Pricing Demand-Side Flexibility With Noisy Consumers: Mean-Variance Trade-Offs

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***Abstract*—We formulate and analyze a demand-side management framework in which an aggregator sets energy prices to flexible consumers who have a noisy demand. The aggregator strategizes between purchasing energy in the day-ahead market, and settling any possible mismatches due to demand variability in the real-time market. Because consumers are sensitive to volatility in their electricity bill, the aggregator sets an upper bound on the variance of the payments made by consumers. We derive the optimal strategies for the consumers and the aggregator in this problem, and show that the aggregator limits dynamic pricing for consumers with too large demand variance, so as to avoid large uncertainty in their payments. On the other hand, consumers with low enough demand variance are charged the same price that would emerge in a noise-free formulation. We also identify two instances of a mean-variance trade-off: one in the consumer payment, which can be made less uncertain at the expense of a higher mean value, and another one in the aggregator's utility, which becomes less uncertain and lower on average as the consumer payment variance constraints become binding. We corroborate our analysis with a numerical case study.**

***Index Terms*—Demand response, mean-variance trade-off, noisy demand, stackelberg game.**

I. Introduction

DEMAND response programs have been increasingly becoming part of the new paradigm of power distribution systems, where consumer price-responsiveness can be leveraged to increase grid flexibility. In these programs, an aggregator can incentivize changes in the consumption pattern of flexible consumers through changes in the electricity prices or any other incentives. It has been shown that demand response can help decrease the effective cost of supplying electricity, thus improving economic efficiency in electricity markets [1]. It can also help the integration of higher amounts of variable renewable generation [2], and serves as an alternative to grid expansion [3], among other benefits.

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In this work, we consider a scenario in which flexible consumers are willing to adjust their demand based on the electricity prices set by an aggregator. We model the interactions between aggregator and consumers as a Stackelberg game in which the aggregator is the leader, and consumers are the followers. The aggregator is responsible for purchasing electricity in the day-ahead energy market to supply to consumers. However, consumers exhibit a price-dependent demand subject to random noise, and therefore their real-time response may be different from what was expected ahead of time. Such variability has been identified in practice, and can be attributed to model error or variability in control response [4]. This consumption uncertainty leads to two implications. First, the aggregator needs to settle any mismatches between the day-ahead supply purchased and the actual consumer demand in the real-time market. With that, the aggregator's day-ahead purchase strategy needs to consider these possible mismatches and the real-time market participation. Second, demand volatility leads to uncertainty in the electricity bill of consumers. Therefore, to attract and retain customer participation, the aggregator bounds the payment variance of each individual consumer when solving his problem.

The literature on demand response is very vast. To avoid a very long literature review, we refer the reader to [5], [6] and the references therein for a comprehensive, but non-exhaustive survey. In this paper, we focus on the specific problem of obtaining the aggregator's optimal strategy to incentivize demand response from heterogeneous price-responsive loads. Various works have adopted a deterministic formulation for this problem. For example, [7] considers a scenario with one aggregator and multiple devices that purchase electricity and can be interruptible or uninterruptible, and [8] models multiple retailers, introducing company-side competition for the supply of energy to consumers. Other streams of work have proposed demand-side management frameworks that consider uncertainty stemming from multiple sources. Process and measurement noise of thermostatically controlled loads are considered in [9], where the trade-off between consumer surplus and retailer profit is characterized. The authors in [10] investigate the impact of net load volatility on energy production costs, and propose a pricing strategy to allocate the cost of volatility among customers. The participation of the aggregator in multiple markets is considered in [11]–[13]. Mixed integer linear programs are proposed in [11], which considers a retailer who owns renewable resources, and in [12], where an additional program is formulated to verify the efficiency of the solution. In [13], an aggregator purchases demand reductions from consumers while taking into account demand uncertainty and information asymmetry in consumers' parameters.

The existing literature also incorporates risk management to solve the aggregator's problem of demand-side participation with flexible and uncertain consumers in different ways. Conditional Value-at-Risk (CVaR) is used in [14] for a retailer who seeks to decrease the likelihood of low-profit outcomes when purchasing power in the wholesale market to serve customers. In [15], a bidding strategy for a utility company is proposed using information-gap decision theory, so that the company can guarantee a certain profit level. The formulation proposed in [16] adopts a chance constraint to ensure, with a certain probability, that the aggregator will purchase enough energy in the day-ahead market to supply consumers, instead of possibly paying a premium in the real-time market. Previous studies, however, have mostly focused on the problem from the aggregator's perspective. As against these works, we propose a formulation which avoids large uncertainty for the end-consumer, which could be a strategy to engage more customers in programs with dynamic pricing.

The body of work which uses such consumer-centric strategies is more scarce. In [17], a utility selection problem is formulated from the perspective of consumers who aim to minimize their consumption costs, while [18] departs from a price-driven approach and uses aggregated consumption data to schedule appliances based on consumer behavior. Much less attention has been given to how volatility in electricity bills impacts customers. The choice-based experiment in [19] suggests that payment volatility significantly impacts a consumer's decision of which energy plan to enroll in. Their results show that customers are less likely to choose a renewable energy plan as the volatility of their monthly payment increases as compared to a non-renewable plan. This indicates that programs that affect the electricity bill of customers should be designed to prevent large payment variations, so as to incentivize consumer participation.

The main contribution of our work is on the formulation and analysis of a demand-side management framework which guarantees a bounded uncertainty on the electricity bill of flexible consumers. Since consumers may be unable to fully control their response, the task of constraining consumer payment variance is delegated to the aggregator. To the best of our knowledge, there are no previous works which adopt this type of consumer-centric strategy, which focuses on limiting payment volatility while placing the responsibility of guaranteeing constraint satisfaction on the aggregator, and not on consumers. We also focus on deriving analytical results, which can provide insights about the solutions that would not be readily observed if only numerical studies were performed.

Our analysis shows that the aggregator's optimal day-ahead purchase depends not only on the level of flexibility of consumers and the day-ahead energy prices, but also on the aggregated consumer noise and on the real-time prices. Further, when comparing our variance-constrained formulation with an unconstrained one, we observe two different mean-variance trade-offs: first, consumers with a binding payment variance constraint will observe a decrease in their payment uncertainty, but their average payment increases; second, if any consumer has a binding payment variance constraint, the aggregator experiences a decrease in his utility variance, and this decreased risk comes at the expense of a lower mean utility. While the first trade-off could have been expected because it occurs on the quantity whose variance we are constraining, the second one is less intuitive, considering that the aggregator experiences uncertainty not only on the payments received by consumers, but also on his participation in the real-time market. We also show that the aggregator limits dynamic pricing to consumers with too large demand noise, so as to avoid large uncertainty in their payments, indicating that consumers with lower noise levels are preferred from the aggregator's perspective.

The remainder of this paper is organized as follows. Our problem formulation is presented in Section II. The optimal strategies of the consumers and the aggregator are derived in Section III, and further analyses of these results are performed in Section IV. We illustrate our proposed framework with a case study in Section V, and Section VI concludes this paper with our final remarks.

