Physic-Informed Neural Network Approach Coupled with Boundary Conditions for Solving 1D Steady Shallow Water Equations for Riverine System

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ABSTRACT

Shallow water equations (SWE) are the governing equations for the open channel flow. The numerical solution is widely considered the most effective approach for solving the SWE in the past few decades. However, numerical solutions are inefficient and need to compromise many aspects, such as order of scheme accuracy, Courant numbers, boundness, etc. In recent years, deep learning (DL) has been one of the rapidly rising techniques that have been widely used in the engineering field. DL models can bridge approximation relations between input and output variables by conducting multiple elementary operations constructed by artificial neural networks. Many researchers achieved success in hydrology and hydraulic problems by using DL models. However, there are still some drawbacks to the previous DL models. These DL models are often purely empirical and not constrained by real physics, which may cause a larger prediction error when test conditions are not included in the training data set. Besides, training this model requires big data, which is mostly expensive in hydrology and hydraulic problems. In this paper, we will introduce a novel and data-free neural network framework that can solve the SWE. The architecture of the framework will be demonstrated in detail, and the framework can be applied to any SWE problems. Additionally, we employed a numerical solver, HEC-RAS, as reference to verify the solution accuracy. As a result, this framework shows great agreement with numerical solutions.

INTRODUCTION

Shallow water equations (SWEs), derived from depth-averaged Navier Stroke equations, are the governing equations for all open channel flow problem by its nature (Brunner 2002; Zhou 1995; Stansby and Zhou 1998). SWEs belongs to hyperbolic partial differential equations (PDEs). Since most PDEs are proven that have no analytical solution, the numerical approaches are widely used to the best method for solving these PDEs (Eivazi et al. 2022; Huang et al. 2022). However, numerical solutions are inefficient and need to compromise many aspects, such as order of scheme accuracy, Courant numbers, boundness, etc. In recent years, many researchers have investigated on new techniques to solve the non-linear PDEs for conventional engineering problems. Deep learning (DL) is one of the rapidly rising and latest techniques that have been

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widely used in the engineering field. Instead of solving PDEs directly, DL can bridge approximation relations between input and output variables by conducting multiple elementary operations constructed by artificial neural networks (ANN). The loss function is normally constructed to describe the difference between ANN outputs and observational ground truth. An optimizer will be used in the models to minimize the loss value. By minimizing the loss function, ideally to zero, through various optimizations, the ANN model can predict the very close result to real solutions.

In water resource engineering problems, may researchers achieved great success by using this end-to-end, black-box DL model (Yin et al. 2022, Zhang et al. 2020, Zahura et al. 2020, Adnan et al. 2021, Tamiru and Dinka 2021). However, this black-box data-driven DL model have several drawbacks. Firstly, it requires a large amount of on-site monitoring data for training. Lack amount of data will significantly harm the model accuracy. However, such amount of on-site monitoring data is mostly very expensive to obtain in most water systems. This type of data-driven model will not even work if there is no data for feeding. Secondly, since this type of ML is a pure black box model, the computation process is almost impossible to be interpreted and transferred to human knowledge. This situation is often described as “data rich, knowledge poor” (Iskhakov and Dinh 2020). Lastly, many of these DL models are not real physics-based models even most of them are using physical parameters. In real physics-based models, the input variables need to be sufficient, which is sometimes hard to achieve in many cases. Besides, since there is not any constraints during the optimization process, the regression results may become nonsense when using insufficient input variables. This problem is more obvious when facing a complicated problem.

To overcome this problem, a new class of deep learning: Physic-informed Neural Networks (PINN) was composed in recent years (Cai et al. 2022). Instead of minimizing L1 or L2 norm from training data, PINN minimizes the L1 or L2 norm of the loss function that is constructed by the governing equations (PDEs mostly). PINN has achieved great success in many fields, including computational fluid dynamics (CFD) (Mao et al. 2020, Yang et al. 2019, Jin et al. 2021), heat transfer (Cai et al. 2021, Bararnia and Esmaeilpour 2022), etc. My previous research is the first paper that coupled PINN with terrain information to solve 1D unsteady SWEs in real applications level. The test case is validated by a hypothetical scenario on an artificial channel and a historical scenario on downstream Cypress Creek, Houston, TX. The PINN showed a great agreement with both water station records and numerical solver (HEC-RAS).

