

# Personalized Freight Route Recommendations with System Optimality Considerations: A Utility Learning Approach

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**Abstract**—Traffic congestion has a negative economic and environmental impact. Traffic conditions become even worse in areas with high volume of trucks. In this paper, we propose a coordinated pricing-and-routing scheme for truck drivers to efficiently route trucks into the network and improve the overall traffic conditions. A basic characteristic of our approach is the fact that we provide personalized routing instructions based on drivers' individual routing preferences. In contrast with previous works that provide personalized routing suggestions, our approach optimizes over a total system-wide cost through a combined pricing-and-routing scheme that satisfies the budget balance on average property and ensures that every truck driver has an incentive to participate in the proposed mechanism by guaranteeing that the expected total utility of a truck driver (including payments) in case he/she decides to participate in the mechanism, is greater than or equal to his/her expected utility in case he/she does not participate. Since estimating a utility function for each individual truck driver is computationally intensive, we first divide the truck drivers into disjoint clusters based on their responses to a small number of binary route choice questions and we subsequently propose to use a learning scheme based on the Maximum Likelihood Estimation (MLE) principle that allows us to learn the parameters of the utility function that describes each cluster. The estimated utilities are then used to calculate a pricing-and-routing scheme with the aforementioned characteristics. Simulation results in the Sioux Falls network demonstrate the efficiency of the proposed pricing-and-routing scheme.

**Index Terms**—Utility learning, Road pricing, Freight, Routing.

## I. INTRODUCTION

Traffic congestion is a major problem in urban areas. According to statistics, in 2017, traffic congestion costed urban Americans an extra 8.8 billion hours and an extra 3.3 billion gallons of fuel to travel, for a total congestion cost of \$166 billion. Not surprisingly, the major congested points are in metropolitan areas where truck traffic mixes with other traffic and along major interstate highways connecting major metropolitan areas [1]. In the United States (U.S.),

transportation's total estimated contribution to U.S. Gross Domestic Product (GDP) was \$1,298.1 billion in 2019. Trucking contributed the largest amount of all the freight modes, at \$368.9 billion [2]. In the European Union (EU), transportation sector is also a major contributor to the economy, representing more than 9% of EU gross value added in 2016 [3]. Therefore, it becomes clear that efficient route planning could have a large positive impact on the global economy.

The Traffic Assignment Problem (TAP) [4] is a key problem for efficient planning in transportation networks. Based on the objective of the assignment process and user behavior assumptions, many assignment models can be classified as User Equilibrium (UE) or System Optimum (SO) [5]. In a UE, drivers act independently in an effort to maximize their own individual utility. On the other hand, in a SO, drivers follow coordinated routing instructions that minimize the expected total cost of the network. It is well known that user optimality does not imply system optimality. The inefficiency between the UE and the SO has been addressed in the literature as the Price of Anarchy (POA) [6]. Recent research efforts have tried to estimate the POA [7], [8] demonstrating that realistic transportation networks suffer from this problem.

Many previous works have tried to address the problem of the inefficiency between an equilibrium flow pattern and a SO solution through pricing schemes. Congestion pricing [9]–[12] is the most frequently studied among these methods. In a congestion pricing scheme, drivers are asked to pay a fee (toll) corresponding to the additional cost their presence causes to the network. Other strategies include the applications of Tradable Credit Schemes (TCS) [13], [14] or tradable travel permits [15] among the drivers of the network. In this work, we propose to address the problem of the inefficiency between an equilibrium flow pattern and the SO through a pricing-and-routing scheme that has a toll-and-subsidy form, similar to [16]–[18]. This scheme has three main characteristics. First, it is budget balanced on average. Second, it guarantees that every truck driver has an incentive to participate in the proposed mechanism and lastly, it drives the network close to the SO solution.

To ensure that every truck driver has an incentive to participate in the proposed mechanism, we first need to estimate the utility function that describes the routing preferences of each driver. A large amount of research has been conducted to study discrete choice modeling under the Random Utility Maximization (RUM) framework [19]. In the RUM framework, the utility to the decision maker of each alternative is

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not completely known. It consists of a deterministic component which is a function of the attributes of the alternative and the characteristics of the decision-maker and a random error term that follows a probability distribution. Depending on whether the error terms are assumed to be multivariate normal or independently and identically Type I extreme value (gumbel) distributed, we get the Multinomial Probit (MNP) model [20] and the Multinomial Logit (MNL) model [21], respectively. For further information related to route choice modeling, we refer the interested reader to [22] and references therein. Recently, there is also a growing interest in studying the connection between traditional discrete choice models and modern machine learning methods, e.g. Deep Neural Networks (DNNs) [23]. Inspired by these research efforts, in this paper, we propose a learning scheme based on the Maximum Likelihood Estimation (MLE) principle that allows us to learn the utility of each cluster of truck drivers using a set of binary route choice questions. A basic characteristic of our method is the use of a model that calculates the difference between the utilities of two alternatives, thus guaranteeing that the transitivity property is satisfied. The transitivity property is important since by only using a binary model and doing a pairwise comparison between 2 routes, we can accurately calculate the utility of each alternative even if the total number of routes per Origin-Destination (OD) pair is greater than 2. Accurately learning the utility function that describes the routing preferences of each cluster of truck drivers helps us provide more personalized route recommendations.

Previous research efforts have tried to incorporate personalization in route planning. In [24], Rogers and Langley used a linear perceptron to learn the drivers' routing preferences and subsequently solved the routing problem using Dijkstra's shortest path algorithm [25]. In [26], Letchner et al. designed a route planner that used real-world GPS data to estimate both time-dependent road speeds and individual driver preferences. In [27], Cui et al. used historical GPS trajectories and a collaborative filtering approach [28] in order to provide personalized travel route recommendations. Using trajectory data and considering three commonly used travel costs, namely travel distance, travel time and fuel consumption, [29] proposed a method to recommend personalized routes to individual drivers. Another work proposed to solve a multi-criteria optimization problem to find the optimal route considering air quality, travel time, and fuel consumption from the source to the destination [30]. Recently, multi-modal transportation recommendations have been also studied [31]. Most of the aforementioned works take into account user optimality only and therefore, the provided solutions may be inefficient for the network. On the other hand, in this paper, we propose a method to learn truck drivers' individual routing preferences and we design a pricing-and-routing scheme that recommends routes that are beneficial for the system optimum.

Perhaps closest to our work is the work of [32]. In their work, Vayanos et al. built a survey that they distributed to drivers of passenger vehicles. Using the results of the survey, they first clustered the drivers into disjoint clusters, and then using the assumption that the utility of the drivers can be described by a linear function, they used a Mixed Integer

Programming (MIP) formulation to learn the parameters of the utility of each cluster. Having learned a linear utility for each cluster, they solved a deterministic traffic assignment problem with an additional constraint to assign the drivers into routes that they will likely follow, i.e. the assigned route will make the utility of the driver not to be much lower than the utility he/she would have if he/she made his/her own routing decisions.