II. Problem Formulation

We model a Stackelberg game in which an aggregator acts as the leader and sets the energy prices for flexible consumers, while these consumers are followers and adjust their demand levels in response to the prices charged. The aggregator is also tasked with purchasing energy in a day-ahead wholesale market to meet consumers' demands. However, consumers have a noisy demand, which can lead to mismatches between the amount of energy purchased and what is actually consumed in real-time. These mismatches are settled by the aggregator in the real-time market, where excess energy can be sold, and shortages can be covered by purchasing more energy. Due to demand volatility, consumers experience uncertainty in the value of their electricity bill. To counteract this undesirable effect, we consider that the aggregator makes his decisions so as to keep the payment variance of every consumer below a certain upper bound.

A. Consumers' Problem

In the Stackelberg game modeled, each consumer $i \in \mathcal{I} := \{1, \ldots, N\}$ receives a price π_i from the aggregator. Given this signal, the consumers will decide on their demand levels so that their utility, given by

$$u_c^i = \gamma_i \ln(\alpha_i + \overline{d}_i) - \pi_i \overline{d}_i, \tag{1}$$

is maximized. In the utility function (1), the first term is consumer i's benefit from consuming $\overline{d}_i$, while the second term is the payment corresponding to this demand. The logarithmic function has been extensively used to model consumer-side utility in demand response problems [8], [20], [21]. Previous works have also considered quadratic [9], and linear [13] functions for similar problems, but the logarithmic function has been shown to lead to proportional fairness [22]. Nonetheless, the general conclusions of our analysis can be extended to a more generic concave utility function.

The parameters γ_i and α_i are particular of each consumer, and thus these agents need not have a homogeneous behavior. We let $\gamma_i > 0$ and $\alpha_i \geq 1$, so that the benefit from consuming is always non-negative. Then, consumer i solves the following problem:

$$\max_{\overline{d}_i} u_c^i \tag{2}$$

$$\text{s.t.} \quad 0 \leq \overline{d}_i \leq D_i. \tag{2a}$$

The consumers solve a deterministic problem when deciding on their demand levels. However, due to lack of full controllability in their electric devices, the actual realization of these demands is noisy. Thus, we let the real-time consumption of customer i be

$$d_i^* = \overline{d}_i^* + \xi_i, \tag{3}$$

where $\overline{d}_i^*$ is the solution to (2), and ξ_i is assumed to be a zero-mean, bounded noise with variance σ_i^2. Further, these noise variables have a continuous, twice differentiable probability distribution function (pdf). These technical assumptions on the noise pdf ensure continuity of the inverse cumulative density function for the aggregated noise, which will be part of our analytical results. We consider the random noises $\{\xi_i\}$ to be independent across consumers. Similar modeling considerations for demand noise have been used in previous works, such as [9], [13]. Understanding the effect of correlation among customers would bring more complexity to our analysis, and is left as a direction for future work. In this context, if the paradigm is shifted to considering customers who act strategically, an interesting approach would be to use a causation-based cost allocation similar to what has been previously proposed for uncertain resources [23], [24].

B. Aggregator's Problem

The aggregator takes into account the uncertainty in consumers' demand when making his decisions. His expected utility function is given by his expected profit:

$$u_a = -\lambda_{DA} \sum_{i\in\mathcal{I}} C_i + \mathbb{E}\left[\sum_{i\in\mathcal{I}} \pi_i d_i^*\right] + \mathbb{E}\left[\lambda^+ \left(\sum_{i\in\mathcal{I}} (C_i - d_i^*)\right)^+ - \lambda^- \left(\sum_{i\in\mathcal{I}} (d_i^* - C_i)\right)^+\right], \tag{4}$$

where $(.)^+$ represents $\max(., 0)$, and the expectation $\mathbb{E}[.]$ is over the sum of the demand noises from all consumers $\sum_i \xi_i$.

The first term in (4) is the cost incurred by the aggregator to purchase energy in the day-ahead market at a price λ_{DA}, where $\sum_i C_i$ is a decision variable for the aggregator, and corresponds to the aggregated purchase for all consumers. The second term is the expected revenue from payments received from the consumers, which are settled based on their actual consumption in real-time (3). This demand is the consumer's best response to the price charged by the aggregator, and its expression will be derived in Section III. The second line expresses the transactions performed in the real-time market. The first term in the expectation indicates the expected revenue from selling energy in the real-time market at a price λ^+ when the actual demand is below the amount that was purchased in the day-ahead market. Conversely, the second term represents the cost of purchasing more energy at a price λ^- when there is a shortage in real-time due to the demands being higher than the amount of energy available.

We assume the aggregator is small relative to the market, and thus behaves as a price-taker. Therefore, all the market prices are assumed to be fixed and known. This assumption follows other works in the literature which investigate the participation of demand response [13], wind [25], and storage resources [26] in the market. The expected value of such prices could also be used without loss of generality, as in [25], as long as they are statistically independent of the demand noise. In such case, the aggregator could use historical data to estimate these expected prices. We further let $\lambda^+ < \lambda_{DA} < \lambda^-$. This avoids trivial results in which the aggregator acts as an arbitrageur by purchasing large amounts of energy in the day-ahead market to resell them in the real-time market, or in which he purchases all energy in the real-time market.

As previously discussed, we consider a situation in which the aggregator wants to avoid high uncertainty in how much consumers expect to pay. The aggregator's problem is to decide how much energy to purchase in the day-ahead market, as well as how much to charge each consumer, so that his own expected profit is maximized. This problem is also subject to the consumers' responses to the prices set:

$$\max_{C_i,\pi_i} u_a \tag{5}$$

$$\text{s.t.} \quad C_i \geq 0 \qquad \forall i \in \mathcal{N} \tag{5a}$$

$$\pi_i \geq 0 \qquad \forall i \in \mathcal{N} \tag{5b}$$

$$\text{Var}[\pi_i d_i^*] \leq \beta_i^2 \qquad \forall i \in \mathcal{N} \tag{5c}$$

$$d_i^* = d_i^*(\pi_i^*) \qquad \forall i \in \mathcal{N}. \tag{5d}$$

The first two constraints in this problem ensure positivity of prices and amount of energy purchased. Through (5c), the aggregator sets an upper bound on each consumer's payment variance. Further, constraint (5d) is the price-dependent noisy demand (3) found through each consumer i's problem (2), which is enforced so that the aggregator's decisions lead to the consumers choosing their best response. Here, we abuse notation to write the price-dependent demand $d_i^*(\pi_i^*)$ as d_i^* to simplify the exposition of the expressions that involve this variable. In this work, we ignore distribution network constraints and focus on the demand-side management problem for noisy consumers. Thus, the prices charged to consumers are not affected by their location in the network, as would be the case in a scenario that uses distribution locational marginal prices. Including such constraints would lead to a less tractable model, limiting the derivation of analytical results, and is left as a direction for future work. Further, we assume the aggregator has full information of the consumers' parameters. Proposing a way to learn consumer response is not in the scope of our paper, but multiple techniques have been shown to be effective for this purpose [13], [27].

