Standard model at 200 GeV

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The Standard Model can be defined quantitatively by running parameters in a mass-independent renormalization scheme at a fixed reference scale. We provide a set of simple interpolation formulas that give the fundamental Lagrangian parameters in the $\overline{\text{MS}}$ scheme at a renormalization scale of 200 GeV, safely above the top-quark mass and suitable for matching to candidate new physics models at very high mass scales using renormalization group equations. These interpolation formulas take as inputs the on-shell experimental quantities and use the best available calculations in the pure $\overline{\text{MS}}$ scheme. They also serve as an accounting of the parametric uncertainties for the short-distance Standard Model Lagrangian. We also include an interpolating formula for the W-boson mass. This paper is based on results obtained with the publicly available computer program SMDR.

I. INTRODUCTION

The Standard Model of fundamental particle physics has reached a high level of experimental maturity with the 2012 discovery of the Higgs boson and the increasingly accurate measurements of its mass, production modes, and decays. Meanwhile, the explorations of the Large Hadron Collider (LHC) at the high-energy frontier have not revealed any substantial and lasting deviations that would compel extensions of the Standard Model, despite many motivations for such new physics.

Given this state of affairs, it is useful to summarize as accurately as possible our quantitative knowledge of the Standard Model in terms of the Lagrangian parameters that define the theory. These defining parameters can then be matched to a larger set of parameters in candidate new physics theories characterized by large mass scales. It is convenient to use the $\overline{\text{MS}}$ renormalization scheme [1,2] based on dimensional regularization [3–7] for this purpose. The Standard Model $\overline{\text{MS}}$ Lagrangian parameters to be evaluated include

$$
\begin{aligned}
\text{Higgs sector:} & \quad \lambda, m^2, \\
\text{gauge couplings:} & \quad g_3, g, g', \\
\text{quark Yukawa couplings:} & \quad y_t, y_b, y_c, y_s, y_u, y_d, \\
\text{lepton Yukawa couplings:} & \quad y_\tau, y_\mu, y_e,
\end{aligned}
\tag{1.1}
$$

where we have neglected the neutrino sector. Also omitted here are the four physical parameters (three flavor-mixing angles and one CP-violating phase angle) associated with the Cabibbo-Kobayashi-Maskawa (CKM) matrix for quarks, which can be considered separately and decouple from the discussion below to a high degree of accuracy due to unitarity of the CKM matrix. For a discussion and numerical values of these four CKM angle parameters in the Standard Model, see the relevant section of the Review of Particle Properties (RPP) [8] published by the Particle Data Group (PDG), and references therein.

Each of the 14 quantities in Eq. (1.1) is a running parameter, dependent on the choice of $\overline{\text{MS}}$ renormalization scale Q, governed by renormalization group equations that are now known [9–35] with some effects through five-loop order. The normalization convention of the Higgs sector parameters is the now-standard one such that the tree-level potential for the canonically normalized real neutral component of the Higgs field is $V = \frac{1}{2}m^2H^2 + \frac{\lambda}{4}H^4$. In particular, the Higgs squared mass parameter m^2 is negative, as required by electroweak symmetry breaking, and can be traded for the vacuum expectation value v for H, defined as the minimum of the all-orders effective potential in Landau gauge, so that the sum of all tadpole diagrams (including the tree-level tadpole) simply vanishes. In practice, the effective potential is known fully at two-loop [36,37] and three-loop [38–40] orders, supplemented by the QCD four-loop contributions [33].

For the purposes of matching to ultraviolet new physics proposals, one should work in a nondecoupling scheme, in which all of the Standard Model particles including the top quark are propagating degrees of freedom. For this reason, we choose as a benchmark $\overline{\text{MS}}$ renormalization scale the value $Q = 200$ GeV, which is somewhat arbitrary but has

the advantages of being a round number, safely above the top-quark mass, and probably[1] well below the scale of new physics. Furthermore, taking a fixed scale (rather than, say, the experimental top-quark mass, which is subject to uncertainty and change) provides for better numerical stability. The results given here can then be evolved to any desired matching scale by the Standard Model renormalization group equations.