In this paper, we study pricing-and-routing schemes that can be specifically applied on trucks. Given that truck drivers routinely use varying routes for the same journey depending on the traffic conditions [17] and the fact that their travel time is already a commodity, make trucks form an ideal candidate subclass of vehicles for coordinated routing [18]. Our main goal is the design of pricing-and-routing schemes that drive the network as close as possible to the SO solution and concurrently guarantee that the expected total utility of a truck driver (including payments) in case he/she decides to participate in the mechanism, is greater than or equal to his/her expected utility in case he/she does not participate. Note that the participation to the mechanism is voluntary and therefore, the mechanism can operate even if some drivers do not participate. Additionally, we prove that the resulting pricing scheme is self-sustainable and the expected total payments made or received by the coordinator of the mechanism, are equal to zero. It is worth mentioning that estimating a utility function for each individual truck driver is computationally intensive. To overcome the computational complexity of this problem, we first divide the truck drivers into disjoint clusters based on their responses to a small number of binary route choice questions and we subsequently propose to use a learning scheme based on the MLE principle that allows us to learn the parameters of the utility function that describes each cluster. The estimated utilities are then used to calculate a pricing-and-routing scheme with the aforementioned characteristics.

Let us now comment on the main differences between the current work and [32]. First, in their work, Vayanos et al. deal with passenger vehicles while on the other hand, we focus on the design of coordination mechanisms that provide routing suggestions only to truck drivers. Second, Vayanos et al. assumed a deterministic model. In our work, we analyze stochastic models and thus the system coordinator has different information from the truck drivers which creates additional opportunities for coordination. Third, instead of using a MIP formulation to learn the utility of each cluster, we use a learning scheme based on the MLE principle. The proposed learning scheme satisfies the transitivity property and allows us to learn utility functions of any form. In contrast, the MIP formulation is limited to the linear case only. Lastly, in their work, Vayanos et al. solved a traffic assignment problem with an additional constraint based on which the assigned route will make the utility of the driver not to be much lower than the utility he/she would have if he/she made his/her own routing decisions. However, in such a solution some drivers may have a lower utility than the one they would have if they made their own routing decisions, which reduces the incentives for their participation. On the other hand, in our approach, we propose a pricing-and-routing scheme that is budget balanced

on average and guarantees that the expected total utility of a truck driver (including payments) in case he/she decides to participate in the mechanism, is greater than or equal to his/her expected utility in case he/she does not participate. Consequently, under the assumption that truck drivers are rational and will be constantly seeking to maximize their individual expected utility, every truck driver has an incentive to participate in the proposed mechanism.

In general, previous research efforts, whether they used utility-based or other approaches to recommend personalized routes to individual drivers, either focused more on the system optimality, e.g. [32], or on the user optimality, e.g. [24], [26], [27]. In this paper, we propose a utility-based pricing-and-routing scheme that drives the network as close as possible to the SO solution and concurrently guarantees that the expected total utility of a truck driver (including payments) in case he/she decides to participate in the mechanism, is greater than or equal to his/her expected utility in case he/she does not participate. To the best of our knowledge, this is the first research work in the field that proposes a method to provide personalized route recommendations to the drivers while taking system optimality into account.

The rest of the paper is organized as follows. Section II deals with the problem formulation. In Section III, we present the proposed methodology. Section IV presents the simulation results related to the clustering of the truck drivers, the utility learning approach and the application of the proposed pricing-and-routing scheme to the Sioux Falls network. Finally, Section V presents the conclusion.

## II. MATHEMATICAL FORMULATION

### A. Problem formulation

The notation used throughout the paper is summarized in Table I. Let  $G = (V, E)$  denote a transportation network, where  $V$  is the set of nodes and  $E$  is the set of road segments of the network, respectively. We assume that the Origin-Destination (OD) demand of the truck drivers is stochastic and follows a probability distribution with finite support. Additionally, we assume a symmetric information model where all truck drivers have the same amount of information and know the number of passenger vehicles at each road segment of the network. Similar models have been used in [16]–[18]. Note that the passenger vehicles are a part of the model we are using but the coordinator does not provide routing suggestions to them.

Previous research efforts, e.g. [33], have shown that there are several factors that can affect drivers' routing decisions. For instance, previous studies showed that on average, drivers take the fastest route for only 35% of their journeys [26]. In this work, we aim to propose a methodology that learns how different factors affect truck drivers' routing decisions and subsequently use this information to efficiently route the truck drivers into the network. The proposed approach is general and can be applied for any number of attributes that can possibly affect drivers' routing decisions. For illustrative purposes, for the rest of this paper, we consider four different factors that could affect the routing decision of a truck driver namely,

TABLE I  
NOTATION

| Variable                      | Meaning                                                                                                                                            |
|-------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|
| $G(V, E)$                     | Network with nodes $V$ and links $E$                                                                                                               |
| $R_w$                         | The set of all paths connecting OD pair $w \in W$                                                                                                  |
| $v$                           | The number of OD pairs in the network                                                                                                              |
| $L$                           | The number of OD road segments in the network                                                                                                      |
| $J_i$                         | Utility of cluster $i$                                                                                                                             |
| $x_r$                         | Vector of attributes of the route $r$                                                                                                              |
| $\theta_i$                    | Learned weights of the attributes of the utility function of cluster $i$                                                                           |
| $p_c$                         | Probability of the demand realization $c$                                                                                                          |
| $\alpha_{w,i,r}^{UE}$         | Proportion of truck drivers belonging to cluster $i$ who want to travel in OD pair $w$ and choose route $r$ at the UE                              |
| $\alpha_{i,r}^{c,w}$          | Proportion of truck drivers belonging to cluster $i$ who want to travel in OD pair $w$ and are assigned in route $r$ during demand realization $c$ |
| $\pi_r^{c,w}$                 | Payment (made or received) of truck drivers who want to travel in OD pair $w$ and are assigned in route $r$ during demand realization $c$          |
| $d_{w,i}^c$                   | Demand of truck drivers belonging to the cluster $i$ who want to travel in OD pair $w$ during demand realization $c$                               |
| $F_{i,r}^w$                   | Expected utility of a truck driver belonging to cluster $i$ who wants to travel in OD pair $w$ using route $r$                                     |
| $X_{lp}$                      | Number of passenger vehicles traversing the road segment $l$                                                                                       |
| $X_{lT}$                      | Number of trucks traversing the road segment $l$                                                                                                   |
| $C_{lT}$                      | Travel time of a truck driver traversing road segment $l$                                                                                          |
| $E[T_{tr}]$                   | Expected total travel time of the truck drivers                                                                                                    |
| $E[T_p]$                      | Expected total travel time of the passenger vehicles                                                                                               |
| $E[T_S]$                      | Expected total travel time of the network                                                                                                          |
| $E[U_{tr}]$                   | Expected total utility of the truck drivers                                                                                                        |
| $O(\alpha)$                   | Objective function to be minimized. Weighted combination of the expected total travel time and the negative of the expected total utility          |
| $\lambda$                     | Weighting factor of the objective function                                                                                                         |
| $\rho$                        | Weighting factor of the objective function                                                                                                         |
| $\mathcal{L}(\theta_i, x, y)$ | Binary cross-entropy loss function                                                                                                                 |
| $s(\theta_i, x_m)$            | Probability according to which we predict that a truck driver belonging to cluster $i$ will pick route 1 or route 2                                |
| $K$                           | The number of clusters in the K-means algorithm                                                                                                    |

the distance, the number of freeway interchanges, the travel time and the 80th percentile of the travel time of a route. Let the vector  $x_r$  contain these four attributes of route  $r$ . In this paper, we assume that different drivers value these four factors differently and therefore, we aim to cluster the truck drivers into  $K$  distinct groups. Note that theoretically, we could have picked any  $g^{th}$  percentile to represent the variability of travel time, without loss of generality. However, since we wanted to include a feature that captures the scenarios where the traffic conditions become worse, we decided to use the 80th percentile of the travel time of a route.