III. Optimal Strategies

In the following results, we present the optimal decisions of the consumers and the aggregator.

Lemma 1: The price-dependent noisy demand of each consumer i is given by

$$d_i^*(\pi_i) = \begin{cases} D_i + \xi_i & \text{if } \pi_i < \gamma_i/(\alpha_i + D_i) \\ \gamma_i/\pi_i - \alpha_i + \xi_i & \text{if } \gamma_i/(\alpha_i + D_i) \leq \pi_i < \gamma_i/\alpha_i \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

so that it is decreasing with the price charged π_i, and becomes zero if $\pi_i > \gamma_i/\alpha_i$.

Proof: In problem (2), the utility u_c^i is concave in the decision variable $\overline{d}_i$. The Lagrangian function for this problem is

$$\mathcal{L}_c^i = \gamma_i \ln(\alpha_i + \overline{d}_i) - \pi_i \overline{d}_i + \underline{\lambda}_i \overline{d}_i + \overline{\lambda}_i (D_i - \overline{d}_i), \tag{7}$$

and the corresponding KKT conditions are

$$\frac{\partial \mathcal{L}_c^i}{\partial \overline{d}_i} = \frac{\gamma_i}{\alpha_i + \overline{d}_i} - \pi_i + \underline{\lambda}_i - \overline{\lambda}_i = 0 \tag{8}$$

$$\underline{\lambda}_i \overline{d}_i = 0 \tag{9}$$

$$\overline{\lambda}_i (D_i - \overline{d}_i) = 0 \tag{10}$$

$$\underline{\lambda}_i, \overline{\lambda}_i \geq 0 \tag{11}$$

If the demand $\overline{d}_i$ is non-binding, we must have $\underline{\lambda}_i = \overline{\lambda}_i = 0$. Using (8), it follows that $\overline{d}_i^* = \gamma_i/\pi_i - \alpha_i$. For the demand bounds (2a) to be satisfied, the price should satisfy $\gamma_i/(\alpha_i + D_i) \leq \pi_i \leq \gamma_i/\alpha_i$. Thus, consumer i's demand function is

$$\overline{d}_i^*(\pi_i) = \begin{cases} D_i & \text{if } \pi_i < \gamma_i/(\alpha_i + D_i) \\ \gamma_i/\pi_i - \alpha_i & \text{if } \gamma_i/(\alpha_i + D_i) \leq \pi_i < \gamma_i/\alpha_i \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Due to noise, this price-dependent demand is given by (3). □

The results on the optimal strategy for consumers show that they set an upper bound on the price charged to begin consumption. For any price beyond this point, the cost of paying for electricity surpasses the benefit derived from consumption. We also note that this price threshold depends on the ratio of the consumer's flexibility parameters γ_i/α_i, so that consumers with higher ratio are more willing to pay higher prices and thus can be seen as more flexible. However, if consumers have the same ratio γ_i/α_i, their consumption level is not necessarily the same, as the optimal demand (6) depends on each parameter separately, being increasing with γ_i and decreasing with α_i.

The optimal strategy of the consumers characterizes how they respond for a given price, and does not take into account the possible variance in their payments. They rely on the fact that, for the demand-side management program proposed, the aggregator sets the prices so that this variance is bounded. In the following result, the consumers' response is used to find the optimal strategies for the aggregator.

Theorem 1: The aggregator's optimal decision on how much energy to purchase in the day-ahead market is given by

$$\sum_{i\in\mathcal{I}} C_i^* = \sum_{i\in\mathcal{I}} \left(\frac{\gamma_i}{\pi_i^*} - \alpha_i \right) + F^{-1} \left(\frac{\lambda^- - \lambda_{DA}}{\lambda^- - \lambda^+} \right), \tag{13}$$

where F^{-1} is the quantile function of the aggregated demand noise, $\sum_i \xi_i$. The optimal price for each consumer i is

$$\pi_i^* = \begin{cases} \sqrt{\lambda_{DA}\gamma_i/\alpha_i} & \text{if } \sigma_i^2 < \frac{\beta_i^2 \alpha_i}{\lambda_{DA}\gamma_i} \\ \beta_i/\sigma_i & \text{otherwise.} \end{cases} \tag{14}$$

Consumers will respond with a non-zero demand if $\pi_i^* < \gamma_i/\alpha_i$. Further, these prices should be no lower than $\gamma_i/(\alpha_i + D_i)$, which is the level that leads to maximum consumer demand D_i.

Proof: We use backwards induction to solve for the aggregator's decisions in this Stackelberg game. Substituting the noisy demands for the non-binding demand case from (6) in the aggregator's utility function (4), we find

$$\begin{aligned} u_a = & -\lambda_{DA} \sum_{i\in\mathcal{I}} C_i + \mathbb{E}\left[\sum_{i\in\mathcal{I}} (\gamma_i - \alpha_i\pi_i + \xi_i\pi_i) \right] \\ & + \mathbb{E}\left[\lambda^+ \left(\sum_{i\in\mathcal{I}} \left(C_i - \frac{\gamma_i}{\pi_i} + \alpha_i - \xi_i \right) \right)^+ \right] \\ & - \mathbb{E}\left[\lambda^- \left(\sum_{i\in\mathcal{I}} \left(\frac{\gamma_i}{\pi_i} - \alpha_i + \xi_i - C_i \right) \right)^+ \right], \end{aligned} \tag{15}$$

which can be shown to be concave in the decision variables for this problem. Making the same substitution in the variance constraint (5c), we can write

$$\begin{aligned} \mathrm{Var}[\pi_i d_i^*] &= \mathrm{Var}[\gamma_i - \alpha_i\pi_i + \xi_i\pi_i] = \pi_i^2 \mathrm{Var}[\xi_i] \\ &= \pi_i^2\sigma_i^2 \leq \beta_i^2 \Rightarrow \pi_i\sigma_i \leq \beta_i. \end{aligned} \tag{16}$$

Then, the Lagrangian for the aggregator's problem (5) is

$$\mathcal{L}_a = u_a + \sum_{i\in\mathcal{I}} \nu_i C_i + \sum_{i\in\mathcal{I}} \mu_i \pi_i + \sum_{i\in\mathcal{I}} \rho_i (\beta_i - \pi_i\sigma_i), \tag{17}$$

from which the KKT stationarity conditions are as follows

$$\begin{aligned} \frac{\partial \mathcal{L}_a}{\partial C_i} = & -(\lambda^- - \lambda^+) F \left(\sum_{i\in\mathcal{I}} \left(C_i - \frac{\gamma_i}{\pi_i} + \alpha_i \right) \right) \\ & - \lambda_{DA} + \lambda^- + \nu_i = 0 \forall i \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{\partial \mathcal{L}_a}{\partial \pi_i} = & -\frac{\gamma_i}{\pi_i^2} (\lambda^- - \lambda^+) F \left(\sum_{i\in\mathcal{I}} \left(C_i - \frac{\gamma_i}{\pi_i} + \alpha_i \right) \right) \\ & - \alpha_i + \frac{\lambda^- \gamma_i}{\pi_i^2} + \mu_i - \rho_i\sigma_i = 0 \forall i. \end{aligned} \tag{19}$$

