Student Strategies Playing Vector Unknown Echelon Seas, a 3D IOLA Videogame

David Plaxco
Clayton State University

Matthew Mauntel
University of New Hampshire

Michelle Zandieh
Arizona State University

We present preliminary results of students' strategies playing Vector Unknown: Echelon Seas [VUES], a 3D videogame intended to support student reasoning about vectors. Our team designed VUES by drawing on theories from Inquiry-Oriented Instruction (IOI), Game-Based Learning [GBL] and Realistic Mathematics Education [RME]. VUES builds from a prior 2D game by giving players vectors with 1, 2, or 3 components, depending on the level. We use codes from our team's prior analysis (Mauntel et al, 2020) to analyze strategies in the 3D game. Early results show that students develop similar strategies during 3D gameplay as other students developed while playing the 2D game. However, we have also found new strategies that we did not witness with 2D gameplay, requiring us to extend our coding scheme. Further, early results emphasized the need for design changes to the 3D game to better support players' progress.

Keywords: Linear Algebra; Mathematical Play; Inquiry-Oriented Instruction; Game-Based Learning; Realistic Mathematics Education

The importance of Linear Algebra in undergraduate STEM students' degree progression is well-established (Stewart et al, in press). In this on-going design-research project, we have iteratively implemented and re-designed a browser-based video game intended to support student experiences with vectors before entering (and early during) a first course in Linear Algebra. Our team has drawn on a combination of design principles from Game-Based Learning (Gee, 2003; Gee, 2005; Gresalfi & Barnes, 2016; Williams-Pierce & Thevenow-Harrison, 2021), Inquiry-Oriented Instruction (Rasmussen & Kwon, 2007; Kuster et al, 2018), and Realistic Mathematics Education (RME; Freudenthal, 1991; Gravemeijer, 1994) to iteratively refine and develop the videogame (Zandieh et al, 2018; Mauntel et al, 2019; Mauntel et al, 2020; Mauntel et al, 2021). We have iteratively re-designed this video game to better support players' algebraic and geometric understanding of linear combinations of vectors and vector equations. In this paper, we present the newest iteration of the video game, discuss early results of clinical interviews with undergraduates playing the video game, and discuss what implications these results have for future iterations of this game and educational game design more generally.

Literature Review and Theoretical Framing

Game-Based Learning (GBL) is a growing area of education research that shows promise for supporting meaningful gains in student thinking outside of high-stakes classroom environments (Gee, 2003; Gee, 2005; Gresalfi & Barnes, 2016). With this project, our team set out to blend best practices from GBL and mathematical curriculum design theory to create a video game based on the Inquiry-Oriented Linear Algebra curriculum (IOLA; Wawro et al, 2013). Our game design leverages the key idea of using vectors as modes of transportation in the Magic Carpet Ride task from the IOLA curricular materials, which use the curriculum design theory of RME (Gravemeijer, 1994; Freudenthal, 1991). Using vectors as modes of transportation provides students and, now, video game players with an experientially real setting for using and understanding linear combinations of vectors. Our team has also incorporated theoretical framing from the Inquiry-Oriented Instruction (IOI) literature (Rasmussen & Kwon, 2007; Zandieh et al, 2017) to inform how student/instructor roles translate into designing a video game interaction.

The current version of the video game, Vector Unknown: Echelon Seas [VUES], takes place in a 3-dimensional environment with the player controlling a pirate avatar who runs between stages and puzzles. In this paper, we focus on Stage 4 of VUES, which we designed as an extension of the original 2-dimensional Vector Unknown game (VU). VUES Stage 4 is subdivided into three different difficulty levels, all of which are played via the same controls and format, but differ based on the number of components in the vectors used (i.e., whether the vectors in the level have 1, 2, or 3 components) and each difficulty level consists of 6 puzzles in which the player helps the pirate aim a grappling hook at an anchor. The player uses the vectors to create a linear combination equal to a goal location by dragging vectors (represented as column vectors in a given collection of cards) into a vector equation and using sliders to adjust scalars in front of each vector. To provide just-in-time feedback (Gresalfi & Barnes, 2016; Williams-Pierce, 2019), the game calculates the appropriate linear combination of those vectors, shows (1) a geometric representation of that linear combination consistent with a "tip-to-tail" representation of linear combinations and (2) a numeric representation of the vector equation with the calculation on the right of the equal sign, labeled as "Answer".

