

Translanguaging in Mathematics Learning: Teacher Instructional and Attentional Supports

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Abstract

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Spaces for Translanguaging in Secondary Mathematics Classrooms

Thesis directed by Associate Professor Victoria Hand

Students from less-dominant linguistic backgrounds generally have less opportunity to participate in classroom mathematical discourse compared to their English-dominant peers. An issue raised by mathematics education researchers concerned with issues of equity and opportunities for students is that status quo classroom practices and norms supported by teachers may be less familiar to students from non-dominant linguistic groups, or even detrimental to their classroom participation. Additionally, students who position themselves as doers of mathematics usually come from dominant cultural and linguistic groups (Abreu & Cline, 2002; Hand, 2012), potentially disposing students to perceive classroom mathematics learning through the lens of dominant cultural norms and practices. Thus, students who do not come from dominant linguistic backgrounds might perceive the mathematics classroom differently than their English dominant peers. However, less research has been conducted on how mathematics teachers attend to or notice norms around language and introduce new ones that encourage a multitude of linguistic practices, therefore heightening student participation. Heightening student participation can have implications for students being more likely to identify with mathematics. Additionally, examining students' participation when using a multitude of linguistic practices or translanguaging is helpful for teachers attending to their own practice to support emerging bilingual students and bilingual students when engaging in mathematical sensemaking.

In this dissertation, I bring together the teacher noticing and translanguaging literatures to explore students' support for mathematical sensemaking in classroom interactions (i.e. spaces for translanguaging). In drawing upon these literatures, I designed a case study of two secondary mathematics teachers in different school contexts to study how they supported spaces for translanguaging in their mathematics classroom over three years. Sources of data were teacher interviews, student interviews, student perception surveys, and classroom video data. The analyses revealed that spaces for translanguaging looked different across classroom contexts. However, closer examination revealed similarities and differences in how both teachers supported these spaces. Results from this work suggest that understanding the noticing and reflective practices of mathematics teachers within different classroom contexts can inform teacher and preservice mathematics teacher practices regarding innovative practices supporting bilingual and emerging bilingual students.

Dedications

I dedicate this dissertation to several people.

To my husband, Wesley Gilmore.

Together, you and I have embarked on this journey and together we continue. Your love and support continue to motivate me to keep reaching, all the while reminding me that that reach is part of the adventure.

To my dad, whose energy has always galvanized me to get things done. You set the wheels in motion with that phone call: “If you want to get your PhD...”

To my mama, whose endless positivity is my reassurance.

To both of you, your love and support has empowered me all my life.

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Chapter 1. Introduction

It is generally known that mathematics can be a gatekeeping subject for many students, especially students from underrepresented groups (Abrue & Cline, 2012; Gutiérrez, 2002; 2009; Hand, 2012; Martin, 2000). As learners from non-dominant backgrounds have less access to resources and support in K-12 mathematics, their success (or not) can have implications for their later professional success in a Science, Technology, Engineering or Mathematics (STEM) career track. More specifically, students from less-dominant linguistic backgrounds generally have less opportunity to participate in classroom mathematical discourse compared to their English-dominant peers (Takechi, 2015; Turner, Dominguez, Empson, & Maldonado, 2013; Turner, Dominguez, Maldonado, & Empson, 2013).

An issue raised by mathematics education researchers concerned with issues of equity and opportunities for students is that status quo classroom practices and norms supported by teachers may be less familiar to students from non-dominant linguistic groups, or even detrimental to their classroom participation. An example of a status quo norm could be grouping students that are not comfortable sharing their ideas in English with students who solely speak English. Additionally, students who position themselves as doers of mathematics usually come from dominant cultural and linguistic groups (Abreu & Cline, 2002; Hand, 2012), potentially disposing students to perceive classroom mathematics learning through the lens of dominant cultural norms and practices. Thus, students who do not come from dominant linguistic backgrounds might perceive the mathematics classroom differently than their English dominant peers. However, less research has been conducted on how mathematics teachers attend to dominant norms around language and introduce new ones that encourage a multitude of linguistic practices, therefore expanding student participation (i.e. encouraging communication

of mathematical ideas across languages, allowing students to multimodally express their mathematical ideas, etc.). Expanding student participation can have implications for students who are normally discouraged from participating as they are more likely to identify with mathematics. Additionally, examining students' participation when they are using a multitude of linguistic practices is helpful for teachers in structuring their own practice to support emerging bilingual students and bilingual students in making meaning of mathematics and expressing their ideas.

However, expanding norms around linguistic practices may not be enough to promote equitable outcomes in mathematics classrooms. How teachers position students in relation to the linguistic practices they bring into the classroom can also impact students' mathematics learning and ultimately, their mathematical identities. I draw from Davies and Harré's (1990) conceptualization of positioning as the discursive process by which selves are located in jointly produced conversations and storylines. An example of instructional positioning in the mathematics classroom is the practice of assigning competence to historically marginalized students, or to publicly affirm that the minoritized student is offering something intellectually productive to the class or team (Gresalfi, Martin, Hand & Greeno, 2008; Jilk, 2016; Turner et al., 2013). When emerging bilingual students are positioned as competent by the teacher, this positioning is more likely to be taken-as-shared by their peers, which in turn shapes future patterns of participation of that student and their peers. Part of the role of teachers is to be reflective of the ways in which students, and in particular emerging bilingual students, are being positioned and positioning themselves around various classroom practices.

A way to understand the practice of assigning competence around students' social and linguistic capital is that it opens up space for translanguaging in the classroom. Classroom spaces

for translanguaging are discursive spaces in which students can access their full range of linguistic practices to engage in classroom learning, thus challenging dominant English-only norms and treating linguistic hybridity as competent (Gutiérrez & Rogoff, 2003). Specifically, by engaging multiple linguistic resources, students are expressing themselves and communicating their ideas no matter which language. For example, a student might draw upon a different linguistic repertoire of practice to engage with an authority figure versus a fellow student (Gutiérrez & Rogoff, 2003). More broadly, studying how multilingual students engage in linguistic maneuvers has implications for refining teachers' attention to the variety of ways students participate in mathematics classrooms as spaces for translanguaging.

Teacher noticing, or “the in-the-moment” decisions teachers make in the course of teaching has implications for equity in the mathematics classroom (Erickson, 2011) and for spaces for translanguaging. Research indicates that teachers who are disposed to advance equity engage in noticing practices that center student positioning, power structures, and status that influence interactions and opportunities within mathematics classrooms (Hand, 2012; Louie, 2017; Turner, Dominguez, Maldonado & Empson, 2013; Wager, 2014). Teachers who are paying less attention to the dimensions listed above may have less success in supporting classroom equity.

Current research on teachers' organization of their classrooms to leverage linguistic repertoires of practice for emerging bilinguals (Aguirre et al., 2013; Fernandes, 2012) can inform our understanding of the noticing practices that might support spaces for translanguaging. For example, attention to students' multiple linguistic resources makes visible the various ways that students make sense of mathematics apart from dominant English language and sense-making processes (Garcia, 2018; Gort & Sembiente, 2015; Hornberger & Link, 2012; Palmer &

Martinez, 2013). Teachers' attention to, or noticing of, students' linguistic resources and their relation to mathematics learning can also challenge deficit perspectives of emerging bilingual students (Aguirre, Turner, Bartell, Kalinec-Craig, Foote, Roth McDuffie & Drake, 2013; Jilk 2016). Teachers can also attend to the cultural significance around concepts in one language to make connection with another.

However, less is known about teacher noticing practices that support spaces for translanguaging as they relate to equity in mathematics classrooms. Mathematics tends to be a more abstract subject and thus less explicitly connected to students' everyday experiences. My dissertation addressed the gaps in the literature surrounding the relationship between teachers noticing practices and what I call spaces for translanguaging in mathematics classrooms (de Araujo et al., 2018; LopezLeiva, Torres & Khisty, 2013). Specifically, I focus on how mathematics teachers attend to classroom interaction and student participation in ways that support the co-construction of spaces for translanguaging with emerging bilingual students. As a result of this focus, the research questions for my study are:

Research Question 1: *How do mathematics teachers co-construct spaces for translanguaging with emerging bilingual students in secondary mathematics classrooms?*

- ***Research Sub-Question 1:*** *What do spaces for translanguaging look like in classroom mathematical activity?*
- ***Research Question Sub-2:*** *Which pedagogical moves support different types of spaces for translanguaging?*
- ***Research Sub-Question 3:*** *How do spaces for translanguaging support student mathematical sensemaking?*

Research Question 2: *How do particular teacher noticing practices support spaces for translanguaging in secondary mathematics classrooms?*

In this dissertation, I analyzed data gathered through Co-ATTEND, a research project sponsored by the National Science Foundation and led by Dr. Victoria Hand at the University of Colorado Boulder and Dr. Elizabeth van Es at the University of California Irvine. For my dissertation, I collected additional data to understand how secondary mathematics teachers supported emerging bilingual students in expressing their ideas in the mathematics classroom. As such, I developed case studies of the teachers' instructional and noticing practices to interpret how teachers supported emerging bilingual students. Furthermore, I focused on how students expressed their mathematical ideas across languages in ways that disrupted school norms and how they were positioned as competent by the teacher and their classmates.

In the following chapters, I present my study of these teachers' classes to better understand how they supported emerging bilingual students in the mathematics classroom. In Chapter 2, I explore the current research on mathematics education, bilingual education, and teacher noticing. In Chapter 3, I explain the study context and data collection in addition to my analytic approach to understanding teacher noticing practices as they relate to language. In Chapters 4 and 5, I present findings about the spaces for translanguaging, the instructional practice and the noticing practices for both teachers in my study. In Chapter 6, I present a cross analysis of the practices of the two teachers. In Chapter 7, I discuss how my findings contribute to the existing literature on mathematics classroom practices, teacher noticing, and languaging practices. Lastly, in Chapter 8, I conclude with implications from the findings of this dissertation for theoretical and practical implications of the integration of mathematics and bilingual education within broader sociopolitical contexts.

In reviewing research on teacher pedagogy and student linguistic resources in the mathematics classroom, I draw upon the following bodies of literature: mathematics classroom practices, translanguaging practices, and teacher noticing.

Chapter 2. Literature Review

In this literature review, I outline bodies of research that contributed to my conceptual framework. I organize my review into three main areas of research: mathematics classroom practices, teacher noticing, and translinguaging practices and describe lines of research within each area.

Within the mathematics classroom literature, I outline sociocultural and sociopolitical perspectives on student and teacher learning with respect to mathematics classroom experiences. Furthermore, I review classrooms as interactional spaces and consider participation structures, discourse, and identity. Within the noticing literature, I detail research on teacher noticing and teacher noticing for equity. In the translinguaging literature, I review research on hybrid language practices and funds of knowledge. Within the translinguaging section, I outline how the sociopolitical perspective is helpful to better understand classroom language practices. These areas of research illustrate the importance of teacher noticing in both attending to and responding to interactions and circumstances around language that are organized in the mathematics classroom. Identifying the ways that teachers attend to opportunities to draw upon students' linguistic resources in classroom mathematical sensemaking may demonstrate how to better support the mathematics learning and mathematical identities of emerging bilingual students. In this research, I utilize Palmer & Martinez's (2013) characterization of bilingualism as "the act of engaging in two or more languages in the home, school, and/or community" (Palmer & Martinez, 2013, p.271) and I refer to emerging bilingual students as students that are learning a second language (García & Kleifgen, 2010). For the scope of this study, I am generally focusing on the Spanish speaking Latinx community.

Sociocultural Perspective on Mathematics Teaching and Learning

In this section, I will outline the sociocultural perspective on learning in the mathematics classroom. The sociocultural perspective is important to consider, as it conceptualizes the classroom as a social and cultural space and acknowledges the complex histories that students bring into them.

The sociocultural perspective derives largely from Vygotsky's notion that all learning is social and cultural, meaning that it is mediated by previous cultural histories and current experiences (Vygotsky, 1978). This means that all students come into the classroom with cultural histories that inform their experiences (Vygotsky, 1978).

Sociocultural perspectives on learning are helpful in understanding complex interactions in mathematics classrooms. One of the ways it is helpful in understanding classroom interactions is by considering the relations between students' culture, community, and past experiences and the construction of classroom communication, norms, and practices. Another way to understand classroom interactions from the sociocultural perspective is by considering community classroom norms around mathematical contributions and productive participation.

Mathematics classrooms as interactional spaces.

The mathematics classroom environment consists of complex interactions. These interactions take place in moment-to-moment activity in which students and teachers are getting organized around (and are organizing) different aspects of the environment. Aspects of the environment that shape moment-to-moment activity include classroom mathematical and social norms, classroom practices, pedagogical practices, the curriculum, and participation structures. These aspects of the environment function together to create a classroom system, which ultimately shapes students' opportunities to learn mathematics. Classroom practices involve

ways of engaging in classroom activities (mathematical and otherwise) that are jointly established by teacher and students over time (Cobb, Stephan & Gravemeijer, 2001). In some mathematics classrooms, these joint practices involve publicly explaining and justifying answers, sensemaking, agreeing, disagreeing, and questioning alternatives (Cobb et al., 2001). Within this type of classroom community, students are engaged in tasks that are authentic to the community of practice of mathematicians and invite students' development of arguments and conjectures (Ball, 2005; Lampert, 1990). However, it is important to keep in mind that while this might be the case for some students, it is not necessary for all.

