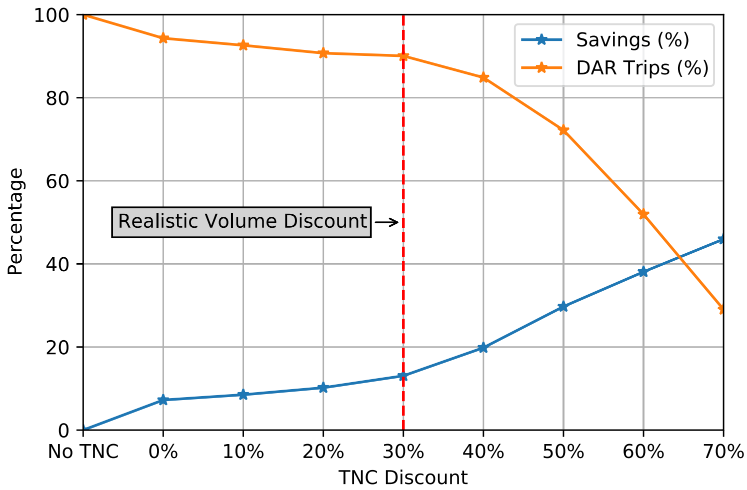


Fig. 19. Sensitivity analysis exploring the impact of TNC unit costs.

evident that relative savings increase as the TNC unit cost decreases. At the same time, fewer trips are being served by DAR as the TNC service becomes more affordable. It is noteworthy to mention that a 30% discount is likely the closest to the cost of the most affordable taxi service in the Maryland area, which would result in 13% savings.

Sensitivity analysis was also conducted on the proposed heuristic (Algorithm 1). By applying the heuristic to the Challenger routes for varying TNC discounts, a recovery of 75%–80% of the savings achieved by the integer program was observed.

It is important to note that the authors did not have access to detailed information about the trip requests, such as whether a wheelchair was required. Therefore, it was assumed that any rider could potentially be served by TNCs, but if a wheelchair was requested, the rider would have to be served by a DAR vehicle or an accessible TNC vehicle. This would result in a slight drop in the expected percentage savings, as approximately 10%–15% of customers would normally request a wheelchair in the study area considered. However, eligible seniors and other customers could still be served by TNCs.

Moreover, TNCs sometimes apply surge pricing to tackle higher demand or a lack of available vehicles. Since the authors did not have access to surge pricing information for TNC trips, it was assumed that TNCs operate in an equilibrium condition. As a result, the savings for the DAR company may be lower than the estimated percentage savings if surge pricing is considered. However, a DAR company outsourcing hundreds of trips per day to TNCs can negotiate a favorable surcharge policy to retain the savings.

6. Conclusions

This paper has presented a two-stage framework for identifying and outsourcing outlier trips in DAR operations to TNCs with the goal of reducing overall service delivery costs. We applied the proposed framework to a dataset obtained from a mid-size paratransit company in Maryland and demonstrated that significant cost reductions of 7%–13% could be achieved, depending on the negotiated TNC cost. These savings were primarily attributed to the reduction in vehicle miles and idle time. Our route level analysis also revealed the underlying mechanisms of outsourcing trips from DAR routes, and we proposed a practical heuristic that can be easily implemented by other DAR companies.

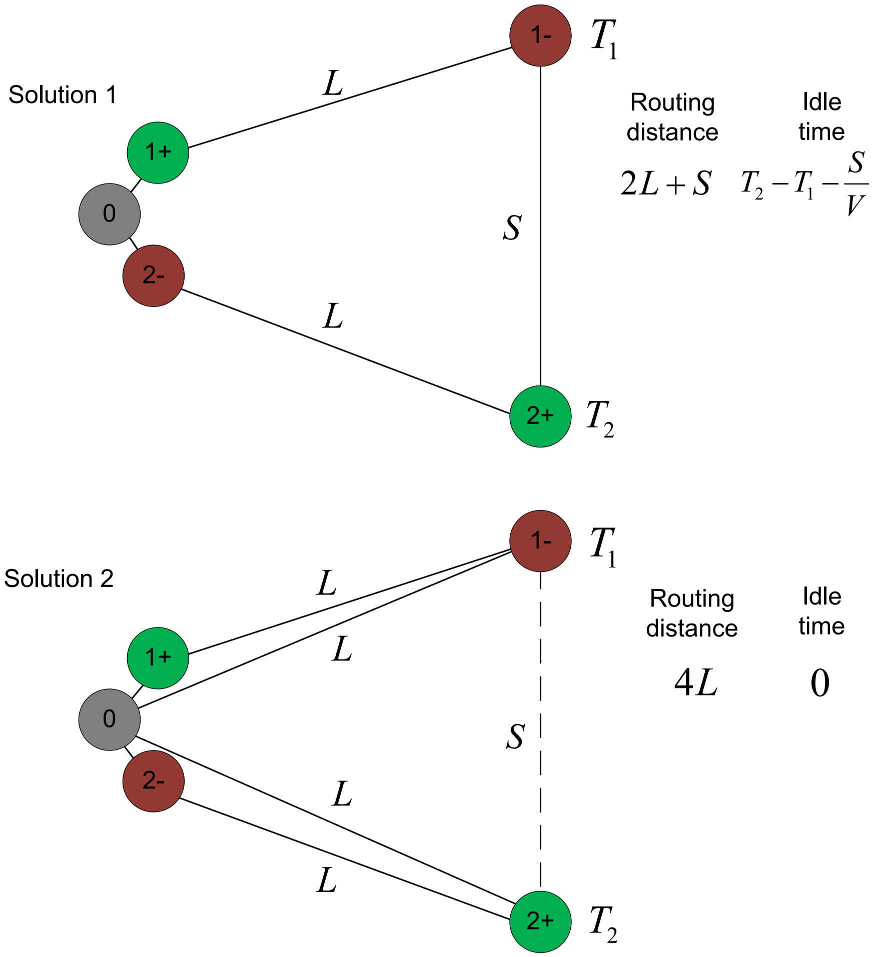


Fig. A.20. Comparison of two solutions.