However, steady flow simulation is more popular in the industry field, and there is no PINN framework to solve the steady shallow water problem. Furthermore, using PINN for solving steady SWEs have several advantages that unsteady PINN solver cannot achieve. In unsteady PINN solver, extrapolation along the spatial direction and temporal direction is the major feature that numerical solver cannot achieve. However, the neural network still needs to be trained again if a completely different event needs to be simulated. Due to complexity of time series data, coupling boundary conditions as input variables for unsteady PINN will become a higher dimensional problem, and it is meaningless because it requires an impossible long period for training the neural network. This problem does not exist in steady PINN solver because the boundary conditions in 1D steady SWEs is scalar, and coupling it as input variable will be much less complicated and more realistic. To achieve this objective, this paper will demonstrate the structure and mechanism of PINN in details. Tenmile Canal downstream of US 41 in Lee County will be used as the case study for validate the model. The numerical solver (HEC-RAS) will be used as the reference to test the model performance.

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METHODOLOGY

Study Area

Tenmile canal is a 20.3 mile stream, which drains an area that starts in Fort Myers and travel south to Mullock Creek. In the past decades, it shows high level of environmental diversity. However, the floods usually occur in many significant stormwater events. During the heavy stormwater event on 08/29/2017 and 04/02/2020, severe flooding scenario was observed, and it caused a number of property loss. The Photograph of flooding situation during the stormwater event on 08/29/2017 is taken by Mark White, South Florida Water Management District (SFWMD) (shown in Figure 1a). Thus, using Tenmile canal, FL as study area is meaningful and representative. The terrain information is shown as RAS map format in Figure 1b. The HEC-RAS used as reference in this paper is built and calibrated by Johnson Engineering, Inc and SFWMD. Same digital elevation model and bathymetry data will be used to extract the cross sectional terrain shape in steady PINN model. Similar to unsteady PINN model, this steady PINN model will also be a data-free model, and will not take any HEC-RAS model to train the neural network.

(a) (b)

Figure 1. (a) Photograph of flooding situation during the stormwater event on 08/29/2017; (b) elevation map of the study area: Tenmile canal, FL

PINN framework

The 1D steady PINN frameworks have three steps to solve the problem: forward, loss function construction, and backward. The overall PINN framework diagram for 1D SWEs used in this paper is shown in Figure 2. In the forward step, a fully connected neural network is employed to predict the output: $\hat{u}$ and $\hat{h}$ based on the input: x and Q. Since the downstream boundary conditions are usually given as friction slope for normal depth, the downstream boundary condition does not need to change to simulate different scenario. That is also the reason that there is only upstream boundary condition, flow rate: Q, used as input variables in

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this work. Once the equation outputs, $\hat{u}$ and $\hat{h}$, are predicted, automatic differentiation can be used to get two first-order partial derivative terms of outputs: $\frac{\partial q}{\partial x}$, and $\frac{\partial h}{\partial x}$. Besides, with the information of DEM data, the numerical methods are employed to get three hydraulic parameters: A, R, P. Since all these elements are available, it is possible to construct the mass and momentum of SWEs. Before constructing the loss function, L2 norm operation is added both on the shallow water equations and boundary conditions to make sure the loss value is always positive in this paper. The L2 norm operation can be expressed as Eq. (1).

$$\|x\|^2 = \sqrt{\sum_{k=1}^{n} x_k^2} \tag{1}$$

where n is the total number of sample points.

In this paper, the loss function is constructed by the summation of four components: loss on the mass equation, loss on the momentum equation, loss on the upstream boundary condition, and loss on the downstream boundary condition. Once the loss function is constructed, the backward process can be conducted. In the backward process, Adam optimizer is employed to optimize the loss function. The final loss function value will be sent back to the beginning of the framework to achieve a closed-loop optimization.

