Two-Dimensional Frequency Comb from a Single Dual-Pumped Microring Dissipative Kerr Soliton

Grégory Moille,[1,2,*] Pradyoth Shandilya,[3] Michal Chojnacky,[1] Christy Li,[1] Curtis Menyuk,[3] and Kartik Srinivasan[1,2]

[1] Joint Quantum Institute, NIST/University of Maryland, College Park, MD, USA; [2] Microsystems and Nanotechnology Division, National Institute of Standards and Technology, Gaithersburg, MD, USA; [3] University of Maryland at Baltimore County, Baltimore, MD, USA

[*]gmoille@umd.edu

Abstract: We present a two-dimensional frequency comb, with distinct fixed repetition-rates in both the azimuthal mode dimension and an orthogonal dimension parametrized by the angular phase-velocity. We experimental demonstrate it using a single integrated microring bichromatically pumped. © 2023 The Author(s)

OCIS codes: (140.3948) Microcavity devices; (190.4390) Nonlinear optics, integrated optics

Frequency combs have been used for metrology, distance ranging, and astronomical instrument calibration [1]. In integrated photonics, they can be created in a microring resonator via the Kerr effect when the resonator is pumped by a continuous wave from a bus waveguide [2]. This process generates a dissipative Kerr soliton (DKS), which is then periodically extracted on the bus waveguide to yield a pulse train. In the frequency domain, the pulse train has a repetition rate ω_{rep} that is related to the angular group velocity (υ_{g}) of the DKS in the resonator, and its detection makes possible ultra-low-noise transduction between the optical and microwave domains [3]. However, the repetition frequency ω_{rep} typically ranges from tens of gigahertz to one terahertz due to the cavity size [2,4] and spans only one dimension corresponding to the resonator azimuthal mode numbers (which we refer to as the μ-domain). Each of these modes supports a corresponding single frequency DKS comb tooth. In this work, we present a new type of frequency comb spanning two dimensions. The first dimension is parameterized by the mode number μ and is related to the DKS υ_{g}, yielding ω_{rep}. The new second dimension is orthogonal to the first, and we refer to it as the σ domain. It is related to the angular phase velocity (υ_ϕ) dispersion, yielding an orthogonal repetition rate $\Omega_\phi \ll \omega_{\text{rep}}$. This otherwise hidden dimension is generated by the bichromatic pumping of a microring resonator. We generate what is effectively a soliton optical-parametric-oscillator (soliton-OPO) in the σ domain, which is the result of cascading $\chi^{(3)}$ nonlinear interactions between the multi-color soliton eigenfrequencies, which then creates the two-dimensional comb. Our experimental demonstration of a two-dimensional microcomb on an integrated photonics chip is supported by a theoretical analysis based on an extended Lugiato-Lefever Equation (LLE). This new orthogonal dimension in the frequency comb has a υ_ϕ repetition rate (Ω_ϕ) of a few gigahertz and is orders of magnitude lower than the group velocity repetition rate of one terahertz. This new kind of frequency comb provides intriguing opportunities for new kinds of nonlinear interaction in cavity solitons along with transduction to low microwave frequencies.