The findings of this study suggest several potential avenues for future research. Analyzing data over several weeks may reveal additional demand trends that could lead to even higher savings by permanently removing underutilized vehicles. Additionally, investigating how savings change with trip density by applying our models to datasets from various regions across the country could provide valuable insights. Finally, exploring the development of machine learning algorithms that can identify spatio-temporal outlier trips without the need to resolve the DARP is an area that warrants further exploration.

Acknowledgments

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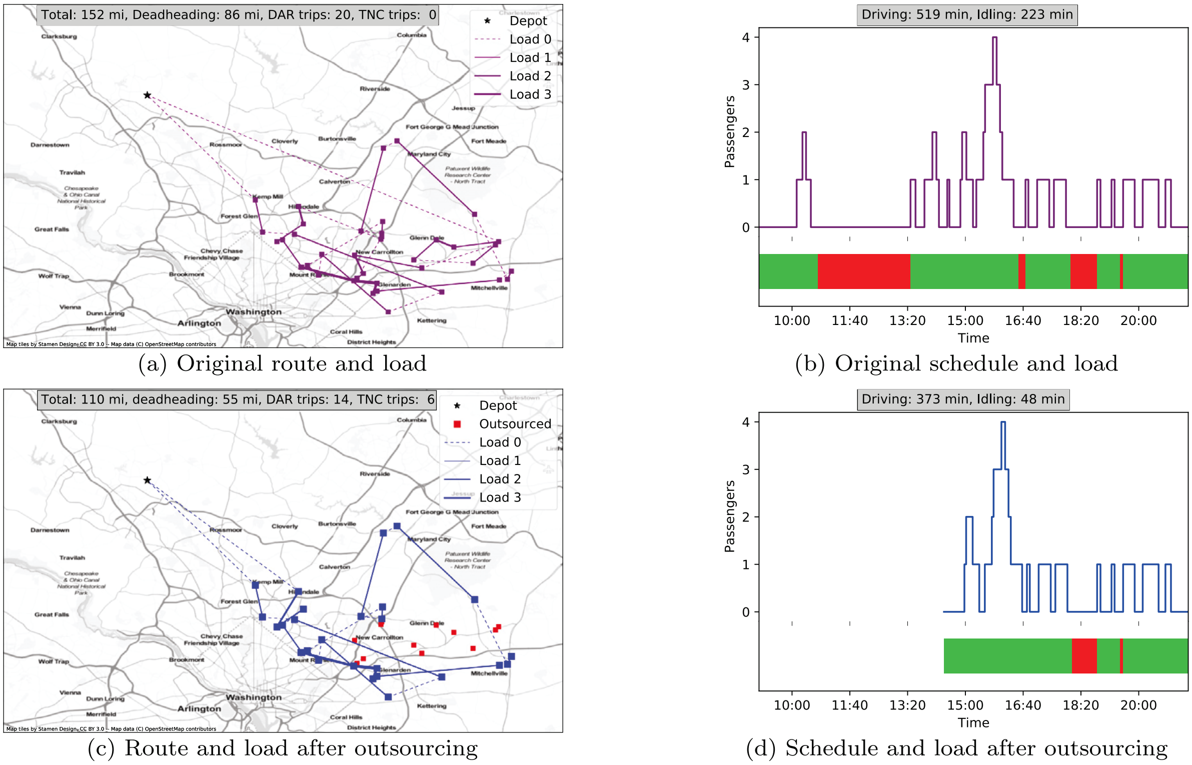
Appendix A. Effect of idle time on vehicle routing

We consider an illustrative DARP instance consisting of only two rider requests, numbered 1 and 2. Rider 1's pickup location is $1+$ and the drop-off location is $1-$. Similarly, the pickup location of rider 2 is $2+$, and the drop-off location of rider 2 is $2-$. The trip distance between the pickup and drop-off locations is L for both riders. The distance between $1-$ and $2+$ is S. Rider 1 must be dropped off at location $1-$ by time T_1 and rider 2 must be picked up at location $2+$ by time T_2. It is also known that $T_1 < T_2$. For simplification, vehicle depot 0, $1+$, and $2-$ are at the same location, which means the distance between each pair of those nodes is negligible. Due to the triangular rule, we know $S \leq 2L$.



(a) Original route and load (b) Original schedule and load

(c) Route and load after outsourcing (d) Schedule and load after outsourcing

Fig. B.21. Outsourcing removes trips before severe vehicle idling and generates $47 saving for route 22.

We examine Solution 1, whose routing sequence is $0 \to 1+ \to 1- \to 2+ \to 2- \to 0$. The routing distance is $2L + S$ and the idle time is $T_2 - T_1 - \frac{S}{V}$, where V is the average vehicle travel speed. Solution 2 consists of two separate routing sequences, namely $0 \to 1+ \to 1- \to 0$ and $0 \to 2+ \to 2- \to 0$. The total routing distance is $2L + S$ and the idle time is 0.

