

Path Planning for Unmanned Aerial Vehicles: Peak Power Minimization

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Abstract— Utilization of moving unmanned aerial vehicles (UAV) has attracted a lot of attention in recent years. Accordingly, path planning to optimize a given utility function, such as mechanical energy, has been the subject of many works. In a prior work, we have proposed path planning schemes to uniformly cover an area for communication coverage and surveillance applications with minimum mechanical energy. As far as energy and power minimization is concerned, an important issue that is sometimes being overlooked is the peak-power that the UAV has to afford to provide the path planning of interest. In this paper, we address this issue and find paths that provide a uniform coverage with minimum peak power. We then compare the results with the case where mechanical energy was minimized. This is done for fixed-wing as well as rotary-wing UAVs. It is observed that depending on the UAV specs, we can expect a mild increase (7% in our case) in peak power in some cases when going from peak power optimized scenario to energy optimized scenario. There are also cases where there is no major difference between the 2 scenarios.

Index Terms: Path Planning, Unmanned Aerial Vehicles, Mechanical Energy, Peak Power, Coverage.

I. INTRODUCTION

Unmanned aerial vehicles (UAV) have attracted much attention in recent years. An important application of UAVs is their deployment as aerial base stations (ABS) which are used to provide communication coverage over a given region [1], [2]. Another similar application is for monitoring and surveillance. In the latter case, the UAV has to necessarily move around to have a more effective monitoring. As such, path planning becomes an important matter to consider. In path planning, different utility functions such as throughput, consumed power, or the traveling distance are optimized subject to different constraints [3], [4], [5], [6]. Given the limitations in battery technology, larger power consumption is directly translated into smaller flying time before the batteries are charged again [1]. As such, a number of works have considered the problem of path planning with respect to consumed mechanical power used for hovering and/or moving in the air [1], [7].

In [7], Zhang et al. provided a comprehensive closed-form formulation for the energy consumption of fixed-wing UAVs. This work was followed by other 2-dimensional path planning frameworks [8], [9], [10], [11], [12] for fixed wing UAVs and ABSs. In these works, the aim is to maximize energy efficiency (EE) which is defined as the user achievable rate divided by the consumed mechanical energy. Recently, the work of [13] provided a closed-form formulation for the energy consumption of rotary-wing UAVs in 2-D arbitrary

flight. The proposed formulation is quite complex, but it can be used to obtain consumed energy of rotary-wing UAVs when traveling on any arbitrary path.

An important issue in deploying UAV's for monitoring or as ABS's, is to provide a uniform coverage on the corresponding area. The work of [14] provides a family of trajectories and speed profiles such that if according to which the ABSs move, a pretty uniform coverage is guaranteed for any arbitrary user in any location of the cell. A major draw-back of this work is to ignore the power/energy analysis of the proposed paths.

In our recent work [15], we took into account the consumed energy in the framework of [14] and provided a path-planning scheme in which while providing a uniform coverage, the mechanical energy is also minimized. Compared to the existing works on path planning with respect to energy minimization such as [8], [9], [10], [11], [12], the work was unique in the sense that the existing works were all user-oriented, i.e., they optimized the UAV path with respect to a specific user and a fixed location. In practice and to be fair to all users, one has to devise a path for the ABS to provide an acceptable service level to all users within a cell.

As a major draw-back of [15], we ignored peak mechanical power imposed to the UAV during path optimization process. This is an important issue that is also ignored in many existing works. In other words, a path that is optimized in terms of energy consumption, might require an instantaneously high power that can not be afforded by the UAV. In this case, we may choose to go with a higher energy path so as to avoid high peak powers.

In this paper, we assume the UAVs are using spiral curves proposed in [14]. This will ensure a pretty uniform coverage over the cell which is preferred in surveillance and communications applications. We then find a path with minimal peak power. Through this derivation, we try to see if there is a trade-off between minimum energy consumption vs minimizing the peak power. In other words, we want to investigate how much increase we face in the peak power, when we design paths with minimum energy.