Figure 1. Images of gameplay in Vector Unknown Echelon Seas.

Using the earlier iteration of the game, Vector Unknown, we characterized the strategies that participants used to play the game (Mauntel et al, 2020, 2021). These strategies focused on two main components of the game, geometric and numeric. Geometric strategies related to the generated geometric display of the linear combination of two vectors. Numeric strategies involved focusing on a vector equation. There were four core strategies each with a numeric and geometric variant. ***Focus on one vector*** involved selecting a vector, scaling it to be as close to the goal location as possible, and then choosing another vector to reach the goal. ***Focus on one coordinate*** involved choosing an x- or y-coordinate in the goal vector and adjusting the scalars on each of the two vectors to first match one coordinate, and then adjusting as necessary to reach the goal. When a student employed ***quadrant-based reasoning*** they chose vectors to reach the goal based upon the quadrant of the goal (geometric) or by the signs of the goal (numeric). ***Button-pushing*** was characterized by adjusting scalars and switching vectors rapidly as a trial-and-error strategy. While not officially characterized as a strategy, participants also tended to choose vectors that had a zero to help them solve the puzzles. Our current work seeks to identify whether and how these identified strategies might emerge as students play the newer version of the game and also what new strategies we might encounter students using.

Methods

One member of our team conducted clinical interviews with five students from a small, four-year undergraduate university in the American Southeast. Each interview lasted between 60 and 120 minutes and consisted of the participant playing at least four puzzles in each difficulty setting as time allowed. Three of the participants had completed an introductory course in linear algebra, but had not yet played any version of the video game. Two participants had. Recently completed a Calculus I course and had not yet taken any course in Linear Algebra. Interviews occurred virtually over Microsoft Teams. Players shared their screens and maximized the window in which they played the game. Each interview was recorded and transcribed using the embedded features of MS Teams. The interviewer guided the participants through gameplay with written prompts and asked follow-up questions to encourage participants to explain their reasoning and to clarify strategies and rationale as they played the game. After data collection, the authors met to discuss analysis methods and narrowed our focus on using one participant's video for an initial coding pass. Our initial analysis is driven by the research question: *How might student strategies while playing VUES be similar or different from the strategies we identified with students playing VU?*

In the next section, we will discuss some of the results from our data collection, exemplifying them with key excerpts from our interview with Kyah (pseudonym, pronounced "KAI–yuh"; a woman, who is African American, majoring in mathematics). We will first briefly discuss some of Kyah's responses during gameplay on Level 2 and explain how we use the codes from prior analysis (Mauntel et al, 2021) to categorize her activity. We then provide examples from Kyah's Level 3 gameplay to highlight how some of her 2-D strategies persisted into her 3-D gameplay. We will then highlight aspects of her 3-D gameplay that did not fall into our existing categories. We expected such strategies would exist and, thus, will necessarily extend our existing taxonomy to include new types of activity that students engage in in the newer iteration of our game.

Results

Kyah spent the first 22 minutes of the interview working through five puzzles of Level 1, which uses vectors from $\mathbb{R}^1$. Across these levels, Kyah correctly articulated numeric strategies for using multiplication and addition to reach the goal and described a general solution for how she could find more than one possible correct solution when asked. When Kyah moved on to Level 2, the 2-dimensional version of Stage 4, the interviewer prompted her to notice the visual aspect of the game which they referred to as blue, red, and green lasers. Kyah at first was surprised that "a whole three different lasers" appeared. She added, "We just need one, right." The interviewer confirmed that "the green is the same as the Final Answer." Kyah then realized "oh, so the green needs to be on the white line." Although Kyah primarily used numeric methods in five puzzles of Level 2, she did explicitly return to the idea of matching the green line to the white line later when playing Level 3.

