



Analysis of Roadway Segment Vulnerability and Risk Factors for Crashes While Driving in the Wrong Direction

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Abstract: Roadway improvements to reduce the frequency of crashes are of the utmost priority to transportation agencies. To a great extent, implementation of improvement programs depends on the reliable identification of roadway segments with high crash risk. Among all crash types, wrong-way driving (WWD) crashes are considered random in nature and are a major safety concern. The Federal Highway Administration defines WWD specifically for high-speed divided highways and access ramps. This definition excludes all other roadway classifications when a crash occurs in the opposing direction to the legal flow of traffic. Screening 5 years of crash data in Minnesota revealed that WWD resulted in crashes on other types of roadway functional classes. This work aimed to (1) introduce a new term/acronym to the literature for driving in the wrong direction (DWD) on all roadway functional classes, (2) apply a set of count data models to estimate the occurrence of DWD crashes, (3) identify roadway geometric features of high-risk segments for DWD crashes, (4) investigate random effects of covariates due to unobserved factors, and (5) calculate elasticity effects of variables. Final models' specifications indicate that the negative binomial (NB) mixed effect model was found to be the best-fit model. Focusing on DWD crashes, we uncovered the factors contributing to higher DWD crash-risk segments: log of average annual daily traffic (AADT), number of lanes, sidewalk, and shoulder type. The change in frequency of crashes is also investigated using marginal effects, and safety interventions for preventing DWD crashes are also discussed. Transportation agencies can use the findings of this research, in terms of contributing factors and their relative effects on DWD crashes, to deploy appropriate countermeasures at high-risk locations. DOI: [10.1061/JTEPBS.0000695](https://doi.org/10.1061/JTEPBS.0000695). © 2022 American Society of Civil Engineers.

Author keywords: Wrong-way driving (WWD); Crash frequency models; Count data; Mixed-effects models.

Introduction

The Federal Highway Administration (FHWA) defines wrong-way driving (WWD) events as those in which vehicles are traveling in the opposing direction to the legal flow of traffic on high-speed access-controlled divided roadways (i.e., freeways and ramps). The current research defines driving the wrong direction (DWD) events as those occurring when a driver enters against the legal flow of traffic on any roadway functional class. The current literature on WWD does not classify crashes as WWD crashes if they occur

while driving in the wrong direction on non-high-speed and undivided roadways. In this study, we introduce DWD, investigate the vulnerability of roadway segments to DWD crashes, and uncover the causal factors in DWD crashes. Studies on DWD crashes are nonexistent in the literature; therefore, we reviewed WWD literature to highlight recent studies because WWD crashes represent a subset of DWD crashes. However, we emphasize that WWD and DWD crashes cannot be used interchangeably.

Due to the installation of many technologies in today's vehicles to improve crashworthiness and ongoing safety improvements of legacy highway systems, crash frequency per 100 million vehicle miles traveled (VMT) has been decreasing in the United States (US) at a steady rate for many years. However, the total number of traffic crashes has not been decreasing at the same rate due to the steady increase in VMT. In addition to millions of traffic injuries and property damages, 37,473 motor vehicle fatalities occurred on US highways alone in 2017 (NHTSA 2019). Due to the nature of DWD crashes (head-on or opposite-direction sideswipe crashes); they are relatively more severe than other types of motor vehicle crashes. One analysis of wrong-way crashes on freeways in Illinois showed that roadway segments close to freeway ramps were the most likely locations for WWD crashes (Zhou et al. 2016). According to the National Transportation Safety Board, 360 WWD-related fatalities were reported annually between 2004 and 2011 in the US (NTSB 2012). The leading contributing factors of WWD crashes are driving under the influence, inattention to driving due to fatigue and distraction, physical and age-related impaired judgment, unfamiliar drivers, infrastructure deficiencies (e.g., poor lighting), limited sight distance, and heavy roadside vegetation (Zhou et al. 2016).

Several past studies (discussed in the "Literature Review" section of this paper) have focused on the identification of causal

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Note. This manuscript was submitted on September 23, 2021; approved on March 10, 2022; published online on May 19, 2022. Discussion period open until October 19, 2022; separate discussions must be submitted for individual papers. This paper is part of the *Journal of Transportation Engineering, Part A: Systems*, © ASCE, ISSN 2473-2907.

factors related to WWD crashes, and crash severity modeling. We found that studies on WWD crash frequency prediction models considering geometric, traffic, and environmental characteristics of roadway segments are very limited. The current study aimed to bridge this gap in the literature. In this research, crash frequency prediction models were developed using 6 years (2010–2015) of observed DWD crashes in the State of Minnesota (MN) transportation network to identify geometric features of roadway segments with relatively high risk for crashes of this type. Data were collected from the Highway Safety Information System (HSIS), which was formally obtained by the HSIS from the Minnesota Department of Transportation (HSIS 2020).