Further, the complementary slackness and dual feasibility conditions are given by

$$\nu_i C_i = 0 \forall i \tag{20}$$

$$\mu_i \pi_i = 0 \forall i \tag{21}$$

$$\rho_i(\beta_i - \pi_i\sigma_i) = 0 \forall i \tag{22}$$

$$\nu_i, \mu_i, \rho_i \geq 0 \forall i, \tag{23}$$

and the inequalities (5a)–(5c) of the original problem must be satisfied. For non-zero prices π_i and purchase amounts C_i, we must have $\nu_i = \mu_i = 0 \forall i$. Then, from (18), we find that

$$F\left(\sum_{i \in \mathcal{I}} \left(C_i^* - \frac{\gamma_i}{\pi_i^*} + \alpha_i\right)\right) = \frac{\lambda^- - \lambda_{DA}}{\lambda^- - \lambda^+}, \tag{24}$$

from which the optimal day-ahead purchase (13) is derived. Substituting (24) in (19), we end up with

$$-\alpha_i + \lambda_{DA}\gamma_i/\pi_i^{*2} - \rho_i\sigma_i = 0 \forall i. \tag{25}$$

If the variance constraint (5c) is binding for a certain consumer i, then $\rho_i > 0$ and $\pi_i^*\sigma_i = \beta_i \Rightarrow \pi_i^* = \beta_i/\sigma_i$. Using (25),

$$\rho_i = \frac{\lambda_{DA}\gamma_i - \pi_i^{*2}\alpha_i}{\pi_i^{*2}\sigma_i} > 0 \Rightarrow \pi_i^* < \sqrt{\lambda_{DA}\gamma_i/\alpha_i}. \tag{26}$$

Then, both conditions hold if

$$\beta_i/\sigma_i < \sqrt{\lambda_{DA}\gamma_i/\alpha_i} \Rightarrow \sigma_i^2 > \frac{\beta_i^2\alpha_i}{\lambda_{DA}\gamma_i}. \tag{27}$$

If this condition does not hold, then constraint (5c) cannot be binding. Thus $\rho_i = 0$, and the corresponding optimal price can be found from (25). Consumers will respond with maximum or zero demand according to the bounds established in the first and last cases of (12), respectively. □

From the optimal amount of day-ahead energy to purchase presented in Theorem 1, we note that the aggregator is only concerned with the overall amount of energy $\sum_i C_i$, rather than the individual allocations C_i. This follows because, when settling deviations in the real-time market, the aggregator pools the demand of consumers, so that the collective deviation from the total demand already purchased is considered. The quantile term in (13) is related to the well-known newsvendor problem. Similarly to the discussion in [28], the numerator in this quantile term represents the opportunity cost of purchasing an extra demand unit in the real-time market due to a low day-ahead purchase, $\lambda^- - \lambda_{DA}$. The denominator is the sum of this so-called underage cost with the overage cost, which in this problem represents the cost of having purchased an extra unit in the day-ahead market, and having to sell it back at a lower cost, $\lambda_{DA} - \lambda^+$.

The expression (13) can be rewritten to find that

$$\text{Prob}\left(\sum_{i \in \mathcal{I}} d_i^* \leq \sum_{i \in \mathcal{I}} C_i^*\right) = \frac{\lambda^- - \lambda_{DA}}{\lambda^- - \lambda^+}, \tag{28}$$

that is, the probability that there will be excess energy to be sold in the real-time market depends on the price ratio given. This probability is increasing with the cost of purchasing energy in the real-time market λ^-, indicating that the aggregator attempts to decrease the likelihood of having a supply shortage when this price is too high. A similar scenario is observed with increasing price to sell back in the real-time market λ^+. Here, the reason is the added value that excess supply would have in the aggregator's utility. Conversely, larger day-ahead prices λ_{DA} lead to lower probability that the total demand will be below the supply already secured.

The price charged to each individual consumer (14) depends not only on their flexibility level, represented by their parameters γ_i and α_i, but also on their individual demand variance σ_i^2. The first price presented corresponds to situations where the consumer variance is low enough for the payment variance constraint (5c) not to be binding at optimality. This situation only holds if the demand variance σ_i^2 satisfies the upper bound established. Note that this threshold becomes tighter for higher λ_{DA}, since the price π_i^*, and thus the payment variance, is increasing with the day-ahead energy price. Thus, this approach limits dynamic pricing as the day-ahead price increases, protecting customers from being exposed to high market prices.[1] Further, the price charged to each individual customer only depends on their own characteristics, being agnostic to the characteristics or behavior of other consumers. Therefore, the proposed pricing scheme leverages consumer heterogeneity and compensates consumers based on the value each one of them brings to the demand response program.

When the demand variance of a consumer is too high, the price charged is capped at the second expression in (14) to ensure that the constraint (5c) is not violated. We remark that, if a consumer is consistently in this category, then they will be charged this flat rate more often as compared to other less uncertain consumers, whose price will vary with the day-ahead energy price. This indicates that a consumer whose response is too uncertain may be exposed to an amount of risk that is beyond the acceptable level, and this volatility in payment can be avoided by limiting dynamic pricing for this consumer in lieu of a static, more traditional approach to pricing. Because of their limitation in following a dynamic pricing scheme, such consumers would not be able to contribute extensively towards tasks such as peak demand reduction. Therefore, the flexibility offered by consumers with high demand variance is not as useful to the aggregator.

IV. Model Implications

In this section, we analyze the optimal strategies derived previously to provide further insights about the interactions between aggregator and consumers.

A. Consumer Participation

We start by analyzing the payment incurred by consumers who have large demand variance.

Corollary 1: When a consumer's demand variance surpasses the threshold $\frac{\beta_i^2\alpha_i}{\lambda_{DA}\gamma_i}$ and the payment variance demand (5c) becomes binding,

1) the per unit price π_i^* charged by the aggregator is lowered to bound the payment variance,
2) consumer i increases the mean value of his demand $\overline{d}_i^*$,
3) the expected payment for consumer i, $\pi_i^*\overline{d}_i^*$, increases.

[1] We note that market prices may surge in extreme weather events. However, such scenarios are beyond the scope of this paper, as they may lead to the violation of the assumption that demand noise variables are independent across consumers.