The majority of Kyah's Level 2 (vectors from $\mathbb{R}^2$) gameplay involved the strategy ***focus on one coordinate***. This worked especially well for her when she had a vector with a 0 in the second coordinate. In this case she could scale a vector with a non-zero second coordinate until its second coordinate matched the second coordinate of the "Goal" vector. Then she could add the vector with the 0 in its second coordinate and scale it until the first coordinate of the "Answer" vector matched the "Goal" as well. For example, Kyah solved Level 2, Puzzle 4 using the linear combination (4)<-1,3> + (-5)<1,0> = <-9,12> by first scaling <-1,3> to get <-4, 12> and then adjusting the scalar on the <1,0> vector. This is consistent with the ***focus on one coordinate*** strategy we identified from analyzing student gameplay in VU (Mauntel et al, 2021). In Puzzle 5, where no vector choice had a 0, Kyah was unable to implement this exact strategy, though she did attempt to modify it. She articulated this clearly when asked, "I'm looking at the Final Answer. I always aim to go for the bottom number first for some reason. Try to get the bottom number … [then] adjust to match the top." It was not until the interviewer said, "I'm looking at the lasers," that she changed her focus, "Oh! I forget about the lasers. … I wasn't even thinking about the lasers." She did agree that the lasers could help. She and the interviewer then worked together, combining geometric and numeric strategies to solve Level 2, Puzzle 5.

When Kyah transitioned to Level 3 (vectors from $\mathbb{R}^3$), she was presented with the vectors: <1,3,1>, <-1,1,1>, <-3,-2,0>, and <-1,2,2> to reach a goal of <-7, 7, 5>. Similar to level 2, we coded Kyah's initial strategy as Numerical Focus on One Coordinate. She moved the vector <1,3,1> into the equation and adjusted the scalar to 1, stating "I was looking at 7s first. I was thinking…oh wait. Actually, I think I should look at the bottom first. I'm going to go with the 5 first." She adjusted the scalar to 5*<1,3,1> to match the z-coordinate of the "Goal". Kyah then said, "[I] need the top number to be negative," and chose the vector <-1,1,1> matching the sign with -7 in the x-coordinate. We coded this as Numerical Quadrant-Based Reasoning because she chose a prospective vector based upon the sign of the location she wanted to travel. She adjusted the scalar on <-1,1,1>, but was unable to get the first pair of vectors to work.

After this, the interviewer suggested that Kyah rotate the camera. Once she tried this, Kyah focused more on the geometric aspects of the game, especially the "lasers" that represented the scaled vectors and their linear combinations. She then adapted the strategy that she used in Level 2 and described her goal by saying "the green line is supposed to go on the white line". Because the prior version of the videogame did not include geometric representation of the resulting linear combination, we did not have a code for this strategy. After describing her intentions, Kyah adjusted the scalars until the green line looked like it overlapped the white line (Figure 2, left). She then rotated the camera view and discovered that the view "can be very misleading" (Figure 2, right). This highlights an important difference between Levels 2 and 3. Solving a puzzle on Level 3 with a geometric strategy requires viewing the vectors from a variety of viewpoints and coordinating these viewpoints to make sense of the linear combination. In this moment, Kyah used this to determine if it was possible to find a solution with these two vectors.