In simulations, we consider both fixed-wing as well as rotary-wing UAVs. It is observed that depending on the UAV specs, we can expect a mild increase in peak power in some cases when going from peak power optimized scenario to energy optimized scenario and the same order of increase in energy if we go the other way around. In this case, we have to compromise between the 2 factors. However, it is interesting to see that in some cases, although the peak power increase is noticeable when going from peak power optimized scenario to energy optimized scenario (7% in our case), the consumed

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energy is not much different and fortunately in this case, no compromise has to be made. Finally, there are also cases where there is no major difference between the 2 scenarios.

This paper is organized as follows: Section II introduces the system model and as well as some preliminary formulations. In Section III, we propose the optimization problems. Section IV presents the numerical results and Section V concludes the paper.

II. PRELIMINARIES AND SYSTEM MODEL

A. Spiral Trajectories

The family of curves below represent a spiral family trajectory:

$$Q(s) = \rho s^k \cos(\zeta s), \rho s^k \sin(\zeta s), \quad s \in [0, 1], \quad (1)$$

where ρ is the radius of cell and k and ζ are constants that determine the shape of the curve. In particular, by setting $\zeta = A/0$, $k = 1$, we come up with a set of curves, each are a radius this from time 0 of the cell. This is referred to as the radial trajectory, which to τ with g give the consumed energy and dividing it by τ will

is the most intuitive path of this family.

Each UAV starts flying from the cell center toward the cell edge over $Q(s)$ in τ seconds. When it reaches the cell edge, it returns to the origin on the same path and continues on curve $-Q(s)$ to reach the other side of the edge and this action repeats continuously.

The instantaneous locations of ABSs along the flying on the spiral trajectory can be obtained by setting $s = \frac{2k}{\tau} \frac{t}{\tau}$ in the above equation:

$$Q(t) = (x(t), y(t)) = \left(\rho \frac{t}{\tau} \cos(\zeta \frac{2k}{\tau} \frac{t}{\tau}), \rho \frac{t}{\tau} \sin(\zeta \frac{2k}{\tau} \frac{t}{\tau}) \right). \quad (2)$$

The velocity and acceleration vectors of the ABSs are obtained simply by differentiating $Q(t)$ with respect to time as follows:

$$V(t) = (x'(t), y'(t)), A(t) = (x''(t), y''(t)). \quad (3)$$

Note that in Radial trajectory, when going from center to the cell edge or vice versa, there is no change of direction, and acceleration is non-zero only when there is a change in the absolute value of velocity, $||V(t)||$. However, in general, even if $||V(t)||$ is fixed, $A(t)$ might be non-zero as moving on a 2-dimensional spiral curve needs constant change of direction.

B. Energy Consumption Model for Fixed-Wing UAV

For a fixed-wing UAV moving on a 2-dimensional plane, the power consumption depends on instantaneous velocity and acceleration as well as the weight. Increasing the velocity in general will result in less power consumption as it generates more lift force in fixed-wing UAVs. On the other hand, higher velocity will increase aerodynamic drag force, which may result in overall increase of power consumption. In [7], a very

useful formulation has been proposed that gives the consumed energy for a fixed-wing UAV that flies for τ seconds:

$$E = \int_0^\tau \left(c_1 V(t)^3 + \frac{c_2}{V(t)} + \frac{A(t)^2 - \frac{(A^T(t) \cdot V(t))^2}{V(t)^2}}{g^2} \right) dt \quad (4)$$

$$+ \frac{1}{2} m V(\tau)^2 - V(0)^2, \quad (5)$$

where $V(t)$ and $A(t)$ denote the instantaneous velocity and acceleration vectors respectively, and c_1 and c_2 are constants related to the UAV specs and are defined in [7] as

$$c_1 = \frac{1}{2} \rho_a C_{D_0} S, \quad c_2 = \frac{2W^2}{(\pi e_0 A_R) \rho_a S}. \quad (6)$$

In the above equations, $W = mg$ is the force of gravity, with m denoting the UAVs mass including all its payload, and g is the gravitational acceleration. Moreover, ρ_a is the air density in kg/m^3 , C_{D_0} is the zero-lift drag coefficient, S is a reference area (e.g., the wing area), e_0 is the Oswald efficiency, and A_R is the aspect ratio of the wing. Integrating

give average consumed mechanical power.