The standard reference for important experimental results in high-energy physics, the RPP [8] published by the PDG, instead (so far, at least) summarizes our knowledge of the Standard Model in terms of what we will refer to as the on-shell quantities. The RPP quantities that are in the most direct correspondence to the Lagrangian parameters in Eq. (1.1) are

$$
\begin{aligned}
\text{fine-structure constant:} \quad & \alpha = 1/137.035999084\ldots \text{and } \Delta\alpha_{\text{had}}^{(5)}(M_Z), \\
\text{Fermi decay constant:} \quad & G_F, \\
\text{5-quark QCD coupling:} \quad & \alpha_S^{(5)}(M_Z), \\
\text{heavy particle physical masses:} \quad & M_t, M_h, M_Z, M_W, \\
\text{running light quark masses:} \quad & m_b(m_b), m_c(m_c), m_s(2\text{ GeV}), m_u(2\text{ GeV}), m_d(2\text{ GeV}), \\
\text{lepton pole masses:} \quad & M_\tau, M_\mu, M_e.
\end{aligned}
\tag{1.2}
$$

It should be noted that in this paper M_Z and M_W are the on-shell masses in the PDG parametrization; these are related to the gauge-invariant complex pole squared masses $s_p = (M_p - i\Gamma_p/2)^2$ by $M = M_p(1+\delta)/\sqrt{1-\delta}$, where $\delta = \Gamma_p^2/4M_p^2$ in each case. (For recent discussions, see Refs. [41,42].) In principle, the hadronic contribution to the fine-structure constant, $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ is not independent and could be determined in terms of the other quantities, but in practice it cannot be perturbatively evaluated and therefore is taken as an independent experimental input. The W-boson mass M_W can also be determined in terms of the others. The other 14 on-shell quantities in Eq. (1.2) are dual to those of the 14 independent $\overline{\text{MS}}$ parameters in Eq. (1.1). This means that one can take the $\overline{\text{MS}}$ parameters as theoretical inputs with the on-shell quantities as outputs, or one can take the on-shell quantities as experimental inputs and view the $\overline{\text{MS}}$ parameters as the outputs.

A great deal of effort has gone into relating the two sets of parameters and to evaluating M_W in terms of the others. For a necessarily incomplete set of earlier references, see [43–106], and papers discussed therein. The present work is based on the computer code SMDR [107,108], which implements calculations in Refs. [42,109–113] in the tadpole-free pure $\overline{\text{MS}}$ scheme. This code contains command-line utilities and library programs for fitting the on-shell parameters in terms of the $\overline{\text{MS}}$ parameters, and vice versa, and for performing the renormalization group running in the Standard Model, implementing the state-of-the-art calculations.

The purpose of the present paper is to provide simple and convenient interpolation formulas that accurately provide the $\overline{\text{MS}}$ parameters in Eq. (1.1) and M_W in terms of the on-shell parameters in Eq. (1.2), obtained by doing a fit to the results[2] of SMDR. The simplicity and accuracy of the interpolation formulas is aided by the fact that the allowed parameters in the Standard Model are now all experimentally restricted to rather narrow ranges.

We now discuss the organization and notation of the interpolation formulas below. First, define a set of benchmark (denoted by subscript 0) on-shell inputs, using the values from the most recent 2022 PDG data:

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[1]In the case of supersymmetry, current bounds on squarks and gluinos are now generally well above 1 TeV, but there is still some room for light weakly interacting superpartners. More generally, despite the LHC's negative search results, the new physics might be only very weakly coupled to the Standard Model.

[2]More specifically, the interpolation formulas in the present paper have a similar functionality to the SMDR command-line invocation calc_fit -Q 200, but with a drastically shorter evaluation time.

$$\alpha_0 = 1/137.035999084, \qquad \Delta\alpha_{\text{had}}^{(5)}(M_Z)_0 = 0.027660,$$

$$G_{F0} = 1.1663787 \times 10^{-5}, \qquad \alpha_{S0}^{(5)}(M_Z) = 0.1179,$$

$$M_{t0} = 172.5 \text{ GeV}, \qquad M_{h0} = 125.25 \text{ GeV}, \qquad M_{Z0} = 91.1876 \text{ GeV},$$

$$m_b(m_b)_0 = 4.18 \text{ GeV}, \qquad m_c(m_c)_0 = 1.27 \text{ GeV}, \qquad m_s(2 \text{ GeV})_0 = 93 \text{ MeV},$$

$$m_u(2 \text{ GeV})_0 = 2.16 \text{ MeV}, \qquad m_d(2 \text{ GeV})_0 = 4.67 \text{ MeV},$$

$$M_{\tau 0} = 1.77686 \text{ GeV}, \qquad M_{\mu 0} = 0.1056583745 \text{ GeV},$$

$$M_{e0} = 0.5109989461 \text{ MeV}. \tag{1.3}$$

The Sommerfeld fine-structure constant α is very accurately known compared to the others, with a fractional uncertainty of 1.5×10^{-10}, so no variation in it will be considered.