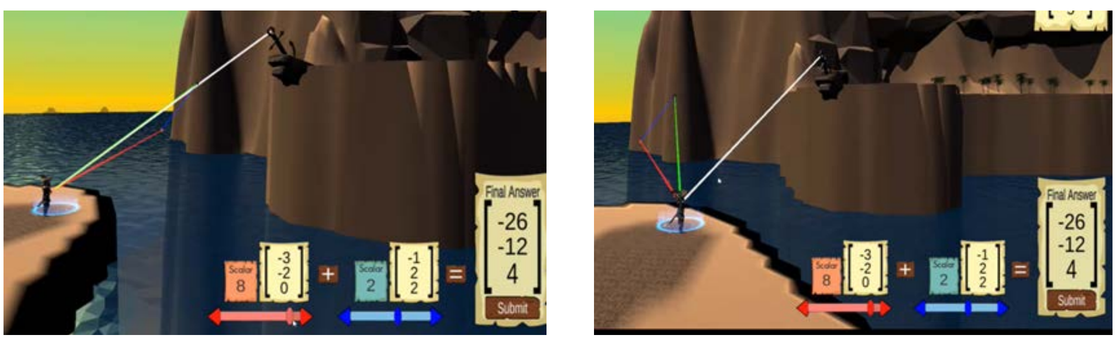

Figure 2. Kyah's "close" linear combination on Level 3 (left) and the same combination with camera moved (right).

Kyah adjusted the scalars several more times and then switched vectors repeatedly with the other two in the list. She arrived at the vectors <1,3,1> as the red vector and <-1,1,1> as the blue vector she tried to figure out how the red and blue vectors affected the green vector. After playing her conclusion was that "the red one is just shifting left and right" and "the blue, that is going to shrink it [the green vector]." From this we can infer that the Kyah's strategy is to try to shift and shrink [or expand] the green line until it lands along the white line. We call this new strategy geometric shrinking and shifting. Kyah experimented with this strategy, but was unable to find a solution and returned to a more numeric strategy.

Kyah had the vector <-3,-2,0> as the red vector and tried several vectors for the blue coordinate. The interviewer asked asked how having a zero in z-coordinate for the red vector affected the coordinate choice for the blue vector. Kyah realized that the scalar for the blue vector had to result in a 5 for the z-coordinate since the red vector contributed nothing to z-coordinate. Kyah tried multiple vectors setting the blue scalar to 5 for the vector <-1,1,1> and then adjusted the scalar on <-3,-2,0> to see if it would work. After trying several scalars for <-3,-2,0> she kept the vector <-3,-2,0>, swapped <-1,1,1> for <1,3,1>, adjusted the scalar for <1,3,1> to 5 so the z-coordinate would work, and finally adjusted the scalar on <-3, -2, 0> to -10 and then increased it to 4 and found that number worked. While Kyah explored geometrically, her final solution was the result of returning to a focus on one coordinate and combining it with a vector with a zero entry in the final coordinate. In the next level there was a vector with two zeros in it, and Kyah employed a similar strategy to reach the goal position by selecting the vector with two zeros, selecting another vector, adjust it to reach the z-coordinate, and then adjusting the vector with two zeros to get the final solution. Thus by this point, Kyah was employing a strategy similar to using vectors with zeros in several coordinates.

Discussion

First, we note that Kyah used several strategies consistent with those we observed students using with previous versions of the game, including Quadrant-Based Reasoning and choosing vectors with zeros in order to Focus on One Coordinate (Mauntel et al, 2021). Further, Kyah extended strategies from Level 2 to Level 3. However, when prompted to notice the geometric aspects of the game, she employed a new strategy, Green Line on White Line, but needed to develop a sense of how to assess whether this strategy was successful and how to manipulate the vectors to achieve this goal. Rotating the camera view allowed her to determine if the strategy was working. This points to a core difference between VU and VUES: adjusting the camera allows a player to solve puzzles in Level 3 of VUES geometrically that was not necessary for players to solve levels in VU because the earlier game only used vectors in $\mathbb{R}^2$. Kyah also found the most success using numeric strategies. We hypothesize this may be due to a key difference between VU and VUES, specifically how the game generates the vectors on any given level. In VU, players could always reach the goal as long as they selected a pair of vectors that were not scalar multiples of each other. In VUES, four vectors are provided and the player is only guaranteed that one pair of the vectors can reach the goal. This means that players could choose linearly independent vectors that do not reach the goal with the available integer scalars. This makes geometric strategies that were employed frequently during VU less effective in VUES.