Clearly, if we minimize the routing distance only, Solution 1 dominates Solution 2, because $S \leq 2L$. In this case, the resulting idle time if Solution 1 is adopted can be arbitrarily large. This supports the claim that if we minimize routing distance only, the idle time can be excessive. If idle time is part of the optimization objective, it is possible to find a threshold for S that one is indifferent between two solutions. In other words, $(2L+S)+b(T_2-T_1-\frac{S}{V}) = 4L$, where b is a positive coefficient translating time cost (hours) to mileage cost (miles). The threshold is given by $b^* = \frac{2L-S}{T_2-T_1-\frac{S}{V}}$. When the cost of idle time exceeds b^*, Solution 2 dominates Solution 1 (see Fig. A.20).

Appendix B. Route-level analysis

B.1. Outsourcing of trips before severe vehicle idling

See Figs. B.21 and B.22.

B.2. Outsourcing of trips after idle time in schedule

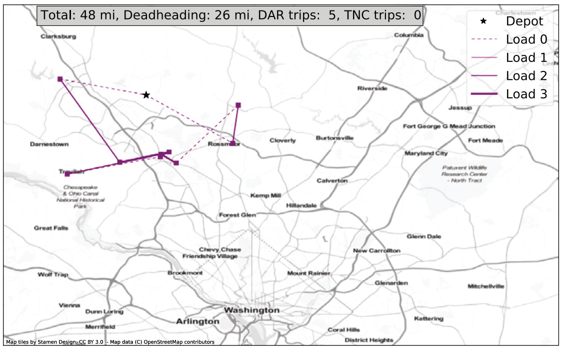
See Figs. B.23 and B.24.

B.3. Outsourcing of spatio-temporal outliers

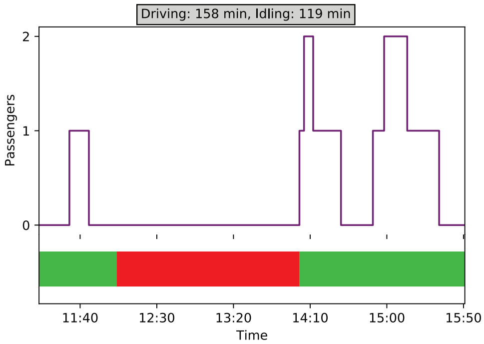
See Figs. B.25 and B.26.

B.4. Outsourcing of spatio-temporal outliers before/after idle time

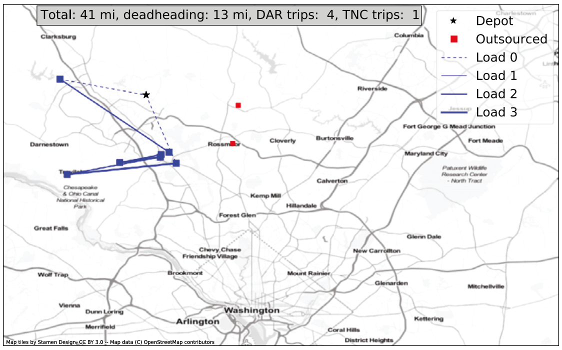
See Figs. B.27 and B.28.



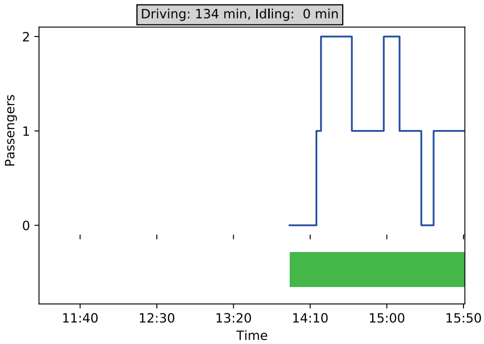
(a) Original route and load



(b) Original schedule and load

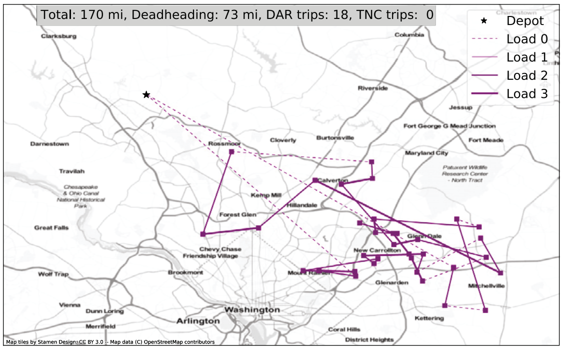


(c) Route and load after outsourcing

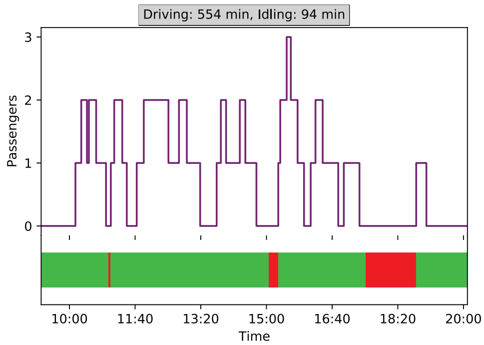


(d) Schedule and load after outsourcing

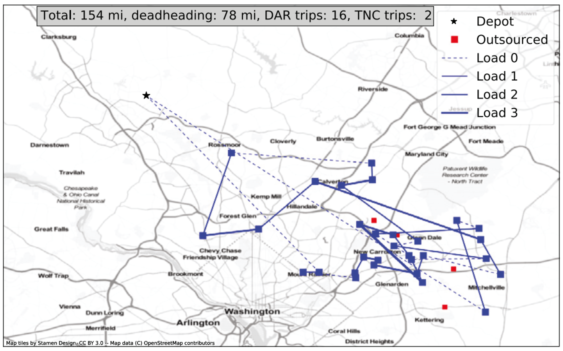
Fig. B.22. Outsourcing avoids 2 h idle time and generates $40 saving for route 32.