For the other on-shell parameters, we next define the following dimensionless quantities:

$$\delta_Z = (M_Z - M_{Z0})/(0.001 \text{ GeV}), \tag{1.4}$$

$$\delta_t = (M_t - M_{t0})/(1 \text{ GeV}), \tag{1.5}$$

$$\delta_h = (M_h - M_{h0})/(0.1 \text{ GeV}), \tag{1.6}$$

$$\delta_S = 1000[\alpha_S^{(5)}(M_Z) - \alpha_{S0}^{(5)}(M_Z)], \tag{1.7}$$

$$\delta_a = 10^4[\Delta\alpha_{\text{had}}^{(5)}(M_Z) - \Delta\alpha_{\text{had},0}^{(5)}(M_Z)], \tag{1.8}$$

as measures of the deviation from the benchmark model. The normalizations of these five quantities are chosen so that a change in the on-shell input by an amount of order the present experimental uncertainty will correspond very roughly to an order 1 change in the corresponding δ. In the interpolation formulas found below, we will often give at least the contributions linear in the five δ's above, even when they are numerically too small to be practically significant, in order to quantitatively illustrate their contributions to the parametric errors.

In the interpolation formulas for the Yukawa couplings at $Q = 200$ GeV, we will also make use of the variations in the fermion masses as needed, parametrized by

$$\Delta_f = \frac{m_f}{m_{f0}} - 1, \tag{1.9}$$

for $f = b, c, s, u, d, \tau, \mu, e$, where the m_f are running $\overline{\text{MS}}$ masses $m_b(m_b)$, $m_c(m_c)$, $m_s(2 \text{ GeV})$, $m_u(2 \text{ GeV})$, and $m_d(2 \text{ GeV})$ for the light quarks and pole (on-shell) masses for the leptons $m_f = M_\tau$, M_μ, and M_e. Also, in a few of the interpolation formulas, we will include the small effect due to a possible deviation in the Fermi decay constant from its benchmark central value, parametrized by

$$\Delta_{G_F} = \frac{G_F}{G_{F0}} - 1. \tag{1.10}$$

The effects of this are typically expected to be very small, since the fractional uncertainty in G_F given in the RPP is about 5×10^{-7}.

The interpolation formulas presented below were obtained by running v1.2 of SMDR with its default choices repeatedly for points in parameter space on grids that cover the plausible allowed ranges and then performing a fit to obtain the coefficients, which were then validated on more parameter space grids. We aim to provide results accurate to well under the experimental and theoretical uncertainties, for deviations of the on-shell inputs by up to 5 times their RPP quoted experimental uncertainties. Contributions quadratic in the deviations therefore will also be included when necessary to achieve relative precision goals for each quantity as stated below. For each output parameter, we will quote a conservative fractional precision, which in this paper refers to the maximum fractional difference between the interpolation formula result and the output of SMDR with default scale-setting choices. The maximum refers to the differences obtained in a scan as all on-shell inputs are varied over ranges such that the total deviation from the experimental central values, added in quadrature, is $\leq 5\sigma$. Here we interpret the uncertainties quoted in the RPP as 1σ, even though a Gaussian distribution of errors may not be the appropriate description. It should be recognized that the actual theoretical uncertainty and the parametric uncertainty are both always much larger than this fractional precision. We have attempted to err on the side of including coefficients even when they are only significant for rather large deviations from the experimental central values.

We now provide the benchmark output results obtained using v1.2 of SMDR with default choices. We give many more significant digits than justified by the theoretical and parametric uncertainties, merely for the sake of reproducibility. The benchmark running $\overline{\text{MS}}$ parameters evaluated at the scale $Q = 200$ GeV are