Under the assumption that the OD demand of the truck drivers follows a discrete probability distribution with finite support, we define the  $g^{th}$  percentile of travel time as the value of  $u$  for which  $P(U \leq u)$  is greater than or equal to  $\frac{g}{100}$  and  $P(U \geq u)$  is greater than or equal to  $1 - \frac{g}{100}$ . In this paper, we assume that the OD demand of the truck drivers is the only source of uncertainty that affects the variability of travel time.

Let  $C_{lT}(X_{lp}, X_{lT}(\alpha))$  be a known nonlinear function representing the travel time of a truck driver traversing the road segment  $l$  when there exist  $X_{lp}$  passenger vehicles and  $X_{lT}(\alpha)$

trucks on it, where  $\alpha$  is a set of variables defined as follows:

$$\alpha = \{\alpha_{i,r}^w : w = 1, \dots, v, i = 1, \dots, K, r \in R_w\} \quad (1)$$

where  $w$  is the index corresponding to a specific OD pair,  $i$  is the index corresponding to a specific cluster of truck drivers,  $r \in R_w$  denotes a specific route among the set of available routes  $R_w$  connecting OD pair  $w$ ,  $K$  is the number of clusters of truck drivers and  $v$  is the number of OD pairs in the network. Therefore,  $\alpha_{i,r}^w$  expresses the proportion of truck drivers belonging to cluster  $i$  with a desired OD pair  $w$  who choose route  $r$  for their trip.

At this point let us clarify the notation used for the set of variables  $\alpha$ . As will be also discussed later, in a UE, the truck drivers make their own routing decisions in an effort to maximize their individual expected utility. Since we have assumed that they know the probability distribution of the demand for the rest of the truck drivers but not the exact realization of it, their routing decisions  $\alpha_{w,i,r}^{UE}$  do not depend on the exact demand realization  $c$ . On the other hand, in a SO solution and in the pricing-and-routing scheme that we discuss in this paper, we assume the existence of a coordinator who knows the exact realization of the demand and provides routing suggestions to truck drivers. Since the coordinator has complete information of the traffic conditions, his/her routing suggestions  $\alpha_{i,r}^{c,w}$  depend on the demand realization  $c$ . Therefore, for the rest of this paper, in the decision variable  $\alpha_{w,i,r}^{UE}$  (or equivalently  $\alpha_{i,r}^w$ ), the truck drivers choose the route  $r$  they want to follow, while in  $\alpha_{i,r}^{c,w}$ , they are assigned to it.

Based on the assumption that the demand of truck drivers is stochastic, let  $d_{w,i}$  be random variables denoting the number of truck drivers belonging to cluster  $i$  with desired OD pair  $w$  and let  $d_{w,i}^c$  be their corresponding values during demand realization  $c$ . Then, the number of trucks traversing the road segment  $l$  is given by:

$$X_{lT}(\alpha) = \sum_{w=1}^v \sum_{i=1}^K \sum_{r \in R_w: l \in r} d_{w,i}^c \alpha_{i,r}^w \quad (2)$$

where on the left side of (2), we omitted the index  $c$  to simplify the notation. Therefore, the expected total travel time of the truck drivers in the network is given by:

$$E[T_{tr}(\alpha)] = E \left[ \sum_{l=1}^L X_{lT}(\alpha) C_{lT}(X_{lT}(\alpha)) \right] \quad (3)$$

where  $L$  is the number of road segments in the transportation network and  $X_{lT}(\alpha)$  is given by (2).

### B. User Equilibrium (UE)

In the absence of cooperation, the users of the network act independently in an effort to maximize their own individual utility. This behavior drives the network to a situation called User Equilibrium (UE).

In this paper, after clustering the truck drivers into  $K$  distinct groups, we aim to learn a utility function  $J_{i,r}^{c,w}(\theta_i, x_r(\alpha))$  for each cluster  $i$ . Note that  $J_{i,r}^{c,w}(\theta_i, x_r(\alpha))$  represents the utility of a truck driver belonging to cluster  $i$  who wants to travel in OD pair  $w$  and is assigned to route  $r$  during demand

realization  $c$ . Additionally,  $\theta_i$  represents the parameters of the utility function of cluster  $i$  that we aim to learn. In case  $J_{i,r}^{c,w}(\theta_i, x_r(\alpha))$  is a linear function of the route attributes  $x_r$ , it can be written in the following form:

$$J_{i,r}^{c,w}(\theta_i, x_r(\alpha)) = \theta_{i1}x_{1r} + \theta_{i2}x_{2r} + \theta_{i3}x_{3r}(\alpha, c) + \theta_{i4}x_{4r}(\bar{\alpha}) \quad (4)$$

where  $x_{1r}$  and  $x_{2r}$  are the distance and the number of freeway interchanges of route  $r$ , respectively. Additionally,  $x_{3r}(\alpha, c)$  and  $x_{4r}(\bar{\alpha})$  are the travel time and the 80th percentile of travel time of route  $r$  during the demand realization  $c$  when the vehicles are routed according to  $\alpha$ , respectively. Note that we use the notation  $\bar{\alpha}$  for  $x_{4r}(\bar{\alpha})$  to denote that the 80th percentile of travel time depends on all the demand realizations and consequently depends on all the values of  $\alpha$ , for all the realizations  $c$ . To simplify the notation, we omit using the notation  $\bar{\alpha}$  when defining  $J_{i,r}^{c,w}(\theta_i, x_r(\alpha))$ .

Based on the assumption that the truck drivers only know the probability distribution of the demand for the rest of the truck drivers and not the exact realization of it, their routing decisions  $\alpha_{w,i,r}^{UE}$  do not depend on the exact demand realization  $c$ . Additionally, it has been shown that there possibly exist many non-equivalent UE solutions [17]. In this work, we calculate a UE solution that minimizes a weighted combination of the expected total travel time of the truck drivers and the negative of their expected total utility. Given the aforementioned, we can calculate a UE solution by solving the following optimization problem with complementarity constraints [34]:

$$\begin{aligned} & \underset{\alpha, \delta}{\text{minimize}} && \lambda E[T_{tr}(\alpha)] - (1 - \lambda) E[U_{tr}(\alpha)] \\ & \text{subject to} && 0 \leq \alpha_{i,r}^w \perp \delta_i^w - F_{i,r}^w(\alpha) \geq 0, \forall w, i, r \\ & && \sum_{r \in R_w} \alpha_{i,r}^w = 1, \forall w, i \end{aligned} \quad (5)$$

where  $\lambda \in [0, 1]$  is a weighting factor,  $\delta_i^w$  is a set of free variables and the notation  $\perp$  means that either  $\alpha_{i,r}^w = 0$  or  $\delta_i^w - F_{i,r}^w(\alpha) = 0$ . Note that the free variables  $\delta_i^w$  are only used in the complementarity constraints. This is a common technique used in many similar problems, e.g. [16]–[18]. Finally,  $F_{i,r}^w(\alpha)$  is the expected utility of a truck driver belonging to cluster  $i$  who wants to travel in OD pair  $w$  using route  $r$  and is given by:

$$F_{i,r}^w(\alpha) = \sum_c p_c J_{i,r}^{c,w}(\theta_i, x_r(\alpha)) \quad (6)$$

where  $p_c$  is the probability of demand realization  $c$ . Additionally,  $E[U_{tr}(\alpha)]$  represents the expected total utility of the truck drivers and at the UE, it is given by:

$$E[U_{tr}(\alpha)] = \sum_c \sum_{w=1}^v \sum_i \sum_{r \in R_w} p_c d_{w,i}^c \alpha_{w,i,r}^{UE} J_{i,r}^{c,w}(\theta_i, x_r(\alpha)) \quad (7)$$

Note that by setting  $\lambda = 1$  in the objective function of (5), we can calculate the UE with the minimum expected total travel time while on the other hand, by setting  $\lambda = 0$ , we can calculate the UE with the maximum expected total utility of the truck drivers.