Ignoring the last term of (5), the instantaneous power is given as

$$P_{\text{inst}} = c_1 V(t)^3 + \frac{c_2}{V(t)} + \frac{A(t)^2 - \frac{(A^T(t) \cdot V(t))^2}{V(t)^2}}{g^2} \quad (7)$$

C. Energy Consumption Model For Rotary-Wing UAV

From [13], the energy consumption of a rotary-wing UAV moving on a 2-dimensional plane that flies for τ seconds can be obtained as

$$E = \int_0^\tau P_{\text{total}}(t) dt, \quad (8)$$

where $P_{\text{total}}(t)$ is the instantaneous total consumed power at time t . It can be obtained by calculating the vertical and horizontal power consumption:

$$P_{\text{total}}(t) = P_{\text{vertical}}(t) + P_{\text{horizontal}}(t). \quad (9)$$

The vertical consumed power can be obtained as

$$P_{\text{vertical}} = P_0 \left(1 + \frac{3V(t)^2}{U_{\text{tip}}^2} \right) + P_{ik} \left(\frac{V(t)^4}{\kappa^2 + 4V_0^2} - \frac{V(t)^2}{2V_0^2} \right)^{\frac{1}{2}}, \quad (10)$$

where κ is defined as the thrust-to-weight ratio, i.e., $\kappa = \frac{T}{W}$ and can be expressed as:

$$\kappa = \frac{r}{1 + \frac{(\rho S_{FP} V(t)^2 + 2m A(t)^2)}{4W^2}}. \quad (11)$$

In the above equation, P_0 and P_i are two constants defined in [16] representing the blade power and induced power in

hovering status, respectively, U_{tip} is the speed of the rotor blade, v_0 is the mean rotor induced velocity in hover, $S_{FP} = d_0 s A$ is the fuselage equivalent flat plate area, $W = mg$ is the force of gravity, with m denoting the UAV mass including all its payload, and g is the gravitational acceleration. It is worth noting that with a maximum speed of $V_{max} = 30 \frac{m}{s}$, κ is approximately equals to 1 for different acceleration and UAV weights. As can be seen, $P_{vertical}$ depends on the instantaneous velocity and has nothing to do with the UAV orientation.

The horizontal consumed power can be modeled as

$$P_{horizontal} = P_{\parallel}(t) + P_{\perp}(t), \quad (12)$$

$$P_{\parallel}(t) = \frac{1}{2} \rho S_{FP} V_{\parallel}^2(t) + m A_{\parallel}(t) V_{\parallel}(t),$$

$$P_{\perp}(t) = \frac{1}{2} \rho S_{FP} V_{\perp}^2(t) + m A_{\perp}(t) V_{\perp}(t),$$

where $V_{\parallel}$ and $V_{\perp}$ are the speed components that are parallel and perpendicular to the UAV head direction, respectively, and they can be expressed as:

$$V_{\parallel}(t) = V(t) \cos \theta_h, \quad V_{\perp}(t) = V(t) \sin \theta_h. \quad (13)$$

$A_{\parallel}$ and $A_{\perp}$ are the acceleration components that are parallel and perpendicular to the UAV head direction and can be defined respectively as follows:

$$A_{\parallel}(t) = A(t) \cos \theta_h, \quad A_{\perp}(t) = A(t) \sin \theta_h, \quad (14)$$

where θ_h is the angle between the acceleration and velocity vector that can be expressed as:

$$\theta_h(t) = \arcsin \frac{\frac{1}{2} \frac{d}{dt} (V(t)^2)}{V(t) A(t)} = \arcsin \frac{(A^T(t) \cdot V(t))^2}{V(t)^2 A(t)^2}. \quad (15)$$

In this equation, θ_h indicates the rotation of the UAV along the roll and pitch axis in the horizontal plane, which can be modeled as

$$\theta_h = \arctan \frac{\tan R_{\theta}}{\tan P_{\theta}}, \quad (16)$$

where R_{θ} is the rotation angle along the roll axis and P_{θ} is the rotation angle along the pitch axis.