Questions

We will continue to analyze Kyah's and the other student's gameplay. During the conference presentation, we anticipate presenting more robust findings from these data, including analysis of other participants' strategies and more focused coding of 3-D game strategies. We expect that the presentation will prepare audience members to engage with the following questions:

1. How might these results inform our game design and GBL design theory?
2. What theoretical and methodological approaches could help us analyze how players move the camera and interact with the computer program's 3-D environment?
3. How does the shift from vectors in $\mathbb{R}^2$ to vectors in $\mathbb{R}^3$ within VUES inform how students use the camera?

Acknowledgments

This material is based on work supported by the United States National Science Foundation (NSF; DUE-1712524). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

References

Freudenthal, H. (1991). *Revisiting mathematics education*. Dordrecht: Kluwer Academic Publishers.

Gee, J. P. (2003). What Video Games have to teach us about Learning and Literacy. New York: Palgrave MacMillan.

Gee, J. P. (2005). Learning by design: Good video games as learning machines. *E-Learning*, *2*(1), 5-16.

Gravemeijer, K. (1994). Educational development and developmental research in mathematics education. *Journal for Research in Mathematics Education*, 443-471.

Gresalfi, M. S., & Barnes, J. (2016). Designing feedback in an immersive videogame: Supporting student mathematical engagement. *Educational Technology Research and Development*, *64*(1), 65-86.

Kuster, G., Johnson, E., Keene, K., & Andrews-Larson, C. (2018). Inquiry-oriented instruction: A conceptualization of the instructional principles. PRIMUS, 28(1), 13-30.

Mauntel, M., Levine, B., Plaxco, D., & Zandieh, M. (2020). Get That Basket! Deciphering Student Strategies in the Linear Algebra Game Vector Unknown. In Proceedings of the 23rd Annual Conference on Research in Undergraduate Mathematics Education.

Mauntel, M., Levine, B., Plaxco, D., & Zandieh, M. (2021). The Characterization and Evolution of Strategies About Vector Equations in the Game Vector Unknown. Digital Experiences in Mathematics Education, 1-24.

Mauntel, M., Sipes, J., Zandieh, M., Plaxco, D., & Levine, B. (2019). “Let’s see” – Students play Vector Unknown, an inquiry-oriented linear algebra digital game. In A. Weinberg, D. Moore-Russo, H. Soto, and M. Wawro (Eds.). In Proceedings of the 22nd Annual Conference on Research in Undergraduate Mathematics Education (pp. 959-965). Oklahoma City, OK.

Rasmussen, C., & Kwon, O. N. (2007). An inquiry-oriented approach to undergraduate mathematics. *The Journal of Mathematical Behavior*, *26*(3), 189-194.

Sierpinska, A. (2000). On some aspects of students’ thinking in linear algebra. In On the teaching of linear algebra (pp. 209-246). Springer, Dordrecht

Stewart, S., Axler, S., Beezer, R., Boman, E., Catral, M., Harel, G., McDonald, J., Strong, D., & Wawro, M. (under review). The Linear Algebra Curriculum Study Group (LACSG 2.0) Recommendations.

Wawro, M., Zandieh, M., Rasmussen, C., & Andrews-Larson, C. (2013). Inquiry oriented linear algebra: Course materials. Available at http://iola.math.vt.edu. This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.

Williams-Pierce, C. (2019), Designing for mathematical play: failure and feedback. *Information and Learning Sciences*, *120* (9/10), 589-610.

Williams-Pierce, C., & Thevenow-Harrison, J. T. (2021). Zones of mathematical play. *Journal of the Learning Sciences*, 1-19.

Zandieh, M., Plaxco, D., & Williams-Pierce, C. & Amresh, A. (2018). Drawing on the Three Fields of Educational Research to Frame the Development of Digital Games for Inquiry Oriented Linear Algebra. In Proceedings from the 21st Annual Conference on Research in Undergraduate Mathematics Education (pp. 1270). San Diego, CA.

Zandieh, M., Wawro, M., & Rasmussen, C. (2017). An example of inquiry in linear algebra: The roles of symbolizing and brokering. PRIMUS, 27:1, 96-124.