### C. System Optimum (SO)

In a System Optimum (SO) solution, drivers make routing decisions in an effort to minimize a total system cost compared to the UE where they are acting in a manner that maximizes their own individual utility. Before presenting the optimization problem through which we calculate the SO solution, let us first define some terms. The number of trucks traversing the road segment  $l$  is given by:

$$X_{lT}(\alpha) = \sum_{w=1}^v \sum_{i=1}^K \sum_{r \in R_w: l \in r} d_{w,i}^c \alpha_{i,r}^{c,w} \quad (8)$$

where the main difference between (2) and (8) is that in the latter,  $\alpha_{i,r}^{c,w}$  depends on the exact demand realization  $c$  which is known by the coordinator. Therefore, the expected total travel time of the truck drivers in the network can be calculated by substituting (8) into (3). Additionally, the expected total travel time of the passenger vehicles is given by:

$$E[T_p(\alpha)] = E \left[ \sum_{l=1}^L X_{lp} C_{lp}(X_{lp}, X_{lT}(\alpha)) \right] \quad (9)$$

where it holds that  $C_{lp}(X_{lp}, X_{lT}(\alpha)) = C_{lT}(X_{lp}, X_{lT}(\alpha))$ .

In this work, we aim to minimize the expected total travel time of the truck drivers and maximize their expected total utility. Therefore, we define the following objective:

$$O(\alpha) = \lambda(\rho E[T_{tr}(\alpha)] + (1-\rho)E[T_p(\alpha)]) - (1-\lambda)E[U_{tr}(\alpha)] \quad (10)$$

where  $\lambda, \rho \in [0, 1]$  are weighting factors and  $E[U_{tr}(\alpha)]$  is given by:

$$E[U_{tr}(\alpha)] = \sum_c \sum_{w=1}^v \sum_i \sum_{r \in R_w} p_c d_{w,i}^c \alpha_{i,r}^{c,w} J_{i,r}^{c,w}(\theta_i, x_r(\alpha)) \quad (11)$$

where the main difference between (7) and (11) is that in the latter,  $\alpha_{i,r}^{c,w}$  depends on the exact demand realization  $c$ . Note that the objective function of (12) is a weighted combination of the expected total travel time of the truck drivers, the negative of their expected total utility and the expected total travel time of the passenger vehicles. This objective function aims to guarantee that by only routing the trucks, the expected total travel time of the passenger vehicles will not be significantly affected.

Based on the aforementioned definitions, we calculate the SO solution of the network by solving the following optimization problem:

$$\begin{aligned} & \underset{\alpha(\cdot)}{\text{minimize}} && O(\alpha) \\ & \text{subject to} && \sum_{r \in R_w} \alpha_{i,r}^{c,w} = 1, \forall c, w, i \\ & && \alpha_{i,r}^{c,w} \geq 0, \forall c, w, i, r \end{aligned} \quad (12)$$

where  $O(\alpha)$  is given by (10).

## III. PERSONALIZED ROUTE RECOMMENDATION

### A. Overview

A System Optimum (SO) solution is not a practical solution since as will be also experimentally shown later, drivers have

an incentive to deviate from this solution in order to increase their individual utility. On the other hand, a UE solution is inefficient for the network. To mitigate this issue, we propose to initially learn individual drivers' utilities. Subsequently, using these utilities, we calculate a pricing-and-routing scheme that minimizes a weighted combination of the expected total travel time and the negative of the expected total sum of utilities while guaranteeing that every truck driver has an incentive to participate in such a mechanism. The participation to the mechanism is voluntary. Note also that the designed mechanism is self-sustainable since the expected total payments made or received by the coordinator are equal to zero.

The proposed approach is shown in Figure 1 and consists of the following steps:

- *Step 1:* Based on drivers' past routing choices, we first divide the drivers into disjoint clusters. In our experiments, we use the K-means algorithm. However, other clustering algorithms can be also used. In case a truck driver participates in the mechanism for the first time, he/she will be asked to answer a small number of binary route choice questions.
- *Step 2:* For each cluster, we learn a utility that is a function of distance, travel time, 80th percentile of travel time and number of freeway interchanges of a route.
- *Step 3:* Having learned a utility function for each cluster of truck drivers, we solve an optimization problem that calculates a pricing-and-routing scheme. The objective function of this optimization problem is given by (10). The constraints of the optimization problem guarantee that every truck driver has an incentive to participate in the proposed mechanism and that the total scheme is budget balanced on average.

In the following sections, we describe each step of the proposed approach in detail.

### B. Clustering

Since learning a utility function for each individual truck driver is computationally intensive, we propose to divide the drivers into disjoint groups based on their past routing choices. To achieve this, we propose to use a clustering algorithm. Several algorithms can be used for clustering. In this paper, we decided to use the K-means algorithm due to its simplicity and speed. However, the choice of the clustering algorithm to be used should be made after careful analysis of the available data.

The K-means algorithm divides a set of  $Q$  samples into  $K$  disjoint clusters  $C$ , each described by the mean  $\nu_i$  of the samples in the cluster. The K-means algorithm aims to create the clusters and choose their centroids (clusters' centers) by minimizing the within-cluster sum-of-squares criterion:

$$\underset{C}{\text{argmin}} \sum_{i=1}^K \sum_{q \in C_i} \|q - \nu_i\|^2 \quad (13)$$

where  $q$  is the vector containing the responses of a truck driver to a set of binary route choice questions. Initially, the algorithm selects  $K$  cluster centers. There are different ways

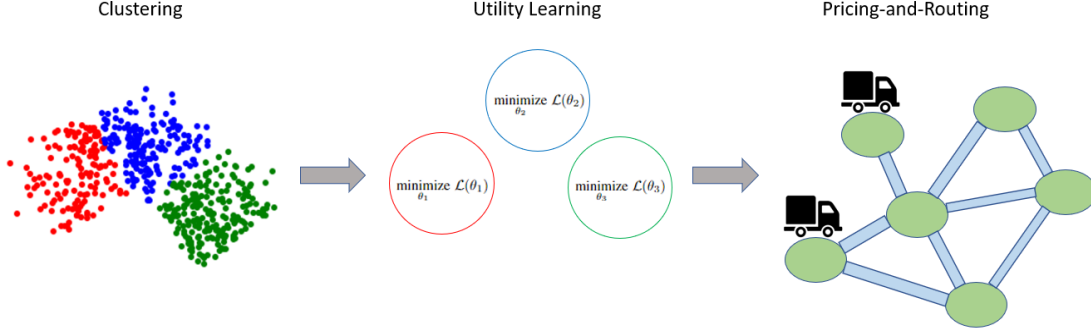


Fig. 1. The proposed personalized route recommendation system.

to initialize the cluster centers. In this work, we use the k-means++ initialization method [35] to speed up convergence. The description of the K-means algorithm is presented in Algorithm 1.

