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A Mathematical Model for Neuronal Activity and Brain Information Processing Capacity

Yu Zheng[1], David Zhu[2], Jian Ren[1], Taosheng Liu[3], Karl Friston[4], Tongtong Li[1]
[1] Department of Electrical and Computer Engineering, Michigan State University, East Lansing, USA
[2] Department of Radiology, Michigan State University, East Lansing, USA.
[3] Department of Psychology, Michigan State University, East Lansing, USA.
[4] The Wellcome Trust Centre for Neuroimaging, University College London, London, UK

Abstract—**Neurophysiological measurements suggest that human information processing is evinced by neuronal activity. However, the quantitative relationship between the activity of a brain region and its information processing capacity remains unclear. In this paper, we introduce an information conservation law for regional brain activation, and establish a mathematical model to quantify the relationship between the information processing capacity, input storage capacity, the arrival rate of exogenous information, and the neuronal activity of a brain region—referred to as the brain information processing capacity (IPC) model. We apply the IPC model to event related fMRI data from a flanker test, designed to determine age-related differences in brain activation. Our analysis demonstrates the predictive validity of the model in terms of providing accurate account of fMRI responses, and shows that for a given cognitive task, higher information processing capacity leads to lower neuronal activity level and faster response. Relying solely on the information conservation law, the IPC model provides a framework for modeling distributed neuronal processing—and can be applied to different data types and scales: i.e., single neurons, brain regions, and networks.**

I. INTRODUCTION

Exploring the information processing and storage capacity limits of the human brain is a central task in cognitive psychology and neuroscience [1]–[3]. In literature, most of the existing research has been focused on the capacity limit of short-term working memory, or how well an individual can manage the complexity of information processing demands when several tasks have to be executed simultaneously [4], [5]. It was shown that our visual short-term memory can maintain representations of three to four objects at any given moment [6]. Along this line, in [7], the information processing capacity was mapped to the computational capacity of a dynamic system and characterized as the total number of linearly independent functions of its stimuli the system can compute.

Previous research in neurophysiology suggests that human information processing is closely related to neuronal activity [8]–[10]. It was shown that the amplitude of neuronal activity is modulated by the input information flow or the number of objects being held in the memory at the time before it approaches an asymptotic limit [8], [9], and automation of cognitive functions can increase the information processing capacity and lead to a decline in the neuronal activity [10].

In literature, mathematical characterization of neuronal activity has been studied extensively, and existing neuronal activity models provide a panoramic coverage on brain dynamics, from single neurons [11], [12], to localized neural populations [13], and to brain networks [14]. However, until today, the quantitative relationship between the activity of a brain unit and its information processing capacity remains unclear.

In this paper, as an effort to address this problem, we aim to model neuronal activity and information processing capacity from an information-theoretic perspective. Our starting point is the information conservation law. It serves as the corner stone and enables us to link the brain information processing and input storage capacities of a brain region, its the neuronal activity, and the arrival rate of the exogenous information all through a first-order differential equation, referred to as the brain information processing capacity (IPC) model. Relying solely on the information conservation law, the IPC model allows us to evaluate the relative information processing and storage capacities, and the brain circuit response time of any brain unit that has information processing and storing capability. Potentially, it provides a framework for multi-scale brain dynamic modeling in terms of information processing, and can be applied to the studies of predictive coding and Bayes-optimal decision-making.

II. THE INFORMATION CONSERVATION LAW

When information arrives at a brain region, there are two possibilities: (i) the region is lossless, i.e., no information is lost; (ii) the region is lossy, i.e., there is an information loss. Here we introduce the information conservation law for both lossless and lossy brain regions.

In an abstract or functional sense, we can say that each brain region has a processing unit and an input storage unit, which can vary across time and/or on-going cognitive activities. Let $I(t)$ denote the overall information fed to a brain region during time period $[0, t]$, $I_p(t)$ the information taken by the processing unit to process during $[0, t]$, and $I_m(t)$ the information saved in the input storage unit of the brain region at time instant t waiting to be processed.