Proof: Proof follows directly from Theorem 1 results on the aggregator's optimal pricing strategy (14), and the consumer's price-dependent function (6). □

We can observe a mean-variance trade-off on the payments performed by consumers with large demand variance. When the aggregator bounds the uncertainty that these consumers may experience in their payments, he adjusts the per unit price so that the mean payment from these customers is higher than if the payment variance constraint were not present. Thus, these consumers incur a financial cost to ensure that their payment variance does not exceed a certain threshold. As compared to a scenario without the payment variance constraint, the increase in payment observed by a consumer with a binding constraint is

$$\sqrt{\lambda_{DA}\gamma_i\alpha_i} - \frac{\alpha_i\beta_i}{\sigma_i}, \tag{29}$$

which is higher for consumers with higher demand variance σ_i, and, as expected, decreases as the upper bound β_i^2 is relaxed.

In the following, we take a closer look at the price charged for consumers with low enough demand variance.

Corollary 2: For a consumer with low enough demand noise that the corresponding variance constraint (5c) is not binding, the price charged π_i^* is the same as in a noiseless case.

Proof: If demands are deterministic, then there are no mismatches between the supply purchased in the day-ahead market and the actual consumer demand, so that $C_i = \overline{d}_i \forall i$. Thus, the aggregator does not participate in the real-time market, and his deterministic utility becomes

$$u_a^d = \sum_{i\in\mathcal{N}} \pi_i \overline{d}_i - \lambda_{DA} \sum_{i\in\mathcal{N}} \overline{d}_i. \tag{30}$$

From the consumers' price-dependent demand curve (12), we can find the inverse demand function

$$\pi_i(\overline{d}_i^*) = \gamma_i/(\alpha_i + \overline{d}_i^*)\forall i, \tag{31}$$

which can be substituted in (30), so that the aggregator solves for $\overline{d}_i \geq 0 \forall i$. The resulting utility can be shown to be concave in these decision variables. Letting μ_i^d be the Lagrange multiplier of the positivity constraint for consumer i's demand, we have the Lagrangian function

$$\mathcal{L}_a^d = \sum_{i\in\mathcal{N}} \left[\frac{\gamma_i \overline{d}_i}{\alpha_i + \overline{d}_i} - \lambda_{DA}\overline{d}_i + \mu_i^d \overline{d}_i \right]. \tag{32}$$

The corresponding KKT conditions are

$$\frac{\partial \mathcal{L}_a^d}{\partial \overline{d}_i} = \frac{\gamma_i\alpha_i}{(\alpha_i + \overline{d}_i)^2} - \lambda_{DA} + \mu_i^d = 0 \forall i \tag{33}$$

$$\mu_i^d \overline{d}_i = 0 \forall i \tag{34}$$

$$\mu_i^d \geq 0 \forall i. \tag{35}$$

For non-zero demands, $\mu_i^d = 0$ and we use (33) to find

$$\overline{d}_i^* = \sqrt{\gamma_i\alpha_i/\lambda_{DA}} - \alpha_i > 0, \tag{36}$$

whose positivity holds for $\lambda_{DA} < \gamma_i/\alpha_i$. Substituting this optimal demand in the inverse demand function (31), we find that the optimal price that leads to this demand is

$$\pi_i^* = \sqrt{\lambda_{DA}\gamma_i/\alpha_i}, \tag{37}$$

which matches the noisy case for low enough σ_i^2. □

Corollary 2 states that consumers will be charged as if they had deterministic load, provided that their demand variance is low enough. This implies that the aggregator handles demand uncertainty mostly by adjusting his purchase in the day-ahead market, instead of finely modifying the price charged to these consumers. From the threshold on σ_i^2 presented in (14), we remark that consumers are less likely to be considered of low enough demand variance at times of peak demand. Thus, flexible consumers can become more valuable during moments in which load reduction is desirable by adopting measures which help them decrease their demand variance, such as using appliances that allow for more precise consumption control. As shown in Corollary 1, this would allow them to achieve lower payments in their electricity bill on average.

B. Aggregator's Outcome

We now evaluate the implications of the proposed model from the aggregator's perspective.

Theorem 2: In a scenario where some of the payment variance constraints (5c) are binding, the optimal utility of the aggregator

- is upper bounded by his optimal utility in a case without these constraints, and
- has lower variance than in a case without these constraints.

Proof: To find the optimal utility of the aggregator, we substitute the noisy demands (6) and the optimal day-ahead purchase (13) in the aggregator's utility function (4). This yields

$$\begin{aligned} u_a = & -\lambda_{DA} \left[\sum_{i\in\mathcal{I}} \left(\frac{\gamma_i}{\pi_i^*} - \alpha_i \right) + F^{-1}\left(\frac{\lambda^- - \lambda_{DA}}{\lambda^- - \lambda^+} \right) \right] \\ & + \mathbb{E}\left[\sum_{i\in\mathcal{I}} (\gamma_i - \alpha_i\pi_i^* + \xi_i\pi_i^*) \right] \\ & + \mathbb{E}\left[\lambda^+ \left(F^{-1}\left(\frac{\lambda^- - \lambda_{DA}}{\lambda^- - \lambda^+} \right) - \sum_{i\in\mathcal{I}} \xi_i \right)^+ \right] \\ & - \mathbb{E}\left[\lambda^- \left(\sum_{i\in\mathcal{I}} \xi_i - F^{-1}\left(\frac{\lambda^- - \lambda_{DA}}{\lambda^- - \lambda^+} \right) \right)^+ \right]. \end{aligned} \tag{38}$$

The terms corresponding to the real-time market transactions are independent of the prices π_i^* charged to consumers. The same is true for the quantile term in the first line. Thus, these terms do not change if any of the constraints (5c) is binding, and we focus on the remaining terms, which we denote $\tilde{u}_a$. In a case without the payment variance constraints, the optimal prices are the same as in the first case in (14). It follows that

$$\tilde{u}_a = -\lambda_{DA} \sum_{i\in\mathcal{I}} \left(\sqrt{\frac{\gamma_i\alpha_i}{\lambda_{DA}}} - \alpha_i \right) + \sum_{i\in\mathcal{I}} \left(\gamma_i - \sqrt{\lambda_{DA}\gamma_i\alpha_i} \right), \tag{39}$$

where we used the fact that the noises are zero-mean. Let $\mathcal{B} \subseteq \mathcal{I}$ be a subset of consumers which have binding payment variance constraint, and are charged as in the second case of (14). For this binding case, we can write

$$\begin{aligned}\tilde{u}_a^b = & -\lambda_{DA}\left[\sum_{i\in\mathcal{I}\setminus\mathcal{B}}\left(\sqrt{\frac{\gamma_i\alpha_i}{\lambda_{DA}}}-\alpha_i\right)+\sum_{i\in\mathcal{B}}\left(\frac{\gamma_i\sigma_i}{\beta_i}-\alpha_i\right)\right]\\ & +\sum_{i\in\mathcal{I}\setminus\mathcal{B}}\left(\gamma_i-\sqrt{\lambda_{DA}\gamma_i\alpha_i}\right)+\sum_{i\in\mathcal{B}}\left(\gamma_i-\frac{\alpha_i\beta_i}{\sigma_i}\right).\end{aligned} \tag{40}$$