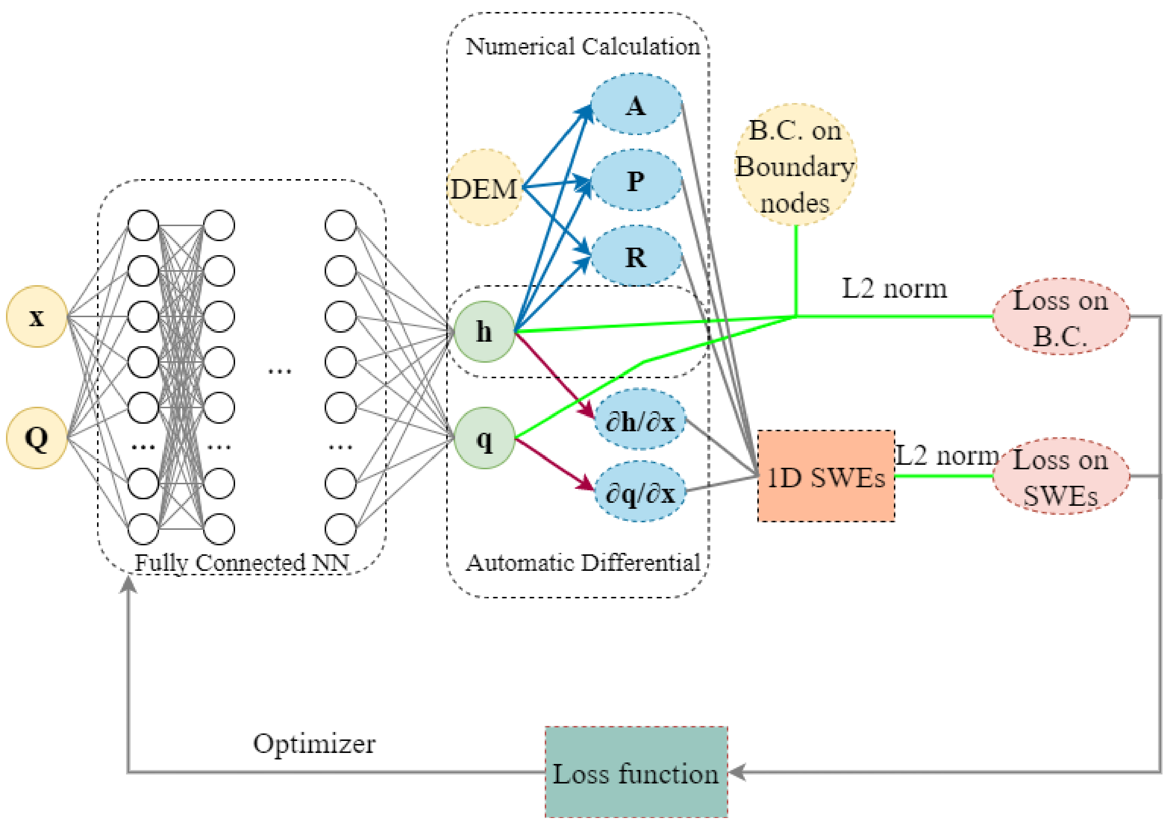

Figure 2. Physic-informed Neural Network (PINN) Framework Diagram for 1D Steady Shallow Water Equations

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Numerical Calculations on Hydraulic Parameters

The hydraulic parameters at each cross section are needed in order to construct the SWEs. These hydraulic parameters are usually hard to obtain analytically because the shape of the cross sections is highly irregular and complex most of the time. To overcome this problem and generalize the framework, a numerical method based on DEM data is used in this work to obtain the hydraulic parameters: A, R, P. From the DEM data, we can get a finite number of sample points depending on the DEM resolution. Each sample point contains two values: [x, y] where x represents the cross-sectional distance from the start point, and y stands for the elevation on these sample points. To simplify the numerical calculation, the x-axis is transformed from the original position to the predicted water depth by subtracting the entire elevation value from the predicted water depth. The transformation for numerical calculations is illustrated in Figure 3.