We start with the lossless situation first. In this case, the information arrival rate is less than or equal to the information processing rate of the brain region and there is no information loss or overflow, we have: *the total information fed to the region equals the sum of the information taken for processing, and the information saved in the input storage unit waiting to be processed*. That is,

$$I(t) = I_p(t) + I_m(t), \tag{1}$$

assuming that they are all in bits. We refer to this as the **information conservation law** at a lossless region.

However, when the information arrival rate is higher than the processing rate, information loss or overflow occurs at the brain region. In this case, we have *the generalized information conservation law*:

$$c(t)I(t) = I_p(t) + I_m(t), \tag{2}$$

where $0 \leq c(t) \leq 1$ denotes the portion of the information that is not lost by the region. If $c(t) = 1$, it means that no information is lost in the region; if $c(t) = 0$, it means that all the information is lost.

It should be emphasized that our input storage unit here holds only the input information waiting to be processed, and is just a functional model used to simplify our analysis. The memory request that occurs during the processing operations is considered to be part of the processing, and happens within the processing unit rather than the input storage unit. As we will see later, the information conservation law—a simple but fundamental principle—serves as the link that bridges the information processing and storage capacities of a brain region, its neuronal activity level, and the input information arrival rate.

III. THE BRAIN INFORMATION PROCESSING CAPACITY (IPC) MODEL

In this section, we derive the brain information processing model based on the information conservation law. We start with the lossless case first, and then extend the results to lossy case.

IPC model in the lossless case Taking derivatives with respect to t on both sides of equation (1), we get

$$H(t) = H_p(t) + H_m(t), \tag{3}$$

where $H(t) = dI(t)/dt$ is the information arrival rate, $H_p(t) = dI_p(t)/dt$ the information processing rate and $H_m(t) = dI_m(t)/dt$ the instantaneous information changing rate in the input storage unit, all in bits per second.

Let $p(t)$ represent the processing capacity, defined as the maximal information processing rate (in bit/s) of the region with respect to a particular cognitive task. Let $m(t)$ represent the input storage capacity, defined as the total amount of input information storage resource (in bits) available to or allocated by the region. Now we try to represent the neuronal activity in terms of information processing and input storage capacities.

Note that the neuronal activity $x(t)$, which measures the activeness or the effort level of the local neuronal population in fulfilling a cognitive task or function, can be understood as the instantaneous percentage of the "workforce" being utilized in the unit. From the information processing perspective, the neuronal population activity level can be defined as the ratio of the instantaneous processing rate and the maximum processing rate:

$$x(t) = \frac{H_p(t)}{p(t)}. \tag{4}$$

Similarly, from the storage perspective, the neuronal activity level can be defined as:

$$x(t) = \frac{I_m(t)}{m(t)}, \tag{5}$$

which is the ratio between the actual input information saved in the input storage unit and the overall allocated storage capacity. Following equations (4) and (5), it can be seen that $0 \leq x(t) \leq 1$.

Consider a single state brain region, that is, a region with one neuronal activity level, which implies that the input storage unit and processing unit have the same neuronal activity level. Moreover, when the task itself has a very short duration, we could assume that the processing capacity and the storage capacity remain approximately unchanged throughout the task. From equations (4) and (5), we have $x(t) = \frac{H_p(t)}{p}$, and

$$\frac{dx(t)}{dt} = \frac{1}{m}\frac{dI_m(t)}{dt} = \frac{1}{m}[H(t) - H_p(t)] \tag{6}$$

It then follows that the neuronal activity of an individual brain region can be modeled as:

$$\frac{dx(t)}{dt} = -\frac{p}{m}x(t) + \frac{1}{m}H(t). \tag{7}$$

As can be seen, we obtained a first-order linear differential equation model that connects the neuronal activity level of a brain region with its information processing, storage capacities and the arrival rate of the input information. We name this model as the **Information Processing Capacity (IPC)** model.