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**Algorithm 1** K-means clustering of truck drivers

---

- 1: Initialize cluster centers using k-means++
  - 2: **repeat**
  - 3:   *Step 1:* Assign each truck driver to the closest cluster center
  - 4:   *Step 2:* Update each cluster center to be the mean of all of the samples (in this case truck drivers) assigned to this cluster
  - 5: **until** the difference between the old and the new centroids becomes less than a threshold
- 

Having clustered the truck drivers into disjoint clusters, in the next section, we show how we can learn a utility function for each cluster of the truck drivers.

### C. Utility learning

For each cluster  $i$  of truck drivers, we solve the following optimization problem:

$$\underset{\theta_i}{\text{minimize}} \quad \mathcal{L}(\theta_i, x, y) \quad (14)$$

where  $\mathcal{L}(\theta_i, x, y)$  is the binary cross-entropy loss function given as follows:

$$\mathcal{L}(\theta_i, x, y) = -\frac{1}{M} \sum_{m=1}^M y_m \log(s(\theta_i, x_m)) + (1 - y_m) \log(1 - s(\theta_i, x_m)) \quad (15)$$

where  $\theta_i$  are the learned parameters of the utility  $J_i$  of cluster  $i$ ,  $M$  is the total number of truck drivers in cluster  $i$  multiplied by the number of route choices they have made in the available dataset,  $y_m$  is a binary variable that represents the route choice that a driver made and can either take value 0 for route 1 or 1 for route 2. Finally,  $s(\theta_i, x_m)$  denotes the probability according to which we predict that a truck driver belonging

to cluster  $i$  will pick route 1 or route 2 and is given by the sigmoid function:

$$s(\theta_i, x_m) = \frac{1}{1 + \exp(-(J_i(\theta_i, x_{m1}) - J_i(\theta_i, x_{m2})))} \quad (16)$$

where  $x_{m1}$  and  $x_{m2}$  are the attributes of route 1 and route 2, respectively. In case we assume a linear model for utility  $J_i$  of cluster  $i$ , then the probability according to which we predict that a truck driver belonging to cluster  $i$  will pick route 1 or route 2 takes the following form:

$$s(\theta_i, x_m) = \frac{1}{1 + \exp(-\theta_i^T(x_{m1} - x_{m2}))} \quad (17)$$

Note that by taking the difference  $J_i(\theta_i, x_{m1}) - J_i(\theta_i, x_{m2})$  in the denominator of (16), we make sure that the transitivity property is satisfied, i.e. if alternative  $a$  is preferred to alternative  $b$  ( $a \succ b$ ) and alternative  $b$  is preferred to alternative  $c$  ( $b \succ c$ ), then alternative  $a$  is also preferred to alternative  $c$  ( $a \succ c$ ). The transitivity property is important since by only using a binary model and doing a pairwise comparison between 2 routes, we can accurately calculate the utility of each alternative even if the total number of routes per OD pair is greater than 2.

### D. Optimization formulation

Having learned the utility  $J_i$  of each cluster  $i$  of truck drivers, we aim to design a mechanism that will provide personalized routing instructions to the truck drivers which at the same time will optimize a total system cost. To achieve this, we introduce a pricing scheme  $\pi_r^{c,w}$  to the system. Depending on demand realization  $c$ , OD pair  $w$  and route  $r$  that a truck driver follows, this pricing scheme determines if the truck driver needs to pay or receive a payment by the coordinator. The participation to the mechanism is voluntary.

Based on the aforementioned, let us formulate the following

optimization problem:

$$\begin{aligned}
& \underset{\alpha(\cdot), \pi(\cdot)}{\text{minimize}} && O(\alpha) \\
& \text{subject to} && B_i^w \geq D_i^w, \forall w, i \\
& && \sum_c \sum_{w=1}^v \sum_i \sum_{r \in R_w} p_c d_i^{c,w} \alpha_{i,r}^{c,w} \pi_r^{c,w} = 0 \\
& && \sum_{r \in R_w} \alpha_{i,r}^{c,w} = 1, \forall c, w, i \\
& && \alpha_{i,r}^{c,w} \geq 0, \forall c, w, i, r
\end{aligned} \tag{18}$$

where  $B_i^w$  and  $D_i^w$  are given by the following equations:

$$B_i^w = \sum_c \sum_{r \in R_w} p_c \alpha_{i,r}^{c,w} \left( J_{i,r}^{c,w}(\theta, x_r(\alpha)) + \pi_r^{c,w} \right) \tag{19}$$

$$D_i^w = \max_c \sum_{r \in R_w} p_c J_{i,r}^{c,w}(\theta, x_r(\alpha)) \tag{20}$$

respectively and  $O(\alpha)$  is given by (10).

The first constraint of (18) guarantees that the expected total utility of a truck driver (including payments) in case he/she decided to participate in the mechanism, is greater than or equal to his/her expected utility in case he/she did not participate. Additionally, the second constraint of (18) guarantees that the total payments made or received by the coordinator are equal to zero and hence, the overall mechanism is budget balanced on average. The following lemma shows that a solution to the optimization problem (18) always exists.

*Lemma 1:* The optimization problem (18) is feasible.

*Proof:* The first constraint of (18) can be equivalently written as:

$$\begin{aligned}
& \sum_c \sum_{r \in R_w} p_c \alpha_{i,r}^{c,w} \left( J_{i,r}^{c,w}(\theta, x_r(\alpha)) + \pi_r^{c,w} \right) \\
& \geq \sum_c p_c J_{i,r}^{c,w}(\theta, x_r(\alpha)), \forall w, i, r
\end{aligned} \tag{21}$$

Let  $\pi_r^{c,w} = 0$  and additionally let  $\alpha_{i,r}^{c,w} = \alpha_{w,i,r}^{UE}$ .

The above values satisfy (21) and this concludes the proof. ■

Lemma 1 proves the existence of a pricing-and-routing scheme that is budget balanced on average and guarantees that the expected total utility of a truck driver (including payments) in case he/she decides to participate in the mechanism, is greater than or equal to his/her expected utility in case he/she does not participate. Additionally, it is worth noting that the pricing scheme  $\pi_r^{c,w}$  is uniform across clusters and it only depends on demand realization  $c$ , OD pair  $w$  and the route  $r$ . Lastly, note that a pricing-and-routing scheme with the aforementioned characteristics always exists and does not depend on the form of the utility function  $J_{i,r}^{c,w}(\theta, x_r(\alpha))$ .

In the next section, we experimentally show that the proposed pricing-and-routing scheme provides a solution that is close to the SO.

#### IV. SIMULATION RESULTS

In this section, we run simulations in order to demonstrate the efficiency of the proposed approach. First, we describe how we generated synthetic data. Using the generated data,

after splitting the data into a train and a test set, we cluster the drivers, learn the utility function of each cluster and then decide the appropriate number of clusters to use. Subsequently, we experimentally show the degree at which the truck drivers would have an incentive to deviate from a SO solution, demonstrating the necessity for a pricing-and-routing scheme. Lastly, we run simulations in a benchmark transportation network and compare the proposed pricing-and-routing scheme with the UE and the SO solutions.

##### A. Data Generation

To cluster the truck drivers into disjoint clusters and learn a utility function for each cluster, we first need to have access to drivers' past routing decisions. As mentioned in section III, in case a truck driver participates in the mechanism for the first time, he/she will be asked to answer a small number of binary route choice questions. In this work, we use 9 route choice questions for training and 5 questions for test<sup>1</sup>. An example of a route choice question is shown in Table II.

