Physics-Informed, Safety and Stability Certified Neural Control for Uncertain Networked Microgrids

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***Abstract*—This letter devises a physics-informed neural hierarchical control for uncertain networked microgrids (NMs) to provide certificated safe and stable control of NMs undergoing disturbances and uncertain perturbations. The main contributions include 1) a learning-based hierarchical control framework for inverter-based resources (IBRs) in NMs under unprecedented uncertainties of renewable energies; 2) a robust control Lyapunov barrier function (rCLBF) to provide provable safety and stability guarantees under uncertain scenarios; 3) an rCLBF-based, physics-informed learning scheme to simultaneously discover the certificates and control policy with explicit safety, stability, and robustness guarantees, enabling certified generalization beyond nominal operating scenarios. The efficacy of the rCLBF-based neural hierarchical control is thoroughly validated in different NMs cases.**

***Index Terms*—Networked microgrids, learning-based control, certified control, microgrid stability, robust control.**

I. Introduction

MICROGRIDS and networked microgrids (NMs) are facing increasing control challenges with the integration of massive inverter-based resources (IBRs). The strongly nonlinear dynamics of IBRs and their reduced inertia can significantly deteriorate the system's stability. The unprecedented uncertainties from renewables also impact the IBR operations and would force the system to unsafe states. Existing model-driven control methods for NMs, either linearization-based [1] or Lyapunov function-based, are found hard to handle the unforeseen large disturbances and uncertain scenarios. Recently, learning-based control has been introduced in power system control [2] to explore their potential for being generalizable to new operating conditions. However, most existing learning-based control employs a posteriori verification module to check the system's stability, which retards the training process. Some learning-based control techniques incorporate certificates to prove the soundness of learned controllers, which may fail under uncertain scenarios unseen in the training process.

To tackle the challenges, this letter devises *physics-informed neural hierarchical control* for uncertain NMs to ensure large-signal stability and nonlinear safety requirements under scenarios with bounded uncertainties. A robust control Lyapunov barrier function (rCLBF) is constructed for NMs under uncertainties from renewables to explicitly and rigorously certify the safety and stability, and an rCLBF-based, physics-informed learning approach is established to train the safety and stability certificates and control strategies simultaneously.

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II. Neural Hierarchical Control of NMs

Mathematically, the hierarchical control for DER $i \in \{1, ..., N\}$ is formulated as:

$$\begin{cases} \omega_i = \omega_i^* - m_{p,i}(P_i - P_i^* + \Delta p_i) + \Omega_i \\ E_i = E_i^* - n_{q,i}(Q_i - Q_i^*) + e_i \end{cases} \tag{1}$$

Here, ω_i, E_i, P_i and Q_i respectively denote the angular speeds, output voltage magnitudes, active power, and reactive power of distributed energy resources (DER) i; superscript * denote the nominal values; $m_{p,i}$ and $n_{q,i}$ denote the droop coefficients; Ω_i and e_i respectively denote the secondary control signals. Specifically, Δp_i represents the impact of uncertainties from renewables on DER's power generation, making the NMs an uncertain dynamic system.

Denote $\boldsymbol{u} = [\Omega_1, \Omega_2, ..., \Omega_N, e_1, e_2, ..., e_N]^T$ as the assembling of the secondary control signals of all DERs. Traditional methods usually design the control rules of $\boldsymbol{u}$ based on linearized system models. Although such methods can guarantee small-signal stability, NMs behaviors under large disturbances can not be certificated, let alone the system's safety and stability under heterogeneous uncertain scenarios.

To bridge the gap, we design a learning-based control policy for $\boldsymbol{u}$ to establish neural network-represented secondary control signals for DERs:

$$\boldsymbol{u} = \pi_\alpha^u(\boldsymbol{X}) \tag{2}$$

Here, π_α^u denotes the *policy neural network* parameterized by α; $\boldsymbol{X}$ denotes the NMs state variables, which are selected as the input of π_α^u (see details in Subsection III-A). By properly training π_α^u, $\boldsymbol{u}$ will be designed to ensure the system's stability and safety even under uncertain scenarios, as discussed in Section III.