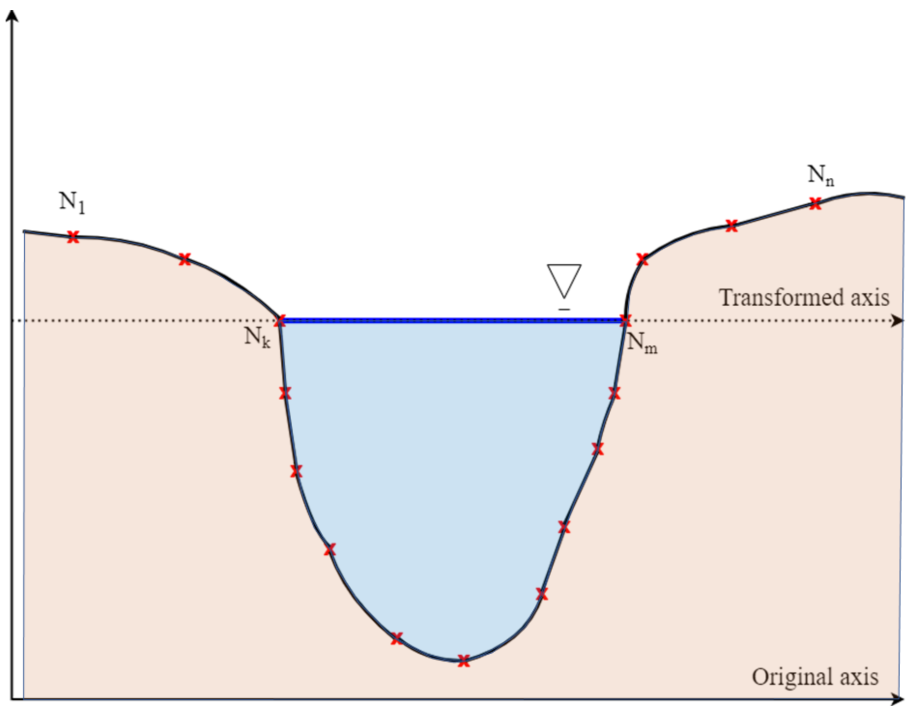

Figure 3. Transformation for Numerical Method to Calculate Hydraulic Parameters

After the transformation, all sample points with positive elevation values can be ignored. The remaining sample points, from N_k to N_m as shown in Figure 3, are used for the numerical calculation. By applying the trapezoidal rule, the cross-sectional wetted area can be expressed as Eq. (2). The wetted perimeters can be written as the summation of the truncated length of all neighboring points, which is expressed as Eq. (3). The hydraulic radius, expressed as Eq. (4), is from simply dividing cross-sectional wetted area by wetted parameters. Since water free surface is always parallel to the x-axis, the top width can be calculated by a simple subtraction between the first and the last used points, which is expressed as Eq. (5).

$$A = -\sum_{i=k}^{m-1} \frac{(y_{i+1} + y_i) + (x_{i+1} - x_i)}{2} \tag{2}$$

$$P = \sum_{i=k}^{m-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad (3)$$

$$R = \frac{A}{P} \quad (4)$$

$$B = x_m - x_k \quad (5)$$

Automatic Differentiation

The differentiation in the conventional numerical methods is approximately calculated by the discretization, and its accuracy are highly dependent on the order of scheme. Furthermore, it could cause many problems during this procedure, such as convergence, conservativeness, boundness, and transportive. One of the benefits of neural networks is that the partial derivatives can be calculated easily by automatic differentiation. Thus, these problems do not exist in this framework. Automatic differentiation used in this framework can calculate the partial derivatives of neural network outputs evaluating their trace of composition. Since the computations in the neural networks consist of a finite set of elementary mathematical operations, the values in each elementary operation are known, and its derivatives can be calculated. Thus, the first-order partial derivatives of the neural network outputs, $\frac{\partial q}{\partial x}$ and $\frac{\partial h}{\partial x}$, can be calculated by forward propagating the derivatives of each elementary mathematical operations. A reserve mode of automatic differentiation is applied to obtain the derivatives of any constructed variables, which is also used during the backward optimization process.

RESULT AND DISCUSSION

Streamwise water stage prediction

The water stage prediction for upstream flow rate of 31 m^3/s from 1D steady PINN model is shown in Figure 4. The model results matches numerical solver (HEC-RAS) very well. If HEC-RAS results is used as ground truth, the mean absolute error for PINN model is 0.0238 m, and the root mean square error is 0.0042 m.