$$g_{30} = 1.1525136966, \tag{1.11}$$

$$g_0 = 0.64683244428, \tag{1.12}$$

$$g_0' = 0.35885152738, \qquad (1.13)$$

$$\lambda_0 = 0.12353343830, \qquad (1.14)$$

$$m_0^2 = -(93.126827678 \text{ GeV})^2, \qquad (1.15)$$

$$y_{t0} = 0.92377763013, \qquad (1.16)$$

$$y_{b0} = 0.0153349059085, \qquad (1.17)$$

$$y_{c0} = 0.00336181598480, \qquad (1.18)$$

$$y_{s0} = 2.8885955612 \times 10^{-4}, \qquad (1.19)$$

$$y_{d0} = 1.4505079604 \times 10^{-5}, \qquad (1.20)$$

$$y_{u0} = 6.6738103560 \times 10^{-6}, \qquad (1.21)$$

$$y_{\tau 0} = 0.0100065524355, \qquad (1.22)$$

$$y_{\mu 0} = 5.8908805223 \times 10^{-4}, \qquad (1.23)$$

$$y_{e0} = 2.7963423115 \times 10^{-6}. \qquad (1.24)$$

Also, the physical W-boson mass in the PDG parametrization is found to be, for this benchmark set of parameters,

$$M_{W0} = 80.352476 \text{ GeV}, \qquad (1.25)$$

where we have used the SMDR default by computing the W-boson pole mass in terms of the running parameters at $Q = 160$ GeV. The values in Eqs. (1.11) will be used in the interpolation formulas below, as they give the results when all of the δ's vanish, by definition.

II. INTERPOLATION FORMULA FOR THE W-BOSON MASS

For the W-boson physical mass in the PDG convention,[3] we find

$$M_W = M_{W0}(1 + c_{M_W}^t \delta_t + c_{M_W}^Z \delta_Z + c_{M_W}^a \delta_a + c_{M_W}^S \delta_S + c_{M_W}^h \delta_h + c_{M_W}^{tt} \delta_t^2), \qquad (2.1)$$

where M_{W0} was given in Eq. (1.25), and the other potentially significant coefficients are

$$c_{M_W}^t = 7.61 \times 10^{-5}, \qquad c_{M_W}^Z = 1.56 \times 10^{-5}, \qquad c_{M_W}^a = -2.29 \times 10^{-5},$$
$$c_{M_W}^S = -8.8 \times 10^{-6}, \qquad c_{M_W}^h = -5.9 \times 10^{-7}, \qquad c_{M_W}^{tt} = 1.3 \times 10^{-7}. \qquad (2.2)$$

This interpolation formula reproduces the results of SMDR (with its default scale-setting choices) to better than 0.1 MeV, which is much smaller than the current theoretical and experimental uncertainties, when the input on-shell parameters are varied such that the total deviation from the central values, added in quadrature, is $\leq 5\sigma$.

The results above are based on the pure $\overline{\text{MS}}$ scheme used by SMDR and can be compared with similar interpolation formula results based on on-shell [80] and hybrid [98] scheme calculations, which both used fits to a much wider range for the Higgs mass. A numerical comparison between the results from these three different approaches was made in Ref. [42] (see in particular Figs. 4.1 and 4.2), showing that they agree well within the theoretical uncertainty due to renormalization scale dependence and supporting a

theoretical error estimate of perhaps ± 4 MeV. This is less than the parametric error, coming principally from the top-quark mass, of about $6.11\delta_t + 1.25\delta_Z - 1.84\delta_a - 0.71\delta_S - 0.047\delta_h + 0.010\delta_t^2$, in MeV, which can be read off from Eq. (2.2). The relatively large uncertainty associated with the top-quark mass is difficult to reduce, since it is due in large part to the problems in connecting hadron collider measurements and simulations to a well-defined short-distance top-quark mass or Yukawa coupling.

III. INTERPOLATION FORMULAS FOR THE $\overline{\text{MS}}$ PARAMETERS

A. Higgs sector

For the Higgs self-coupling λ at $Q = 200$ GeV, we find

$$\lambda = \lambda_0(1 + c_\lambda^h \delta_h + c_\lambda^t \delta_t + c_\lambda^Z \delta_Z + c_\lambda^S \delta_S + c_\lambda^a \delta_a + c_\lambda^{tt} \delta_t^2 + c_\lambda^{tS} \delta_t \delta_S + c_\lambda^{hh} \delta_h^2$$
$$+ c_\lambda^{ht} \delta_h \delta_t + c_\lambda^{SS} \delta_S^2 + c_\lambda^{hS} \delta_h \delta_S + c_\lambda^{ttt} \delta_t^3 + c_\lambda^{ttS} \delta_t^2 \delta_S + c_\lambda^b \Delta_b + c_\lambda^{G_F} \Delta_{G_F}), \qquad (3.1)$$