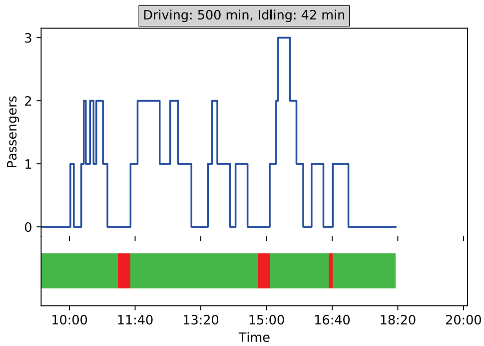
(a) Original route and load



(b) Original schedule and load

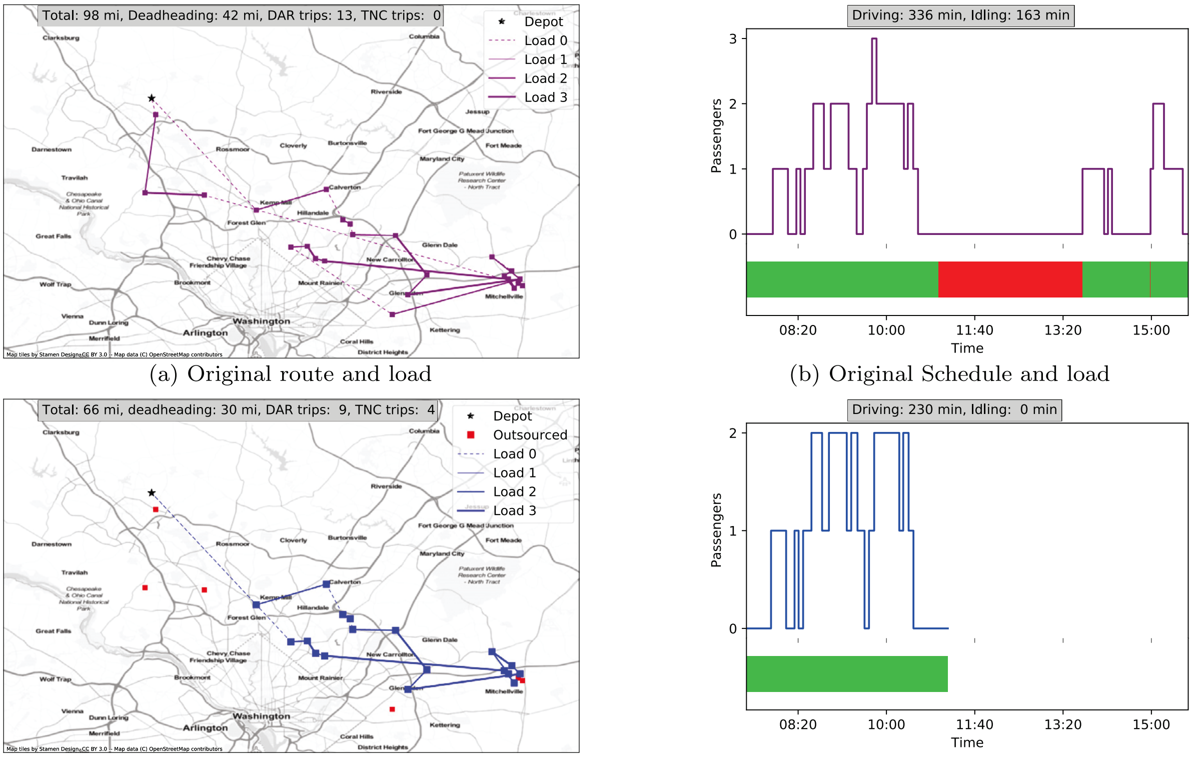


(c) Route and load after outsourcing



(d) Schedule and load after outsourcing

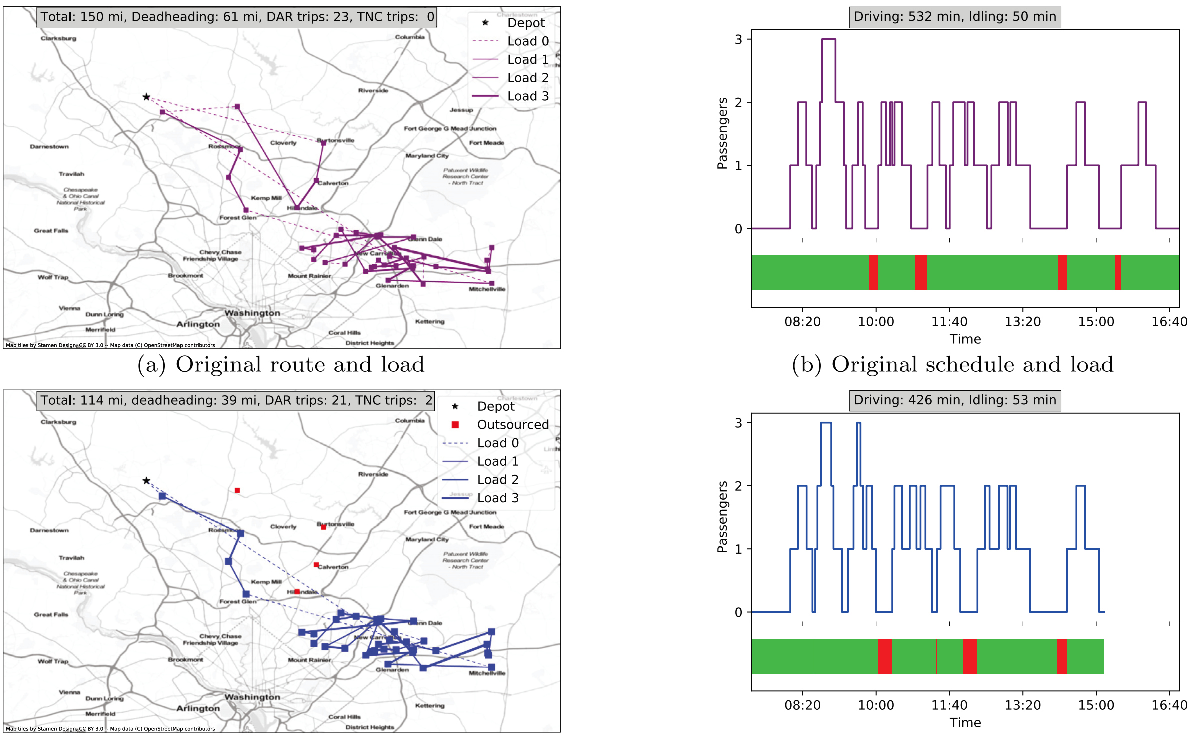
Fig. B.23. Outsourcing removes trips after idling and generates $10 saving for route 45.



(a) Original route and load (b) Original Schedule and load

(c) Route and load after outsourcing (d) Schedule and load after outsourcing