Comparing the unconstrained (39) and the binding (40) cases, we find that $\tilde{u}_a \geq \tilde{u}_a^b$, as detailed below

$$\begin{aligned}& -\lambda_{DA}\sum_{i\in\mathcal{B}}\left(\sqrt{\frac{\gamma_i\alpha_i}{\lambda_{DA}}}-\alpha_i\right)+\sum_{i\in\mathcal{B}}\left(\gamma_i-\sqrt{\lambda_{DA}\gamma_i\alpha_i}\right)\\ & \geq -\lambda_{DA}\sum_{i\in\mathcal{B}}\left(\frac{\gamma_i\sigma_i}{\beta_i}-\alpha_i\right)+\sum_{i\in\mathcal{B}}\left(\gamma_i-\frac{\alpha_i\beta_i}{\sigma_i}\right)\\ & \Rightarrow -2\sum_{i\in\mathcal{B}}\left(\sqrt{\lambda_{DA}\gamma_i\alpha_i}\right)\geq -\sum_{i\in\mathcal{B}}\left(\frac{\lambda_{DA}\gamma_i\sigma_i}{\beta_i}+\frac{\alpha_i\beta_i}{\sigma_i}\right)\\ & \sum_{i\in\mathcal{B}}\left(\sqrt{\frac{\lambda_{DA}\gamma_i\sigma_i}{\beta_i}}-\sqrt{\frac{\alpha_i\beta_i}{\sigma_i}}\right)^2\geq 0.\end{aligned} \tag{41}$$

For the utility variance, we note from (38) that the terms in the first line have zero variance, while the real-time market terms may have non-zero variance, but are independent of the prices charged to consumers. Thus, we focus on the term in the second line, whose variance is given by

$$\mathrm{Var}\left[\sum_{i\in\mathcal{I}}(\gamma_i-\alpha_i\pi_i^*+\xi_i\pi_i^*)\right]=\sum_{i\in\mathcal{I}}\pi_i^{*2}\sigma_i^2, \tag{42}$$

since the random noises are independent. From Corollary 1, we know that the per unit price π_i^* charged to consumers with binding payment variance constraint is lower than if this constraint is non-binding. It follows that, if constraint (5c) is binding for a subset $\mathcal{B}\subseteq\mathcal{I}$ of consumers, then the aggregator's utility will have a lower variance as compared to an unconstrained case.

Similarly to consumers, the aggregator also observes a mean-variance trade-off due to the payment variance constraints. While the uncertainty in his utility is decreased with the addition of these constraints, allowing him to avoid high volatility, he also observes a decrease in his expected utility. We also highlight that the expected real-time transactions at the optimal solution remain unaffected as these constraints become binding, as they only depend on the energy market prices and the noise probabilistic model. Thus, the change in the aggregator's utility is due to the variations in his day-ahead purchase and in the payments received from consumers. The analysis presented in Theorem 2 leads to the following results on the aggregator's utility as the number of consumers becomes large.

Corollary 3: The aggregator's expected profit decreases as the number of consumers with binding payment variance constraint becomes larger.

Proof: Proof follows from Theorem 2. As shown in (41), the gap in the aggregator's expected profit in the constrained case, as compared to the unconstrained one, is increasing with the number of consumers in the set with binding constraints. □

C. Comparison With a Uniform Pricing Approach

The proposed pricing strategy considers each customer's flexibility and noise levels, as well as their individual upper limits on payment variance. However, current policies may impose barriers against such price discrimination. In previous works, the evaluation of the effects of price discrimination across consumers have mostly focused on discriminatory pricing strategies for excess energy. In this context, it has been shown that maintaining fairness in programs with discriminatory pricing, such that the price distribution reflects the contribution of each agent in the system, is important to keep the customer participation in the program from diminishing [29]. Further, these schemes can also benefit the central entity responsible for managing energy trades in a smart community [30]. For a scenario where only base demand is considered, early works analyzed price discrimination while categorizing consumers into different classes depending on their demand patterns [31], and it has been shown that a conventional uniform pricing approach may fail to incentivize different groups to participate in demand response [32].

To complete our analysis, we compare our proposed strategy with a scenario with no price discrimination across different consumers, so that the aggregator charges the same price regardless of how flexible or noisy each consumer is individually, i.e. $\pi_i = \pi\forall i$. In this uniform price scenario, the price-dependent demand function of each consumer (12) remains the same, only now the price charged π is the same across all customers. Further, the payment variance constraint (5c) is modified to become

$$\mathrm{Var}[\pi d_i^*]\leq\beta_i^2\forall i\Rightarrow\pi\leq\min(\beta_i/\sigma_i). \tag{43}$$

Following these changes, the Lagrangian of aggregator's problem under a uniform pricing strategy becomes

$$\mathcal{L}_a^u = u_a^u+\sum_{i\in\mathcal{I}}\nu_i^u C_i+\mu^u\pi+\rho^u(\min(\beta_i/\sigma_i)-\pi), \tag{44}$$

where u_a^u is the expected profit of the aggregator (15) with $\pi_i = \pi\forall i$. For non-zero uniform price π and purchase amounts C_i, the same expression for the optimal day-ahead purchase (13) can be derived, but with $\pi_i=\pi\forall i$. As for the pricing strategy, for $\Gamma=\sum_i\gamma_i$ and $A=\sum_i\alpha_i$, the optimal uniform price is

$$\pi^*=\begin{cases}\sqrt{\lambda_{DA}\Gamma/A} & \text{if } [\min(\beta_i/\sigma_i)]^2>\lambda_{DA}\Gamma/A\\ \min(\beta_i/\sigma_i) & \text{otherwise.}\end{cases} \tag{45}$$

These results indicate that the uniform price charged is limited by the customer with the lowest payment variance threshold-to-noise ratio, β_i/σ_i. Further, this customer also limits the overall adoption of dynamic pricing. As a consequence, consumers who are able to reliably contribute towards grid flexibility might not receive enough incentive to do so when a uniform price is adopted. When consumers are homogeneous in their maximum demand and their flexibility parameters, that is $D_i = D\forall i$, $\gamma_i=\gamma\forall i$, and $\alpha_i=\alpha\forall i$, the uniform pricing leads to consumers having the same expected demand, and thus the same expected

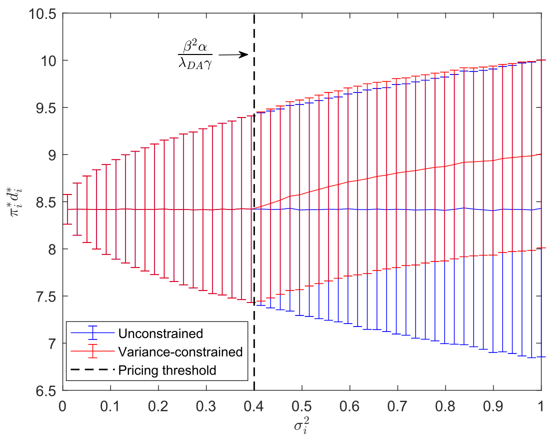

Fig. 1. Consumers' payments as a function of their demand variance.