One of the critical methodological aspects of statistical modeling is the correlation that potentially exists between roadway segments (within-segment correlation) considered in the modeling process, and the different types of crashes (DWD in this context). The possible presence of segment-level unobserved factors, as well as potential unobserved heterogeneity in the effects of key factors, may influence the outcomes in terms of the level of risk posed by one segment type versus another for the different types of crashes. Ignoring unobserved factors—which may produce within-segment correlation and possible unobserved heterogeneity in the effects of covariates on outcomes—can lead to biased estimates of the variable parameters. The effects of unobserved factors can affect other controlled observed factors, which could lead to change in their impact. This issue, known as unobserved heterogeneity, is commonly addressed in the literature by utilizing random-parameter modeling techniques versus fixed-parameter methods (Mannering et al. 2016). This restrictive method of fixed-parameter models assumes that the effects of explanatory variables are similar across all observations (i.e., roadway segments in the current study). The random-parameter method utilizes simulation techniques in the estimation process to estimate both the mean and standard deviation of parameters, as compared with mean-only parameters in the fixed-parameter method (Anastasopoulos and Mannering 2009; Bogue et al. 2017). Such deviations in the parameter estimates, if found to be statistically significant, provide credible indications that variables behave differently across observations and, therefore, the utilization of fixed-parameter methods would be improper. There are several methodological approaches to account for unobserved heterogeneity. In the context of count data models, these approaches can be broadly summarized into one of the following types: (1) random parameter count models, (2) finite mixture models, and (3) Markov switching models (Mannering et al. 2016). Random parameter count models are by far the most widely adopted approach in the literature (e.g., Barua et al. 2016; Mitra and Washington 2012). Finite mixture models identify latent classes within observations and vary parameters for each latent class (e.g., Buddhavarapu et al. 2016). Markov switching models account for temporal correlation in data arising from aggregation over time (e.g., Malyskina and Mannering 2010). Notably, the issue of temporal instability has also been highlighted in recent years. Statistical analysis of crash data assumes that the model parameters are temporally stable and do not change over time; this assumption overlooks the effect of variables that may change over time (Mannering 2018). Researchers have addressed this issue to model crash severities and vehicle ownership decisions by utilizing random parameter models that account for unobserved heterogeneity in means and variances of the random parameters (Se et al. 2021; Hamed and Al-Eideh 2020).

In summary, the current work aimed to fulfill three main objectives and contribute to the existing literature on WWD crashes: (1) introduce DWD to the existing literature, (2) investigate and identify statistical approaches to model DWD crashes,

and (3) identify roadway geometric features of high-risk segments for DWD crashes by accounting for random effects due to unobserved factors.

The remainder of this article is organized as follows. The next section presents a literature review, followed by the methodology section describing the formulation of the statistical modeling frameworks utilized in this study. The data section discusses the dataset used and the final estimation sample assembly process, as well as descriptive statistics of different variables available for statistical modeling. The modeling results and findings section presents a detailed overview of the estimation results and the models' measures-of-fit and comparison analysis. Then we discuss the results of the variables' elasticities. Finally, we present conclusions and limitations of the study along with major findings, and future scope of research based on the limitations of this study, respectively.

Literature Review

A comprehensive review of WWD crashes on freeways and expressways in 1971 documented that roadway geometry and configuration are major factors in the occurrence of WWD crashes. Most WWD events occur when a vehicle enters exit ramps. Partial interchanges were two times riskier than full interchanges (i.e., full cloverleaf) (Friebele et al. 1971). Other significant factors associated with WWD crashes cited in this study include limited line-of-sight on exit ramps; distracted driving under the influence of alcohol (distracted driving and driving under the influence can occur simultaneously or independently); lighting conditions; nighttime driving; presence of two-way frontage roads; insufficient signing and pavement markings; absence of divergent roadways to redirect wrong-way entry; and absence of technology-based warning systems (Friebele et al. 1971). Using the Fatality Analysis Reporting System (FARS), a study found that driving under the influence represents approximately 60% of WWD crashes (NTSB 2012). Several studies reported that most WWD crashes occur on urban roadways during weekend nights (Cooner et al. 2004; FDOT 2015; Finley et al. 2014; Zhou et al. 2016). Young male drivers under the influence of drugs/alcohol were overrepresented in the literature of WWD crashes on freeways (Baratian-Ghorghi et al. 2014; Finley et al. 2014; Tamburri and Theobald 1965; Zhou et al. 2016). However, in all WWD crashes, older drivers were overrepresented (Baratian-Ghorghi et al. 2014; FDOT 2015; NTSB 2012).

Different types of countermeasures have been implemented by state transportation agencies at high-risk WWD crash locations. The National Cooperative Highway Research Program (NCHRP) Report 881 identified the most frequent countermeasures to be DO NOT ENTER and WRONG WAY signs, wrong-way arrow markings, flashing red LEDs on WRONG WAY signs, centerline in median openings, and stop or yield lines (Finley 2018). As most WWD crashes were related to drunk/distracted driving, many of the infrastructure-based solutions were dynamic and caught drivers' attention, thereby reducing the frequency of WWD crashes. Several studies reported a high number of WWD crashes during early morning hours (Cooner et al. 2004; Copelan 1989). Analyzing WWD crashes in California, Copelan (1989) recommended countermeasures such as the installation of high retroreflective WRONG-WAY pavement markings; proper placement of high retroreflective WWD signs; geometric modifications to freeway ramps; modifications of partial-interchanges to full-interchanges; installation of specialized pavement lighting (similar to those found in airport runways/taxiways); detector and surveillance cameras; and modifications to the design of ramps and interchanges in terms of spacing and configuration, to reduce driver confusion. Vaswani (1976) presented