where λ_0 was given in Eq. (1.14), and the other coefficients are

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$$c_\lambda^h = 1.6823 \times 10^{-3}, \qquad c_\lambda^t = -1.488 \times 10^{-4}, \qquad c_\lambda^Z = -3.5 \times 10^{-7},$$
$$c_\lambda^S = -2.2 \times 10^{-7}, \qquad c_\lambda^a = 3.4 \times 10^{-7}, \qquad c_\lambda^{tt} = 1.528 \times 10^{-5},$$
$$c_\lambda^{tS} = -4.02 \times 10^{-6}, \qquad c_\lambda^{hh} = 7.0 \times 10^{-7}, \qquad c_\lambda^{ht} = -6.1 \times 10^{-7},$$
$$c_\lambda^{SS} = 3.0 \times 10^{-7}, \qquad c_\lambda^{hS} = 6.4 \times 10^{-8}, \qquad c_\lambda^{ttt} = 1.9 \times 10^{-7},$$
$$c_\lambda^{ttS} = -7.6 \times 10^{-8}, \qquad c_\lambda^b = 4.5 \times 10^{-5}, \qquad c_\lambda^{G_F} = 0.95. \tag{3.2}$$

This formula, based on a fit to the best available calculation of the physical Higgs boson mass [42,109], agrees with the results of SMDR to better than 10^{-6} fractional precision in λ as the input parameters are varied over ranges with a total deviation, added in quadrature, of 5σ from their central values. Again, the theoretical and parametric errors are much larger than this fractional precision, with the top-quark mass giving the largest contribution to the error budget other than the Higgs boson mass itself.

For the running Higgs squared mass parameter at $Q = 200$ GeV, we find

$$m^2 = m_0^2(1 + c_{m^2}^h \delta_h + c_{m^2}^t \delta_t + c_{m^2}^S \delta_S + c_{m^2}^Z \delta_Z + c_{m^2}^a \delta_a + c_{m^2}^{tt} \delta_t^2 + c_{m^2}^{tS} \delta_t \delta_S + c_{m^2}^{hh} \delta_h^2 + c_{m^2}^{ht} \delta_h \delta_t), \tag{3.3}$$

where m_0^2 was given in Eq. (1.15), and the other significant coefficients are

$$c_{m^2}^h = 1.4319 \times 10^{-3}, \qquad c_{m^2}^t = 2.337 \times 10^{-3}, \qquad c_{m^2}^S = -1.052 \times 10^{-4},$$
$$c_{m^2}^Z = -5.7 \times 10^{-7}, \qquad c_{m^2}^a = 5.4 \times 10^{-7}, \qquad c_{m^2}^{tt} = 2.02 \times 10^{-5},$$
$$c_{m^2}^{tS} = -2.45 \times 10^{-6}, \qquad c_{m^2}^{hh} = 5.8 \times 10^{-7}, \qquad c_{m^2}^{ht} = -4.3 \times 10^{-7}. \tag{3.4}$$

This formula provides agreement with the output of SMDR to a fractional precision of better than 10^{-5}.

B. Gauge couplings

For the $SU(3)_c$ $\overline{\text{MS}}$ gauge coupling g_3 evaluated at $Q = 200$ GeV, we obtained the following interpolation formula:

$$g_3 = g_{30}(1 + c_{g_3}^S \delta_S + c_{g_3}^t \delta_t + c_{g_3}^{SS} \delta_S^2 + c_{g_3}^h \delta_h + c_{g_3}^Z \delta_Z + c_{g_3}^a \delta_a), \tag{3.5}$$

where g_{30} was given in Eq. (1.11), and the coefficients are

$$c_{g_3}^S = 3.7875 \times 10^{-3}, \qquad c_{g_3}^t = -3.98 \times 10^{-5}, \qquad c_{g_3}^{SS} = -1.07 \times 10^{-5},$$
$$c_{g_3}^h = 2.5 \times 10^{-8}, \qquad c_{g_3}^Z = 2.7 \times 10^{-9}, \qquad c_{g_3}^a = -2.0 \times 10^{-9}. \tag{3.6}$$

Note that the top-quark mass is significant here because we are relating the five-quark QCD coupling $\alpha_S^{(5)}(M_Z)$ to the Standard Model QCD coupling g_3 with the top quark not decoupled. The three terms proportional to δ_S, δ_t, and δ_S^2 are sufficient to obtain a fractional precision compared to SMDR of better than 10^{-5}, but the linear deviation coefficients $c_{g_3}^h$, $c_{g_3}^Z$, and $c_{g_3}^a$ are also listed in order to illustrate the small size of the parametric errors.