III. Physics-Informed Learning for Neural Hierarchical Control

A. rCLBF Certificates for Uncertain NMs

This subsection establishes the rCLBF certificates for an uncertain NMs system to provide theoretical safety and stability certificates under uncertain operating scenarios.

The uncertain NMs system under neural hierarchical control (i.e., (1)) is functionally formulated as:

$$\forall \theta \in \Theta \quad : \quad \dot{\boldsymbol{X}} = \boldsymbol{f}(\boldsymbol{X}, \theta) + \boldsymbol{g}(\boldsymbol{X}, \theta)\pi_\alpha^u(\boldsymbol{X}) \tag{3}$$

The rationale of modeling the NMs as a set of ordinary differential equations (ODE) in (3) is detailed in [3]. Basically, considering the load and branch dynamics in NMs, such an ODE-based formulation is rigorously equivalent to the original differential algebraic equation (DAE)-based model [3]. In (3), $\boldsymbol{X} \in \boldsymbol{\chi} \subseteq \mathbb{R}^n$ denotes the NMs states; $\pi : \mathbb{R}^N \to \mathbb{R}^{2N}$ denotes the neural control policy in (2); $\boldsymbol{f}$ and $\boldsymbol{g}$ are functions describing the NMs dynamics and are assumed to be locally Lipschitz; θ denotes the uncertain parameters of NMs (e.g., Δp_i in (1)) and Θ denotes the set of possible values of the uncertainties. We assume that Θ is a convex hall.

To theoretically certify the stability and safety of NMs in arbitrary uncertain scenarios, we introduce the robust control Lyapunov barrier function (rCLBF) theory [4].

Denote $\boldsymbol{x} = \boldsymbol{X} - \boldsymbol{X}_e(\theta)$ where $\boldsymbol{X}_e(\theta)$ denotes the equilibrium point of the NMs under uncertainty θ. A function $V : \boldsymbol{\chi} \to \mathbb{R}$ is defined as a rCLBF for the NMs in (3), if it satisfies:

$$\forall\theta \in \Theta : \quad V(\boldsymbol{x}_{\text{goal}}) = 0, \quad V(\boldsymbol{x}) > 0 \quad \forall \boldsymbol{x} \in \boldsymbol{\chi} \backslash x_{\text{goal}} \tag{4a}$$
$$\inf_u L_f V + L_g V u + \lambda V \le 0 \quad \forall \boldsymbol{x} \in \boldsymbol{\chi} \backslash x_{\text{goal}} \tag{4b}$$
$$V(\boldsymbol{x}) \le c \quad \boldsymbol{x} \in \boldsymbol{\chi}_{\textbf{safe}} \tag{4c}$$
$$V(\boldsymbol{x}) > c \quad \forall \boldsymbol{x} \in \boldsymbol{\chi}_{\textbf{unsafe}} \tag{4d}$$

The *stability certificate* is guaranteed by the control Lyapunov function requirements in (4a) and (4b), where $\boldsymbol{x}_{\text{goal}}$ denotes a stable goal point under control input u; $\boldsymbol{\chi}_{\text{safe}}$ and $\boldsymbol{\chi}_{\text{unsafe}}$ denote a set of safe states and a set of unsafe states separately; $L_f V$ and $L_g V$ are respectively the Lie derivatives of V along $\boldsymbol{f}$ and $\boldsymbol{g}$ (i.e., the NMs dynamic models in (3)); $\lambda > 0$ is the convergence rate. The *safety certificate* is guaranteed by the barrier function requirements in (4c) and (4d), where c denotes a hyperparameter describing the safe level. The *robustness characteristic* is guaranteed because (4) is required for an arbitrary uncertain scenario $\theta \in \Theta$.