Additionally, classroom mathematics communities often uphold norms for what counts as mathematical contributions and productive mathematical participation (Lampert, 1990). In her research study of her practice on the development of mathematics classroom culture and how students come to participate in it, Lampert (1990) asserts that the teacher models productive mathematics participation through articulating her mathematical arguments. Furthermore, in learning how to articulate mathematical arguments, students take ownership of the mathematics, offering their reasoning, instead of the teacher's. Cobb, Wood, & Yackel (1993) explain that since mathematics learning is a social activity, mathematical meaning is being negotiated in moments of social interaction. However, in navigating classroom mathematics as a social activity, classroom participants are constrained by individuals' interpretations of mathematical reasoning (Cobb, Wood & Yackel, 1993). For example, in their study on elementary mathematics classrooms, Cobb, Wood, & Yackel (1993) found that the teacher chose which points of individual student and group mathematical interpretation to highlight to the rest of the classroom community. Thus, teachers must attend to how classrooms are interactional spaces of mathematical sensemaking so that all students are invited to participate. Feeling invited to

participate might emerge over time as students learn the classroom patterns of interaction, or the classroom participation structures.

Mathematics classrooms: participation structures.

Analysis of the participation structures of mathematics classrooms is important to understand how space for sensemaking is made available or not in classroom interaction. Classroom participation structures are the patterned ways that individuals come to organize their interactions with signs and tools over time. They also involve the formation of interactional roles and the enactment of roles between participants (Cazden & Beck, 2003; Tabak & Baumgartner, 2004). For example, a common participation structure in mathematics classrooms is student presentation of mathematical ideas. The type of participation structure within which students present their ideas can vary widely. One participation structure might support students in making guesses about mathematical ideas and trying out ideas, while another might be formed around students presenting work correctly. The ways that mathematics teachers facilitate classroom participation structures may shape their own and students' perceptions of the mathematical abilities of students from different cultural and linguistic backgrounds. Perceptions of students' mathematical abilities can be shaped by societal views based on broader social and linguistic stereotypes and hierarchies (Fernandes, 2012; Egalite, Kisida & Winters, 2015; Nasir & Shah, 2011). Similar to students experiencing gender bias in mathematics and science classrooms (Radovic, Black, Salas & Williams, 2017; Rosenthal, London, Levy, Lobel, 2011) emerging bilingual students experiencing linguistic bias (i.e. where English is the language assigned value in the classroom) in classrooms may begin to view themselves as members of a group who does not "do" mathematics (Abrue & Cline, 2003; Fernandes, 2012) and therefore are not invited to participate in the mathematics classroom. Attending to instances in which students feel uninvited

to participate, and how this is related to other aspects of the mathematics classroom such as classroom interaction and participation structures can enable teachers to shift their instruction towards a more inclusive environment (Hand, 2012) with more opportunities for students to engage in discourse.

Mathematics classrooms as discursive spaces.

Furthermore, a sociocultural perspective considers student learning as a trajectory of participation in practices of mathematical discourse or the process of meaning making and thinking (Boaler & Greeno, 2000; Moschkovich, 2007; 2015). Classroom interactional spaces consist of students, the teacher, and other aspects of the classroom (Greeno, 1997) co-constructing and engaging in discursive spaces. As such, students' conceptual understanding can be considered as an outcome of their discursive interactions around mathematical sensemaking (Greeno & Sande, 2007). Students in classrooms where their teacher is disposed to facilitate discursive interactions are given more opportunities to contribute towards their mathematical sensemaking (Gresalfi et al., 2008; Staples, 2007; Turner et al., 2013). In supporting whole classroom mathematical sensemaking, mathematics teachers engage in specific instructional moves to facilitate productive interactions (Staples, 2007). Staples (2007) explored how the high school mathematics teacher in her study positioned students to work collectively and contribute their ideas to each other before voicing them to the whole group. Staples (2007) found that mathematics teacher facilitation of student participation was important, yet the role of the teacher needed further explanation as it related to collaborative inquiry by the whole class. The teacher facilitation of these opportunities for students to voice understandings may have implications for whose voice is heard in the classroom and who is positioned as an expert (Turner et al., 2013).

For example, students enter the mathematics classroom with sociocultural experiences from outside of the classroom and encounter the practices and norms of the mathematics classroom. The discourse practices in students' homes and communities may be positioned as more or less competent in particular classroom communities (Nasir, Hand, and Taylor, 2008). If there is a blend of students in the classroom from a range of backgrounds and schooling experiences, groups of students may be positioned relative to their perceived mathematical ability, either by the teacher or students by each other, based on their everyday practices and the degree to which these overlap with practices valued in the classroom (Nasir, 2002; Nasir & Hand, 2008; Nasir et al., 2008). In capitalizing on opportunities to draw out student conceptions, teachers can validate student sensemaking through everyday practices.

The teacher facilitation of these opportunities for students to voice understandings may have implications for whose voice is heard in the classroom and who is positioned as an expert (Turner et al., 2013). For example, teachers who attend primarily to students' abilities to describe mathematical ideas in English may form deficit perspectives of the mathematical capacities of bilingual students (Fernandes, 2012). Thus, teachers must attend to the type of discursive spaces for mathematical sensemaking that are being constructed so that all students are invited to participate. On the other hand, students might reject classroom patterns and participation structures if they feel as though they contradict their identity.

Mathematical identity.

Being positioned as mathematically weak over time can negatively shape students' identities as mathematics learners. The sociocultural perspective holds that learning and identity are inextricably related and shift through participation in social activity (Lave & Wenger, 1991). According to Greeno and Sande (2007) individuals participate in multiple communities, in

trajectories of participation, and as they participate, they develop identities which are shaped by the norms, practices and values of the various communities in which they participate. This means that the identities they are developing in one set of communities may conflict with the identity they are developing in the mathematics classroom. Again, these identities are a reflection of differences in the norms, practices and values of these different communities and these differences play out in the moment-to-moment interactions in the classroom.

Langer-Osuna & Esmonde (2017) describe identity as “a construct that people use to capture something they intuitively grasp about themselves and other humans” (p. 637). From this, I assert that mathematics identity is a way that people grasp themselves and other humans in relation to their ability to do mathematics successfully (Cobb et al., 2009) or their decision regarding whether or not to participate in what they perceive to be the practices of doing mathematics (Boaler & Greeno, 2000). How teachers instruct and facilitate students in relation to the resources they bring into the classroom can impact student participation in mathematical activity. However, how students perceive those opportunities to participate is what might ultimately impact students’ identifying with mathematics. In better understanding and attending to the extent to which students identify with mathematics, teachers can have a better sense of how to adapt (or not adapt) their practice.

Mathematics teacher noticing.

Understanding the noticing practices of mathematics teachers is important in gauging how teachers make sense of complex classroom environments (Jacobs, Lamb & Phillips, 2010; Lampert, Franke, Kazemi, Ghouseini, Turrou, Beasley, Cunard & Crowe, 2013) and the instructional decisions that result. Since no one can be aware of all stimuli in a space at any given time, teacher noticing gives insight into the rationale and/or impetus behind the

instructional decisions teachers are making. These instructional decisions are informed by teacher reflection on their own practice (Hiebert, Morris, Berk, & Jansen, 2007; Lampert, 2010) that highlight how they make sense of classroom interactions. Jacobs, Lamb & Phillip (2010) assert that noticing highlights the complex interactions in a classroom and “underscore the idea that teachers see classrooms through different lenses depending on their experiences, educational philosophies, cultural backgrounds, and so on and that particular kinds of experiences can scaffold teachers’ abilities to notice in particular ways” (p.171). In other words, the noticing practices of mathematics teachers from different backgrounds and experiences are important in order to understand how teachers make sense of complex classroom environments (Jacobs, Lamb & Phillips, 2010) across different contexts.

Additionally, teachers across different contexts have different dispositions towards noticing that influence their teaching. These dispositions or pedagogical commitments (Erickson, 2011), are shaped by their participation in sociocultural communities (Hand, 2012). Essentially, what teachers foreground and background in their noticing depends heavily on their identities and previous experiences. Since teachers' identities and previous experiences are all contributing factors to their teaching experience, van Es & Sherin (2008) assert that how teachers notice is just as important as what they notice (*italics added*). Mathematics teachers can be supported to reflect on their noticing in ways that have implications for equity (van Es & Sherin; 2009; Louie, 2017). For example, a teacher might notice that a student for whom English is not a first language is hesitant to participate. Given opportunities to reflect on this noticing in a variety of ways, the teacher may come to see that one interpretation of the student hesitation centers problems with the student alone, while another one focuses on whether the student feels like a member of the classroom community.

In sum, sociocultural perspectives on learning are helpful in considering complex classroom interactions. Furthermore, these classroom interactions involve participation structures where spaces for sensemaking are made available or not in students feeling invited to participate. Feeling invited to participate can opportune students to engage in mathematical discourse which can shape students' identities as mathematics learners. Teacher attention to these opportunities can further illustrate these complex classroom interactions.

Sociopolitical Perspectives on Mathematics Education

A sociopolitical lens is also helpful to understanding the complexity of mathematics classrooms, particularly in relation to issues of power. Gutiérrez (2002; 2009) explains that mathematics classrooms often reflect the status quo distribution of power in society, which are reflected in broader sociopolitical hierarchies. This means that as a social and cultural process mathematics learning is embedded in these hierarchies. As a result, teaching mathematics is not politically neutral (Gutiérrez, 2009). For example, mathematical ability has become a proxy for intelligence, which means that students who are viewed as less capable mathematically may also be perceived (and perceive themselves) as less intelligent. This has important implications for how teachers organize their classroom instruction to either reinforce or disrupt this idea. For instance, Gutiérrez (2009) has conceptualized dimensions of mathematics teaching and learning that relate to the sociopolitical perspective. On one axis is Achievement and Access, which refers to opportunities for students to engage deeply in dominant mathematical content and to achieve success as a result (Gutiérrez, 2009). The other axis is Identity and Power, which refers both to seeing oneself reflected in the classroom mathematical community, as well as employing mathematics to transform society more broadly (Gutiérrez, 2009). Gutiérrez argues that attention to the second axis, Power and Identity, can help students develop into critical citizens so they can

eventually change the status quo into being more equitable in order to “change the game” of mathematics teaching and learning (Gutiérrez, 2009; Lubienski & Gutiérrez, 2012). The idea of “changing the game” of mathematics teaching and learning helps explain how students can be empowered to participate more fully in mathematics classrooms.

A way to envision how students can “change the game” of mathematics teaching and learning (Gutiérrez, 2009; Lubienski & Gutiérrez, 2008) is through validating students’ mathematical achievement and identity. Student identity as a doer of mathematics (Abrue & Cline, 2003; Cobb, Gresalfi & Hodge, 2009) can be the result of how teachers balance content, respect students and create community or not. Furthermore, developing identities are also shaped by classroom microcultures (Cobb et al., 2009). By microcultures in the mathematics classroom, Cobb et al. (2009) are referring to the nature of mathematical activity and what it means to know and do mathematics. Cobb et al. (2009) conceptualize the relationship between classroom microcultures and student identity through Martin’s (2000) interpretive framework on identity, which is comprised of sociohistorical, community, school and intrapersonal levels of identity (Martin, 2000; Cobb et al., 2009). The sociohistorical level encompasses the racial hierarchy of mathematical ability whereas the community level involves the context where mathematics teaching and learning occur (Martin, 2000). The school level denotes a concern for negotiation of norms in classrooms, while the intrapersonal level focuses on what it means for students to know and do mathematics in the classroom and to what extent they come to identify with classroom math activities (Martin, 2000). These levels of identity (Martin, 2000) all need to be considered when envisioning how to validate more students in the mathematics classroom. One way to envision this validation is through attention to equitable teaching.

Mathematics teacher noticing for equity.