TABLE II  
AN EXAMPLE OF A ROUTE CHOICE QUESTION USED TO LEARN DRIVERS' ROUTING PREFERENCES.

|                                      | Route 1 | Route 2 |
|--------------------------------------|---------|---------|
| Distance (miles)                     | 45.0    | 55.0    |
| Travel time duration (min)           | 55.0    | 50.0    |
| 80th percentile of travel time (min) | 69.0    | 65.0    |
| # of freeway interchanges            | 7       | 4       |

As a first step in the data generation process, we generate the variables  $\theta$  that describe the drivers' routing preferences. To do this, we draw samples from a Gaussian mixture model assuming 3 components consisting of isotropic Gaussian distributions. In our experiments, we generated 200 samples per cluster. Note that the term 'clusters' here refers to the Gaussian distributions of the Gaussian mixture model. The cluster centers and the standard deviations of the corresponding covariance matrices were assumed to be:

$$\mu_{\theta_1} = [-0.08, -0.12, -0.06, -0.02]$$

$$\mu_{\theta_2} = [-0.15, -0.15, -0.15, -0.05]$$

$$\mu_{\theta_3} = [-0.04, -0.12, -0.12, -0.04]$$

and

$$\sigma_{\theta_1} = [0.03, 0.04, 0.02, 0.01]$$

$$\sigma_{\theta_2} = [0.03, 0.04, 0.04, 0.01]$$

$$\sigma_{\theta_3} = [0.02, 0.04, 0.02, 0.02]$$

respectively, where  $\theta_i$  are the parameters of cluster  $i$ . Note that for each cluster, there are 4 parameters that correspond to distance, travel time, 80th percentile of travel time and number of freeway interchanges, respectively. The cluster centers were chosen with the following intuition. The first Gaussian distribution includes the truck drivers who consider travel time as the most important factor, followed by the distance of the route. The second Gaussian distribution describes the

<sup>1</sup>The train and test route choice questions can be found in this link: <https://bit.ly/3o8fZif>

truck drivers who treat both distance, travel time and the 80th percentile of travel time as equally important factors when making a routing decision. Lastly, the third Gaussian distribution describes the truck drivers who consider the travel time and the 80th percentile of travel time as the most important factors when making a routing decision.

Assuming that the truck drivers' utility can be described by a linear function, e.g. like the one described by (4), using the generated  $\theta$  in (17), we can subsequently generate the responses of the drivers to the route choice questions. Given that we generated a total of 600 samples from the Gaussian mixture model and we used 9 route choice questions for training and 5 questions for test, we got a total of 5400 data points for training and 3000 for test.

### B. Clustering and Utility Learning

Given the responses of the drivers to the route choice questions, we can cluster the drivers into disjoint clusters using the K-means algorithm as described in Section III-B and then use the learning algorithm as described in Section III-C in order to learn the parameters  $\tilde{\theta}_i$  of each cluster  $i$ . However, an important factor in this procedure is the number of clusters  $K$  that we assume in the K-means algorithm. To apply the K-means algorithm, we used the Scikit-learn package in Python [36].

Before describing the way we determine  $K$ , let us first mention some additional implementation details regarding the learning algorithm described in Section III-C. Assuming that the function that describes the utility of each cluster of truck drivers is linear, e.g. like the one described by (4), we build a linear model that we train using projected gradient descent with a fixed learning rate  $\eta = 0.001$ . The projected gradient descent is implemented as follows:

$$\begin{aligned}\xi_k &= \tilde{\theta}_k - \eta \nabla \mathcal{L}(\tilde{\theta}_k) \\ \tilde{\theta}_{k+1} &= \Pr_{R^+_{\geq 0}}(\xi_k)\end{aligned}\quad (22)$$

where  $\mathcal{L}(\tilde{\theta}_k)$  is given by (15) and  $Pr$  is the projection to the positive orthant. In our case, the projection operator ensures that at each iteration, the learned  $\tilde{\theta}_k$  is non-positive. Additionally, in case there is class imbalance in the data, we modify (15) as follows:

$$\begin{aligned}\mathcal{L}(\theta_i, x, y) &= -\frac{1}{M} \sum_{m=1}^M \epsilon_{0,i} y_m \log(s(\theta_i, x_m)) \\ &\quad + \epsilon_{1,i} (1 - y_m) \log(1 - s(\theta_i, x_m))\end{aligned}\quad (23)$$

where  $\epsilon_{0,i}$  and  $\epsilon_{1,i}$  are weights for the classes 0 (Route 1) and 1 (Route 2), respectively. After clustering the truck drivers using the K-means algorithm, for each cluster  $i$ , we determine the values of  $\epsilon_{0,i}$  and  $\epsilon_{1,i}$  using the following formulas:

$$\begin{aligned}\epsilon_{0,i} &= \frac{\max(\# \text{ of samples in class 0}, \# \text{ of samples in class 1})}{\# \text{ of samples in class 0}} \\ \epsilon_{1,i} &= \frac{\max(\# \text{ of samples in class 0}, \# \text{ of samples in class 1})}{\# \text{ of samples in class 1}}\end{aligned}$$

Note that the loss function described by (23) is the weighted cross-entropy loss function [37]. There are several ways to

pick the weights  $\epsilon_{0,i}$  and  $\epsilon_{1,i}$ . However, a common approach is to give more weight to the minority class.

At this point, let us describe how we choose the number of clusters  $K$  in the K-means algorithm. First, we cluster the drivers based on their responses to the 9 route choice questions that are used for training, as described in Section IV-A. In this work, we experiment with  $K = 1, 2, 3, 4, 5$  and 10. Subsequently, for each cluster  $i$  of truck drivers, we train a linear model using the 9 training route choice questions and we learn the parameters  $\tilde{\theta}_i$  using (23) and (17). Having learned the parameters  $\tilde{\theta}_i$  for each cluster, we test the performance of our method using the 5 test route choice questions. To measure the performance of our learning approach, we use the Area Under the Receiver Operating Characteristic curve (AUROC) [38] and the Area Under the Precision-Recall curve (AUPR) [39]. The results are presented in Table III.

TABLE III  
AUROC AND AUPR FOR DIFFERENT NUMBER OF CLUSTERS  $K$ .

| K  | AUROC | AUPR |
|----|-------|------|
| 1  | 0.75  | 0.79 |
| 2  | 0.86  | 0.90 |
| 3  | 0.90  | 0.94 |
| 4  | 0.89  | 0.94 |
| 5  | 0.94  | 0.96 |
| 10 | 0.94  | 0.96 |

As can be observed from Table III, choosing  $K = 5$  or  $K = 10$  gives us the highest AUROC and AUPR scores. In general, we expect that as the number of clusters  $K$  increases, the AUROC and AUPR on the test set will also increase until they reach a point at which the performance will start degrading. Additionally, as the number of clusters  $K$  increases, the computational complexity increases as well. To demonstrate this, in Table IV, we report the computational time needed to solve (18) in the Sioux Falls network [40] for different number of clusters  $K$ . To calculate the solution of (18), we used the fmincon optimization solver implemented in the MATLAB Optimization Toolbox [41].