engineering solutions implemented in Virginia to reduce WWD crashes, which included the removal of pavement flares informing drivers about new geometric configurations. Wang (2018) recommended that stop-line positioning, narrower turning radius, nontraversable medians, and narrower median widths can reduce WWD crash risk at partial cloverleaf interchanges. While physical barriers (e.g., spike barrier) had been considered as a solution to avoid wrong-way entry, none of the states' transportation agencies found it practical (Copelan 1989). Beyond many of the solutions presented in the California study, a Texas study recommended installing WWD signs at a lower height than the standard 7-ft height for urban signs, and developed a checklist for engineers and field crews to review WWD risks (Cooner et al. 2004). A 2013 survey, which included state transportation officials who participated in the first National WWD Summit, identified that the geometric modification of ramp configurations was the most effective countermeasure to reduce WWD crashes (Pour-Rouholamin et al. 2014). In addition to crash data, an Alabama study conducted onsite data collection (e.g., sight distance measurement) and applied the Haddon matrix to identify WWD crash contributing factors (Wang and Zhou 2017). Azzeh et al. (2016) conducted a simulation-based study to identify drivers' decision making at WWD scenarios, and reported that the installation of multiple countermeasures reduced driver confusion. A study in North Carolina developed a toolbox consisting of signs, markings, and geometric modifications of existing facilities to reduce WWD risks (Carter et al. 2018).

Although many studies investigated WWD crashes, several studies applied statistical modeling techniques to investigate the causal factors of WWD crashes. Jalayer et al. (2018) applied an ordered probit model to identify significant crash contributing factors using WWD crash data from Illinois and Alabama. The authors identified significant factors included seatbelt use, vehicle age and type, airbag deployment status, crash type, and a set of engineering and nonengineering (i.e., education and enforcement) measures were proposed. Das et al. (2018a) applied multiple correspondence analysis (MCA) methods to investigate WWD crashes in Louisiana, and measured the relative association of different crash factors to understand the combined contribution to WWD crash occurrences. Jalayer et al. (2017) also applied MCA methods to model WWD crashes in Illinois and Alabama, and identified the most significant contributing factors. Another study used frequent pattern mining (FPM) to identify significant factors in WWD on freeway ramps and median crossover crashes on undivided highways (Das et al. 2018b). A binomial logistic regression model was developed for WWD and non-WWD crashes in Florida to identify the odds of occurrence and the associated significant crash contributing factors (Ponnaluri 2016). The model findings were integrated with a perception survey of roadway users and transportation professionals. It was found that, in addition to freeways, arterials were also susceptible to WWD. Because urban areas received significant attention for roadway safety improvements, roadway lighting at high-risk locations, especially in rural areas, was recommended. A similar study was conducted using WWD and non-WWD crashes in Alabama (Zhang et al. 2017). Applying the Poisson regression model using different WWD event data (including non-crash WWD events), high-risk locations for WWD were identified for safety improvements in Central Florida (Sandt and Al-Deek 2018). Sandt and Al-Deek (2018) developed an optimization algorithm to assist transportation agencies in selecting locations with high WWD crash risk for installation of technology-based countermeasures. A study on WWD crashes on undivided highways recommended the installation of infrastructure-based countermeasures (such as centerline rumble strip) on rural two-lane highways (Kusano and Gabler 2013). A French study compared

the characteristics of WWD and non-WWD crashes and recommended interventions that were effective for cognitively impaired drivers (Kemal 2015). Moreover, driver-related factors, which were common in WWD crashes, were also a primary cause of many other crash types (e.g., drunk driving), and countermeasures to reduce the causal factors were recommended in addition to infrastructure improvements. A high percentage of fatalities resulted from WWD compared with other crash types: In WWD crashes, 63% of drivers were alcohol-intoxicated versus only 5.6% intoxication rate in other crash types. It was also reported that Native Americans were over-represented in WWD crash fatalities in New Mexico; a potential reason noted was the higher drinking and driving rate in the Native American population.

Methodology

Frequency estimation models are considered to be a form of parametric model. Parameters of covariates are estimated from observed frequency/count data, which present the influence of each independent variable on the frequency of a certain event (e.g., DWD crash frequency in the current study). Due to the nonnegative nature of crash frequency events, log-linear models are suitable compared with linear regression models. The covariate estimates of log-linear models are multiplicative, where parameter estimates in regression models are additive.