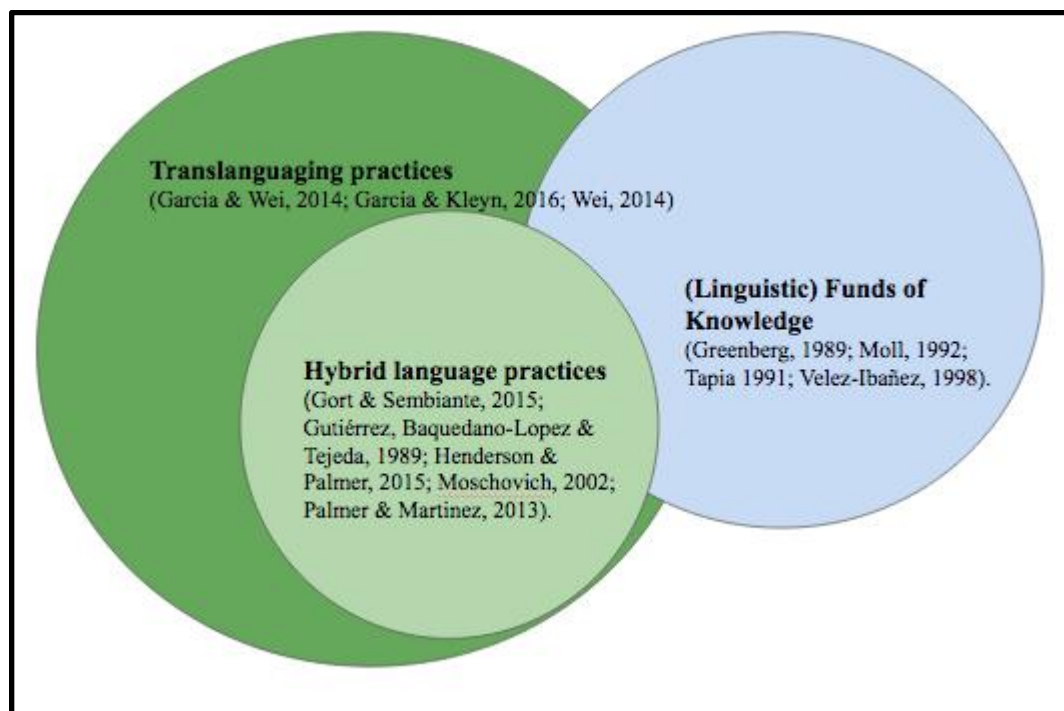
Research on noticing for equity can help us understand more about sociopolitical aspects of mathematics teaching. Teacher noticing for equity not only involves attention to student thinking and understanding, it also places an emphasis on attending to student strengths and facilitating mathematical connections to students' current understandings (Aguirre et al., 201; Jilk, 2016; Turner et al., 2012). For example, van Es, Hand, & Mercado (2017) identify teacher classroom practices in their study as they relate to noticing practices and draw connections between the two that support greater classroom equity. In attending to whether students understand the mathematical task at hand, all teachers in the study attended to the connections between students' histories inside and outside of the classroom and how they could be considered within the classroom via task design, available resources, and students' emotional states (van Es, Hand & Mercado, 2017). Furthermore, these teachers attended to issues of student status and positioning, and student energy and flow of the classroom interactions. Louie (2017), on the other hand, in her research study on teacher noticing for equity, cautions us that moment-to-moment noticing is shaped by broader ideologies, and these ideologies may cause teachers to fail to notice aspects of their teaching practices and classroom activities despite the desire to create particular classroom spaces. Thus, as suggested above, teachers noticing is often a reflection of dominant ideologies. According to Hand (2012), teachers who attend differently to class math activity will "provide a more even playing field for nondominant learners" (Hand, 2012, p.235) and that by learning to notice inequities in the classroom, teachers dispositions shift towards issues of culture and power in math learning.

In summary, the sociopolitical perspective is important in that it highlights how issues of power influence classroom hierarchies, identity, and status in the mathematics classroom.

Teacher noticing for equity is a way to attend to these aspects.

Emerging Bilingual Students in Mathematics Classrooms

In this section, I consider the role of the teacher as a major influence on how students, particularly emerging bilingual students, come to learn mathematics and identify as being “good” at it. Additionally, I consider perspectives of language that have taken place in education and mathematics education research by outlining language practices, students’ (linguistic) funds of knowledge, and translanguaging practices.

Figure 2. 1*Literature Review, Language Practices*

Additionally, I explain below how I conceptualize translanguaging as encompassing hybrid language practices and partly encompassing (linguistic) funds of knowledge as a reflection of these shifts in language perspectives. Examining shifts in how language is viewed as a resource (or not) in the mathematics classroom is helpful in considering how teachers may leverage language in their own practice to create space for emerging bilingual students to make meaning of mathematics and express their ideas.

Teachers' roles with emerging bilingual students.

The teacher's role in the classroom is vital to students' experiences (Khisty & Chval, 2002; Lopez, 2014). For example, in showcasing how a particular teacher avoided interacting with a specific bilingual student, de Araujo et al. (2018) explained how that teacher positioned that student to be excluded by his classmates, which may have affected how he identified as a

doer of mathematics. Lopez (2014), in his study of Mexican high school students reporting their experiences in elementary mathematics, found that their teachers were key social agents in presenting students with opportunities to form positive identities in the mathematics classroom. Additionally, through a case study exploring mathematics teaching practices of monolingual teachers teaching bilingual, Latinx learners, Willey et al. (2017) demonstrate the need to understand direct teacher interactions with students as a precursor to developing a meaningful relationship or engaging in mathematics learning (Razfar, Licon Khisty & Chval, 2011).

As described above, status quo classroom practices and norms supported by teachers may be less familiar to students from non-dominant groups, or even detrimental to their classroom participation. Students who position themselves as doers of mathematics usually come from dominant cultural groups (Abreu & Cline, 2002; Hand, 2012), potentially disposing teachers to perceive classroom mathematics learning through the lens of dominant cultural norms and practices. For instance, a monolingual English-speaking teacher following a mathematics curriculum rubric for evaluating student presentations may discount a mathematical explanation that mixes English and Spanish due to the explanation not being entirely in English. As a result, students outside the margins of the dominant language group may not feel like a doers of mathematics if their language is not valued as a resource in the classroom.

Mathematics identity of emerging bilingual students.

More broadly, bilingual student identity as a doer of mathematics (Abrue, & Cline, 2003; Cobb, Gresalfi & Hodge, 2009; Martin 2000) is related to the degree to which these students feel invited to participate in the classroom. Furthermore, as described above, developing identities are also shaped by classroom microcultures (Cobb et al., 2009). What it means to know and do mathematics can be connected to feelings of what it means to be “smart” in math class. For

example, a Spanish dominant student learning English that is given a standardized mathematics test in English, might not score as highly as their English dominant peers. As a result, that emerging bilingual student might perceive that English is the language in which to do mathematics and discount the variety of ways they might have problem solved, had the test been in both languages. Broadly speaking, this example points to a need for a consistent shift in orientations towards language as a resource.

Language as a resource and hybrid language practices.

Palmer & Martinez (2013; 2016) affirm that language is still not generally viewed as a resource within the classroom, especially for emerging bilingual students (Garcia & Sylvan, 2011; Gort, 2012; Hornberger, 2003). There is a call for the relation between learning and language to shift from a deficit perspective to one in which languages other than English are viewed as a resource in the field of education and more specifically, mathematics education (Barwell, 2005a; Civil & Hunter, 2015; de Araujo et al., 2018; Dominguez, 2011; Gutiérrez, 2002; Palmer & Martinez, 2013).

Although the focus on language as a resource is important, it alone is not enough to address historically maintained and reproduced unequal power relations in schools. This power imbalance has resulted from deficiency-based models that invalidate the linguistic and cultural heritages of minority families (Perez, Vasquez & Burial, 2010). Historically, bilingual education has been viewed from a monolingual perspective, meaning that languages and students' biliteracy practices are viewed as separate entities and should be separated in practice. However, classroom interactions and circumstances are complicated whether the classroom is composed of monolingual or multilingual participants. For example, one aspect of complexity in monolingual classrooms is that dominant social norms can impact instructional designs. More specifically,

one of these norms that can impact instructional designs is that monolingual teachers are asked to question students' bilingualism in the form of second language accommodations (Palmer & Martinez, 2013). In their research on dual immersion language program implementation, Henderson and Palmer (2015) assert that instructional designs are influenced by macro or outside factors and social norms such as school program implementation and standardized testing.

Palmer and Martinez (2016) argue for a bi/multilingual perspective, which is a shift towards viewing language as a hybrid practice in which languages are employed meaningfully and purposefully (Palmer and Martinez, 2013) in moments of interaction. Understanding hybrid language practices is important in considering how to support flexible language uses (Gort & Sembiente, 2015; Gutiérrez, Baquedano-Lopez & Tejeda, 1989; Henderson & Palmer, 2015; Moschovich, 2002; Palmer & Martinez, 2013). Hybrid language practices involve the mixing of language to make meaning and establish connections (Palmer & Martinez, 2013). This hybridity is composed of dominant discourse practices of classrooms used fluidly with linguistic practices from students' homes and local communities. Historically, hybrid language practices have been described in a variety of ways, as codeswitching (Cacoullos & Travis, 2018), polylinguaging (Jorgensen et al. 2011), codemeshing (Canagarajah, 2011) and Spanglish (Henderson & Ingram, 2018). Although there is not a general consensus on what to name or how to conceptualize hybrid language practices, there is generally a consensus that they capitalize on mixing language to make and express meaning.¹

¹ Currently, hybrid language practices can be used synonymously with translanguaging (Garcia, 2009). However, I view translanguaging from a sociopolitical perspective so I view hybrid language practices as different than and encompassed by translanguaging.

Employing hybrid language practices in the classroom may shape the likelihood that emerging bilingual students will fully engage with concepts and activities. Gort and Sembianti's (2015) study of a preschool teacher and her two co-teachers in an elementary dual immersion classroom found that the teachers' language practices supported emerging bilingual students' participation. This participation involved formalized language performance across subjects (including mathematics) since it enabled the co-construction (with students) of discursive spaces that integrated academic language and content (Gort & Sembianti, 2015). To perceive language as a discursive space requires that teachers attend to the multiple ways that students are using different language practices in the classroom. Specific to the mathematics classroom, leveraging language as a hybrid practice and resource has the potential to create more space for students to participate.

Hybrid language practices in the mathematics classroom.

Research on hybrid language practices can help us understand more about how to support emerging bilingual students in the mathematics classroom. Several studies have found that using hybrid language practices validates an asset orientation towards language as a resource, which can support greater multi-modal dimensionality in students' mathematical meaning-making and expression (LópezLeiva, Torres & Khisty, 2013; Martinez, 2014). For example, LópezLeiva et al., (2013) explored how language was used in small groups engaging in mathematical problem solving during an after-school program. LópezLeiva et al. (2013) identified patterns across groups in how students' leveraged multidimensionality or their experiences, languages and hybrid mixing of languages in their problem solving. In other words, students brought all their resources to bear when they engaged in mathematical reasoning. LópezLevia et al. (2013) found that the more educators formalized what counted as mathematical reasoning, the more students'

multidimensionality in expression was limited. This research also shows that drawing upon student ideas across languages helps expand students' conceptual understanding (de Araujo et al., 2018; LópezLeiva, Torres, & Khisty, 2013). Again, teachers must be attuned to students' multidimensional use of language. Finally, though his study took place in an English classroom, Martinez (2014) contends that teachers modeling how students can apply or utilize hybrid language practices in unfamiliar contexts is important.

For students navigating unfamiliar classroom contexts, it is important to consider how students can be obligated to speak in one language or the other in the mathematics classroom. An obligation to speak a preferred language can affect who has ownership in mathematical meaning making. To not allow for language choice reinforces dominant power structures, mainly the assumption that only one language is needed in the classroom. When making sense of mathematical situations, Dominguez (2010), found that his emerging bilingual participants were more disposed to share knowledge in Spanish than in English. This negotiation of language was exemplified in how student problem solving in Spanish took the form of discussing, arguing, taking risks, and learning with other students. English was the language to do more traditional schoolwork (Dominguez, 2010). Thus, the use of both English and Spanish provided the space needed to develop and articulate mathematical concepts and mathematical language. In honoring how students articulate mathematical concepts, and attending to student resources, teachers can expand student opportunities to express their understanding and thinking, thus providing more openings for students to participate in class.

Together, these studies suggest that it is important to understand the role of hybrid language practices in mathematics classrooms, yet there is scant research on how high school mathematics teachers leverage hybrid language practices with their emerging bilingual students.

Additionally, research indicates that hybrid language practices alone do not necessarily capture the cultural components of students' participation in classroom mathematics learning.

(Linguistic) Funds of knowledge in the mathematics classroom.

Mathematics teachers can also engage students' funds of knowledge as resources in the classroom. Funds of knowledge are historically accumulated and culturally developed bodies of knowledge and skills that are essential for well-being (Greenberg, 1989; Moll, 1992; Tapia 1991; Velez-Ibañez, 1998). This theory stems from a historic shift from deficit thinking in the dominant culture to acknowledging the repertoires of knowledge of other groups. The teacher can attend to and access student funds of knowledge to inform the development of meaningful contexts for students in the classroom that relates to their lives outside of school (Moll, 1992). Accessing students' funds of knowledge is different from hybrid language practices in that funds of knowledge consider identity and language while hybrid language practices consider the mixing of languages. Furthermore, attending to funds of knowledge as a resource is powerful in that it can move marginalized individuals such as emerging bilingual students into empowering spaces where they can create academic, bicultural identities (Gort & Sembiente, 2015; Palmer & Martinez, 2013; Takeuchi, 2015). Drawing on students' funds of knowledge by leveraging students' linguistic repertoires and identities is a potential teaching practice that can support students in powerful mathematical meaning-making.

Gutiérrez (2002) examined how high school mathematics teachers drew upon bilingual students' funds of knowledge in Calculus classes. She found that the teachers honored the diversity of their Latinx students, got to know them, avoided deficit thinking and provided opportunities for mathematical discussions. Not only did teachers respect students' identities, they allowed students to take language ownership (Gutiérrez, 2002) whether they were bilingual

or multilingual. Furthermore, the teachers that Gutiérrez (2002) was studying allowed for students to discuss mathematics in the language of their choice. Specifically, students were provided with opportunities to engage in sensemaking around mathematics in the language that best fostered their ability to learn it. Thus, it is most important for educators to get to know their students and the funds of knowledge they bring into the classroom (Dominguez, 2011; Gutiérrez, 2002; Moll 1992) This is in addition to student classroom language preferences in order to facilitate available resources in the mathematics classroom.