III. THE PATH PLANNING PROBLEM

In this part, we state 2 problems. In the first one, we find a path that minimizes the peak power and in the second one which was already stated in [15], we find a path that minimizes the consumed energy, both over a flight period of τ .

To do so, given that we are constrained to move on spiral paths to provide the uniform coverage, we replace the path in (2) into (7) for the case of fixed-wing and in (9) for the case of rotary-wing UAV.

For a given τ , the problem is simplified into:

$$\min_{\zeta, k} P_{peak}. \quad (17)$$

For the time t which results in the peak power, and a fixed τ , (17) is not necessarily convex in ζ and k . However, given

the closed-form formulation, optimal values for ζ and k can be readily obtained.

With similar arguments, we have the following optimization problem to minimize the consumed energy as already stated in [15]:

$$\min_{\zeta, k} E. \quad (18)$$

It is important to remind that once E is obtained, the average power $\bar{P}$ can be calculated as E/τ . We will report this instead of E in the simulation result so that a better comparison with P_{peak} can be made.

IV. SIMULATION RESULTS

A. Fixed-wing UAV

For simulations, we consider 3 types of fixed-wing UAVs. A light one with weight $m_1 = 4.5$ kg and a heavier one with $m_2 = 63.5$ kg, and a third one with $m_3 = 10$ kg. We set the values of c_1 and c_2 according to [15]. We consider a cell of radius $\rho = 4000$ m and set $\tau = 1800$ seconds.

In Table I, we have reported the optimized values for ζ and k for the 3 UAV's. For each weight, we obtain the path with minimum peak power and report the peak power. For each weight, we also report the path with minimum energy (or equivalently average power) from our prior work [15].

As can be seen, the peak power has improved in all 3 cases compared to the paths already optimized for energy consumption in [15]. However, depending on the UAV specs, we see different figures. For the lighter UAV, we see 7% increase in P_{peak} while the average power is almost the same. So in this case, our choice is obvious. For the medium weight UAV, we see about 4% difference in P_{peak} . The difference in average power is about the same percent and thus there is a mild trade-off here between peak power reduction and energy minimization. For the heavy UAV, all 4 values, i.e., average power and peak power in 2 scenarios are almost the same and we can go with either path.

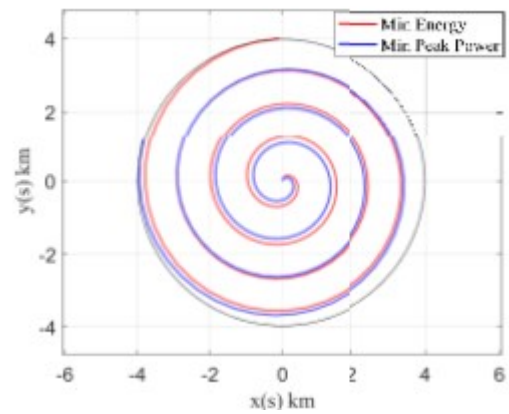


Fig. 1: The spiral paths for fixed-wing UAV : min. energy vs min. peak power.

In Fig. 1, we have plotted the optimal path for minimum peak power and the one for minimum energy from [15] for the 2nd UAV in Table I. As can be seen, 2 paths are pretty different

m (Kg)	Optimized Utility	P_{peak} (Watts)	$\bar{P}$ (Watts)	ζ	k
4.5	Peak Power	20.9	19.94	2.47π	0.8
4.5	Energy	22.34	19.8	2.81π	0.885
10	Peak Power	104.03	101.4	7.6π	1
10	Energy	108	98	8π	0.85
63.5	Peak Power	1186.5	1162.9	6.2π	0.9
63.5	Energy	1187	1159.4	6.033π	0.933

TABLE I: Average and peak powers: energy-optimized vs peak-power minimization scenarios for fixed-wing UAVs.

where the blue path performs better in terms of mechanical energy while the red path results in minimum peak power.