Different forms of Poisson and negative binomial (NB) models have been used for frequency modeling (Washington et al. 2010). The equal mean and variance property is a prerequisite of applying the Poisson model to any frequency dataset. The NB model is suitable for frequency data, which exhibits the assumption of unequal mean and variance (known as overdispersion or underdispersion of frequency data). One typical characteristic of crash frequency data is the excessive zero occurrences of crash frequency, which over-represents zeros in the standard NB model. Modeling frameworks such as the zero-inflated, hurdle, and zero-inflated mixed effect models are specifically developed to address the "excessive zeroes" issue in frequency data. For example, Yang et al. (2016) applied the zero-inflated negative binomial (ZINB) model to estimate transit trip frequency (Yang et al. 2016). Unlike Poisson and NB models, hurdle and zero-inflated models assume crashes as a phenomenon following a dual-state process, and therefore model zero and non-zero crash frequencies (therefore inferring a location is either safe or unsafe, respectively) as different processes. The dual-state assumption, particularly in ZINB for crash modeling, has been criticized by researchers for lack of clear distinction between the states (Lord et al. 2005, 2007). In their research, Lord et al. (2007) presented a simple yet intuitive example to demonstrate this. For example, assuming two adjacent segments have identical geometry, their state is determined by highway geometry alone. In such a case, if one segment is classified as unsafe, what makes it so? If two different states are assumed, it would be more appropriate to analyze them independently rather than together using a single model (Lord et al. 2007). In the case of missing data, there is also a possibility that the dual-state assumption will lead to erroneous inferences, resulting in the classification of unsafe locations as safe. As a result, the use of single, less complex models (Poisson and NB) is often more suitable for crash modeling (Lord et al. 2007). Therefore, in the current study, to model the occurrence of DWD crashes, account for unobserved heterogeneity in the data, compare the relative fit of fixed versus mixed effects count models and select the best model, both fixed and mixed effects models based on Poisson and NB regression were employed. Goodness-of-fit measures were used to compare models and select the best fit for the current DWD

dataset. In the following subsections, a description of each of the models' formulations is discussed.

Poisson Model

According to the Poisson model, the probability of occurrence of a certain frequency event (y_i) with respect to the vector of covariates (X_i) is expressed as

$$P(Y_i = y_i | X_i) = \frac{e^{-\lambda_i} \times \lambda_i^{y_i}}{y_i!}, \quad y_i = 0, 1, 2, \dots \quad (1)$$

The distribution mean (λ_i) is a function of the covariate vector and expressed as

$$E(y_i | X_i) = \lambda_i = \exp(X_i' \beta) \quad (2)$$

where X_i , X_i' , and β = vector of independent variables, vector of exogenous variables, and corresponding coefficient, respectively.

Poisson Mixed Effect Model

The Poisson model with fixed effects can be extended to incorporate random effects by estimating a coefficient for event y_{ij} (event y_i belonging to group j) and a random component u_j for every group j . The extended model capable of incorporating fixed and random effects is called the Poisson mixed effect model. The probability is expressed as

$$P(Y_{ij} = y_{ij} | X_{ij}) = \frac{e^{-\lambda_{ij}} \times \lambda_{ij}^{y_{ij}}}{y_{ij}!}, \quad y_{ij} = 0, 1, 2, \dots \quad (3)$$

The distribution mean (λ_{ij}) is a function of the covariate vector and expressed as

$$E(y_{ij} | X_{ij}) = \exp(X_{ij}' \beta + u_j) \quad (4)$$

where X_i , X_{ij}' , β , and u_j = vector of independent variables, vector of exogenous variables, corresponding coefficient, and the random effect component, which is normally distributed with zero mean, respectively.

NB Model

According to the NB model, the probability of occurrence of a certain frequency event (y_i) depends on the mean (λ) and parameter of dispersion ($\theta > 0$), and is expressed as

$$P(Y = y | \lambda, \theta) = \left(\frac{\theta}{\theta + \lambda} \right)^\theta \times \frac{\Gamma(\theta + y)}{\Gamma(y + 1)\Gamma(\theta)} \times \left(\frac{\lambda}{\theta + \lambda} \right)^y \quad (5)$$

where the gamma function (Γ) is defined as

$$\Gamma(t) = \begin{cases} \int_0^\infty X^{t-1} e^{-X} dx & \text{for positive noninteger } t \\ (t-1)! & \text{for positive integer } t \end{cases} \quad (6)$$

where $v(\lambda + \lambda^2/\theta)$, θ , and λ are the variance, overdispersion parameter and mean of the NB distribution, respectively.

NB Mixed Effect Model

Using the same notation for events and groups introduced earlier for the Poisson models, the probability of occurrence of an event in a group (y_{ij}) according to the NB model depends on the means (λ_{ij}) and the dispersion parameter θ , as follows:

$$P(Y = y_{ij} | \lambda_{ij}, \theta) = \left(\frac{\theta}{\theta + \lambda_{ij}} \right)^\theta \times \frac{\Gamma(\theta + y_{ij})}{\Gamma(y_{ij} + 1)\Gamma(\theta)} \times \left(\frac{\lambda_{ij}}{\theta + \lambda_{ij}} \right)^{y_{ij}} \quad (7)$$

The gamma function (Γ) is defined as identical to the fixed NB model. The variance, overdispersion parameter, and means of the distribution are $v(\lambda_{ij} + \lambda_{ij}^2/\theta)$, θ , and λ_{ij} , respectively.

Data

In the current study, data on DWD crashes were collected from all reported crashes in the State of Minnesota for the years between 2010 and 2015 (HSIS 2020). Table 1 presents the descriptive statistics of all continuous and categorical independent variables considered for modeling. A total of 2,787 DWD crashes were observed during that period. The crash data were coupled with additional features such as exposure and highway geometry to form a complete database.