The sections above have described hybrid language practices and students' funds of knowledge in supporting emerging bilingual students in (mathematics) classrooms. In the next section, I focus on research that explicitly explores classroom practices and translanguaging.

Sociopolitical perspective on translanguaging.

In this section, I discuss translanguaging as a sociopolitical construct. Translanguaging involves “the construction and use of original and complex interrelated discursive practices that cannot be easily assigned to one or another traditional definition of a language, but that make up the speakers' complete language repertoire” (Garcia & Wei, 2004, p. 22). In order to translanguage, a person draws upon his or her linguistic repertoire in order to accomplish an end goal (Garcia & Kleyn, 2016). By linguistic repertoire, I draw upon Garcia and Kleyn's reference (2016) to the existing linguistic practices and forms of knowing that a person can draw upon in their communication practices. However, not only is drawing upon one's linguistic repertoire important, what distinguishes translanguaging from other hybrid language practices such as codeswitching is its embodiment of language as a sociopolitical act of rupturing dominant language ideologies. I discuss this in more detail below as well as the differences between translanguaging and other language practices.

Translanguaging ruptures the idea of learning occurring in a monolingual space (Garcia & Sylvan, 2011). In the United States, English continues to be the valued currency of language and English monolingualism is the norm. Garcia (2009) names the promotion of English monolingualism as a hegemonic practice that denies the complexity existing between multiple languages. Translanguaging is a potential counter to this as it acknowledges that the distinctions between using languages are blurred.

Specific to the classroom space, translanguaging is transformative as it attempts to rupture the hierarchy of language practices positioning English as more valuable. For example, a student that is translanguaging might switch between languages, use gestures, and or diagrams to better communicate their ideas. A teacher taking on a transformative stance (Garcia, Johnson, & Seltzer, 2017) is supporting opportunities for students from less and more dominant linguistic backgrounds to communicate since the student is expressing ideas outside of a monolingual norm.

Furthermore, the translanguaging process is different from other languaging practices in that it encompasses a new way of being, acting, and languaging in a different social, cultural, and political context. For example, codeswitching is a hybrid language practice where students engage in mixing languages throughout an interaction. While encompassing the idea of codeswitching, translanguaging expands on this but refers to the process in which bilingual students make sense and perform bilingually in the classroom via reading, writing, note taking, discussing, and gesturing (Garcia & Sylvan, 2011). Thus, translanguaging can account for a more multimodal form of communication. This multimodal form of communication, when viewed from a translanguaging lens, can rupture dominant language ideologies. This rupture gives way to a new way of being, acting, and languaging that constructs a new social, cultural,

and political space in which students who might not have traditionally participated, do (Garcia & Sylvan, 2011).

Translanguaging and mathematics classrooms from a sociopolitical perspective.

I use the notion of “spaces for translanguaging” to acknowledge the mathematics classroom as an interactional space. According to Wei (2011), a translanguaging space is a social space, coordinating meaningful performances that brings together histories, contexts, ideologies, cognitive capacities, and physical capacities. I understand translanguaging as a discursive space, in which participants are communicating and making sense of their activities. Since traditional schooling has generally maintained dominant language policies, a translanguaging space allows multilingual individuals to integrate language practices that have formerly been practiced separately (Garcia & Li, 2014). For example, Turner et al. (2013) introduced the notion of privileging Spanish which “is an effective form of restructuring opportunities for emerging bilingual students to take on agentive problem-solving roles, such as presenting their solution strategies” (p. 214). For example, by privileging Spanish or inviting students to center it as their language of mathematical meaning making, the teacher is inviting students into a space for translanguaging.

Teachers can also plan for a translanguaging space (García, Ibarra Johnson, & Seltzer, 2017; Garcia & Kleyn, 2016; Wei, 2011). Garcia and Kleyn (2016) suggest having teachers note places in curriculum and assessments that have opportunities for cultural connections and multilingual resources (such as textbooks in another language) in class. In the process of planning for this space, one of the teachers’ goals should be oriented towards helping students make connections between mathematics and translanguaging (Phakeng & Moschkovich, 2013). In planning for translanguaging spaces, teachers can support complexities between language and

mathematics by utilizing any combination of home and other languages as resources (Phakeng & Moschkovich, 2013; Garcia & Kleyn, 2016; Wei, 2011). Although this may create unique challenges for the monolingual teachers working with multilingual students, Phakeng & Moschkovich (2013) maintain that teachers do not have to be multilingual to develop these supports. According to Phakeng & Moschkovich (2013) what is important is the teacher's understanding of the complexities involved for students working across languages and how to support them learning mathematics in English.

de Araujo et al. (2018) identified teacher practices that provided students opportunities to engage in multimodal (verbal and nonverbal) communication, or what I understand to be translanguaging. One instructional practice to support students in mathematical discourse was eliciting or engaging students in verbal interactions. A second, revoicing, involved reiterating students' ideas whereas a third, recognizing, involved valuing students' resources (de Araujo, 2018). Modeling occurred both independently and in conjunction with the above-named practices. In attending to opportunities to elicit, revoice, recognize, and model, teachers supported students in mathematical sensemaking (de Araujo et al, 2018), thus creating opportunities for translanguaging in their classroom.

In addition to teachers creating opportunities for students to translanguage, those teachers can engage in translanguaging with students as well. For example, in Garcia and Leiva's study (2014) of the flexible use of linguistic resources in a bilingual classroom, planning for translanguaging allowed the teacher to involve and give voice to her students, reinforce ideas, manage the classroom, and extend and ask questions. As a result of this study, Garcia & Leiva (2014) argue that translanguaging opens opportunities and possibilities of student and teacher participation.

Another example of how teachers can create opportunities for translanguaging in the classroom could be a teacher using emerging bilingual students' familiarity with cognates to design word problems. Cognates are words that look familiar across languages. In this case, the teacher would be attending to and utilizing what students know (cognates) to opportunities for translanguaging where students can engage in mathematical sensemaking.

In summary, by attending to opportunities to leverage language in the mathematics classroom (Phakeng & Moscovitch, 2013; Staat, 2009), teachers can invite a culture of translanguaging. Garcia & Kleyn (2016) elaborate that this culture involves teachers and students as co-learners in the classroom. More specifically, in her research study teaching Somali students, Staat (2009) learned the cultural significance behind mathematical terms in the Somali language which she connected to teaching mathematics to her emerging bilingual students in English. Furthermore, Staat (2009) learned these terms by interviewing her students and eliciting the cultural and historical meanings of words in Somali. Thus, by interviewing students and attending to the unfamiliar cultural component of mathematical terms within the Somali language, Staat (2009) positioned herself as a co-learner with students in the classroom. By participating as such, Staat (2009) more fully understood how students were grappling with concepts that could be attributed to cultural differences. In a language such as Somali, there is a metaphorical relationship between a concrete action and mathematical term (e.g. iskudofa means 'to hit something against the other,' or multiply). As a co-learner, Staat (2009) learned from her students about opportunities to present examples highlighting connections between languages (e.g. iskudofa is like algebraic notation where $2x$ is two times x), thus expanding upon her students' understanding. More broadly, teachers can position themselves as co-learners in the

classroom like Staat (2009) by attending to opportunities to draw connections between languages and mathematical concepts.

Creating spaces for translanguaging through planning for and creating a culture of translanguaging is a way to further rupture dominant language ideologies in the mathematics classroom. Not only can students translanguage in these spaces, the teacher can as well. Furthermore, teacher attention to opportunities for these spaces can inform these transformative practices within them, which I discuss below.

Teacher noticing for spaces for translanguaging.

Attending to the opportune time for highlighting connections between language and mathematics involves complex pedagogical maneuvers by the teacher. Teacher noticing and teacher noticing for equity (Aguirre et al., 2013; Fernandes, 2012; Jilk, 2016; Thomas et al., 2015; Turner, Drake, McDuffie, Aguirre, Bartell & Foote, 2012; Wager, 2014) can potentially support these multifaceted pedagogical moves. Noticing for spaces for translanguaging might involve attending to students' linguistic funds of knowledge (Aguirre et al., 2013), the design of a mathematical task with attention to language being used (Fernades, 2012), and students' (linguistic/communicative) strengths (Jilk, 2016; Louie, 2017). For example, in their study on how pre-service teachers attended to emerging bilingual students' experiences outside of school, Aguirre et al. (2013) found that pre-service teachers could pair meaningful mathematical connections with language connections across students' communities. More specifically, in one mathematics class, a pre-service teacher took her students to "Las Socias," a neighborhood food market frequented by Latinx families (Aguirre et al., 2013) where students interviewed the owner about how she used mathematics in her work. In drawing upon relatable examples to use in mathematics class, teachers were accessing students' linguistic funds of knowledge, thus

creating a potential space for translanguaging that connected familiar contexts with abstract mathematical concepts.

Fernandes (2012) provides another example of how teacher noticing can highlight potential spaces for translanguaging with mathematical task redesign. In his study, Fernandes (2012) chose tasks for his middle school mathematics preservice teachers to administer to emerging bilingual students. These tasks involved problems that had different modes of presentation, were linguistically challenging in a variety of ways, but also allowed students to draw upon a variety of ways to explain their answer. Pre-service teachers interviewed emerging bilingual students while they completed the task, then wrote a report on what they noticed was linguistically challenging for them when they solved it. Additionally, the pre-service teachers came up with strategies such as adapting the task to isolate linguistic challenges and providing concrete materials and drawings. In noticing linguistic challenges in these mathematical tasks, and coming up with potential strategies for task redesign, pre-service teachers were also drawing upon potential connections to make the tasks more accessible and meaningful. Furthermore, noticing opportunities to modify tasks such as those above could also create a potential space for translanguaging for students by designing tasks that incorporate multimodal methods of problem solving and sharing mathematical ideas.

The discussion above regarding research on mathematics teaching and learning, teacher noticing, and mathematics classrooms and emerging bilingual's literature suggests that the field has yet to understand how spaces for translanguaging emerge in secondary mathematics classrooms and are related to teachers' instructional practices. Clearly, the teacher plays a powerful role in facilitating kinds of classroom interactions that open these spaces. What do mathematics teachers attend to in classroom activity that supports this type of facilitation? This

was the focus of my dissertation research on teacher noticing practices that support spaces for translanguaging in mathematics classrooms. In the section that follows, I describe how this research focus is informed by study of mathematics teacher instructional moves, translanguaging practices, and teacher noticing.

Chapter 3. Methodology

In this chapter I describe the methodology that I employed in this qualitative research study. To better understand teacher practices around language, I conducted a case study of two mathematics teachers that were involved in a larger research project for which I was a research assistant. I divided this case study into two phases. Phase 1 occurred during the 2017-2018 and 2018-2019 school years with two teacher participants at two different high schools. Phase 2 occurred during the 2019-2020 school year with the same two teacher participants at the same two high schools. Below, I describe my case study design.

Study Design

In this section, I outline my reasoning for utilizing a qualitative approach to the study of noticing for translanguaging. I also highlight how a case study was particularly productive for the study of teacher noticing and practice.

Phase 1 and Phase 2 of my study were qualitative in nature since I sought to better understand complex classroom interactions as social activity between the teacher and students. According to Yazan (2015), qualitative research is helpful to better understand how people make sense of their world and experiences within it. In the case of the classroom, for example, the subtleties in teacher attention to classroom interactions would have been difficult to capture with quantitative data, thus, a qualitative approach was best to study teacher noticing in order to better understand the nuances in these classroom interactions.

Using a case study approach illuminated my understanding (Yazan, 2015) of spaces for translanguaging and how teachers supported them. Furthermore, using a case study design allowed for flexibility when unexpected changes occurred to my original dissertation plan

(Yazan, 2015). ² Specific to the focus of my research, teacher noticing, Louie's work (2017) was insightful in considering the importance of a case study approach in examining how specific teachers attend, interpret, and respond to students' activity in the mathematics classroom. In her case study, Louie (2017) studied the noticing of a mathematics teacher around the subtleties in her attention to students' "smartness" when watching video clips of her teaching. Furthermore, using a case study illuminated potential teacher strategies to attend to students' strengths while considering more equitable and ambitious noticing (Louie, 2017). These case study approaches informed my own since both of my research phases were organized around two teachers' classrooms to better understand their pedagogical moves in classroom spaces -- resulting in (or not in) translanguaging spaces --and how these spaces related to the teachers' noticing practices and to students' opportunities for mathematical sensemaking.

In this section, I emphasized the importance of a case study approach to better understand teachers' practices around language and mathematical sensemaking. I also described how I drew upon Louie's case study approach in considering my analysis of teachers supporting spaces for translanguaging. In the next section, I detail my own positionality within this project.