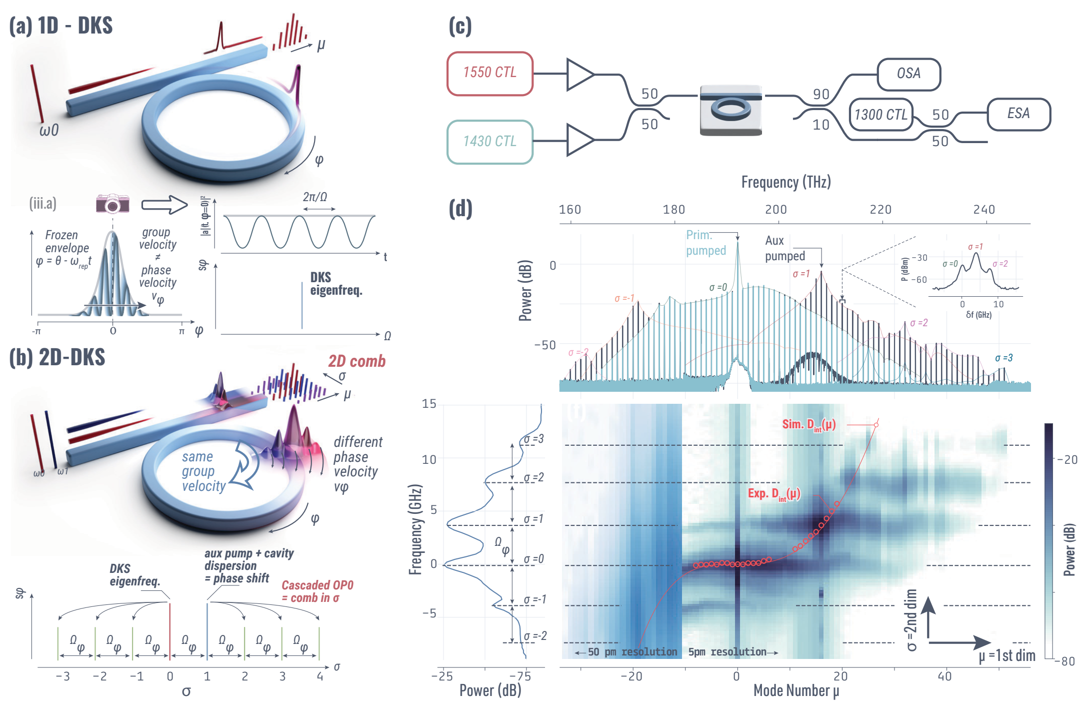

Fig. 1. Singly-pumped dissipative Kerr soliton (DKS), yielding a one-dimensional frequency comb in the mode number (μ) domain. The single pump creates a single cavity soliton that propagates azimuthally through the resonator. Periodic extraction by the bus waveguide yields a pulse train, thereby creating a frequency comb. **(b)** Bichromatic pumping makes possible a single cavity soliton in which both components travel at the same υ_{g}. The difference between υ_ϕ of the two components leads to two different oscillation frequencies, which can then cascade into a soliton-OPO, analogous to modeal optical parametric oscillation/amplification instead here instead of continuous waves each frequency are temporal wave containing a range of μ components. This interaction creates a second comb dimension along σ, with a repetition rate Ω_ϕ. **(c)** Experimental setup for dual-pumping of a 23 µm microring resonator made of Si_3N_4 embedded in silica. **(d)** Experimental demonstration of a two-dimensional microcomb. We show the experimental spectrum in the upper subplot. The comb teeth associated with each value of σ are in different colors and are extracted based on their frequency offset. The two-dimensional comb can also be showcased by extracting the power around $\Omega = [-\omega_{\text{rep}}/2, \omega_{\text{rep}}/2]$ for each mode μ, yielding the power mapping in the bottom subplots. The bottom left subplot represents the integrated power as the mode number μ varies, highlighting the second orthogonal dimension σ, where multiple distinct peaks are separated by the same value of $\Omega_\phi \approx 3.8$ GHz. In this case, the two-dimensional comb spans about 70 mode numbers (μ) and six teeth in the σ dimension. We also plot the simulated (solid red line) and measured (red circles) integrated dispersion $D_{\text{int}}^{\text{NL}}$ agreeing well with the theory presented.

Before describing the theoretical model for the two-dimensional frequency comb, we first recall that a soliton wavepacket that is generated by a single pump has two associated angular velocities. The first is the angular group velocity υ_g governing the wave envelope and determines the repetition rate. The second is the angular phase velocity υ_ϕ different from υ_g yielding the carrier-envelope-offset (ceo) in the pulse train that is extracted in the bus waveguide. Most of the work of modeling and understanding cavity solitons has been done through the mean field approximation with the LLE and therefore discards — with good reasons — the angular phase velocity. Indeed, in the azimuthal coordinate system $\phi = \theta - \omega_{\mathrm{rep}} t$ that moves with the soliton at ω_{rep} (essentially *freezing* the soliton envelope), the temporal profile of the DKS at any given azimuthal angle ϕ corresponds to a perfect sine function due to the mismatch between group and phase velocity. Therefore, the soliton has a single, well-defined frequency in this coordinate system [fig. 1(a)].

When a cavity is bichromatically pumped, both external driving fields must be on resonance with a cavity mode to enter the cavity, which yields a mismatch of $\upsilon_\phi \equiv \Omega_\phi$ due to the intrinsic dispersion of the cavity, with $\Omega_\phi = \omega(\mu_{\mathrm{aux}}) - (\omega_0 + \omega_{\mathrm{rep}}\mu)$, where μ_{aux} is the secondary pumped mode, $\omega(\mu)$ are the resonant optical frequencies, and ω_0 is the primary pumped frequency. As a result, strong cross-phase modulation binds together the components of the wavepacket that are driven by both pumps [5] and thereby locks v_g so that it is the same for both components. In the same fashion as for a singly-pumped system, one can recast the coordinate system to move with the wavepacket and retrieve the soliton frequencies. However, in this case, the difference between the υ_ϕ of the two pumps yields two distinct soliton oscillation frequencies that are separated by Ω_ϕ [fig. 1(b)], and the soliton has two colors.