On the one hand, it can be proved that any control policy $\pi_\alpha^u(x) \in \{u | L_f V + L_g V u + \lambda V \le 0\}$ will be both safe and stable[1] when executed on NMs specified by $\boldsymbol{f}$ and $\boldsymbol{g}$ with uncertain parameters $\theta \in \Theta$ [4]. On the other hand, as Θ is a convex hull of the uncertain parameters, the learned controller is safe and stable in any scenarios from the convex hull as long as the vertex scenarios are guaranteed [4]. Correspondingly, the rCLBF-based neural hierarchical control of NMs, if satisfying (4), can provably stabilize the uncertain NMs without any safety violation under uncertain scenarios in Θ.

B. Physics-Informed Co-Learning of rCLBF and the Control Policy

This subsection develops a physics-informed training process to learn rCLBF (i.e., $V(x)$ in (4)) and the neural control policy ($\boldsymbol{u}$ in (2)) in a simultaneous manner.

Denote $\pi_\beta^V(\boldsymbol{x})$ as the *rCLBF certificate neural network*. The rCLBF is designed as $V(\boldsymbol{x}) = h^T(\boldsymbol{x}) h(\boldsymbol{x})$, where $h(\boldsymbol{x})$ is the activation function of the last hidden layer of π_β^V. The following loss function is constructed to train π_β^V and π_α^u simultaneously:

$$\min_{\alpha,\beta} \mathcal{L} = \mathcal{L}_{rCLBF} + \gamma \mathcal{L}_u \tag{5}$$

where each loss term is defined as:

$$\mathcal{L}_{rCLBF} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 \tag{6a}$$
$$\mathcal{L}_1(\beta) = V(x_{\text{goal}})^2 \tag{6b}$$
$$\mathcal{L}_2(\alpha,\beta) = \sum_{i=1}^{n_s} \frac{1}{n_s} \Bigg(\frac{a_3}{N_{\text{train}}} \sum_x \sigma(\epsilon + L_f V(x) + L_g V(x) \pi_\alpha^u(x) + \lambda V(x)) \Bigg) \tag{6c}$$
$$\mathcal{L}_3(\beta) = \sum_{i=1}^{n_s} \frac{1}{n_s} \Bigg(\frac{a_1}{N_{\text{safe}}} \sum_{x \in \chi_{\text{safe}}} \sigma(\epsilon + V(x) - c) \Bigg) \tag{6d}$$
$$\mathcal{L}_4(\beta) = \sum_{i=1}^{n_s} \frac{1}{n_s} \Bigg(\frac{a_2}{N_{\text{unsafe}}} \sum_{x \in \chi_{\text{unsafe}}} \sigma(\epsilon + c - V(x)) \Bigg) \tag{6e}$$
$$\mathcal{L}_u(\alpha) = ||u(x) - u_{\text{nominal}}||^2 \tag{6f}$$

where γ, $a_1 \sim a_3$, ϵ, λ and c (i.e., the safe level) are positive hyperparameters of the algorithm; n_s denotes the number of uncertain scenarios to ensure the control robustness. Specifically, $\mathcal{L}_{rCLBF}$ is the rCLBF-related loss function [4] to enforce (4) to be satisfied by $\mathcal{L}_1 \sim \mathcal{L}_4$, where $\sigma(x) = \max(x, 0)$ denotes the ReLU function, and the strict inequality satisfaction is ensured by ϵ. $\mathcal{L}_u$ provides a training signal for the policy neural network π_α^u, where u_{nominal} is a nominal controller(e.g., the LQR policy).

Consequently, π_β^V and π_α^u will be jointly trained by minimizing (5). The small weight applied to $\mathcal{L}_u$ ensures that the training process prioritizes satisfying the Equation (4) conditions. By every I epoch, new training samples will be generated using the currently trained π_α^u. Fig. 1 summarizes the overall architecture for the training process.