Researcher Positionality

I identify as a White female who learned Spanish as a second language outside of school. Although I am interested in teacher language practices specific to emerging bilingual students, I am not a bilingual expert. My interest in translanguaging in mathematics classrooms stems from my role as a former high school Math and Spanish teacher and current math educator with

² Due to receiving few permission forms from students during Phase 2 of my study, I was not able to collect very much data in one teacher's classroom. Additionally, due to the stay at home order with COVID-19, I had limited time to access video data. I discuss this more below.

students from various language backgrounds. During my time as a teacher, while I did not teach many students who were labeled emerging bilingual, language practices in the classroom and ways to help students learn mathematical language were always on my mind. Additionally, I was interested in how language was positioned as capital and how access to new language empowers students. For example, in the United States especially, an English dominant student learning Spanish generally has more language capital than a Spanish dominant student learning English. Similarly, teaching Spanish as a foreign language is thought of as different from teaching English as a second language. This notion of language as capital shapes my own dissertation research in considering which students' linguistic backgrounds and experiences are automatically positioned as valuable. Furthermore, while teaching high school, I noticed that many of my pedagogical practices to develop language (i.e. Spanish or mathematics in English) were very similar. This curiosity around the similarities in teaching Spanish and mathematics were the impetus for exploring teacher practices (instructional and noticing) around language in my dissertation study.

Next, I outline the first phase of my case study.

Phase 1 Study

In this section, I outline Phase 1 of my dissertation study. Below, I first introduce the context of the larger study, provide my reasoning for selecting the two high schools for my case study, provide information about the participants, describe my data collection, and outline my original approach to analysis. Within the first part of my analysis, I explain how the first phase informed the second phase of my study. My research question for this first phase was: What are the noticing practices that support mathematics teachers' implementation of translanguaging practices?

Phase 1 Context

My research project was conducted within the context of a larger research project. Below, I outline its research goals, describe its participants, and discuss my role within the project.

My project was part of a larger NSF funded research grant focused on mathematics teacher noticing for equity (CORE 17-0048). The Co-ATTEND project took place during the 2017 to 2020 school years in a large urban school district in the Midwest. Co-ATTEND was designed as a community-based research partnership amongst university researchers, high school mathematics teachers, and community organization leaders. The Co-ATTEND project investigated the following research questions:

- 1. What is the relation between the dispositions, noticing practices and mathematics instruction of secondary mathematics teachers?*
- 2. What activities, tools and frameworks support teachers in learning to notice equitably?*
- 3. How can a community-based design model enhance the conceptualization of noticing for equity and the development of tools/ frameworks for noticing for equity?*

The research that was central to the CO-ATTEND project was designed to be comprised of a number of collaborative research activities (Hiebert, Gallimore, & Stigler, 2002; Reiser, Spillane, Steinmuller, Sora, Carney, & Kyza, 2000) in order to bring more critical conversations to the forefront between its members (Horn, 2007; Horn & Little, 2010). These critical conversations between the “noticing research team” broadly construed (i.e., university scholars, community leaders, teachers) occurred in noticing interviews, video club meetings, research meetings, and summer institutes.

Teachers and community leaders were recruited for the Co-ATTEND project in various ways. Teachers from the school district were recruited based on recommendations by district personnel that they were exceptional at equity-oriented mathematics instruction. Community

organization leaders from the neighborhoods about the teachers' schools were recruited based on working with youth and recommendations from current education professionals. I participated as a graduate research assistant on the project and conducted both phases of my dissertation study on the noticing practices of two of the research team's teachers. I conducted my preliminary observations, interviews, and collected artifacts in conjunction with the IRB approved protocol.

Phase 1 Participants

Below, I introduce the teacher participants of my study, first Gabe then Jeannette. Next, I describe the context for each teacher, and what led them to participate in this smaller study.

The first teacher of my study, Gabe, self-identified as a white male who was English dominant but spoke some Spanish. Gabe conducted the class primarily in English and had been teaching mathematics for eleven years at the time of this project. Gabe taught at The Academy, which was in the northern sector of the school district. The school became a full time Early College offering high school and career-oriented classes in 2003 (Anonymized, 2018).

At the time of the first study, the student school population was 92% Hispanic, 2% African American, and 4% White students (Anonymized, 2018). Students in grades nine through twelve took courses that gave them experience in the careers they wanted to pursue and or credit towards a college degree.

During Phase 1 (2017-2018 and 2018-2019 school years) Gabe taught Advanced Algebra to juniors and Math 2 to ninth graders respectively. Both Advanced Algebra and Math 2 classes reflected the demographics of the school population. The 2018-2019 Math 2 course was an integrated curriculum consisting of Algebra, Statistics, Probability, and Geometry topics.

Gabe had been attending professional development opportunities specifically focused on working with bilingual students in math. He was interested in this dissertation project to learn

about mathematical language development in order to improve upon his practice. Gabe was a participant-researcher in the larger project, and a key collaborator on my research study.

Additionally, Gabe was a teacher lead at his school. Being a teacher lead consisted of observing his teacher colleagues and offering them feedback on their practice. Because of this, Gabe did not have to teach as many courses as other teachers at his school.

I asked Gabe to be part of this case study not only because of his commitment to equity, but because of my interest in the context of an Early College school. At this Early College, the students were taking high school as well as college courses. This meant that Gabe was also focused on preparing students for their college math courses at the Early College. Having a teacher who was preparing students for college math courses in my study was interesting in considering whether there might be additional differences to consider with respect to teacher language practices when compared with the second teacher's practice at a more traditional high school.

The second teacher, Jeannette, self-identified as Latina and spoke English and Spanish. However, she did not identify as bilingual in both languages. Jeannette had been teaching mathematics for twenty-three years at the beginning of this study and taught at The High School, which was in the downtown area of an urban school district. The High School became a dual immersion program in the second year of Phase 1, which occurred during the 2018-2019 school year.

At the time of this first study, the school demographics were 90.1% Hispanic, 4.8% African American, 2.5% White, and 3.6% Other (Anonymized, 2018). Housing grades six through twelve, The High School partnered with the College Board in order to facilitate school and classroom environments that prepared students for college (Anonymized, 2018).

Throughout her involvement in Phase 1 during the 2017-2018 and 2018-2019 school years, Jeannette taught Integrated Math 1 to freshmen high school students. The demographics of both her Math 1 classes reflected the demographics of the school population. Math 1 was the first high school level integrated curriculum that included Algebra, Statistics, Probability, and Geometry and followed the College Preparatory Mathematics curriculum (CPM).

I asked Jeanette to participate in my study because of my interest in how she already organized her language practices to teach emerging bilingual students in her mathematics classroom. Additionally, Jeanette wanted to encourage other math teachers to become more cognizant of their practices to develop mathematical language.

From my research with Co-ATTEND, I noticed that Jeannette and Gabe's classrooms offered considerably different mathematics classroom contexts. Their teaching looked quite different, as did the activity in their classrooms. Additionally, Jeannette's experiences and perspectives were very different from Gabe's. Finally, when they joined the Co-ATTEND project, they both expressed a strong commitment to equitable practice, which was also fruitful for studying spaces for translanguageing in their classrooms.

Phase 1 Data Collection

Data for this first part of the study was collected over the 2017-2018 and 2018-2019 school years in the form of classroom video observations, semi-structured baseline noticing interviews, and reflective noticing interviews.

Baseline interviews were conducted in the Spring of 2018 through the Co-ATTEND project (Appendix A). The purpose of this hour long, initial interview was to discern how teachers understand, prioritize, and design for developing mathematical language in the classroom. The interview consisted of asking teachers about their school year and goals for it,

beliefs around equitable teaching, positionality, and challenges and constraints. I also prompted Jeannette and Gabe to explain how they understood language, mathematical language, and language with respect to emerging bilingual students.

I conducted Gabe's interview with the principal investigator of the Co-ATTEND project to learn the baseline interview protocol and conducted Jeannette's baseline interview on my own. I audio recorded the interviews, and later made content logs of them with another research assistant on the team.

Another data source was video observations of classroom activity in Jeannette and Gabe's classrooms. The purpose of these videoed classroom observations was to describe the kinds of tasks, structures and activities the teachers organized, and how they positioned language within each of these. Additionally, classroom observations were important to better understand the context of each classroom with respect to each teacher's noticing.

Video recording began when at least half of the class returned their video permission forms. The teachers and I then scheduled video observations a few weeks in advance of each observation. Scheduling video observations a few weeks in advance was important since both teachers assigned seating in student groups. Each teacher made sure to tell me ahead of time where to place the camera (generally in the back of the room) to make sure that students who had given video permission were in sight of it (and those who did not give permission were not in view). To acclimate students to their new seating charts and to the presence of the video camera in the room, we organized a couple of informal observation days where the camera was turned off but was set up in the room. When I was videotaping, I would change the camera angle or zoom in if there was a particularly interesting classroom interaction occurring. The tradeoff in

setting up the camera this way was that I was not able to capture video of interesting interactions amongst students who had not turned in their permission forms.

A third source of data were fieldnotes (Appendix B) I made each day I visited the classrooms. I attempted to collect field notes of classroom activities. I generally collected field notes using my computer and sat at the back of each teacher's classroom. I would also get up in each class and circulate around the room, helping students with their math work. In addition to fieldnotes, I collected the assigned mathematical tasks, which helped me know where students were in the curriculum.

After each video observation, I conducted a noticing interview (Appendix C) with the teacher where the teacher watched video clips of his or her teaching. The goal of the interview was to better understand how the teacher was attending to and interpreting classroom interactions around emerging bilingual students' mathematical language. The noticing interview was different from the baseline interview since the noticing interviews focused on teachers' attention and instructional decision-making in the moment. These noticing interviews were semi-structured in that excerpts of videoed classroom interactions that involved aspects of language were selected ahead of time to show to the teachers. Teachers could also select video clips they wanted to watch with me. During a noticing interview, teachers watched video clips of these excerpts and were asked about their noticing around 1) classroom interactions, 2) students' mathematical work, 3) their role in supporting students' linguistic resources, and 4) their practices to support students' mathematical sensemaking. The noticing interviews were audio-recorded and transcribed. Below is a table showing my data collected per teacher in Phase 1.

Table 3. 1*Data Collected per Teacher, Phase 1*

Teacher	Video Observations	Baseline Interviews	Noticing Interviews
Gabe	9	1	9
Jeannette	8	1	8

In this section, I have provided the data sources I used for this first phase. Furthermore, I have discussed how each data source was relevant to my research study. Next, I turn to the following section to outline my analytic approach.

Phase 1 Analysis

In the section below, I outline my process for analyzing the data in relation to Phase 1 research. I analyzed the following data sources: baseline interviews, video observations, and teacher noticing interviews. These data gave me a sense of the ways that teacher noticing practices supported translanguaging practices.

First, I analyzed each teacher's initial baseline interview. I analyzed the baseline interviews by developing content logs of them and inductively coded them for teacher beliefs, math instructional practices, and non-math instructional practices. Based on the coding, I identified themes and wrote memos of them for each teacher. I then made a memo identifying themes about how each teacher described their classroom teaching practices and teaching beliefs in general and specific to their teaching practices to develop mathematical language for emerging bilingual students. I then wrote another memo identifying similarities and differences in themes across each teacher's baseline interview. Identifying these themes (e.g. developing relationships with students, empowering students, leveraging resources, awareness of social injustices, etc.) was useful to triangulate with teachers' practice from video observations and reflections from

noticing interviews. Writing ongoing memos was helpful in keeping a log of emergent patterns throughout my analysis process not only for baseline interviews, but for all other interviews, my coding, etc. (Miles, Huberman, & Saldaña, 2007). Below are two examples of how I made memos around the emerging themes per each teacher's baseline interview.

Developing relationships with her students. Throughout her baseline interview, Jeannette reflected on the importance of her students knowing that she cared about them (Interview, 4/13/2018). She focused on this by setting a goal to know something about each of her students outside of mathematics. Furthermore, Jeannette emphasized the importance of her classroom being a place where students were welcome outside of class time. This was evident in that it was common to see groups of students electing to have lunch in her classroom, or just stopping by to say hello.

Awareness of social injustices. In his baseline interview (3/15/2018), Gabe spoke holistically of social injustices and how they related to students in the mathematics classroom. He saw himself as a person of privilege as a White male but viewed his role as a teacher as one to help disrupt oppressive norms such as school testing. Interestingly, Gabe named dominant ideologies around language and Whiteness as constraints in math class. With respect to emerging bilingual students, Gabe explained how he viewed current norms of testing as inequitable since emerging bilingual students had to take tests in English.

After analyzing the baseline interviews, I analyzed the teachers' video to use in conjunction with noticing interviews. I did this by identifying translanguaging practices in classroom video observations to discuss with the teacher. In doing so, I coded the video observation for teacher or student translanguaging practices with deductive codes such as

presenting in multiple languages, using tools, using cognates, and leveraging students' expertise (Garcia & Kleyn, 2016).

Third, I deductively coded the noticing interviews. Similar to the video observations, some examples of how I deductively coded was for how the teacher attended to opportunities for themselves to engage in translanguaging practices such as presenting in multiple languages, using tools, using cognates, and leveraging students' expertise (Garcia & Kleyn, 2016). Examples of other deductive noticing codes were attention to student energy and emotional state (Louie, 2017; van Es, Hand & Mercado, 2017; Jilk, 2016). Next, I employed inductive coding of noticing practices by generating themes around the teachers' noticing from their noticing interviews. I generated themes by writing memos about each teacher's noticing practices per interview. Additionally, I wrote memos about each teacher's noticing practices as a whole, then wrote another memo comparing patterns in their noticing practices. Examples of these inductive codes were teacher attention to student biliteracy practices and attention to student use of English and Spanish. Next, I wrote memos about each teacher's noticing interviews compared with their baseline interviews to identify relationships between these translanguaging practices and noticing practices. In the next section, I present my preliminary findings of these analyses.