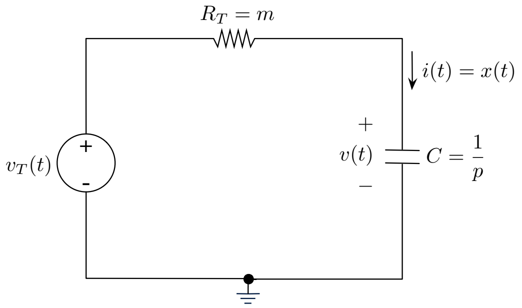
Following equations (6) and (7), we can see that the **physical meaning** of the IPC model lies in that: (i) the gap between the information arrival rate $H(t)$ and the information processing rate $H_p(t)$ directly influences the changing rate of the neuronal activity level. When $H(t) > H_p(t)$, that is, when the arrival rate is larger than the processing rate, then the neuronal activity level increases; otherwise, it decreases. This observation is consistent with existing findings where it was shown that the amplitude of neuronal activity is modulated by the input information flow or the number of objects being held in the memory at the time before approaching an asymptotic limit [8], [9], and automation of cognitive functions can increase processing capacity and lead to a decline in the neuronal activity [10], only that the processing capacity there was defined as the number of objects that can be held in our visual short-term memory. (ii) On the other hand, larger input information storage capacity m can alleviate the demand on neuronal activity level when the arrival rate $H(t)$ increases. This reflects the self-adjustment capability of the brain.

In the special case when $I(t) = \alpha u(t)$ is a step input, where $u(t)$ is the unit step function, we have $H(t) = \alpha\delta(t)$, where $\delta(t)$ is the Dirac delta function, and then

$$x(t) = \min\{\frac{\alpha}{m}e^{-\frac{p}{m}t}u(t), 1\}.$$

We can see that, $x(0_-) = 0$ and $x(0) = \min\{\frac{\alpha}{m}, 1\}$, that is, there is an abrupt change of the neuronal activity level at $t = 0$. This is consistent with the existing findings that the brain activity level changes abruptly instead of smoothly.

Following the circuit theory, the IPC model in equation (7) can be affiliated with a resistor-capacitor (RC) circuit as shown is Fig. 1, where V_T and R_T denote the $Th\acute{e}venin$ equivalent voltage and resistance, respectively. $x(t)$ can be regarded as the current that goes through the RC circuit (instead of being the voltage across the capacitor), since $x(t)$ experiences an abrupt change at $t = 0$, and the voltage across the capacitor is a continuous variable and cannot have abrupt changes. Compare equation (7) and the differential equation corresponding to the RC circuit, if $V(t) = I(t)$, then we have $R_T = m, C = 1/p$, and the time constant of the circuit is $T_c = R_T C = m/p$, which implies that $x(t)$ converges or decays essentially to zero after about $5T_c$. Note that the time constant of the circuit is related to the response of the system, we can see that higher information processing capacity p leads to shorter response time.



$$i(t) = C\,\frac{dv(t)}{dt},\; R_T C\,\frac{dv(t)}{dt} + v(t) = v_T(t)$$

$$R_T C\,\frac{di(t)}{dt} + i(t) = C\,\frac{dv_T(t)}{dt}$$

Fig. 1. The corresponding circuit of the IPC model.

Recall that based on experimental observations of the neuronal networks, the neurons constantly add up the excitatory and inhibitory input in time and over the area of the dendrites receiving synaptic contacts. Recent studies indicate that excitation–inhibition balance (E–I balance) is a form of homeostatic plasticity that helps to maintain neuronal activity within a narrow, safe range [15].

To reflect the synaptic integration of the excitatory and inhibitory neuronal activity, let $I_E(t) = I(t)$ denote the excitatory input information as before, and $I_I(t)$ the inhibitory control signal, then we have $V_T = I_E(t) - I_I(t)$. We start with a simple case where $V_T(t) = \alpha u(t) - \beta u(t - T_0)$. The physical meaning here is that, under a stimulus, the excitatory input information lasts a period of T_0 seconds before it is intervened by an inhibitory control signal. The first transition in $V_T(t)$ happens at $t = 0$, is considered as the recognition or interaction of the internal dynamics with the input stimulus; the latter is considered as an operation to clear the previous input data and prepare for new input, and is driven by a negative feedback control mechanism to prevent the brain from excessive activation [16].