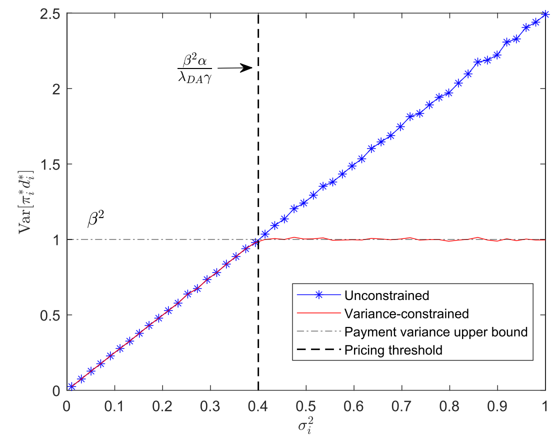

Fig. 2. Variance of consumers' payments as a function of their demand variance.

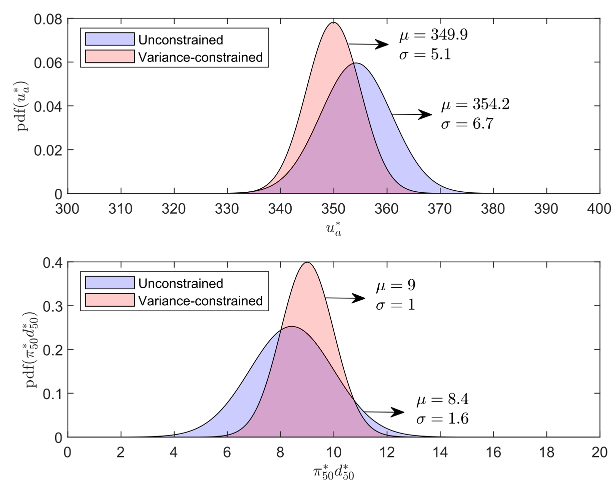

Fig. 3. Fitted probability density function for the aggregator's utility (top) and payments from a consumer with binding payment variance constraint in the constrained problem (bottom).

payment. In this scenario, the only difference among the customers would be on the real-time noise realizations, but the uniform pricing would guarantee the desired bounded payment variance for every consumer.

V. Case Study

For our numerical analysis, we consider a scenario with 50 consumers. We let the day-ahead energy price be $\lambda_{DA} = \$0.25/\mathrm{kWh}$, and the real-time prices be $\lambda^+ = \$0.20/\mathrm{kWh}$ for selling and $\lambda^- = \$0.30/\mathrm{kWh}$ for buying. Further, let the consumers have the same flexibility parameters, $\gamma_i = 10 \forall i$ and $\alpha_i = 1 \forall i$, so that we can analyze the effect of their different demand noises in the results.[2] The parameters used are illustrative, as their estimation is not in the scope of this paper. We again refer the reader to [13], [27] and the references therein for examples of literature that focus on parameter learning and estimation. The following results represent the outcome of 50000 noise realizations. We reiterate, however, that the decisions are taken ex-ante, based on the noise probabilistic model, and each realization gives a possible final outcome given the decisions taken.

We begin by comparing the payments performed by consumers in an unconstrained case where the aggregator does not enforce the payment variance constraint (5c) versus when this constraint is present with $\beta_i^2 = 1 \forall i$. The noise variables ξ_i are assumed to follow a zero-mean Gaussian distribution truncated in the interval $[-2500, 2500]$. We set the demand variance of the consumers to be distributed uniformly from $\sigma_i^2 = 0.01$ to $\sigma_i^2 = 1$. The results are plotted in Fig. 1, where the error bars are centered at the mean of all the realized values for each consumer, and the end caps mark the intervals within one standard deviation of the mean. In the unconstrained case, in blue, we notice that the payment variance increases with the consumer demand variance. Further, the mean payment remains flat, which follows because the consumers have the same flexibility parameters. Then, on average, consumers who are equally flexible will incur the same payments, even though they have different response uncertainty.

The results for the variance-constrained case match the unconstrained one for consumers who are below the threshold which indicates a binding payment variance constraint. For consumers beyond this threshold, the price adjustment made by the aggregator is such that their mean payments increase. However, their payment variance decreases to satisfy the desired upper bound, as can be seen in Fig. 2. In this plot, the instances where the payment variance is slightly above the β^2 threshold are due to the sample size of the simulation, and they diminish as more realizations are added.

The mean-variance utility trade-off for the aggregator can be observed in Fig. 3 (top). For this plot, the realizations calculated for the unconstrained and variance-constrained cases were fitted to a Gaussian distribution, whose probability density functions

[2]The different noise levels can be representative of different types of consumer. For example, industrial consumers have better capability for demand control than residential customers, and thus can be less noisy. In reality, these consumers would also have different flexibility parameters. However, better insights can be drawn from the analysis if not all parameters are varied at the same time. Nonetheless, the general results are also observed in the case of heterogeneous consumers, as was shown in our analytical findings.

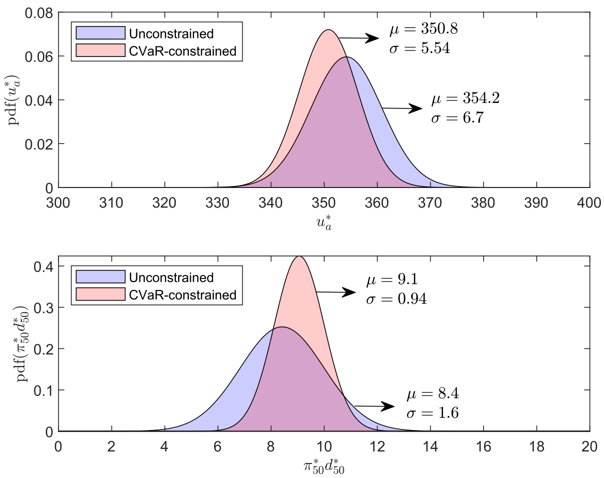

Fig. 4. Fitted probability density function for the aggregator's utility (top) and payments from a consumer with binding payment CVaR constraint in the constrained problem (bottom).

are presented. As discussed in Section IV-B, the fact that the payment variance constraints are binding for some consumers leads to a decrease in the mean utility of the aggregator as compared to an unconstrained case. However, the aggregator can achieve a decrease in the variance of his utility, avoiding undesirable volatility. In Fig. 3 (bottom), we can observe the trade-off in the payment from a consumer whose payment variance constraint is binding in the constrained problem. The fitted Gaussian distribution for the unconstrained case is shown to have a lower mean and higher variance than the one corresponding to the constrained scenario, so that the consumer is subject to a higher mean payment in order to achieve a lower payment uncertainty. We note that other risk metrics can be used in the proposed model, and the insights still hold for other convex metrics. To illustrate this, Fig. 4 shows the results achieved for the same analysis if the payment variance constraint (5b) is substituted with its Conditional Value at Risk (CVaR) counterpart, given by

$$\text{CVaR}_{\delta_i}[\pi_i d_i^*] = \mathbb{E}\left[\pi_i d_i^* | \pi_i d_i^* \geq \text{VaR}_{\delta_i}[\pi_i d_i^*]\right] \leq \beta_i \forall i,$$

where $\text{VaR}_{\delta_i}[\pi_i d_i^*]$ is the value at risk. In the case considered, the CVaR was limited to be at most 11 for the worst 5% scenarios (i.e. the 5% highest payments) for all consumers.