TABLE IV  
COMPUTATIONAL TIME (IN SECONDS) TO SOLVE (18) FOR DIFFERENT NUMBER OF CLUSTERS  $K$ .

| K  | Computational time |
|----|--------------------|
| 1  | 1.8                |
| 2  | 4.5                |
| 3  | 43.2               |
| 4  | 253.8              |
| 5  | 764.6              |
| 10 | 3215.8             |

As can be observed from the results of Table IV, the computational time needed to solve (18) increases as the number of clusters increases. This can be justified by the fact that the set of decision variables  $\alpha(\cdot)$  in the optimization problem (18) depends on  $i$ . Additionally, note that for each cluster  $i$ , we first need to learn a different set of parameters  $\tilde{\theta}_i$  that we will later use in (18). This will further increase the computational complexity of the proposed method.



Based on the aforementioned observations, we choose  $K = 5$  as the appropriate number of clusters for the rest of the experiments. Then, for each cluster, we train a linear model using the 9 training route choice questions and we learn the parameters  $\tilde{\theta}_i$  using (23) and (17). These parameters are shown below:

$$\begin{aligned}\tilde{\theta}_1 &= [-0.843, -0.529, -0.636, -0.148] \\ \tilde{\theta}_2 &= [-0.686, -1.225, -1.189, 0] \\ \tilde{\theta}_3 &= [-0.770, -0.912, -0.319, -0.537] \\ \tilde{\theta}_4 &= [-0.654, -1.310, -0.765, -0.340] \\ \tilde{\theta}_5 &= [0, -0.868, -1.004, -1.059]\end{aligned}\quad (24)$$

where each element of a set of parameters  $\tilde{\theta}_i$  corresponds to distance, travel time, 80th percentile of travel time and number of freeway interchanges.

Having clustered the truck drivers into  $K = 5$  disjoint clusters and by using the learned parameters  $\tilde{\theta}_i$  given by (24), in the following sections, we compute the UE, the SO and a Pricing-and-Routing scheme by solving the optimization problems (5), (12) and (18), respectively.

### C. Necessity of a Pricing-and-Routing Scheme

In this section, we experimentally show the degree that truck drivers have an incentive to deviate from the SO generated routes. In our experiments, we use the Sioux Falls network which is a benchmark in the transportation research field consisting of 24 nodes and 76 links [40]. The Sioux Falls network<sup>2</sup> is shown in Figure 2. In our experiments, we

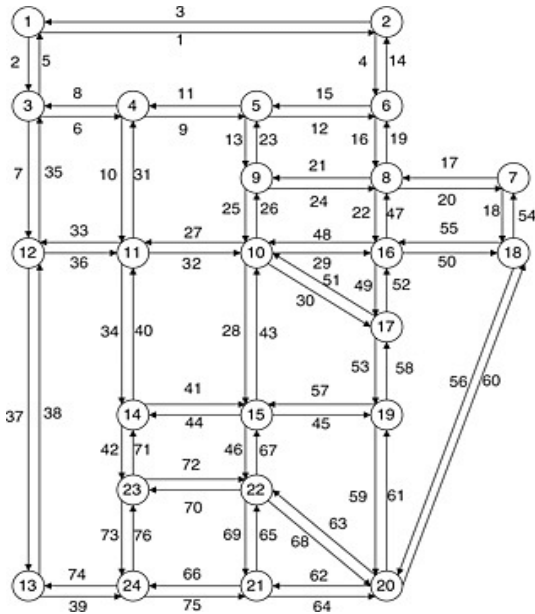


Fig. 2. The Sioux Falls network.

assumed that the cost of each route corresponds to travel time

<sup>2</sup>The distance of each link can be found in this link: <https://bit.ly/31W8Oah>

and can be described by a Bureau of Public Roads (BPR) function [42] of the form:

$$C_{lT}(X_{lp}, X_{lT}) = \gamma_a + \gamma_b \left( \frac{X_{lp} + 3X_{lT}}{\gamma_c} \right)^4$$

where  $\gamma_a, \gamma_b$  and  $\gamma_c$  are constants<sup>3</sup>.

After solving an equilibrium assignment problem for the passenger vehicles, we calculated the number of passenger vehicles at each link of the Sioux Falls network. Similar to [18], these numbers were assumed to remain constant<sup>4</sup>. For the truck drivers, we assumed that they want to travel in 6 available Origin-Destination (OD) pairs, namely  $(E_1, E_7), (E_1, E_{11}), (E_{10}, E_{11}), (E_{10}, E_{20}), (E_{15}, E_5)$  and  $(E_{24}, E_{10})$  and that they follow the 5 least congested routes per OD pair. Their demand was assumed to take one of the 5 equiprobable values, namely  $d_1, d_2, d_3, d_4$  and  $d_5$ <sup>5</sup>. In other words, for the purpose of the simulation experiments, we assumed that the demand of the truck drivers follows a discrete uniform distribution. However, the proposed methodology in this paper can be applied in the case that the demand of the truck drivers follows any probability distribution with finite support. Lastly, the values of the weighting factors of the objective function (10) were chosen to be  $\lambda = \rho = 0.9$ .

Using the aforementioned, we solved the optimization problem (12) that gives us the SO solution. To demonstrate the necessity for a pricing-and-routing scheme, in Table V, we present results that show the percentage by which the truck drivers could increase their utility in case they decided to deviate from the SO solution.

TABLE V  
PERCENTAGE BY WHICH THE TRUCK DRIVERS COULD INCREASE THEIR UTILITY IN CASE THEY DECIDED TO DEVIATE FROM THE SO SOLUTION.

| OD pair / Cluster | 1    | 2   | 3   | 4   | 5    |
|-------------------|------|-----|-----|-----|------|
| 1                 | 4.0  | 0.3 | 6.0 | 0.3 | 2.0  |
| 2                 | 10.0 | 0   | 7.7 | 3.9 | 2.5  |
| 3                 | 5.0  | 0   | 7.0 | 0.5 | 3.5  |
| 4                 | 8.9  | 0.1 | 6.2 | 4.3 | 4.6  |
| 5                 | 8.3  | 0   | 7.6 | 7.3 | 7.1  |
| 6                 | 9.6  | 2.8 | 6.8 | 6.5 | 12.0 |

More specifically, Table V measures:

$$\frac{I_1 - I_2}{I_2} * 100\%$$

where  $I_1$  and  $I_2$  are given by:

$$\begin{aligned}I_1 &= \max_{r \in R_w} \sum_c p_c J_{i,r}^{c,w}(\tilde{\theta}, x_r(\alpha^{SO})) \\ I_2 &= \sum_c \sum_{r \in R_w} p_c \alpha_{c,w,i,r}^{SO} \left( J_{i,r}^{c,w}(\tilde{\theta}, x_r(\alpha^{SO})) + \pi_r^{c,w} \right)\end{aligned}$$

<sup>3</sup>The values of  $\gamma_a, \gamma_b$  and  $\gamma_c$  can be found in this link: <https://bit.ly/3ibHNi1>

<sup>4</sup>The number of passenger vehicles at each link of the network can be found in this link: <https://bit.ly/3zBJoDy>

<sup>5</sup>The demand values of the truck drivers can be found in this link: <https://bit.ly/3zJ3ImS>

respectively. As can be observed from the results of Table V, most truck drivers have an incentive to deviate from the SO solution regardless of the OD pair or the cluster they belong. This result was expected since it is known in the literature that system optimality does not imply user optimality. Therefore, the SO solution is not a practical solution and a pricing-and-routing scheme is needed in order to guarantee the participation of the truck drivers.