Based on the superposition property of linear circuits, the overall neuronal activity can be represented as $x(t) = x_E(t) - x_I(t)$, where

$$x_E(t) = \frac{\alpha}{m} e^{-\frac{p}{m}t} u(t)$$

is the excitatory activity and

$$x_I(t) = \frac{\alpha}{m} e^{-\frac{p}{m}(t - T_0)} u(t - T_0)$$

is the inhibitory activity. Note that driven by spontaneous neuronal activities, both the excitatory inputs and inhibitory control signals could occur multiple times [17].Therefore, we can further extend the excitatory and inhibitory activity to

$$x_E(t) = \sum_{i=0}^{M} \frac{\alpha_i}{m} e^{-\frac{p}{m}(t - T_{E,i})} u(t - T_{E,i})$$

and

$$x_I(t) = \sum_{i=0}^{M} \frac{\alpha_i}{m} e^{-\frac{p}{m}(t - T_{I,j})} u(t - T_{I,j}),$$

respectively. In general, the actual representation of $x(t)$ is determined by the input information arrival rate $H(t)$, which is related to the cognitive task, and the way the information is encoded by the brain.

IPC model with information loss When information loss is involved, following the Generalized Information Conservation Law in equation (2), the IPC model can be generalized as

$$\frac{dx(t)}{dt} = -\frac{p}{m} x(t) + \frac{c}{m} H(t), \tag{8}$$

where $0 \leq c \leq 1$ denotes the portion of the information that is not lost by the region. This generalized model may help us evaluate the information losses in brain regions, especially those involved in faulty decision making due to information overflow or abnormal conditions such as Alzheimer's disease or seizures.

IV. SIMULATION RESULTS

We applied the IPC model in (3) to analyze neuronal activity and information processing capacity based on experimental fMRI data obtained from a flanker test, which was used to study the aging-associated decline in selective attention and executive functions [18]. Twenty-three young adults and twenty-six older adults participated in this study. In the experiment, the subjects were presented with three conditions:

- The Congruent (C) condition (">>>>>>>" or "<<<<<<<");
- The Incongruent (IC) condition (">>><>>>" or "<<<><<<");
- The Neutral condition ("□□□ > □□□" or "□□□ < □□□").

Each trial was presented for 2.5 sec, during which time the subjects were asked to identify the direction of the central arrowhead and press the corresponding button for each trial. The rapid event-related design was chosen so that the subjects'

TABLE I

RESULTS FOR RIGHT MFG, HERE p/α AND m/α DENOTE THE RELATIVE INFORMATION PROCESSING CAPACITY AND STORAGE CAPACITY, RESPECTIVELY; $T_c = m/p$ DENOTES THE TIME CONSTANT OF THE REGIONAL BRAIN CIRCUIT.

Right MFG	m/α	p/α	$T_c = m/p$
Old IC	2.5436	7.2636	0.35018
Young IC	4.0838	14.169	0.28822
Old C	3.5179	11.382	0.30907
Young C	6.9985	27.527	0.25424

general attentiveness level was kept relatively invariant. In [18], it was reported that compared to young adults, older adults had more difficulty responding to Incongruent cues during the flanker task. For comparison purpose, here we focus on Congruent(C) and Incongruent (IC) conditions.