Next, we simulate a scenario in which all 50 consumers are homogeneous and have a demand variance of $\sigma_i^2 = 0.6^2 \forall i$. We then vary the upper bound on the payment variance constraint β^2 from 0.3 through 0.6 in increments of 0.1, and also consider an unconstrained case. For all of the β^2 scenarios, we evaluated the aggregator's average utility by breaking down each component of his revenues and costs, as shown in Table I. As this constraint is relaxed, the aggregator spends less in day-ahead purchases, but his income from consumer payments also reduces. We also note that the real-time trades are the same regardless of the level of payment variance of the problem. This corroborates the findings in Theorem 2, where it was shown that the real-time transactions depend only on the demand noise statistics and the energy prices both in the day-ahead and in the real-time

TABLE I
BREAKDOWN OF AGGREGATOR'S OPTIMAL UTILITY (AVERAGE OF ALL RUNS) WITH INCREASING β^2

β^2	Day-Ahead Purchases	Consumer Sales	Real-Time Sales	Real-Time Purchases
0.3	-155.2	462.7	0.34	-0.51
0.4	-124.4	454.4	0.34	-0.51
0.5	-106.1	447.3	0.34	-0.51
0.6	-93.57	441.1	0.34	-0.51
$\rightarrow \infty$	-66.5	420.9	0.34	-0.51

markets. Therefore, the aggregator's strategy is to adjust his day-ahead purchases to increase supply availability when the payment variance constraint is tight. This extra cost is balanced with higher revenue from consumers' payments, which increases on average in such situations. For the parameters in this case study, we can use the expression (28) to find that the theoretic outcome of the aggregator's strategy is to achieve a probability of excess day-ahead purchase of

$$\text{Prob}\left(\sum_{i \in \mathcal{I}} d_i^* \leq \sum_{i \in \mathcal{I}} C_i^*\right) = \frac{0.30 - 0.25}{0.30 - 0.20} = 0.5.$$

Numerically, 49.7% of the simulation trials yielded an excess supply, which had to be sold in the real-time market, while 50.3% of the outcomes had a supply shortage.

Lastly, to illustrate how the flexibility coming from consumers with low demand noise is more useful to the aggregator, we consider a scenario with increasing day-ahead prices. We again let the real-time prices for selling and buying be 20% lower and higher than the day-ahead price, respectively. We maintain the same flexibility parameters from consumers throughout all cases considered, so that, for each λ_{DA}, we can observe the differences in the results due to demand variance only. We start with a scenario in which all 50 consumers have a demand variance of $\sigma_i^2 = 1$. Then, we simulate cases in which 30%, 50%, 70%, and 100% of the consumers have $\sigma_i^2 = 0.1^2$, while the others still have the variance $\sigma_i^2 = 1$.

For the first case, the demand noise of all consumers exceed the threshold in (14) for all day-ahead price scenarios, and thus they are considered to be high-noise and receive a flat energy price. The aggregator's optimal day-ahead purchase for this case is shown in Fig. 5, and we notice that this amount is the highest one across all scenarios. As we increase the number of low-noise consumers, the average day-ahead purchase starts decreasing, and we can also notice a more pronounced correlation between the day-ahead prices and the amount of energy purchased by the aggregator. This is due to the presence of more low-noise type consumers, who are charged the dynamic price. With that, the aggregator is able to incentivize higher consumption when the day-ahead price is low, and lower demand, with respect to the average, at times of high prices. These results indicate that consumers with low demand noise are better suited to respond reliably to the proposed dynamic pricing, and thus the aggregator benefits from having more consumers of this type.

The results of this case study show that the proposed consumer-centric approach benefits high-noise consumers by reducing their payment volatility. At the same time, low-noise

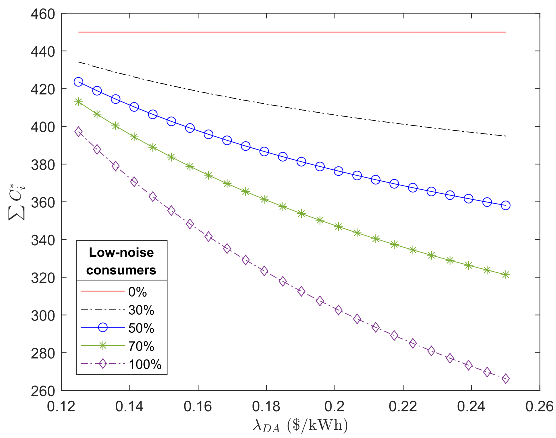

Fig. 5. Aggregator's optimal purchase in the day-ahead market as a function of the day-ahead energy price for increasing percentage of low-noise consumers.

consumers are able to participate more actively in dynamic pricing strategies. This shifts the responsibility of how much exposure to dynamic prices certain types of customers should have from the customers themselves, who may not always be well informed about the implications of market price fluctuations, to the aggregator. These insights would also apply for other scenarios, such as having consumers with different flexibility parameters. The only difference would be the fact that each consumer would have a different threshold indicating whether they should receive a flat rate or a dynamic price, as showed in Theorem 1.

VI. Conclusion and Future Work

We propose a demand-side management framework in which an aggregator sets energy prices to flexible consumers with noisy demand response. The aggregator decides on the energy supply to be purchased in the day-ahead market, and settles any mismatches between supply and realized demand in the real-time market. To avoid high volatility in the electricity bills of consumers, the aggregator sets an upper bound on their payment variance. We show that the aggregator's optimal pricing decision is to establish a threshold-based price, so that consumers with low enough demand variance receive a price that is a function of the day-ahead energy price, while consumers with too high demand variance are charged a constant price. When any of the payment variance constraints is binding, the consumer with the binding constraint will experience lower payment uncertainty and higher mean payments, while the aggregator observes a decrease both in his utility variance and mean.

Avenues for future work include extending the proposed formulation to incorporate randomness from distributed renewable generation, so that customers have the additional decision about whether to supply to the network or consume their generation locally. Moreover, allowing customer flexibility to be correlated, considering network constraints, and analyzing scenarios with information asymmetry and strategic consumers constitute interesting directions for further investigation, along with the possibility of using a distributed implementation for the Stackelberg game.

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