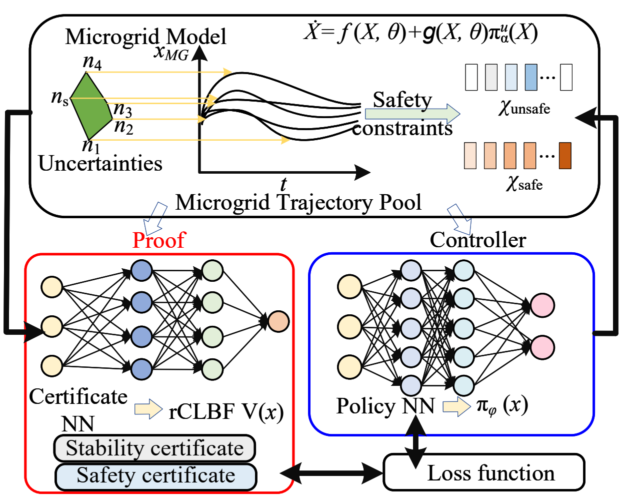

Fig. 1: Architecture of the physics-informed training process of the safety-and-stability-certified neural hierarchical control

C. Algorithm Flow

The pseudo-code for the proposed method is given in Algorithm 1. The hyperparameters to be pre-determined are c, λ, ϵ and the size of the rCLBF neural network π_β^V and the control policy neural network π_α^u. A nominal controller is utilized

[1]By "safe and stable", we mean [4]: for every initial state x_0: $|| \lim_{t\to\infty} x(t) - x_{\text{goal}}|| = 0$; For all $t_2 \ge t_1 \ge 0$, $x(t_1) \in \boldsymbol{\chi}_{\text{safe}}$ implies $x(t_2) \notin \boldsymbol{\chi}_{\text{unsafe}}$.

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to generate NMs trajectories for initialization. Training data are sampled from the state space covered by the trajectories. To improve the training performance and broaden the space covered by training samples, we specify fixed percentages of training points sampled from the goal, safe, and unsafe regions and use the learned controller to regenerate new training samples to improve the training performance after several epochs.

Algorithm 1: Learning controller with certificates

▷ ***Require***: NMs parameters set Θ, learning rate α, batch size H, epochs I per episode and total epochs K
▷ ***Input***: NMs initial states, safe region, unsafe region and system model (3) and initial weights φ for network
▷ ***Training samples generation***:
$\dot{\boldsymbol{x}} = \boldsymbol{f}_\theta(\boldsymbol{x}) + \boldsymbol{g}_\theta \boldsymbol{u}_{\text{nominal}} \longrightarrow x_{\text{goal}}, x_{\text{safe}}, x_{\text{unsafe}}$
for *current_epoch* = 1 to K **do**
  for i = 1 to n_s **do**
    Calculate descent loss with uncertain parameters:
    $L_{f_{\theta_i}} V(x) + L_{g_{\theta_i}} V(x) u(x) + \lambda V(x)$
  end
  Calculate total loss of all the batches
  $Loss = \mathcal{L}_{CLBF} + \beta \mathcal{L}_u$
  Update weights in the neural network by passing Loss to Adam optimizer
  Output $V(x)$ and $\pi_\varphi(x)$
  if *current_epoch* $\%\ I == 0$ **then**
    Update training samples :
    $\boldsymbol{x} : \dot{\boldsymbol{x}} = \boldsymbol{f}_\theta(\boldsymbol{x}) + \boldsymbol{g}_\theta \pi_\varphi(x)$
  end
end
▷ ***Output***: Neural controller $u(x)$

IV. Case Study

The rCLBF-enabled neural hierarchical control method is tested in a typical microgrid system [1] and a 6-microgrid NMs system [5]. Hyperparameters in (6) are set as $a_1 = a_2 = 100$, and $a_3 = 1$, $\gamma = 10^{-5}$, $\lambda = 1.0$ and $\epsilon = 0.01$. We construct fully-connected neural networks with 2 hidden layers, 64 units at each layer, and Tanh activation functions for all cases.