Being in a third-order nonlinear medium, these oscillation frequencies can cascade – generating additional colors at equal frequency separation Ω_ϕ (*i.e* at different equal υ_ϕ mismatch). This cascading is analogous to the cascading that occurs in an optical parametric oscillator (OPO) in which new frequencies can be generated from azimuthal mode mixing, as long as energy and momentum are conserved. However, instead of being continuous wave components, each of these colors contains a range of azimuthal mode components μ and are essentially temporal waves with photon energy all at the same level. The system is thus a *soliton-OPO*. When the multi-color soliton is periodically extracted by the by the bus waveguide, the different colors appear in the microcomb as a constant $\Delta\omega_{\mathrm{ceo}} = \Omega_\phi$ shift, which can then be resolved experimentally.

Assuming a single v_g for all wavepacket colors, one can recast the system into a set of coupled mean-field nonlinear-Schrödinger equations such that:

$$\frac{\partial a_\sigma(\phi,t)}{\partial t} = -\left(\frac{\kappa}{2} - i\sigma\Omega_\phi\right)a_\sigma + i\sum_\mu A(\mu,t)D^{NL}_{\mathrm{int}}(\mu)\mathrm{e}^{i\mu\phi} + ig_0\sum_{\alpha,\beta} a_\alpha a^*_\beta a_{\alpha-\beta+\sigma} + \delta_0\sqrt{\kappa_{\mathrm{ext}}}F\mathrm{e}^{i\delta\omega_0 t} + \delta_1\sqrt{\kappa_{\mathrm{ext}}}F\mathrm{e}^{i\mu_p\phi + i\delta\omega_p t} \quad (1)$$

where κ is the loss rate, κ_{ext} is the external coupling rate, g_0 is the nonlinear Kerr coefficient, μ is the azimuthal mode number normalized to the main pump, ϕ is the azimuthal coordinate, and σ is the υ_ϕ related mode number referenced to the original DKS, so that the auxiliary pump color correspond to $\sigma = +1$. Ω_ϕ is the repetition rate of the σ dimension, $A_\sigma(\mu,t) = \mathrm{FT}_\mu[a_\sigma(\phi,t)]$ is Fourier transform modal field, $D^{NL}_{\mathrm{int}}(\mu) = \omega(\mu_{\mathrm{aux}}) - (\omega_0 + \omega_{\mathrm{rep}}\mu)$ is the nonlinear integrated dispersion and can be different from the linear one as $\omega_{\mathrm{rep}} \neq D_1$ for non-symmetric D_{int}, and δ_X is the Kronecker delta and is 1 for $\sigma = X$ and 0 otherwise. It is interesting to note that for $\sigma \neq [0,1]$, all the color-wavepackets creating the comb teeth in the σ dimension are only parametrically driven by the soliton-OPO interaction.

We experimentally demonstrate a two-dimensional comb using an integrated 23 μm microring resonator made of 770 nm Si_3N_4 with a ring width of 1125 nm and embedded in SiO_2. We drive the resonator using a 150 mW on-chip primary pump at 192 THz (1560 nm) and a 65mW on-chip auxiliary pump at 209 THz (1432 nm) [fig. 1(c)]. The experimental spectrum showcases new dispersive-wave-like (DW-like) features, consistent with previous observation [6,7]. Looking closely at the spectrum, we notice that each of these DW-like features exhibits a different fixed frequency offset from the primary pumped DKS — consistent with the theoretical model developed of the soliton-OPO cascading in the σ dimension. To process the microcomb, we recall that each azimuthal mode number consists of any energy within $\Omega = [-\omega_{\mathrm{rep}}/2, \omega_{\mathrm{rep}}/2]$ centered around a single mode μ. From the measurement of the ω_{rep} one is able to index the frequency comb in the μ-dimension. We can then extract the two-dimensional power profile of the frequency comb [fig. 1(d)]. The interpretation of this figure is as follows. First, if only a single DKS was present, in the two-dimensional profile there would only be a single line in μ at $\Omega_\phi = 0$, corresponding to $\sigma = 0$. However the soliton-OPO interaction results in multiple horizontal lines that are parallel to the μ-axis, with different values of Ω, which indicates that each of these colors has the same ω_{rep} inside the microcavity and is locked to all the others. We note that even a small variation of ω_{rep} would be noticeable once μ becomes large. The locking of the angular group velocity of the wavepacket colors allows us to retrieve υ_{phi} from the constant shift of ω_{ceo}. An arithmetic average along μ highlights the frequency comb teeth in the σ-domain (left inset to fig. 1d). A total of six different σ components are visible, while only two pumps lasers are driving the resonator. The spectral separation between these σ comb teeth, $\Omega_\phi = 3.8$ GHz, is equal across all of them. Microwave measurements have also been performed, confirming that Ω_ϕ is indeed uniform across the σ dimension.

To conclude, we present a new kind of frequency comb that exhibits two-dimensional behavior, by harnessing the phase velocity dispersion that enables soliton-OPO cascading. Apart from new studies exploring new kinds of cavity solitons with synthetic dimensions and new pathways for nonlinear interactions in microresonators, potential applications could include transduction of optical signals to low microwave frequencies.

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