Phase 1 Findings

In this section, I present an annotated summary of the preliminary results of my analysis of the data for Phase 1. My analysis revealed that Gabe and Jeannette engaged in particular noticing practices to support their translanguaging practices such as attending to opportunities for 1) groupwork, 2) leveraging student expertise, and 3) student energy. Although Gabe and Jeannette had similar goals for their bilingual and emerging bilingual students in having more access to mathematical discourse, understanding their noticing practices is important since their

reasoning about their instructional choices was different. These differences in reasoning were in part, due to the context of each teacher's practice and their disposition towards noticing (Wager, 2014). I present a summary of the noticing practices that came up most often per teacher around risk taking and leveraging student expertise and the similarities in noticing practices between the two teachers. Additionally, I describe how these findings led me to design Phase 2 of my dissertation.

Across his noticing interviews, a pattern emerged that Gabe was attending to noticing opportunities to leverage groupwork and students' strengths within groupwork to signal his next instructional moves around translanguageing. For example, while students worked in groups, Gabe attended to their roles and how students were explaining their work to each other across English and Spanish. In addition to consistently attending to groupwork, Gabe attended to body language within groups or whole group. For example, Gabe would attend to students' body language signaling student energy such as frustration or confidence and gauged students' engagement as directly influenced by adaptations in his teaching. These teaching adaptations could include translanguageing practices.

Like Gabe, one of the noticing practices that came up most often for Jeannette was attending to student energy in informing her instructional moves around translanguageing. This practice took the form of noticing student body language and what the classroom environment looked like when students were comfortable. She clarified that students demonstrated their comfort by moving across languages to demonstrate their understanding of the mathematical concepts.

A commonality in both teacher's noticing was that both teachers attended to how they leveraged language as a resource in the classroom. For example, Gabe attended to when students

seemed comfortable speaking Spanish and English and would codeswitch if students seemed uncomfortable. Jeannette, on the other hand, attended to when students seemed to buy-in (or not) to a mathematical task and would redesign problems to better reflect student interests. Other examples of how both teachers attended to opportunities to leverage language as a resource included using highlighters, using visuals, using tools, using cognates, and drawing upon student language practices across English and Spanish. In leveraging language, both teachers were encouraging more students to participate.

As a result of my preliminary analysis, I decided to continue this study to better understand classroom interactions where not only teachers engaged in translanguaging practices, but students, specifically emerging bilingual students also engaged in them in order for spaces for translanguaging to occur. To more thoroughly examine these spaces, I designed a second phase of my research.

Phase 2 Study

In this section, I give an overview of the second phase of my research study. This phase occurred during the 2019-2020 school year. Again, I describe how my study was organized and outline my additional data collection and analysis processes, which again took place in the context of the Co-ATTEND research project. The research questions for this phase of research were the following:

Research Question 1: *How do mathematics teachers co-construct spaces for translanguaging with emerging bilingual students in secondary mathematics classrooms?*

- ***Research Sub-Question 1:*** *What do spaces for translanguaging look like in classroom mathematical activity?*
- ***Research Question Sub-2:*** *Which pedagogical moves support different types of spaces for translanguaging?*
- ***Research Sub-Question 3:*** *How do spaces for translanguaging support student mathematical sensemaking?*

Research Question 2: *How do particular teacher noticing practices support spaces for translanguaging in secondary mathematics classrooms?*

Phase 2 Context

The context for this study was the same as that of Phase 1. The study occurred in the same district.

Phase 2 Participants

In the second part of the study, Gabe taught Math III to tenth graders. These tenth graders were the same cohort of students as the year before with the similar racial demographics to the previous year. Gabe and I had decided to continue studying the same class from the 2018-2019 school year since it would be interesting to document the shifts in his practice with the same class over two school years.

Jeannette, on the other hand, taught a new Math 1 class. The demographics of this new class were also like her class the previous school year.

Phase 2 Data Collection in Gabe's Classroom

In this section, I outline my timeline of data collection in Gabe's classroom from August through January of the 2019-2020 school year. Data collection took the form of classroom video observations, reflective noticing interviews, student interviews, and student surveys. In addition to outlining my data collection, I discuss how each data source informs my research questions.

At the beginning of the semester, Gabe and I agreed that I would come twice per week throughout the semester for observations. In September 2019, I distributed video permission forms and informally observed in Gabe's classroom until most students returned signed permission forms. Informal observations did not involve video or fieldnotes. During informal observations I sat at the back of the classroom but would also circulate around the room, working

with students on what they were doing in class. I observed for the duration of each ninety-minute class on Wednesdays and forty-five minutes on Fridays. Conducting informal observations without the video camera was meant to help students become accustomed to me as a participant and researcher in the class.

Once most students returned their permission forms in October 2019, I began video observations one to two times per week in Gabe's class. I continued to video until the following January in 2020. The video data helped me identify when spaces for translanguaging emerged, the practices that supported the space, and how students up that space. I placed the camera at the back of the room in order to best capture the whole class activity and would zoom in or change the angle if there was a particularly interesting interaction occurring. Additionally, I adjusted the camera angle so students who did not agree to be videoed or did not turn in their permission forms were out of sight of the camera. The tradeoff in setting up the camera to focus on the whole class was that I did not capture as many small group interactions as I would have liked.

I also took fieldnotes of classroom activity and noted particular sections of the lesson that seemed connected to adaptations that Gabe was making to support students linguistically. Additionally, written fieldnotes for each observation were useful to my goal to capture classroom interactions off camera. In my field notes, I also attended to whom the teacher interacted, as well as interesting moments of classroom activity that I could ask the teacher about during noticing interviews. After each observation, I reflected on classroom interactions that potentially emerged into spaces for translanguaging. In addition to observing the teacher, I collected the assigned mathematical tasks, which helped me know where students were in the curriculum.

I conducted five semi-structured noticing interviews with Gabe which took place from October 2019 through January 2020. The interviews occurred after his lesson for the day. In

these interviews, I asked him about his noticing practices with respect to students' mathematical sensemaking, participation, and supporting linguistic repertoires. These noticing interviews were semi-structured in that I selected particular excerpts of classroom interactions that involved potential aspects of translanguaging ahead of time to show to Gabe. Similar to Phase 1, during noticing interviews, I posed questions about his attention to: (1) what he was noticing that led him to engage in a particular instructional practice, (2) engaging students' participation, and (3) supporting students' multiple linguistic resources. These interviews helped me document Gabe's noticing practices around spaces for translanguaging. During the interviews, Gabe would attend to his interactions with his students that appeared in the clip, which helped me better understand both what drew his attention and how he interpreted his interactions with them. Additionally, the interviews helped give me context for understanding how students normally interacted with their classmates and teacher. I audio recorded the noticing interviews and transcribed them.

Transcribing the reflective interview helped me better understand and capture Gabe's attention to his teaching moves and to better support my understanding of the classroom context. Classroom observations, attention to mathematical tasks, and noticing interviews continued through the end of January.

In addition to teacher interviews, I also conducted student interviews in January 2020 with three emerging bilingual students from Gabe's class. Gabe had selected these three students ahead of time and the students and I coordinated a time to meet in the school library. I developed a structured interview protocol for these student interviews (Appendix D) and interviews lasted within a time frame of fifteen to twenty minutes. The interview questions attempted to solicit students' perceptions about how they felt supported in their mathematics class in general and linguistically. During this interview, students had time to elaborate and reflect on their answers.

During student interviews, I gave students the same math problems they had been working on that week in class and posed questions about: (1) how they each solved the task, (2) how they used language in their mathematics class, and (3) what about their mathematics class encouraged them to participate (or not). These interviews helped me understand dominant language norms in the classroom (according to students). They also gave me a sense of how students potentially translanguage to make sense of mathematics. Finally, students shared the support they felt as math learners. I audio recorded the student interviews and transcribed them. Transcribing the students' interviews helped me better understand and capture students' interpretation of their participation in mathematics class and to better support my understanding of the classroom context.

Additionally, in January 2020, Gabe administered the anonymized online student surveys to his class (Appendix E). The goal of the survey was to better understand his students' perceptions of their identity as doers of mathematics. The survey consisted of twenty-two Likert Scale items that asked students questions around their level of 1) confidence communicating ideas, 2) self-confidence, and 3) participation. While students took the survey, I was also conducting a video observation during that time but paused to answer students clarifying questions around the survey. I analyzed the surveys using Excel.

Below, is a table of my data collection in Gabe's classroom.

Table 3. 2*Data Collected in Gabe's Classroom*

Data Source	Number Collected
Video Observations	15
Noticing Interviews	5
Student surveys	25
Student interviews	3

In sum, I conducted fifteen video observations, five noticing interviews, three student interviews and administered a survey to twenty-five students. Next, I turn to my data collection process in Jeannette's classroom.

Data Collection in Jeannette's Classroom

Despite intending to follow the same data collection process in Jeannette's classroom as in Gabe's, I was unable to do so due to human subject constraints. I describe my data collection process below.

I observed Jeannette twice per week from August through September of the 2019-2020 school year. Starting in October, I began observing Jeannette three times per week. In September 2019, I distributed video permission forms to students. I also began to informally observe her classroom. This lasted until November 2019. Again, informal observations did not involve video. During informal observations I sat at the back of the classroom but would also circulate around the room, working with students on what they were doing. I observed for the duration of each class ninety minutes on Tuesdays and Thursdays, and fifty minutes on Fridays. Conducting informal observations without the video camera was meant to help students become accustomed to me as a participant and researcher in the class.

In Jeannette's classroom, by mid-December, only eight students turned in permission forms.³ Thus, I was only able to observe formally four times and conduct three video observations in total. Since I could only video six students, I videoed those students and focused on their small group interactions. However, due to high levels of student absenteeism, I could not sufficiently capture those six students' interactions over time. As a result, I ended up adapting my dissertation plan and simply re-reviewed video data from Jeannette's classroom from 2018-2019.

I also did not conduct noticing interviews with Jeannette about her 2019-2020 classroom. Since Gabe and Jeannette were familiar with viewing each other's' videoed teaching through the larger Co-ATTEND project, I asked Gabe if he would be comfortable with Jeannette reviewing his videoed observations as a source of triangulation for his data. He agreed, so I conducted one 30-minute noticing interview and one 45- minute noticing interview with Jeannette in which she reviewed clips of what I had selected from Gabe's teaching in 2018-2019 and 2019-2020. These interviews with Jeannette took place in January 2020 after school. During the noticing interviews with Jeannette, I posed questions about: (1) what she was noticing around Gabe's instructional practice, (2) how this instance was related to student participation, and (3) Gabe's role in supporting students' multiple linguistic resources. These interviews helped triangulate my analysis around spaces for translanguaging in his classroom, the way they were co-constructed, and the noticing practices that were potentially supporting them.

Below is a table showing the data I was able to capture in Jeannette's classroom.

³This low number of students that turned in permission forms might have had something to with the federal crackdown on undocumented students and their families. This crackdown might have influenced students and their families giving video consent.

Table 3. 3*Data Collected in Jeannette's Classroom*

Data Source	Number Collected
Video Observations	4
Noticing interviews	3

Although I intended to have a similar set of data from Jeannette's classroom to Gabe's, I was unable to do so. Instead, I was able to capture a bit more video data from Jeannette's classroom and conducted interviews with her that helped inform my conceptualization around spaces for transanguaging in Gabe's classroom. In further conceptualizing these spaces, I turn to my analytic approach below.

Phase 2: Data Analysis

In the section below, I outline my process for analyzing the data in relation to my research questions. I analyzed the following data sources: baseline interviews, student surveys, fieldnotes of classroom observations, videoed observations, mathematical tasks, teacher noticing interviews, and student interviews. I describe how the data informed my research questions, and accordingly, the type of analysis I conducted. These data gave me a sense of the ways that spaces for transanguaging are supported by particular teacher noticing practices. I outline my analytic process in the order that I approached my research questions, beginning with research question two.

Research Question 2: *How do particular teacher noticing practices support spaces for transanguaging in secondary mathematics classrooms?*

In order to study teachers' noticing practices, I drew upon teacher noticing interviews, not only from the 2019-2020 school year, but from the 2017-2018 and 2018-2019 school years, for both Gabe and Jeannette. The content logs had already been developed for the Phase 1

noticing interviews. I transcribed the 2019-2020 noticing interviews through a transcription service.

Upon completing transcripts of the 2019-2020 noticing interviews, I coded them using MAXQDA software. I began by deductively coding with my Phase 1 codes, starting with Gabe's noticing interviews, and moving on to Jeannette's. I was paying attention both to what the teachers' said about the characteristics of their noticing, as well as their instructional practices. In total, I generated approximately 64 codes and subcodes. Examples of the new deductive codes I included were (Garcia & Kleyn; Jilk, 2016; Louie, 2017; van Es, Hand & Mercado, 2017):

Teacher noticing of:

- Biliteracy practices
- Missed opportunities that relate to spaces for translanguaging
- Intentionally not noticing

Like Phase 1, as I was coding the noticing interviews, I also developed conceptual memos for each teacher that summarized my analysis from the deductive coding. I then developed a memo that compared themes that emerged among the inductive codes from baseline and noticing interviews. Developing memos helped me organize low inference summaries around teacher attention and practices. For example, developing memos helped me organize distinguishable patterns in teacher language support such as teacher attention to opportunities to draw upon student biliteracy and teacher attention to opportunities to draw upon their own biliteracy.