#### D. Pricing-and-Routing

In this section, we run simulations in the Sioux Falls network. More specifically, using the learned parameters  $\tilde{\theta}_i$  from (24), we calculate the UE, the SO and the pricing-and-routing scheme by solving the optimization problems (5), (12) and (18), respectively.

Before presenting the results of the simulation experiments, let us first explain the choice of UE and SO as benchmarks for our comparison. It has been previously proved that as the number of GPS routing apps is increased, the observed traffic assignment converges to a UE [43]. Additionally, using simulation experiments based on real data of the Los Angeles network, it was shown that the minimum travel time of a driver can be achieved whenever all drivers in the network use GPS routing apps [43]. Therefore, the UE can be seen as an optimistic version of the real traffic conditions. On the other hand, the SO is the best possible traffic assignment that could be achieved in the network.

In the simulations, we use the same network parameters as the ones described in Section IV-C. Note that we consider 6 available OD pairs for truck drivers to retain computational tractability since as the number of OD pairs increases, the computational time increases as well. For realistic transportation networks with tens, hundreds or thousands of OD pairs, more computationally efficient solutions need to be studied.

To solve these problems, the `fmincon` optimization solver implemented in the MATLAB Optimization Toolbox was used. Since `fmincon` solves optimization problems with local optimality guarantees, in this section, we compare local minima between the approaches. Note also that the UE solution was obtained with a constraint tolerance of  $4 \cdot 10^{-3}$ , while the SO and the pricing-and-routing scheme solutions were obtained with a constraint tolerance of  $10^{-6}$ . In our experiments, we increased the constraint tolerance when calculating the UE solution compared to the default of  $10^{-6}$  in order to accelerate the computation of the solution. The results are shown in Table VI.

TABLE VI  
SIMULATION RESULTS OF THE SIOUX FALLS NETWORK.

|             | UE       | SO       | Pricing-and-Routing |
|-------------|----------|----------|---------------------|
| $E[T_{tr}]$ | 33025.5  | 30141.3  | 30169.7             |
| $E[U_{tr}]$ | -69688.2 | -60681.5 | -60996.0            |
| $E[T_S]$    | 51381.4  | 47557.9  | 47603.8             |
| $O(a)$      | 35371.5  | 32050.1  | 32106.2             |

As can be observed from the results presented in Table VI, the pricing-and-routing scheme achieves a significant reduction in the expected total travel time of the truck drivers and

the expected total travel time of the network compared to the UE, while simultaneously increasing the expected total utility of the truck drivers. Furthermore, the pricing-and-routing scheme achieves a performance that is close to the SO solution. However, in contrast with the SO, the pricing-and-routing scheme guarantees that the expected total utility of a truck driver (including payments) in case he/she decided to participate in the mechanism, is greater than or equal to his/her expected utility in case he/she did not participate through the first constraint of (18). Lastly, as already mentioned, the expected total payments made or received by the coordinator are equal to zero, thus making the mechanism self-sustainable.

#### E. Computational Complexity with respect to OD pairs

In this section, we present results that demonstrate how the proposed methodology scales with respect to the number of OD pairs. Using the same network parameters as the ones described in Section IV-C, in Table VII, we report the computational time needed to solve (18) for varying number of OD pairs.

TABLE VII  
COMPUTATIONAL TIME (IN SECONDS) TO SOLVE (18) FOR DIFFERENT NUMBER OF OD PAIRS.

| OD pairs | Computational time |
|----------|--------------------|
| 2        | 5.3                |
| 3        | 44.5               |
| 4        | 148.2              |
| 5        | 355.1              |
| 6        | 764.6              |

As can be observed from the results of Table VII, as the number of OD pairs increases, the computational time needed to solve (18) increases very fast. This can be explained by the fact that both the set of decision variables  $\alpha(\cdot)$  as well as the set of decision variables  $\pi(\cdot)$  depend on the OD pairs  $w$ . Additionally, the number of constraints of (18) also increases with the increase of the number of OD pairs. Therefore, even though the proposed methodology can be applied to medium size networks, more computationally efficient solutions need to be studied for larger networks.

#### F. Degree of Personalization

In this section, we present results that demonstrate the degree of personalization that the pricing-and-routing scheme can provide to the truck drivers. Using the same network parameters as the ones described in Section IV-C, in Table VIII, we report the percentage of truck drivers who have a higher total utility (including payments) compared to the UE at each demand realization and for each cluster.

As can be observed from the results of Table VIII, at most demand realizations and for most clusters, the percentage of truck drivers who have a higher total utility compared to the UE is relatively high. Note also that even though at some demand scenarios and for some clusters, some truck drivers may have a lower utility, it is guaranteed through the first constraint of (18) that on average, the expected total utility of

TABLE VIII  
PERCENTAGE OF TRUCK DRIVERS WHO ARE BETTER-OFF COMPARED TO  
THE UE AT EACH DEMAND SCENARIO AND FOR EACH CLUSTER.

| Scenario / Cluster | 1    | 2    | 3    | 4    | 5    |
|--------------------|------|------|------|------|------|
| 1                  | 99.0 | 97.2 | 68.4 | 62.5 | 95.5 |
| 2                  | 60.8 | 80.2 | 78.1 | 73.8 | 72.6 |
| 3                  | 72.2 | 69.9 | 73.3 | 66.1 | 71.1 |
| 4                  | 74.6 | 99.3 | 73.0 | 69.0 | 93.9 |
| 5                  | 88.5 | 96.8 | 41.6 | 95.5 | 89.0 |

a truck driver (including payments) in case he/she decided to participate in the mechanism, is greater than or equal to his/her expected utility in case he/she did not participate. Therefore, under the rationality assumption, every truck driver has an incentive to participate in the proposed mechanism.

## V. CONCLUSION

In this paper, we proposed a pricing-and-routing scheme for trucks that can be applied in a general transportation network. The proposed approach consists of 3 steps. In the first step, we cluster the truck drivers into disjoint clusters based on their responses to a small number of binary route choice questions. In the second step, we propose to use a learning scheme based on the Maximum Likelihood Estimation (MLE) principle that allows us to learn the parameters of the utility function that describes each cluster. In the third step, the estimated utilities are used to calculate a pricing-and-routing scheme that satisfies certain properties. The pricing-and-routing scheme guarantees that the expected total utility of a truck driver (including payments) in case he/she decided to participate in the mechanism, is greater than or equal to his/her expected utility in case he/she did not participate. Additionally, it satisfies the budget balance on average property and optimizes a total system-wide cost.

There are several potential extensions of this work. First, pricing-and-routing schemes similar to the one that we propose in this paper, belong to the category of route-based schemes. Route-based schemes can be applied to trucks that constitute a subclass of vehicles with certain characteristics. However, different approaches that can be applied to passenger vehicles need to be studied. Second, the current form of the proposed approach does not allow it to scale to very large transportation networks with tens, hundreds or thousands of OD pairs. This is mainly because the proposed scheme is route-based. Additionally, distributed optimization methods are also a promising direction for scaling the idea of a pricing-and-routing scheme into large transportation networks. Third, the proposed approach is studied for transportation networks that are in an equilibrium state. A potential future direction is the study of dynamic pricing-and-routing schemes that can be applied in real-time. Lastly, future research efforts also need to focus on the problem of the design of stable infrastructure that will allow the application of similar pricing-and-routing schemes in a real world scenario. Nevertheless, we believe that the proposed method can form the foundation and initial step in the design of personalized pricing-and-routing schemes that also take system optimality into account.

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