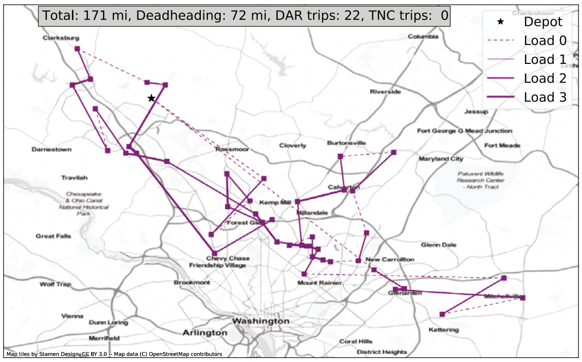
Fig. B.24. Outsourcing avoids large idle time and generates $23 savings for route 65.



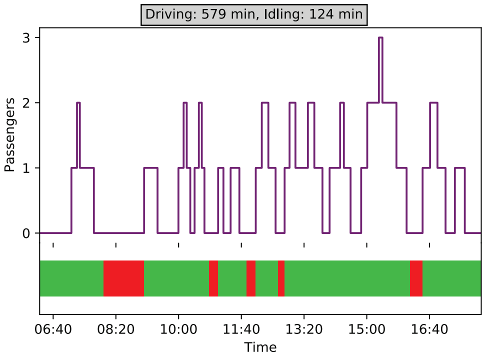
(a) Original route and load (b) Original schedule and load

(c) Route and load after outsourcing (d) Schedule and load after outsourcing

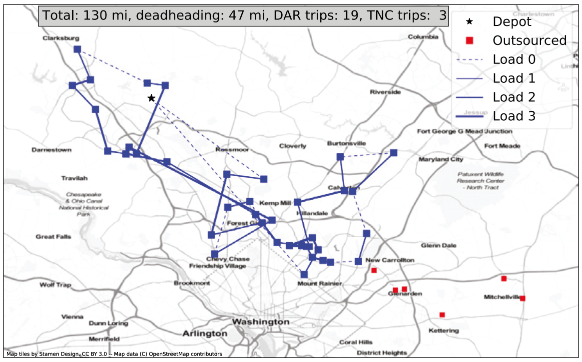
Fig. B.25. Outsourcing removes spatio-temporal outliers and generates $8 saving for route 36.