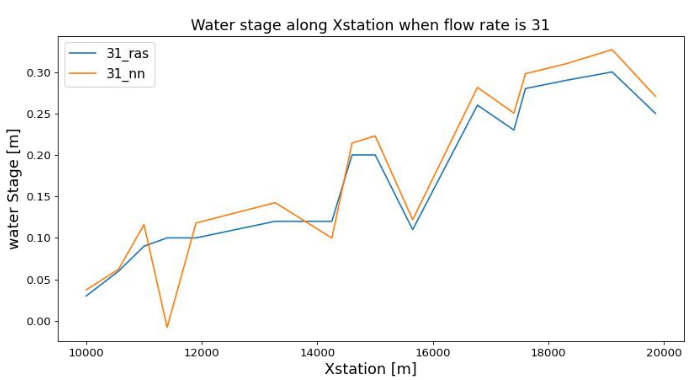

Figure 4. Water stage prediction along streamwise direction when upstream flow rate is 31 m^3/s.

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Extrapolation on upstream boundary conditions

The upstream flow rate used in training phase is ranged from 15-31 m^3/s. By the feature of neural networks, we could extrapolate results from different upstream flow rate as the test case. The water stage prediction for upstream flow rate of 32 m^3/s is shown in Figure 5. As Figure 5 shows, the accuracy of extrapolation is not very different from the trained case, which matches the expectation because the upstream boundary flow rate is coupled as the input variables in our framework.

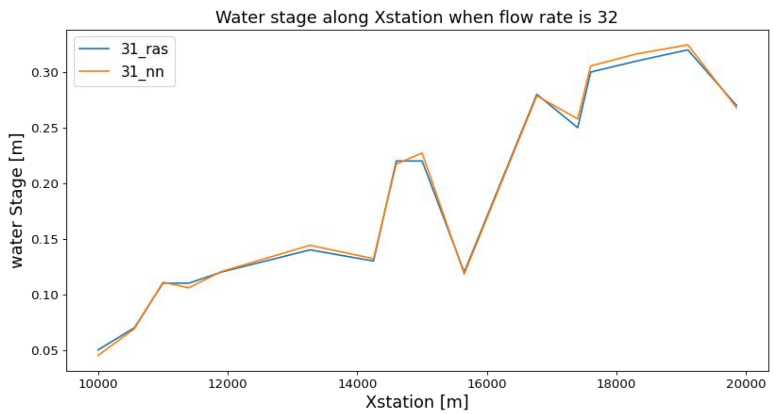

Figure 5. Extrapolated water stage prediction along streamwise direction when upstream flow rate is 32 m^3/s

CONCLUSION

In conclusion, this paper presented and tested a Physic-informed Neural Network (PINN) for solving the 1D steady shallow water equations with coupled upstream flow rate as input variable. The framework is tested by using Tenmile Canal downstream of US 41 in Lee County, FL, and the results showed a good agreement with numerical solver. The results also matched very well with HEC-RAS solutions in terms of extrapolation. One of the advance of this framework compared to other PINN models is that it converts the upstream boundary conditions into the input variables in the neural network so that it achieve the situation called "one training for all scenarios". In the future work, this framework are suggested to be applied on more complex 2D shallow water equations and it is able to visualize the 2D flood area.

NOMENCLATURE

Shallow water equations	SWEs
partial differential equations	PDEs
Machine learning	ML
Deep learning	DL
Artificial neural networks	ANN
Physic-informed Neural Network	PINN
Computational fluid dynamics	CFD
Digital elevation model	DEM

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Wetted cross sectional area	A
Wetted perimeter	P
Hydraulic radius	R
Top width of river	B
Water depth	h
Predicted water depth	$\hat{h}$
Cross sectional velocity	u
Predicted cross sectional velocity	$\hat{u}$
Cross sectional flow rate	Q
Mean squared error	MSE
Exponential linear unit	ELU
Boundary conditions	B.C.
Hydrologic Engineering Center's River Analysis System	HEC-RAS
Hydrologic Engineering Center's Hydrologic Modeling System	HEC-HMS
Mean absolute error	MAE

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