During my second round of coding I used MAXQDA Creative Coding to reorganize my codes and explore new relationships among my data. I used the Creative Coding tool to create visuals that mapped code frequencies and overlaps in code. Based on these overlaps and frequencies, I created a second codebook. I developed memos about code similarities and

differences per teacher, and how I condensed codes and came up with new ones. In addition to using the Creative Coding tool, I also used Jeannette's noticing interviews to inform my second round of coding. An example of a new code that emerged in the second round of coding was teacher attention to students feeling comfortable speaking Spanish in the classroom. More specifically, this code had to do with teacher attention to how comfortable students seemed engaging with the mathematics, the teacher, or the class in Spanish. This code emerged after I interviewed Jeannette around Gabe's video clips and she noticed that students seemed comfortable engaging in mathematical sensemaking across languages. This led me to consider whether Gabe was attending to that level of comfort as well, thus adding this to my codes.

Research Question 1: *How do mathematics teachers co-construct spaces for translanguaging with emerging bilingual students in secondary mathematics classrooms?*

Research Sub-Question 1: *What do spaces for translanguaging look like in classroom mathematical activity?*

The video data and noticing interviews helped me identify when spaces for translanguaging emerged in classroom activity, and when they did not. I operationalized classroom spaces for translanguaging as involving 1) emerging bilingual students being positioned or positioning themselves as competent mathematics learners (Gresalfi, Martin, Hand, & Greeno, 2008; Hand, 2010), 2) dominant classroom norms being ruptured (Garcia & Li, 2014; Gutiérrez, 2009; Hand, 2012;), 3) and students speaking across languages when engaged in mathematical meaning making or communicating mathematical ideas (Garcia & Kleyn, 2016).

I identified potential spaces for translanguaging in the video data from Gabe and Jeannette's classrooms by first reviewing my Phase 1 codes from noticing interviews that were connected to translanguaging. Next, I identified the video clips associated with these codes and reviewed them based on the criteria above. I also reviewed classroom instances connected to

language that I had marked in my fieldnotes for further review during classroom observations, and again applied my criteria.

In the reviewing process, I focused first on transitions between where a space for translanguaging was or was not occurring. By transitions, I mean where the teacher or student made a bid to translanguage and whether another classroom participant accepted that bid and engaged in translanguaging. Second, in reviewing video clips I identified instances where students were positioned as competent and they appeared comfortable expressing their ideas across languages (or not). For example, a dominant classroom norm that might have been ruptured was the teacher walking away from the front of the classroom to let a student teach the class and that particular student codeswitching while at the front of the room sharing mathematical ideas.

Once I identified potential spaces for translanguaging in the data, I used multimodal discourse analysis to describe in detail the moment-to-moment classroom activity surrounding the potential space for translanguaging. I created multimodal transcripts to better understand not only the verbal interactions of teachers and students, but also the nonverbal ones (Rogers 2017; Johnstone, 2018; Ochs, 1979). This analysis helped me understand whether a space for translanguaging was co-constructed, and the classroom norms and practices that might have supported it. In addition to accounting for how students were using linguistic resources to co-construct a space for translanguaging, I focused my multimodal analysis on gestures, proximity, gaze, movement, and verbiage specific to the translanguaging practices.

My analysis revealed 5 video observations including spaces for translanguaging in Gabe's video data, and 4 video observations including spaces for translanguaging and potential spaces in Jeanette's video data.

Research Question Sub-2: Which pedagogical moves support different types of spaces for translanguaging?

Once I had identified my potential spaces for translanguaging in video, I coded the multimodal transcripts to help me better understand the context of the teachers' classrooms. Furthermore, coding the multimodal transcripts helped me to better understand how the teachers engaged in classroom practices that reflected their understanding of language. In identifying interactions where individuals felt invited to take up spaces for translanguaging, I drew upon deductive codes from the literature. Examples of my deductive codes included eliciting, revoicing, and recognizing, modeling and valuing emerging bilingual students' resources (de Araujo et al., 2018). Other examples of inductive coding involved drawing on students' funds-of-knowledge either implicitly through the design of classroom tasks, or explicitly in classroom mathematical conversations (Gort & Sembiante, 2015; Palmer & Martinez, 2013; Takeuchi, 2015). Another was to position an emerging bilingual student as having contributed to classroom mathematical knowledge or having an important contribution to make (Lampert, 1990; Turner et al., 2013). Examples of inductive codes I drew upon were bids for translanguaging such as inviting students to present in multiple languages. Inviting students to present in multiple languages was also an example of recognizing and valuing emerging bilingual students' resources.

To complement the analysis of teacher pedagogical moves, I analyzed student surveys from Gabe's classroom. I analyzed the surveys to better understand students' perceptions of the classroom culture and this identity they were developing within it. This related to my analysis of supports for spaces for translanguaging in understanding how students saw themselves in the

classroom community and whether they felt empowered to participate within it (Gutiérrez, 2009).

My survey consisted of twenty-two Likert style response items that I reverse scored then collapsed dichotomously (1 for agree, 1 for disagree). I then calculated the descriptive statistics for the total score and total score for each item. For example, Items 22 (I do not let anyone know if I do not understand a problem in math class.), 4 (I have different kinds of opportunities to participate in math class.), and 18 (It does not affect my future if I am not good at math) had the lowest difficulty for students. This meant that these items had the highest probability of students agreeing with the item. Item 4 (I have different opportunities to participate in math class) was especially interesting to consider from the students' perspective about the culture of Gabe's classroom. I used this data to triangulate my findings about Gabe's instructional practices.

In the student interviews (again with Gabe's students), I looked for similarities and differences around student perceptions of their participation, support, and language use in mathematics class. Furthermore, I analyzed the interviews to better understand how the students described their use of Spanish in their mathematics classroom. I first used open coding to analyze the student interviews with respect to student perceptions. I then developed memos around the themes that were emerging. For example, all three students interviewed affirmed that their student peers encouraged them to participate in their mathematics class and that they felt a familiarity and trust in working with their classmates and teacher.

Research Sub-Question 3: How do spaces for translanguaging support student mathematical sensemaking?

To study the relationship between spaces for translanguaging and student mathematical sensemaking, I drew both on video observations and field notes. The multimodal transcripts that

I made earlier enabled me to use discourse analysis to examine the verbal and nonverbal interactions that students engaged in as they participated in mathematics (Moschkovich, 2015). When students were co-constructing a space for translanguaging, I documented how students engaged in mathematical sensemaking. For example, if students were engaging in a space for translanguaging during groupwork, I reflected on what was occurring during groupwork and why that was important in supporting students' sensemaking (Moschkovich, 2015).

In the next three chapters, I report on the findings that emerged from my analysis of the data from the two teachers' classrooms. Chapter 4 consists of my findings around spaces for translanguaging in Gabe's classroom whereas Chapter 5 consists of my findings in Jeannette's. In Chapter 6, I present my cross analysis of the similarities and differences in the support for spaces for translanguaging within each teacher's classroom.

Chapter 4. Findings: Gabe's Classroom

Over the span of two years, both Gabe and Jeannette supported spaces for translanguaging among emerging bilingual students in secondary mathematics classrooms. For both teachers, these spaces were supported through noticing and instructional practices. These teacher noticing and instructional practices supported students' mathematical sensemaking and learning.

In the following chapters, I provide vignettes of classroom interaction that illustrate what (potential) spaces for translanguaging looked like in the context of each of these two teachers' mathematics classrooms. The six vignettes I provide are representative of the types of spaces for translanguaging that I identified in my analysis of Gabe and Jeannette's videoed observations. Importantly, while there was the potential for a space for translanguaging to emerge in each vignette, this did not occur each time. I selected these excerpts as spaces for translanguaging as they challenged dominant norms for schooling and resulted in emerging bilingual students being better able to communicate their ideas (Garcia & Kleyn, 2016; Wei, 2011; Garcia & Li, 2014). Additionally, emerging bilingual students appeared to be comfortable speaking across their language repertoires and were often positioned as competent mathematics learners. Throughout these examples, I argue that the spaces for translanguaging that did emerge supported student mathematical sensemaking.

For each vignette, I describe the classroom scene in which the interaction took place. I then analyze the multimodal transcripts of the interaction from the video data and demonstrate how a space for translanguaging was co-constructed (or not) by the classroom participants. After this analysis, I identify instructional and noticing practices that supported the potential space

both in the moment and over time. Additionally, I explore the degree to which the space supported students' mathematical sensemaking.

Spaces for Translanguaging in Gabe's Classroom

In this chapter, I first describe how translanguaging occurred in Gabe's mathematics classroom through three vignettes of classroom activity. Additionally, I present the different pedagogical moves and noticing practices that supported different types of spaces for translanguaging in the instance of the example and over time. As a reminder, I operationalized spaces for translanguaging as follows: (Garcia & Kleyn, 2016; Garcia & Li, 2014; Wei, 2014)

- 1) A dominant norm of schooling is ruptured, resulting in students from nondominant linguistic backgrounds being able to better voice and communicate their ideas.
- 2) Students are comfortable speaking a language other than English when they are engaged in mathematical meaning-making or communicating mathematical ideas.
- 3) Emerging bilingual students are positioned as competent mathematics learners.

From further analyzing the noticing practices and instructional moves within those video clips, I argue that Gabe's practices broadened students' discursive opportunities. As a result, students were able to combine academic language and mathematical content to embody multiple ways of using language.

I present three vignettes of classroom activity to demonstrate various instances where Gabe and the students co-constructed spaces for translanguaging. These spaces were opportuned through Gabe using reform mathematics practices to translanguage, thus further supporting students to develop mathematical language.

Vignette 1: Valerie's Spanish Presentation

The first vignette comes from a class session in Fall 2018 where students were tasked with developing and delivering presentations about famous mathematicians and their

contributions to present day society. As Gabe specified in his noticing interview after class (Interview, 11/28/2018), the aim of the activity was to make connections between mathematics and someone who had used mathematics to make an impact on society. Students were required to give an overview of the mathematician's origins, schooling, professional life, and contributions to mathematics. Although students did not engage in mathematical sensemaking in this example specifically, the way in which their teacher facilitated these presentations may have shaped how students saw themselves in the mathematics classroom. For example, studies indicate that when teachers require students whose first language is not English to present in English, students are sometimes discouraged to participate in class (Fernandes, 2012). In contrast, in this example, Gabe invited a student to give her presentation in the language of her choice, which had the potential to support her future participation.

The ninth-grade focal student in this excerpt, Valerie, had just moved to the United States from Mexico the year before and was raised in a Spanish-dominant home. Although all of Valerie's schooling was in English at The Academy, she spoke predominantly Spanish daily in mathematics class to her peers and to Gabe, whether they responded in English or Spanish.

The vignette opens with Valerie and her classmates sitting in groups facing the board after a student has given their presentation. Valerie is seated next to Julia, who is another bilingual English-Spanish student. When it was Valerie's turn to give her presentation, she was very hesitant, possibly since her classmates had all presented in English before her. Below I provide excerpts of the multimodal transcript around Valerie's presentation and illustrate how this example constitutes a space for translanguaging.

Figure 4. 1*Vignette 1, Multimodal Transcript Part 1*

Time:	Gestural, Proximity, Gaze, Movement	Verbal
29:15	Gabe is pulling up a presentation. He gestures for Valerie to come up to the board. There is a four second pause while Valerie is looking at Gabe.	Gabe: "You've got this."
29:35	Valerie emphatically nods before looking back down at her table. Valerie scoots her chair under her table as Julia gets up to give her presentation. Julia gives her presentation.	Gabe: "Do you want Julia to go first?"

This first interaction between Gabe, Julia, and Valerie sets the stage for the space for translanguaging that emerged. Gabe signals for Valerie to come to the board to give her presentation. There was a four second period in which Valerie simply looked at Gabe. Gabe then improvised the potentially awkward situation by asking if Julia should go up before Valerie, to which Valerie nodded. Interestingly, Gabe did not ask Julia if she would like to present before Valerie. This suggested that Gabe was more focused on Valerie's comfort level than Julia's, perhaps because of Julia's status as an exited emerging bilingual student⁴. Also interesting was that Valerie did not answer Gabe with a simple "yes" or "no" when he asked her if she wanted Julia to present first. Valerie did not speak verbally to Gabe, which suggested her preference to communicate through body language. The fact that Gabe noticed Valerie's hesitation and

⁴I use the term exited emerging bilingual to describe students' status as emerging bilingual students who have achieved high enough proficiency on English language assessments, thus allowing them to exit the official programming for emergent bilingual students.

improvised an instructional support to have her friend present were precursors to a possible space for translanguaging.

Figure 4. 2

Vignette 1, Multimodal Transcript Part 2

Time	Gestural, Proximity, Gaze, Movement	Verbal
34:13	Gabe goes up to the board. Julia goes back to her seat and gestures at Valerie who is leaning forward and laughing. Valerie continues laughing as Julia gestures towards the board.	Gabe: "One more before a quick break." Gabe: "Everyone is doing it."
34:37	Gabe brings up Valerie's presentation and clasps his hands. Julia holds up five fingers to Valerie. Gabe continues clasping one hand over the other as he and the student look at each other. Gabe gestures outwards as he looks at the student. Gabe shakes his head and points to the screen. He waves his hands in a circular motion.	Gabe: "Está bien?"