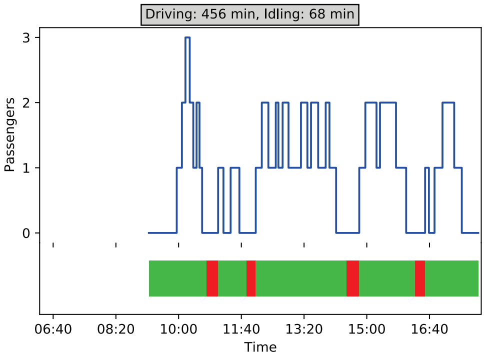
(a) Original route and load



(b) Original Schedule and load

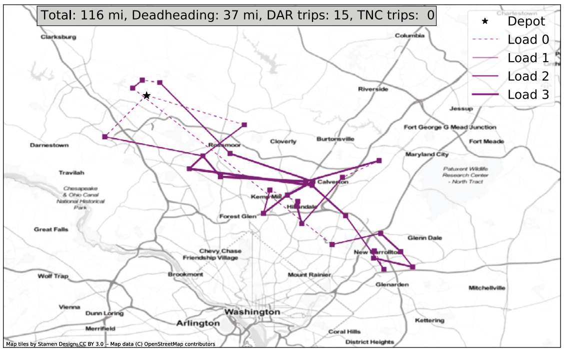


(c) Route and load after outsourcing

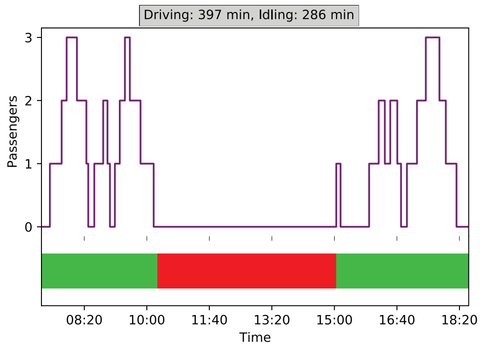


(d) Schedule and load after outsourcing

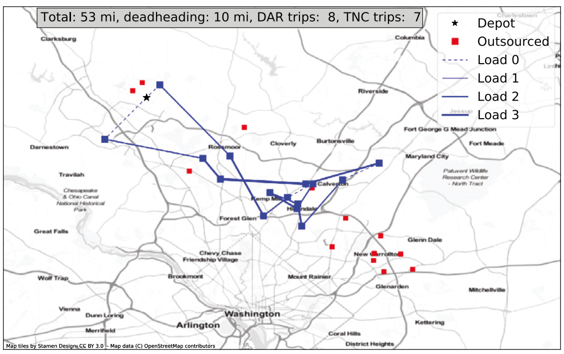
Fig. B.26. Outsourcing removes trips far from depot and generates $24 savings for route 48.



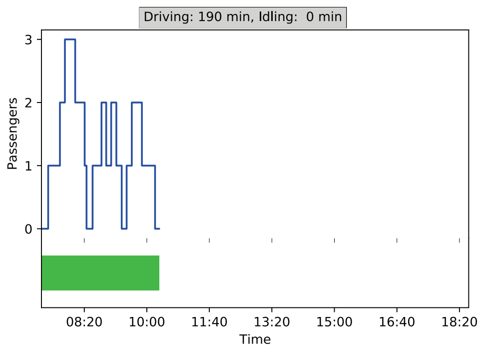
(a) Original route and load



(b) Original schedule and load

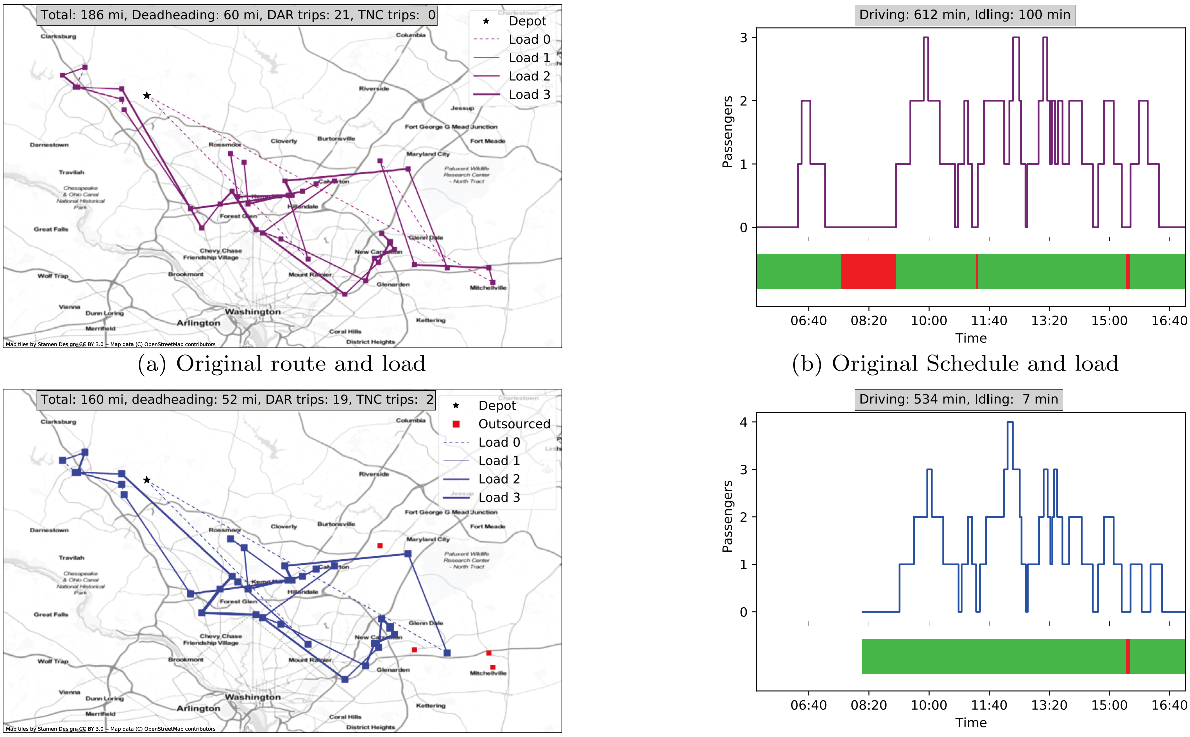


(c) Route and load after outsourcing



(d) Schedule and load after outsourcing

Fig. B.27. Outsourcing avoids zig-zagging and generates $17 saving for route 56.



(a) Original route and load (b) Original Schedule and load

(c) Route and load after outsourcing (d) Schedule and load after outsourcing

Fig. B.28. Outsourcing avoids zig-zagging and generates $9 savings for route 35.

Table C.10
Comparison of results between the two day of operations.