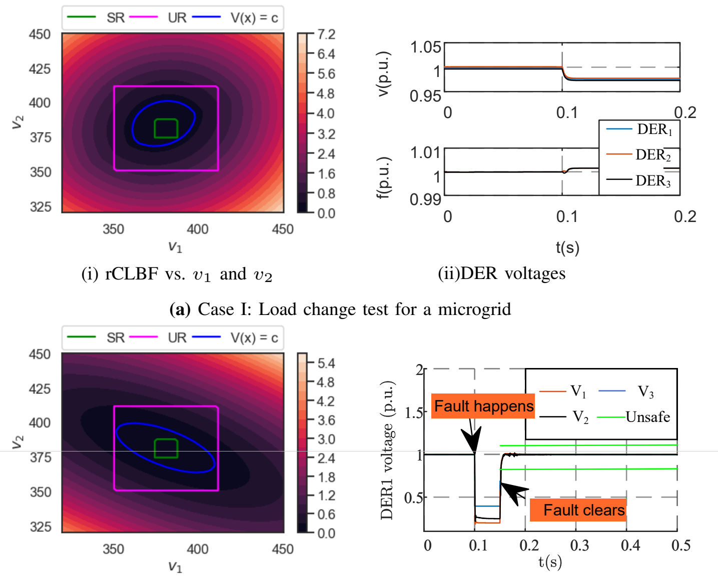
A. Validity of rCLBF-based Neural Hierarchical Control

We first validate the efficacy of the rCLBF-based neural hierarchical control to guarantee systems' safety and stability under typical large disturbances.

Fig. 2 studies the performance of the devised method for the microgrid case [1]. Case I is a load change scenario. Fig. 2(a) illustrates the contour plot of the rCLBF $V(x)$ vs. the voltages of DER1 and DER2, as well as the safe (SR) and unsafe regions (UR). It shows that the safe region is enclosed in the safe level set ($c = 0.5$), which ensures (4c)-(4d) are satisfied with the safety requirement. Case II is a short-circuit fault scenario, where the fault happens at 0.1s and is cleared after 0.05s. Fig. 2(b) shows that the DER voltages are also in the safe region after the fault clearance.

B. Scalability of rCLBF-based Neural Hierarchical Control

Fig. 3 studies the method's performance for the NMs case [5] under short-circuit faults. The counter-plot in Fig. 3(a) again shows that the safe level set (c=0.5) encloses the safe region, which ensures the system's safety as long as the NMs



(i) rCLBF vs. v_1 and v_2 (ii)DER voltages

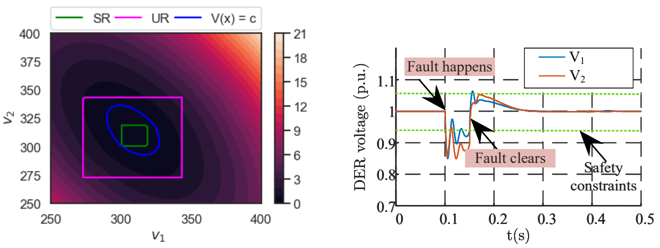
(a) Case I: Load change test for a microgrid

(i) rCLBF vs. v_1 and v_2 (ii) DER voltages

(b) Case II: Short-circuit fault test for a microgrid

Fig. 2: Performance of the rCLBF-based neural controller for a microgrid

enter a safe region. The NMs dynamics in Fig. 3(b) also validate the effectiveness of the neural controller to guarantee the NMs' safety and stability after the fault clearance.