For the $SU(2)_L$ gauge coupling g, we find

$$g = g_0(1 + c_g^t \delta_t + c_g^a \delta_a + c_g^Z \delta_Z + c_g^S \delta_S + c_g^h \delta_h + c_g^{tt} \delta_t^2 + c_g^{tS} \delta_t \delta_S + c_g^{G_F} \Delta_{G_F}), \tag{3.7}$$

where g_0 was given in Eq. (1.12), and the other coefficients are

$$c_g^t = 5.735 \times 10^{-5}, \qquad c_g^a = -2.295 \times 10^{-5}, \qquad c_g^Z = 1.558 \times 10^{-5},$$
$$c_g^S = -5.97 \times 10^{-6}, \qquad c_g^h = -8.5 \times 10^{-7}, \qquad c_g^{tt} = 1.9 \times 10^{-7},$$
$$c_g^{tS} = -7.8 \times 10^{-8}, \qquad c_g^{G_F} = 0.71. \tag{3.8}$$

This formula provides a fractional precision of better than 10^{-6} as the input on-shell parameters are varied with $\leq 5\sigma$ total deviation from their central values, added in quadrature.

For the $U(1)_Y$ gauge coupling, we find

$$g' = g'_0(1 + c^t_{g'}\delta_t + c^a_{g'}\delta_a + c^Z_{g'}\delta_Z + c^S_{g'}\delta_S + c^h_{g'}\delta_h), \tag{3.9}$$

where g'_0 was given in Eq. (1.13), and the other coefficients are

$$c^t_{g'} = -2.609 \times 10^{-5}, \qquad c^a_{g'} = 7.714 \times 10^{-5}, \qquad c^Z_{g'} = -4.70 \times 10^{-6},$$
$$c^S_{g'} = 3.29 \times 10^{-6}, \qquad c^h_{g'} = 2.6 \times 10^{-7}. \tag{3.10}$$

This formula again provides a fractional precision of better than 10^{-6} compared to SMDR.

C. Top-quark Yukawa coupling

For the top-quark Yukawa coupling at $Q = 200$ GeV, we find

$$y_t = y_{t0}(1 + c^t_{y_t}\delta_t + c^S_{y_t}\delta_S + c^h_{y_t}\delta_h + c^{tt}_{y_t}\delta_t^2 + c^{SS}_{y_t}\delta_S^2 + c^Z_{y_t}\delta_Z + c^a_{y_t}\delta_a), \tag{3.11}$$

where y_{t0} was given in Eq. (1.16), and the other coefficients are

$$c^t_{y_t} = 6.352 \times 10^{-3}, \qquad c^S_{y_t} = -7.76 \times 10^{-4}, \qquad c^h_{y_t} = -2.36 \times 10^{-6},$$
$$c^{tt}_{y_t} = 8.9 \times 10^{-7}, \qquad c^{SS}_{y_t} = -1.23 \times 10^{-6}, \qquad c^Z_{y_t} = -1.6 \times 10^{-7},$$
$$c^a_{y_t} = 2.2 \times 10^{-8}. \tag{3.12}$$

The five terms proportional to δ_t, δ_S, δ_h, δ_t^2, and δ_S^2 are sufficient to obtain a fractional precision better than 10^{-5}, and the linear deviation coefficients $c^Z_{y_t}$ and $c^a_{y_t}$ are also included in order to show their small contribution to the parametric error budget.

D. Yukawa couplings of light quarks

In the interpolation formulas for light-quark Yukawa couplings in the present subsection, the quantities δ_a, δ_h, and δ_Z make a relatively insignificant difference and are therefore omitted.

For the bottom-quark Yukawa coupling at $Q = 200$ GeV, we find

$$y_b = y_{b0}(1 + c^b_{y_b}\Delta_b + c^{bb}_{y_b}\Delta_b^2 + c^{bS}_{y_b}\Delta_b\delta_S + c^S_{y_b}\delta_S + c^t_{y_b}\delta_t + c^{SS}_{y_b}\delta_S^2 + c^{SSS}_{y_b}\delta_S^3), \tag{3.13}$$

where y_{b0} was given in Eq. (1.17), and the other coefficients are

$$c^b_{y_b} = 1.185, \qquad c^{bb}_{y_b} = 0.075, \qquad c^{bS}_{y_b} = -3.3 \times 10^{-3}, \qquad c^S_{y_b} = -6.125 \times 10^{-3},$$
$$c^t_{y_b} = -2.4 \times 10^{-5}, \qquad c^{SS}_{y_b} = -2.1 \times 10^{-5}, \qquad c^{SSS}_{y_b} = -1.5 \times 10^{-7}. \tag{3.14}$$