Dataset	Trips	Outsourced		Stage-1	Stage-2	Total	% Savings
Solution method 1: Exact Solution and Improvement							
Day 1	1,057	63	(5.96%)	$1,290	$467	$1,757	7.24%
Day 2	897	58	(6.47%)	$1,168	$368	$1,536	7.19%
Solution method 2: Custom Heuristic and Improvement							
Day 1	1,057	35	(3.31%)	$1,122	$290	$1,412	5.81%
Day 2	897	28	(3.12%)	$946	$213	$1,159	5.43%

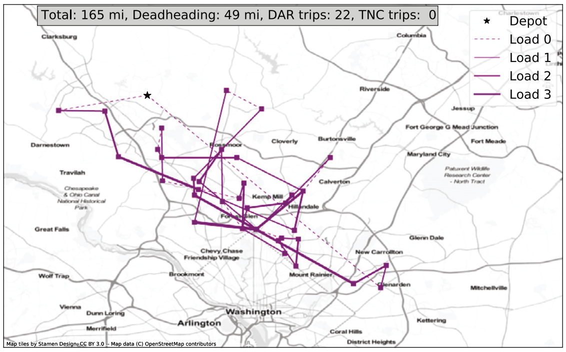
B.5. Changes in trip sequence without outsourcing

See Figs. B.29 and B.30.

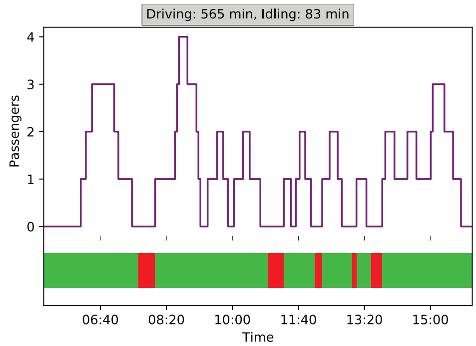
Appendix C. Another day of operations

To ensure the consistency of model performance, our proposed two-stage approach is tested on another day of operation (day 2). In order to ensure that the data for day 2 are significantly different than the data for day 1, a paired t-test is conducted between the origin–destination demand of the cities for the two days, aggregated on 2,809 OD pairs containing pick-up and drop-off locations. The results show a significant difference ($p < 0.05$) in the origin–destination demand for the two days, implying that the spatial distribution of demand varied significantly.

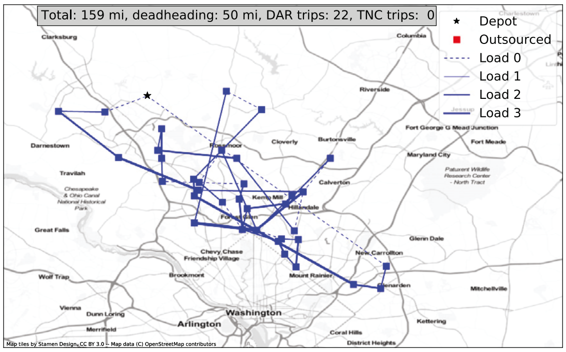
The results based on the two-stage framework for the two days are summarized in Table C.10. Although the absolute savings of day 2 are smaller than those of day 1 due to fewer trips served, the percentage of outsourced trips and relative savings are consistent. The distribution and characteristics of DAR and TNC trips after the optimization of routes are presented in Fig. C.31.



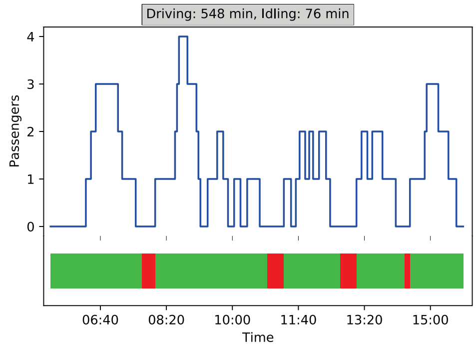
(a) Original route and load



(b) Original schedule and load

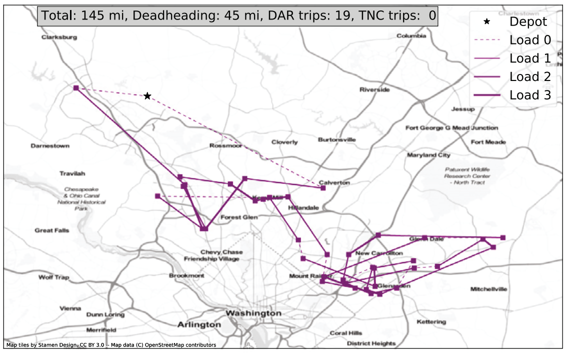


(c) Route and load after outsourcing

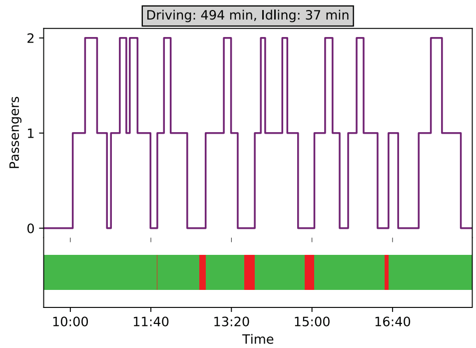


(d) Schedule and load after outsourcing

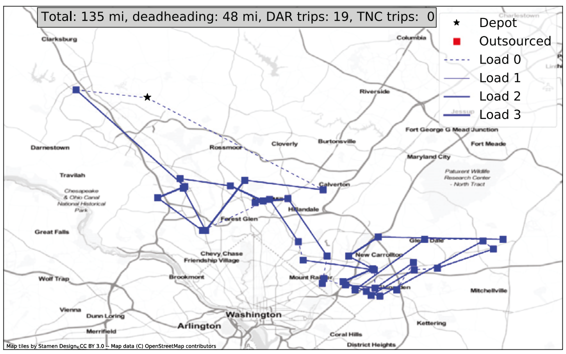
Fig. B.29. Changes in trip sequence reduces driving time and generates $16 savings for route 2.