Recall that the BOLD signal (denoted as $y(t)$) is generally modeled as a convolution $y(t) = x(t) * h(t)$, where $h(t)$ is the hemodynamic response function. Our analysis was conducted in three steps:

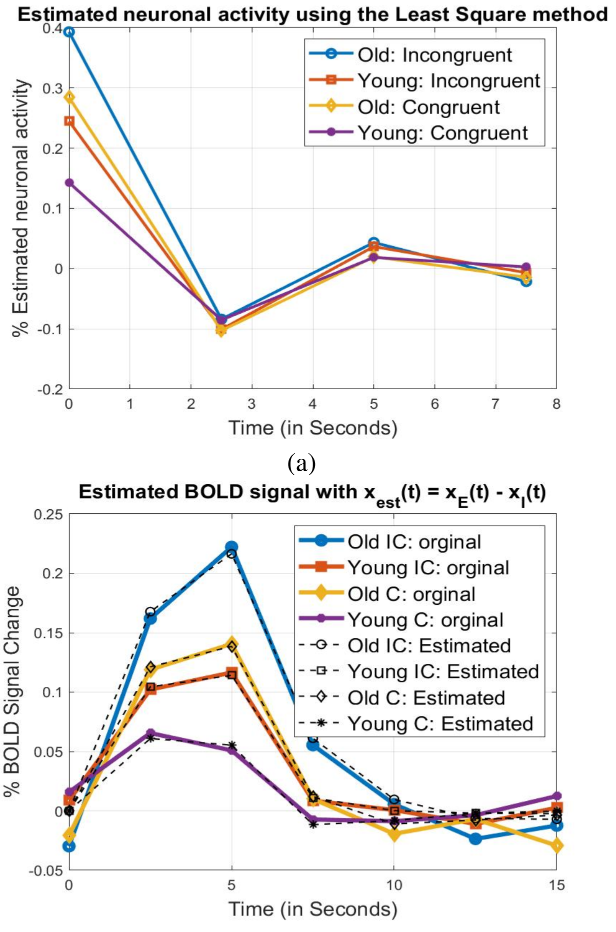
- *Step 1:* Estimating the neuronal activity x(t) from the BOLD signal.
- *Step 2:*Estimating the relative information processing capacity p/α, input storage capacity m/α, and the time constant T_c. Due to the simplicity of the experiment tasks, we approximated the information arrival rate corresponding to the Congruent and Incongruent conditions as $H(t) = \alpha_C \delta(t)$ and $H(t) = \alpha_{IC} \delta(t)$, respectively, and estimated the desired parameters based on $x(t) = x_E(t) - x_I(t)$.
- *Step 3:* With the parameters estimated in the previous step, we can reconstruct the estimated neuronal activity $x_{est}(t)$ and hence the BOLD signal based on IPC model. We can then evaluate the performance by comparing the estimated BOLD signal and the original BOLD signaI.

We analyzed all the active regions identified by FreeSurfer, including the middle frontal gyri (MFG), the inferior frontal gyrus (IFG), the inferior occipital gyru (IOG)s, the middle occipital gyrus (MOG) and the superior frontal gyrus (SFG). The simulation results for the right MFG was shown in the table and Fig. 2. The results for other regions are similar.

From the simulation result, it can be seen that: (i) Under the same cognitive task, higher information processing capacity leads to lower neuronal activity and smaller time constant in individual brain regions. (ii) Within each group, the Incongruent task imposes a higher information arrival rate, and hence higher neuronal activity than the Congruent task. (iii) The younger group has higher relative information processing capacity and a smaller time constant (or faster response) than the older group under the same task. It can also be observed that the IPC model can predict the BOLD signal with high accuracy.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we first introduced the information conservation law, and then established and verified a mathematical model for brain information processing capacity, named the IPC model, which characterizes the relationship of the neuronal activity of a brain region to its information processing and input storage capacities, and the arrival rate of the input information. Our analysis indicated that for a given cognitive task, higher information processing capacity leads to lower neuronal activity level and faster response. This observation is consistent with the findings reported in literature that high-capacity individuals generally have lower neuronal activity [8]. The IPC model can be applied to any data type from which neuronal activity can be extracted. Moreover, relying on the information conservation law, the IPC model can serve as a unified framework for multiscale modeling of brain dynamics, from neuron to region, and to the whole brain. It links neuronal activity to information processing capacity and provides a new perspective in computational brain analysis.



(a)

(b)

Fig. 2. Results for right MFG: (a) Estimated neuronal activity; (b) True BOLD signals versus the estimated signals based on the IPC model.

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