The dataset consists of 252,757 unique roadway segments for the entire MN roadway system, which includes both zero and non-zero DWD crash segments. The segments included different functional classes, including principal arterials (including freeways and expressways), minor arterials, collectors, and local system roadways. Some additional features were added to the dataset such as distance to upstream and downstream junctions (i.e., merge/diverge points). These limits were determined based on detailed mileposts and junction descriptions for each segment as provided in the dataset. These mileposts specify the beginning and end of a roadway segment.

Model Results and Findings

This section presents a comparison of candidate DWD frequency prediction models based on goodness-of-fit measures and marginal effects of significant variables in the best-fit model.

Model Comparison

Four candidate count estimation models were considered in this study to estimate the DWD crash frequency on MN roadway segments, namely, Poisson model [Model #1], Poisson mixed effect model [Model #2], NB model [Model #3], and NB mixed effect model [Model #4]. A performance comparison of these four models, including goodness-of-fit measures for each model, is presented in Table 2. Coefficients of all statistically significant variables, including t -value of each variable (presented in parenthesis of coefficient value) of each model are also listed in Table 2. The goodness-of-fit measures, calculated to compare the performance of all models, are log-likelihood (LL) values of the null model (constants-only model with no covariates), LL of the converged model (full model with all parameters), log-likelihood ratio (LR) test statistic for each set of null/converged models, the Akaike information criterion (AIC), and the Bayesian information criterion (BIC) for all models (Akaike 1987). It is expected that the mean and variance are equal to apply the Poisson model (Model #1) to any count datasets for frequency estimation. Thus, statistical verification of equal mean and variance is a prerequisite of applying the Poisson model, and many count datasets might not exhibit this characteristic. The NB model is used to address the frequent presence of overdispersion property in count data. NB is a generalized form of Poisson's model and includes an additional model parameter (θ) to account for overdispersion property.

Table 1. Descriptive statistics of variables considered for modeling

Categorical variables					
Variable	Count	%	Variable	Count	%
Control-of-access			Divided		
None	248,369	98.26	Yes	16,471	6.52
Partial	2,789	1.10	No	236,286	93.48
Full	1,599	0.63	Curbed		
Median			Yes	66,351	26.25
Yes	15,210	6.02	No	186,406	73.75
No	237,547	93.98	Parking type		
Left Shoulder type			None	144,421	56.00
Paved	28,048	11.10	Diagonal	453	1.00
None	206,273	81.61	Mixed	400	1.00
Gravel	18,436	7.29	Parallel	107,483	42.00
Surface type			Urban Municipality		
Paved	151,643	60.00	Urban	107,015	42.34
Unpaved	101,114	40.00	Rural	145,742	57.66
Right shoulder type			Lane width		
Paved	30,351	12.01	<12 ft	78,259	30.96
None	203,762	80.62	=12 ft	108,152	42.79
Gravel	18,644	7.38	>12 ft	66,346	26.25
Additional lanes			Functional class		
Acceleration/Deceleration	484	1.00	Local	184,387	72.95
Escape	1,798	0.71	Collector	29,020	11.48
None	250,475	98.00	Principal arterial	17,971	7.11
Sidewalk			Minor arterial	21,379	8.46
Yes	33,996	13.45			
No	218,761	86.55			
Continuous Variables					
Variable	Min.	1st Qu.	Mean	3rd Qu.	Max.
AADT	5.0	82.0	2,262.0	832.0	196,333.0
Log of AADT	0.7	2.0	2.6	2.9	5.3
Left shoulder width (ft)	0	0	1.7	3.0	29.0
Right shoulder width (ft)	0	0	2.0	3.0	29.0
Total number of lanes	2.0	2.0	2.2	2.0	10.0
Speed limit (mph)	10.0	30.0	40.1	55.0	70.0
Downstream junction (mi)	0	0	1.8	1.0	8.5
Upstream junction (mi)	0	0	3.3	7.3	10.2

The signs and magnitude of variable coefficients for the models and mixed models in Table 2 are similar, which indicates consistency in the effects of the estimated variables, although some variables were not statistically significant across all the models. For example, the indicator variable for lane width = 12 ft was statistically significant only in the Poisson model. It is noteworthy that indicator variables for the presence of median and divided roadways were correlated with a correlation coefficient of 0.8. Therefore, only the indicator variable for divided roadways was included in four models. Variables that were statistically significant in estimating DWD crash frequencies for the four candidate models considered in this study include log of average annual daily traffic (AADT), principal arterial, minor arterial, collector, number of lanes, lane width, escape lanes, acceleration/deceleration lanes, sidewalk, parallel parking, curb, left shoulder width, left shoulder gravel, speed limit and divided. The AADT variable had a positive relationship with DWD crash frequency in all models. The reason for this relationship is that a higher number of vehicles on a roadway increases the possibility of a higher number of distracted or intoxicated drivers attempting improper maneuvers and resulting in a DWD crash. Many studies found that AADT and crash frequency have a positive relationship. For example, Ponnaluri (2016) reported that a higher AADT value increased the odds of WWD crashes. In addition to other geometric and traffic variables, a crash frequency estimation model using crash data in Central Florida