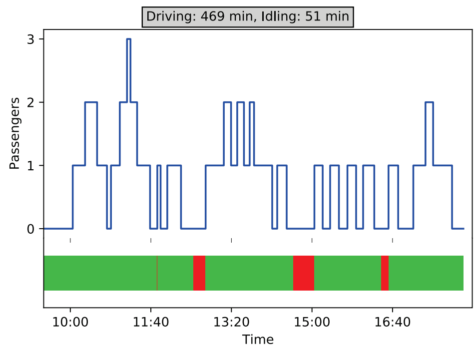
(a) Original route and load



(b) Original Schedule and load

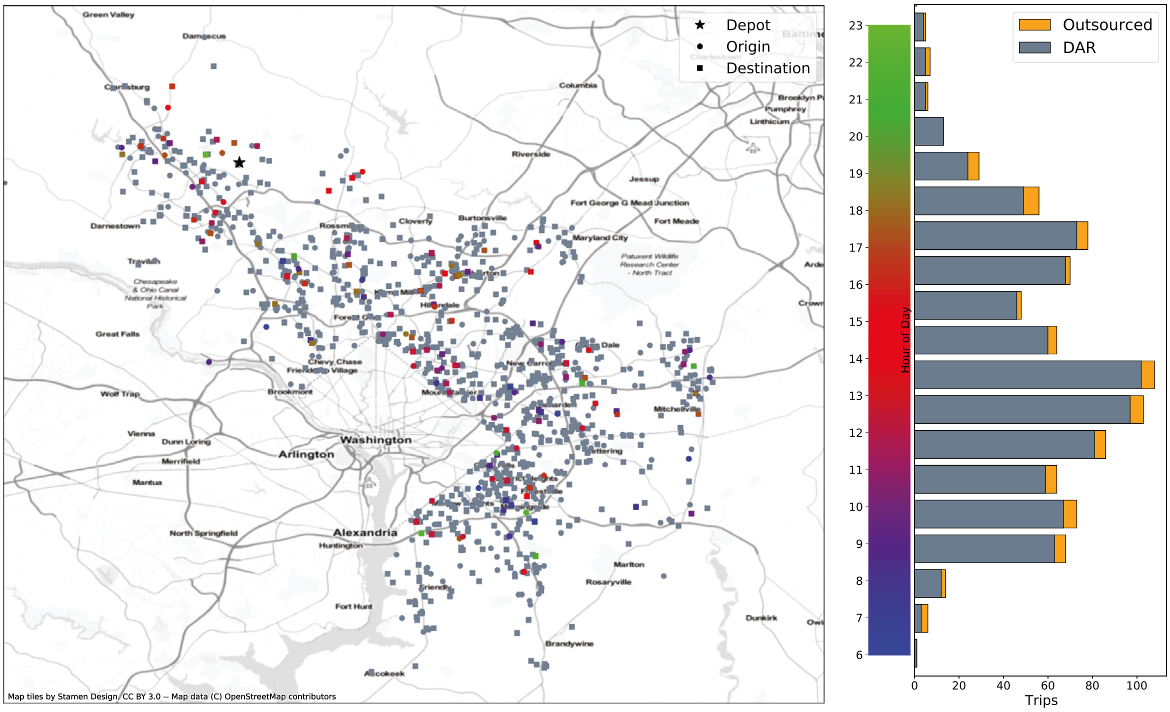


(c) Route and load after outsourcing

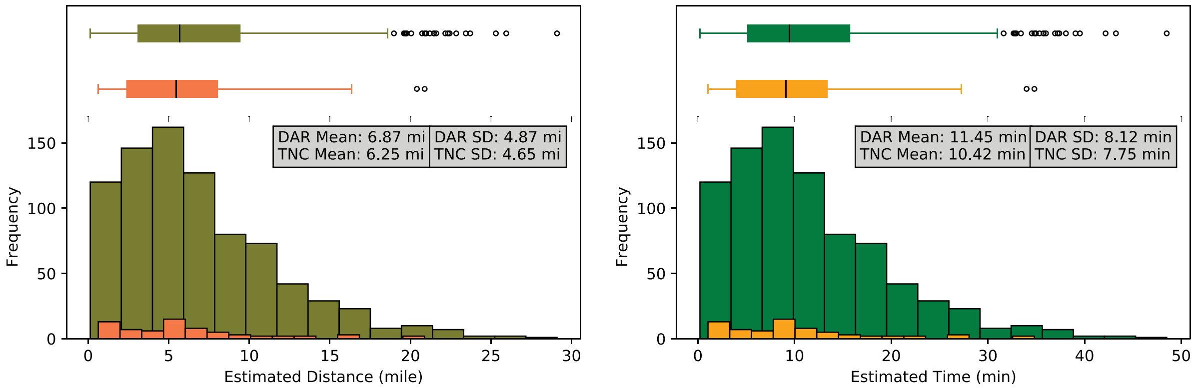


(d) Schedule and load after outsourcing

Fig. B.30. Changes in trip sequence reduces total routing time and generates $14 savings for route 19.



(a) Spatio-temporal distribution of trip origins and destinations for both DAR and TNC



(b) Histogram of distance and duration for DAR and TNC trips

Fig. C.31. Distribution of DAR and TNC trips on day 2.

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