This agrees with the results of SMDR to a fractional precision of better than 10^{-4}. For the charm-quark Yukawa coupling at $Q = 200$ GeV, we obtain

$$y_c = y_{c0}(1 + c^c_{y_c}\Delta_c + c^{cc}_{y_c}\Delta_c^2 + c^{cS}_{y_c}\Delta_c\delta_S + c^S_{y_c}\delta_S + c^{SS}_{y_c}\delta_S^2 + c^{SSS}_{y_c}\delta_S^3$$
$$+ c^b_{y_c}\Delta_b + c^{bS}_{y_c}\Delta_b\delta_S + c^t_{y_c}\delta_t), \tag{3.15}$$

where y_{c0} was given in Eq. (1.18), and the other coefficients are

$$c^c_{y_c} = 1.415, \qquad c^{cc}_{y_c} = 0.078, \qquad c^{cS}_{y_c} = -3.0 \times 10^{-3},$$
$$c^S_{y_c} = -0.01746, \qquad c^{SS}_{y_c} = -2.34 \times 10^{-4}, \qquad c^{SSS}_{y_c} = -6.5 \times 10^{-6},$$
$$c^b_{y_c} = -0.027, \qquad c^{bS}_{y_c} = -1.6 \times 10^{-3}, \qquad c^t_{y_c} = -1.5 \times 10^{-5}. \tag{3.16}$$

This result for the charm-quark Yukawa coupling agrees with SMDR to a fractional precision of better than 10^{-4}.

For the strange, down, and up Yukawa couplings, the interpolation formulas have a simpler, universal form, due to the fact that the "on-shell" input parameters from the RPP are actually running $\overline{\text{MS}}$ parameters determined at a common scale of $Q = 2$ GeV, so that the same QCD corrections apply to all three in the same way. For the Yukawa couplings at $Q = 200$ GeV, we find

$$y_q = y_{q0}(1 + \Delta_q)(1 + c^S_{y_q}\delta_S + c^{SS}_{y_q}\delta^2_S + c^{SSS}_{y_q}\delta^3_S + c^b_{y_q}\Delta_b + c^t_{y_q}\delta_t), \tag{3.17}$$

where the coefficients in all three cases ($q = s, d, u$) are approximated well by

$$\begin{aligned}
c^S_{y_q} &= -0.01089, & c^{SS}_{y_q} &= -7.93 \times 10^{-5}, & c^{SSS}_{y_q} &= -1.2 \times 10^{-6}, \\
c^b_{y_q} &= -0.0128, & c^t_{y_q} &= -1.5 \times 10^{-5},
\end{aligned} \tag{3.18}$$

and y_{s0}, y_{d0}, and y_{u0} were given, respectively, in Eqs. (1.19)–(1.21). These formulas agree with those obtained by SMDR to a fractional precision of better than 10^{-4}.

E. Yukawa couplings of leptons

For the tau-lepton Yukawa coupling at $Q = 200$ GeV, we obtain

$$\begin{aligned}
y_\tau = y_{\tau 0}(1 &+ \Delta_\tau + 0.5\Delta_{G_F} + c^t_{y_\tau}\delta_t + c^S_{y_\tau}\delta_S + c^a_{y_\tau}\delta_a + c^h_{y_\tau}\delta_h + c^Z_{y_\tau}\delta_Z \\
&+ c^{tt}_{y_\tau}\delta^2_t + c^{tS}_{y_\tau}\delta_t\delta_S),
\end{aligned} \tag{3.19}$$

where $y_{\tau 0}$ was given in Eq. (1.22), the coefficients of Δ_τ and Δ_{G_F} are very close to 1 and 0.5 as indicated, and the other coefficients are

$$\begin{aligned}
c^t_{y_\tau} &= -1.252 \times 10^{-5}, & c^S_{y_\tau} &= 2.63 \times 10^{-6}, & c^a_{y_\tau} &= -1.83 \times 10^{-6}, \\
c^h_{y_\tau} &= 1.74 \times 10^{-6}, & c^Z_{y_\tau} &= -1.8 \times 10^{-7}, & c^{tt}_{y_\tau} &= -6.9 \times 10^{-7}, \\
c^{tS}_{y_\tau} &= 1.3 \times 10^{-7}.
\end{aligned} \tag{3.20}$$

This interpolation formula gives agreement with SMDR to a fractional precision of better than 10^{-7}.