found that AADT has the highest relative effect on the likelihood of crash occurrence on a roadway segment (Abdel-Aty and Wang 2006). Number of lanes also has a positive effect on DWD crash frequency, as more lanes generally carry higher traffic volume. A positive relationship of AADT and the number of lanes with DWD crash frequency suggests that transportation agencies should consider further investigation for installing DWD-related countermeasures at high-volume roadways with a higher number of lanes. Potential countermeasures in this regard can be a combination of signage and pavement markings that are more visible, use of intelligent transportation systems, and geometric modifications on ramps. For example, countermeasures such as the use of lowered signs displaying “Wrong Way” or “Do Not Enter” and use more visible pavements markings to delineate exit ramps, use of flashing beacons with detectors to warn wrong-way drivers, and avoidance of left side exit ramps have been adopted by agencies across the country (Sandt et al. 2015).

Results from Poisson and NB models suggest that higher functional class roadways were less likely to experience DWD crashes than lower functional classes. While the higher functional class of roadways (e.g., principal arterials) generally carries higher traffic volume, fewer DWD events occur on higher functional class roadway segments because of the high level of access control. Lane width of 12 ft or less was found to negatively affect DWD crash frequencies. The likely reason for this finding is that wider lanes

Table 2. Model results

Variables	Poisson model (Model 1)	Poisson mixed effect model (Model 2)	NB model (Model 3)	NB mixed effect model (Model 4)
Count estimation				
Intercept	−11.21 (−61.19) ^{***}	−11.63 (−67.33) ^{***}	−11.43 (−59.14) ^{***}	−11.68 (−66.88) ^{***}
Log (AADT)	1.85 (33.11) ^{***}	1.96 (40.25) ^{***}	1.88 (32.05) ^{***}	1.97 (39.58) ^{***}
Principal arterial	0.24 (2.54) [*]	—	0.20 (2.01) [*]	—
Minor arterial	0.66 (8.48) ^{***}	0.46 (7.06) ^{***}	0.61 (7.42) ^{***}	0.45 (7.03) ^{***}
Collector	0.65 (9.15) ^{***}	0.49 (7.64) ^{***}	0.63 (8.42) ^{***}	0.49 (7.63) ^{***}
Number of lanes	0.18 (9.36) ^{***}	0.14 (5.15) ^{***}	0.19 (8.06) ^{***}	0.14 (5.15) ^{***}
Lane width <12 ft	−0.17 (−1.86) ^{****}	—	—	—
Lane width = 12 ft	−0.10 (−2.13) [*]	—	—	—
Escape lanes	−1.01 (−5.26) ^{***}	−1.07 (−4.21) ^{***}	−1.03 (−5.18) ^{***}	−1.11 (−4.31) ^{***}
Acceleration lanes	−0.76 (−2.72) ^{**}	−0.67 (−2.09) [*]	−0.74 (−2.55) [*]	−0.70 (−2.15) [*]
Sidewalk	0.44 (8.89) ^{***}	0.40 (7.79) ^{***}	0.44 (8.37) ^{***}	0.41 (7.57) ^{***}
Left shoulder gravel	0.38 (5.62) ^{***}	0.37 (5.28) ^{***}	0.38 (5.23) ^{***}	0.37 (5.06) ^{***}
Speed limit (mph)	0.004 (2.30) [*]	0.009 (4.65) ^{***}	0.006 (2.78) ^{**}	0.010 (4.73) ^{***}
Divided	−1.38 (−20.13) ^{***}	−1.27 (−16.60) ^{***}	−1.40 (−18.57) ^{***}	−1.29 (−15.86) ^{***}
Log (θ)	—	—	0.33 (11.22) ^{***}	0.51 (10.21) ^{***}
Random parameters				
Log (AADT)	—	0.186	—	—
Minor arterial	—	0.464	—	0.359
Collector	—	0.710	—	0.539
Number of lanes	—	0.156	—	0.135
Escape lanes	—	1.069	—	1.037
Acceleration lanes	—	0.867	—	0.754
Measures-of-fit				
Degrees of freedom	14	17	13	17
LL (null model)	−15,572.73	−14,240.99	−15,572.73	−14,055.6
LL (converged model)	−12,599.35	−12,472.54	−12,413.78	−12,361.50
LR stat (null/converged)	5,946.71	3,536.93	5,162.89	3,388.22
AIC	25,227	24,979	24,854	24,757
BIC	25,373	25,156	24,989	24,935

Note: Significant levels: *0.05, **0.01, ***0.001, and ****0.1.

(e.g., two-lane roadways with extra lane width for roadside parking/bike lane) encourage drivers to travel at higher than the assigned segment speed limit when there are no/limited parked vehicles/bike/pedestrian activities. The presence of acceleration lanes was found to be associated with decreased DWD crash frequency, as drivers are more careful at a merge junction (e.g., intersection, ramp) accompanied by acceleration lanes. Additional lanes (e.g., acceleration lane) on certain segments may contribute to narrower mainline lanes due to possible right-of-way limitations, and therefore drivers are more cautious and travel at lower speeds to stay within narrow lanes. Furthermore, additional lanes are more likely to be on the right side of the roadway, which reduces the chances of vehicles crossing a median into opposing traffic—however, this may not be the case for all segments, as additional lanes may also exist on the left sides of some segments.