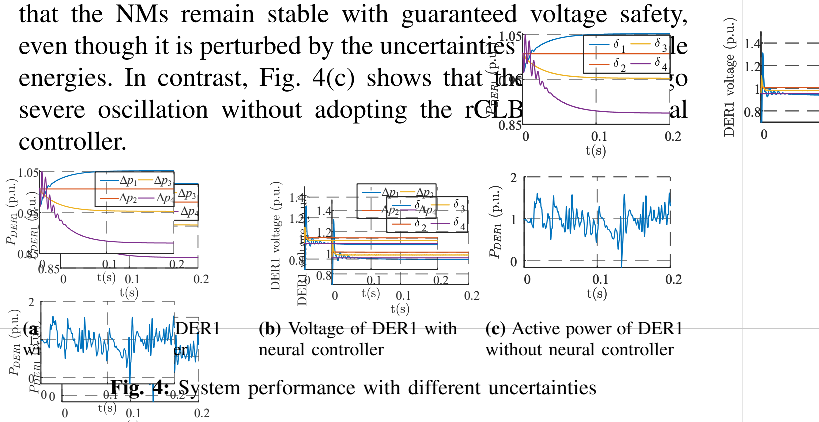


(a) rCLBF vs. v_1 and v_2 **(b)** DER voltages with short-circuit fault

Fig. 3: Performance of the rCLBF-based neural controller for NMs

C. Robustness of rCLBF-based Neural Hierarchical Control

We then validate the robustness of the proposed method in the NMs case [5] under uncertainties. Fig. 4(a) and Fig. 4(b) respectively present the active power and output voltage of DER1 under different uncertain scenarios. It can be observed that the NMs remain stable with guaranteed voltage safety, even though it is perturbed by the uncertainties of renewable energies. In contrast, Fig. 4(c) shows that the NMs undergo severe oscillation without adopting the rCLBF-based neural controller.

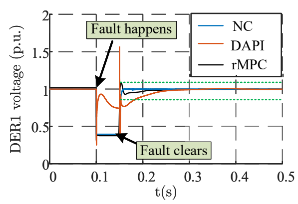


(a) Active power of DER1 with neural controller **(b)** Voltage of DER1 with neural controller **(c)** Active power of DER1 without neural controller

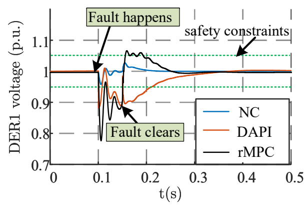
Fig. 4: System performance with different uncertainties

D. Comparison with Existing Methods

We compare the rCLBF-based neural control against DAPI control [3], i.e., a typical model-driven hierarchical control, and min-max robust model predictive control(rMPC) [6], which also provides safety guarantees. A short-circuit fault occurs at 0.1 s and is cleared after 0.05. Table I summarizes the dynamic performance of the controllers in Fig. 5. The NMs with neural-certificated controller always operates in a safe region and gets stable in a very short time after the fault clears (safety rate[2] is 100%). The results show that the certificated safe operation is guaranteed using neural-certificated control and has a better safety rate and dynamic performance against DAPI and rMPC methods.



(a) Performance for the microgrid case



(b) Performance for the NMs case

Fig. 5: DER voltage under a short-circuit fault using different control methods (NC: the devised rCLBF-based neural hierarchical control)

TABLE I: Comparison of controller performance under parameter variation

Test	Algorithm	Safety rate	Coverage time(ms)
MG	NC	100%	10
	rMPC	85%	100
NMs	NC	100%	50
	rMPC	66.7%	100

V. Conclusion

This letter develops an rCLBF-based, physics-informed neural hierarchical control for NMs. The unique feature of the devised method is its provable safety and stability certificates for real-time NMs control and its capability of handling the fast dynamics and uncertain scenarios in the simultaneous design of rCLBF and the control policy. The next step is to further validate the controller performance under more complicated operating scenarios, such as reconfiguration operations and cyber-physical hybrid operations.

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[2]The safety rate is defined as the ratio of the safe period to the period of the system getting stable after the fault clears.