The Yukawa couplings for $\ell = \mu, e$ at $Q = 200$ GeV are written in the common form:

$$\begin{aligned}
y_\ell = y_{\ell 0}(1 &+ \Delta_\ell + 0.5\Delta_{G_F} + c^t_{y_\ell}\delta_t + c^S_{y_\ell}\delta_S + c^a_{y_\ell}\delta_a + c^h_{y_\ell}\delta_h + c^Z_{y_\ell}\delta_Z \\
&+ c^{tt}_{y_\ell}\delta^2_t + c^{tS}_{y_\ell}\delta_t\delta_S + c^c_{y_\ell}\Delta_c + c^b_{y_\ell}\Delta_b),
\end{aligned} \tag{3.21}$$

where $y_{\mu 0}$ and y_{e0} were given in Eqs. (1.23) and (1.24), respectively. For the muon, the other coefficients are

$$\begin{aligned}
c^t_{y_\mu} &= -1.3105 \times 10^{-5}, & c^S_{y_\mu} &= 2.17 \times 10^{-6}, & c^a_{y_\mu} &= -2.84 \times 10^{-6}, \\
c^h_{y_\mu} &= 1.73 \times 10^{-6}, & c^Z_{y_\mu} &= -1.78 \times 10^{-7}, & c^{tt}_{y_\mu} &= -6.93 \times 10^{-7}, \\
c^{tS}_{y_\mu} &= 1.26 \times 10^{-7}, & c^c_{y_\mu} &= -3.3 \times 10^{-5}, & c^b_{y_\mu} &= -4.1 \times 10^{-6}.
\end{aligned} \tag{3.22}$$

For the electron, the coefficients are

$$\begin{aligned}
c^t_{y_e} &= -1.312 \times 10^{-5}, & c^S_{y_e} &= 2.87 \times 10^{-6}, & c^a_{y_e} &= -4.72 \times 10^{-6}, \\
c^h_{y_e} &= 1.73 \times 10^{-6}, & c^Z_{y_e} &= -1.78 \times 10^{-7}, & c^{tt}_{y_e} &= -6.93 \times 10^{-7}, \\
c^{tS}_{y_e} &= 1.26 \times 10^{-7}, & c^c_{y_e} &= -8.1 \times 10^{-5}, & c^b_{y_e} &= -1.4 \times 10^{-5}.
\end{aligned} \tag{3.23}$$

The fractional precisions, compared to the results from SMDR, are less than 10^{-9}. Since the present fractional uncertainties in M_μ and M_e are about 2×10^{-8} and 6×10^{-9}, respectively, we see that for each lepton, the bottleneck for obtaining the most accurate possible Yukawa coupling in the ultraviolet is not the uncertainty in the corresponding lepton mass, but rather the uncertainty associated with the top-quark mass, which is difficult to reduce as we have already mentioned.

IV. OUTLOOK

In this paper we have presented simple interpolation formulas that provide the fundamental Lagrangian parameters for the Standard Model, given the corresponding on-shell experimental values as inputs. (The three physical angles and *CP*-violating phase associated with CKM mixing are omitted, having a tiny effect on these results due to CKM unitarity, and can be obtained from Ref. [8] and sources referenced therein.) These results are an alternative to a more time-consuming and complicated evaluation using e.g., the computer code SMDR, on which our results are based. The structure of the interpolation formulas has been designed so as to avoid any numerically significant loss of precision and are made to provide results at the $\overline{\text{MS}}$ renormalization scale $Q = 200$ GeV as a reference. For convenience, we have included as Supplemental Material [115] a simple interactive command-line PYTHON code sm200.py implementing the interpolation formulas above. We intend to update our results in the preprint version of this paper and in that code as new theoretical refinements and experimental measurements become available.

Besides satisfying basic curiosity about the fundamental parameters of the Standard Model, the results given here will have applications in matching to various candidate ultraviolet completions of the Standard Model, provided that the mass scales associated with new physics are sufficiently high that nonrenormalizable terms in the effective theory can be neglected or corrected for. The results also can be viewed as providing the parametric error budget for the defining couplings of the Standard Model Lagrangian.

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[115] See Supplemental Material at http://link.aps.org/ supplemental/10.1103/PhysRevD.107.013010 for a simple interactive command-line PYTHON code sm200.py implementing the interpolating formulas found in this Paper.