Divided highways negatively contribute to higher risks for DWD crash frequencies. This finding is intuitive and is likely due to the lower chances of distracted/intoxicated drivers crossing the center lane/lines into opposing traffic. This finding is well supported by current highway design standards because medians and barriers (e.g., cable barrier, raised or depressed median) are widely used to separate opposing traffic, especially on high-speed roadways. Due to the associated higher costs of constructing rigid barriers, medians, or limited right-of-way, lower functional class roadways are usually undivided. Left gravel shoulder was found to be associated with increased risks for higher DWD crash frequencies. This result is intuitive, as gravel shoulders are typically not striped, and especially if wide enough, can lead distracted drivers

to enter a gravel shoulder, either intentionally or unintentionally, which ultimately can contribute to a cross-over DWD event.

These findings provide valuable information on safety-oriented strategies that could be adopted to mitigate DWD. Since DWD is less likely to occur on higher functional class roadways, safety countermeasures can be allocated and prioritized on lower functional class roadways. Similarly, locations and segments where drivers are more confident driving at higher speeds could be provided with better signage and markings to avoid DWD behavior. As acceleration lanes decrease the possibility of DWD, highway ramps and exits can be prioritized to be on the right side. Our findings suggest that clear demarcation of opposing lanes using markings and medians can reduce DWD crashes. Because lower functional class roadways typically are undivided highways and are also more likely to experience DWD crashes, better signage and marking and use of loop detectors and beacons in place of medians can alert DWD drivers and potentially prevent crashes.

Model Validation and Measures-of-Fit

To validate the models, root mean squared error (RMSE) was calculated between the predicted number of crashes and the observed number of crashes. The calculated RMSE is presented in Table 3. As the cumulative RMSE is often skewed by the prediction errors across different observed counts, the table presents RMSE for different counts observed in the highway segments (min = 0, max = 8). The incorporation of mixed effects in both Poisson and NB regression results in lower RMSE and, therefore, better prediction. Among the mixed models 2 and 4, Model 2 fits better

Table 3. RMSE for model predictions

DWD counts	Poisson model (Model 1)	Poisson mixed effect model (Model 2)	NB model (Model 3)	NB mixed effect model (Model 4)
0	0.0798	0.0860	0.0804	0.0791
1	0.9497	0.9347	0.9495	0.9404
2	1.8996	1.8253	1.9010	1.8550
3	2.8936	2.7609	2.8906	2.8314
4	3.7608	3.2306	3.7742	3.4832
5	4.8325	4.4757	4.8302	4.6621
6	5.9613	5.8943	5.9606	5.9032
7	6.7899	4.9353	6.8108	6.1964
8	7.7680	6.6816	7.7765	7.3044

for larger DWD crash counts as suggested by a smaller value of RMSE for DWD counts greater than 0. On the contrary, Model 4 fits 0 counts (no crash occurrences) better.

The calculated goodness-of-fit measures for four candidate DWD crash frequency models include LL of the null and converged models, LR test statistics (null/converged), AIC, and BIC values for all models. Additionally, adding more variables could increase the goodness-of-fit, and could result in an overfitted model. The AIC and BIC measures control for the model's overfitting by introducing a penalty term in their calculation (Akaike 1987; Schwarz 1978). The lowest BIC value model is the best-fit model considering the overfitting issue. Based on the goodness-of-fit values presented in Table 2, the NB mixed effect model has the lowest AIC and BIC values (24,757 and 24,935, respectively) and therefore outperformed all other proposed models. In the following subsection, the elasticity effects of significant variables in the NB mixed effect model are discussed.

Marginal Effects of Significant Variables

Table 4 summarizes the elasticity values of statistically significant independent variables of the NB mixed effect model. The 95% confidence interval for the elasticities is also provided in parentheses. Elasticity values explain the contribution of independent variables on the outcomes of the dependent variable. In the context of this research, the marginal effect represents the unit change (increase/decrease) in DWD crash frequency due to the unit change (increase/decrease) of an independent variable (e.g., AADT) in its corresponding measuring unit. For example, the marginal effect of AADT indicated that a unit increase in the value of the log of AADT increased DWD crash frequency by about 1.96, which is the highest value among all independent variables.

Table 4. Marginal effects of NB mixed effect model

Variables	Marginal effects
Log (AADT)	1.961 [1.871, 2.066]
Minor arterial ^a	0.448 [0.322, 0.573]
Collector ^a	0.498 [0.370, 0.625]
Number of lanes	0.143 [0.088, 0.200]
Escape lanes ^a	−1.115 [−1.622, −0.608]
Acceleration lanes ^a	−0.698 [−1.325, −0.071]
Sidewalk ^a	0.413 [0.306, 0.519]
Left shoulder gravel ^a	0.375 [0.230, 0.519]
Speed limit (mph)	0.010 [0.006, 0.014]
Divided ^a	−1.292 [−1.452, −1.133]

^aIndicator variables.