B. Rotary-wing UAV

For simulations, we consider 3 types of rotary wing-UAVs. A light one with weight $m_1 = 1.27$ kg and a heavier one with $m_2 = 20$ kg, and a third one with $m_3 = 10$ kg. We set the values of P_0 , P_i , U_{tip} , v_0 and S_{FP} for each UAV according to [16], [17] and [18]. We consider a cell of radius $\rho = 4000$ m and set $\tau = 2500$ seconds.

In Table II, we have reported the optimized values for ζ and k for the 3 UAV's. For each weight, we obtain the path with minimum peak power and report the peak power. For each weight, we also obtain the path with minimum energy (or equivalently average power).

As can be seen, the peak power has improved compared to the paths optimized for energy consumption for all 3 cases. For the lighter UAV, we see about 5% reduction in P_{peak} while we are consuming almost the same energy in in both cases. As such, the choice is obvious. For the other UAVs, the results are very close and as such, we can go with either path. In Fig.

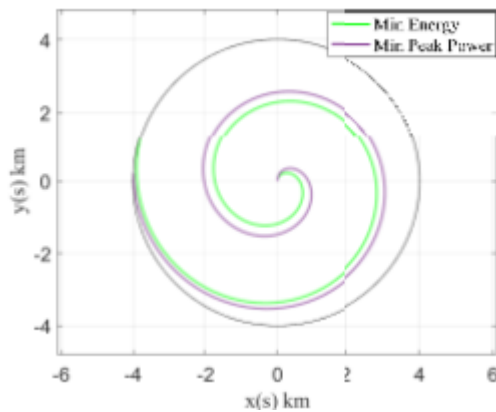


Fig. 2: The spiral paths for rotary-wing UAV: min. energy vs min. peak power.

2, we have plotted the optimal path for minimum peak power and the one for minimum energy for the 2nd UAV in Table II. As can be seen, 2 paths are pretty different where the green path performs better in terms of mechanical energy while the purple path results in minimum peak power.

m (Kg)	Optimized Utility	P_{peak} (Watts)	$\bar{P}$ (Watts)	ζ	k
1.27	Peak Power	30.9	25.52	2.27π	0.75
1.27	Energy	32.33	25.4	2.4π	0.9
10	Peak Power	785.87	711	5.79π	0.8
10	Energy	801.4	708.9	5.8π	0.9
20	Peak Power	6017.9	5882.2	3.52π	0.79
20	Energy	6063.4	5876.8	3.6π	0.95

TABLE II: Average and peak powers: energy-optimized vs peak-power minimization scenarios for rotary-wing UAVs.

V. CONCLUSION

In this paper, we proposed a UAV path planning scheme based on spiral paths which can provide a fairly uniform coverage over the cell with minimum peak power among all existing paths. To investigate if there is any trade-off between minimum peak power and minimum energy, we also obtained the consumed energy for the very same path. We then obtained the peak power for the path with minimum energy. This was done for both types of UAV: fixed-wing and rotary wing. Simulation result show very high dependency on the UAV specs. There are cases where we have to compromise between a mild increase in peak power to have the minimum energy or a mild increase in energy if we choose to go with minimum peak power, depending on the technical limitations we have. On the other hand, there are cases where both scenarios provide very close values for peak power and consumed energy. The most interesting case was for the lightest chosen fixed-wing UAV where by choosing the path with minimum peak power, we face 7% improvement in peak power compared to the path with minimal energy while the increase in energy compared to the minimum possible energy was below 1%. So in this case, it makes much more sense to go with the path of minimum peak power even if we care more about the consumed energy.

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