Similarly, with a unit increase in the number of lanes, DWD crash frequency increased by 0.143. The effect of speed limit on crashes was negligible compared with other predictors. This was expected because its coefficient in the ZINB model was also small. To be specific, there was only about 0.01 increase in DWD crashes with a unit increase in the posted speed limit. In the case of indicator variables, the elasticity is interpreted in terms of the base category. For example, the frequency of DWD crashes increased by 0.448 on minor arterials compared with other roadways. The increase in DWD on collector roads was approximately 0.5 compared with other roads. On roadway segments with escape and acceleration, however, DWD crashes decreased by about 1.12 and 0.7, respectively. Similarly, the presence of sidewalks on roadway segments, which, as previously mentioned, indicates lower functional class roadways, increases DWD crashes by 0.413. The frequency of DWD crashes increased by 0.375 on roadway segments with a graveled left shoulder. Similarly, on divided highways, the frequency of DWD crashes decreased by 1.292 compared with nondivided highways.

Conclusions

Transportation agencies have considered WWD events a major safety issue because these events are associated with higher fatality rates and higher injury severity levels than other crash types. WWD has been defined by FHWA as well as in the safety literature as a vehicle entering the opposing direction of the legal flow of traffic exclusively on high-speed roadways (i.e., freeways) and ramps. Screening of crash data in Minnesota revealed that crashes while driving in the wrong direction (DWD) occurred on all roadway functional classes. Therefore, the current study adopted the new term, with the acronym DWD, to refer to those events taking place on any roadway functional class, while preserving FHWA's definition of WWD events. While several studies investigated WWD crash severity and developed crash severity models, none of the studies developed DWD crash frequency models to determine high-risk roadway segments for such types of crashes. The contributions of this research are (1) introduction of a new acronym, DWD, to the literature while preserving WWD as defined by FHWA, (2) determination of the applicability of different count data models for DWD crash frequency estimation, and (3) identification of significant crash contributing factors, and the relative effect of each contributing factor on DWD crash frequency outcomes for different roadway segments. In this study, four frequency estimation models were proposed: Poisson model, Poisson mixed effects model, negative binomial (NB) model, and NB mixed effect model. Based on four goodness-of-fit measures (i.e., log-likelihood values, LR tests, AIC, and BIC), the NB mixed effect model was found to best fit the current DWD dataset. It was found that AADT, number of lanes, presence of sidewalk, left-side gravel shoulder, and speed limit had positive effects on DWD crash frequencies. Factors that had a negative effect on DWD frequencies included presence of escape lanes, presence of acceleration/deceleration lanes, and divided roadways. Elasticity values of significant independent variables of the NB mixed effect model indicated that AADT had the highest positive effects. Variables with the highest negative effects on segment risks for DWD crashes included the presence of escape lanes and divided roadways. Transportation agencies can use these findings in terms of significant contributing factors and their relative effects on DWD crash frequency to identify roadway segments of different roadway functional classes with high DWD crash risk, and to deploy appropriate countermeasures at such locations accordingly.

Limitations and Future Study

The current study has a few notable limitations that can be investigated in future studies. First, our analysis did not consider the influence of weather conditions, and relied solely on the effect of highway geometry. DWD might be more likely to occur during adverse weather or poor lighting conditions (e.g., during heavy rain or in the absence of adequate lighting) when visibility is an issue. Second, our dataset does not consider the underlying causes behind DWD. Drivers under the influence, distracted, or inexperienced may be more likely to be involved in a DWD event. If a considerable share of DWDs is a result of these underlying causes rather than driving error and geometry, it would be imperative to consider them in the modeling process. Additionally, our models did not include temporal and spatial correlation. The inclusion of these correlations may provide better insight into DWD crashes. Future research can also include a broader range of datasets that include multistate DWD crash data to increase confidence that the study results can be used nationwide. Additionally, WWD/DWD crashes are considered rare compared with all other types of crashes. Future research could use more combined years of DWD crash frequency data to increase sample size and gain more in-depth insights into the effects of the considered variables. Finally, utilizing interaction terms between the different roadway geometric features will allow even more in-depth insight into the effects of certain features, for example, the interaction between the roadway division indicator variable as a function of another variable such as speed or urban municipality. Researchers can also account for temporal instability in future studies.

Data Availability Statement

Some or all data, models, or code used during the study were provided by a third party. Direct requests for these materials may be made to the provider as indicated in the Acknowledgments. Crash, traffic, and roadway data of Minnesota were collected from the Highway Safety Performance System (HSIS).

Acknowledgments

This study is supported by the Tennessee Department of Transportation. Crash, traffic, and roadway data of Minnesota were collected from the Highway Safety Performance System (HSIS), <https://www.hsisinfo.org/>. The authors confirm contributions to the paper as follows: study conception and design, M. Osman, K. Dey, and S. Mishra; data collection, M. Osman, S. Mishra, and S. El Said; analysis and interpretation of results, M. Osman, K. Dey, S. Mishra, S. El Said, and D. Thapa; draft manuscript preparation, M. Osman, K. Dey, S. Mishra, and D. Thapa. All authors reviewed the results and approved the final version of the manuscript.

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