After Julia presented and went back to her seat, she gestured at the board, indicating that it was Valerie's turn. Although Julia did not say anything, the implication was also a public affirmation that Valerie could do the presentation and a move of accountability that she needed to complete the task. Additionally, Gabe said to the whole class, "One more before a quick break," indicating not only did Valerie have time to mentally prepare while her peer was presenting but she would have time after her presentation to mentally decompress.

Here, we see Gabe ultimately giving Valerie a directive to present when he says, "everyone is doing it." At the same time, however, Gabe clasped his hands in a way that he was almost beseeching Valerie to go to the board. Again, Gabe was communicating that Valerie, like everyone else in the class, was accountable for the task, while at the same time recognizing her vulnerability. In the next line (timestamp 34:37), we see Gabe, who was not fluent in Spanish,

also codeswitching to Spanish to ask Valerie if she was comfortable with the presentation, “¿Está bien?” In this way, he shifted his normal mode of communication (Noguerón-Lui & Warriner, 2014) to use Spanish. By using Valerie’s first language, I argue that he was continuing to open a space for translanguaging for Valerie to co-construct with him. Again, Valerie did not respond to Gabe verbally.

The vignette continues when Valerie starts to move to the board, while Julia is telling Gabe in Spanish that Valerie does not want to present because of her lack of confidence.

Figure 4. 3

Vignette 1, Multimodal Transcript Part 3

Time	Gestural, Proximity, Gaze, Movement	Verbal
34:49	Valerie stands up and tugs down her shirt as she scoots her chair under her. Valerie approaches the board and waits as Gabe finishes what he is doing at the front. When he turns he gestures with his pen for her to come up. Gabe walks to the back of the room. Valerie’s head is not facing the class as she laughs at something that her friend says. She turns back to the board when Gabe asks if she is ready (hands have been in her pockets this whole time).	Gabe: “It’ll be over before you know it.”

In this episode, Valerie is moving to the board. Gabe once again reminds Valerie that she is accountable for the presentation by pointing his pen at the board for her to come up. While she waits for Gabe to finish what he is doing at the front, Valerie is still not facing the class as she laughs at something her friend says. This pause to laugh might have been what Valerie needed to continue co-constructing a space for translanguaging with Gabe.

As Gabe walked towards the back of the room, he further positioned Valerie as the authority on the topic on which she was to present. When Gabe reassured her, “It’ll be over before you know it,” Valerie turned to face the class.

Figure 4. 4

Multimodal Transcript Part 4

Time	Gestural, Proximity, Gaze, Movement	Verbal
35:25	<i>Valerie’s slides are in Spanish and projected on the board.</i> Valerie makes a blegh noise and sticks out her tongue. Valerie’s hands are still in her pockets and she reads from her slides on the board, she does not look at the class. While she presents, the class is quiet and looking at her presentation. On the next slide, Estudios, Valerie turns towards her friend for help with pronunciation of a word. Valerie continues reading from the board quietly. She is struggling over saying the words in Spanish as well. She finishes reading from the last slide and turns back to the class, motioning that she is done before looking down and smiling. Valerie gives a small nod while still looking down. The class claps and she goes back to her seat.	Gabe: “One more before break, Calm the energy. Valerie, when you’re ready.” Valerie starts: “Mi presentación es de...” but she has a hard time saying Katherine. Valerie: “Ella nació el 26 de agosto mil novecientos dieciocho.” Nació en (pause). “No sé como se dice (<i>West Virginia</i>)... Gabe: “Está bien?”

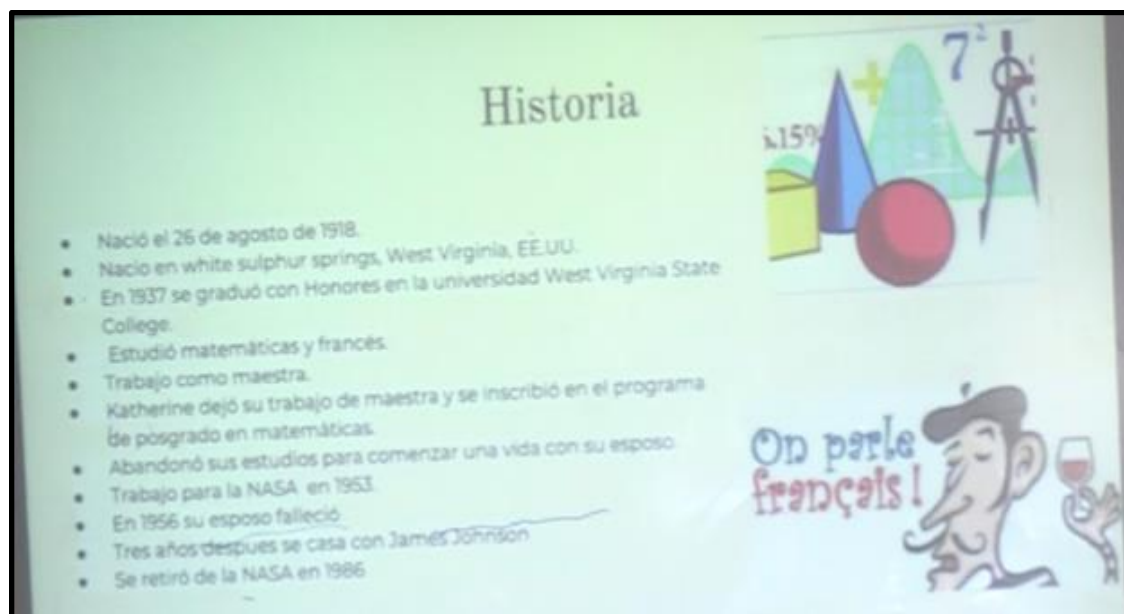
Gabe positioned Valerie as an authority on her topic and of the instructional space as he asked the class to calm down so they could fully attend to her presentation. Valerie began her presentation, “Mi presentación es de...” but had a difficult time pronouncing the name of the person she was presenting about (Katherine Johnson), made a blegh noise, and stuck out her tongue. This indicates that Valerie was not completely comfortable in the space for translanguaging that she was co-constructing through her presentation. It is also important to

point out that the whole time she presented, Valerie's hands remained in her pockets, and she did not look at anyone nor directly face the class. Instead she quietly read her presentation written in Spanish from the board. Again, this reinforces the claim that Valerie continued to be nervous throughout the presentation. In turning towards her friend for help with pronunciation in front of the class, however, Valerie indicated that she was comfortable asking for help navigating English in order to better communicate her ideas.

As she finished reading her presentation, Valerie turned back to the class, motioning that she was done before looking down and smiling, indicating some level of relief and satisfaction. Again, Gabe asked her, "¿Está bien?" to which Valerie gave a small nod while still looking down. In this case, though Gabe's codeswitch marked the end of her presentation, it also indicated a level of Gabe's continued vulnerability in that his attempts to connect with her were not. As Valerie returned to her seat, the class clapped, showing the same affirmation for her presentation as everyone else's.

Figure 4. 5

A Slide from Valerie's Presentation



Valerie was the only student in Gabe's class to give her presentation in Spanish and to have written her presentation in Spanish for her slides. By being invited to present her ideas about Katherine Johnson's contribution to everyday mathematics in Spanish, and by developing and giving this presentation, Valerie and Gabe were rupturing a dominant classroom norm of many mathematics classrooms that students should present their work in the dominant English language. Equally important to evidence for a space for translanguaging was the response by the rest of the class, who sat quietly listening to her presentation. The class did not make any indication that they were surprised or uncomfortable with her Spanish presentation.

In my analysis of the multimodal transcript of the scene, I illustrate how a space for translanguaging was co-constructed between Valerie, Julia, and Gabe. I described how Valerie was initially very hesitant to present when it was her turn, so Gabe improvised and had Valerie's friend, Julia, go before Valerie. After Julia presented, both Julia and Gabe continued to hold

Valerie accountable to present. Gabe made a bid to co-construct a space for translanguaging through codeswitching to Spanish. Although Valerie would not respond verbally to Gabe, she participated in the space by getting up to give her presentation. In moving to the back of the room, Gabe ultimately positioned Valerie as the authority on her presentation topic. Although it was clear that Valerie continued to feel uncomfortable throughout her presentation in that she did not face the class and read in Spanish from her slides, she ultimately gave her presentation. In allowing Valerie to present in the language of her choice, Gabe and Valerie ruptured a dominant classroom norm of presenting in English. This rupture was further supported in that the class did not indicate they were uncomfortable while Valerie presented.

In the next section, I analyze the instructional practices that supported the space for translanguaging that was co-constructed. I make explicit these instructional practices through the multimodal transcript of the video clip as well.

Vignette 1: Gabe's Instructional Practices

There were several instructional practices that supported the space for translanguaging that Valerie, Gabe, and Julia co-constructed. These instructional practices included 1) public affirmation of Valerie's competence, and 2) modification of the instruction to accommodate her while continuing to hold her accountable to present. To illustrate these instructional practices, I refer to the multimodal transcript above.

Throughout the episode, Gabe employed multiple ways to position Valerie as competent (Gresalfi et al., 2008; Hand, 2010). For example, after gesturing for Valerie to come to the front of the class (see multimodal transcript timestamp 29:15), Gabe encouraged her, "You've got this." Additionally, in walking to the back of the room (multimodal transcript timestamp 34:49), Gabe positioned Valerie as the authority on which she was about to present.

Gabe also modified his instruction on the fly to support Valerie in giving her presentation. For example, he allowed her friend, Julia, to present before her, as Gabe explains in his noticing interview (Interview, 11/28/2018), to give her extra time to mentally prepare to present. He also asked her if she was ready to give the presentation, instead of assuming that she would come up on her own. Another instance was Gabe codeswitching to Spanish with, “¿Está bien?” to try and make Valerie feel more comfortable prior to presenting. These strategies seemed successful in that Valerie did give her presentation.

Although Valerie was hesitant to give her presentation, Gabe continued to hold her accountable until she gave it. In the multimodal transcript (timestamp 34:13), after Julia presents, Gabe holds her accountable by using the directive, “Everyone is doing it” and gestures for her to come to the front.

In this section, I have discussed the instructional practices related to this space for translanguaging. Next, I outline the noticing practices that supported Gabe’s instruction in this episode.

Vignette 1: Gabe’s Noticing Practices

In this section, I identify Gabe’s noticing practices that supported the instructional practices identified above. My analysis revealed noticing practices that included Gabe attending to 1) student engagement indicating self-advocacy, 2) student engagement signaling resistance, 3) student vulnerability, and 4) student channels of communication. In order to illustrate Gabe’s noticing practices, I provide excerpts from the transcript of Gabe’s noticing interview that occurred after this classroom event.

Table 4. 1

Summary of Vignette 1 by Gabe's Noticing & Instructional Practices

Vignette	Instructional Moves	Noticing Practices
1 Valerie gives her presentation in Spanish.	1) Gabe publicly affirming Valerie's competence	1) Student engagement indicating self-advocacy
	2) Gabe modifies his instruction to accommodate Valerie while continuing to hold her accountable to present.	2) Student engagement signaling resistance
		3) Student channels of communication

First, Gabe gave context for Valerie's language support in class. He describes how he was attending to whether Valerie was advocating for what she needed in the situation. We see this in the transcript excerpt below of Gabe's noticing interview.

Figure 4. 6

Gabe's Noticing Interview Transcript Excerpt 1

1. **G:** Soo, Valerie is, um, like, she's been in the United States for a year and a half. So, um, Valerie's just past the point of, uh, getting like full Spanish language um, supports. Um, and so we've actually as a team had a few conversations um, around how to support her. Ah, so (pause), she mostly responds in class in Spanish, um (pause), Valerie uses a Spanish textbook and and has requested...
2. **M:** CPM?
3. **G:** CPM, yep.
4. **M:** Ok, [in Spanish?
5. **G:** In Spanish] yep. And she'd been pretty good about advocating for herself and um, I hadn't really (pause) nothing leading up to it yesterdayyy, ah, as we were preparing for the project really came up about um (pause), her being nervous about getting in front of the classroom...
6. **G:** And I hadn't specified anything about you need to ah do your presentation in English or I hadn't even said, you know it's ok if you do your presentation in Spanish. I hadn't, I hadn't given any of that direction, um, I just kind of communicated ah with the

ELA service teacher that we have and said I'm happy with whatever, however she goes up and does the presentation cause the point's not to talk in English, the the the point of the project is to make connections between math and um, you know somebody that has used math to make an impact.

In this portion of transcript, Gabe noticed that Valerie did not self-advocate for more support with this presentation. In line 1, Gabe gives a brief description of Valerie's experience as a Spanish dominant, new student to the United States. Although Valerie was no longer receiving full Spanish language supports, her teachers had been in communication with each other around how to best support her. In lines 1 and 5, Gabe specifies that Valerie had a history of using and self-advocating for Spanish resources such as her textbook. However, in line 5, Gabe elaborates that although the class had time the previous day to prepare for their presentations, nothing had emerged about Valerie needing additional support until it was time for her presentation.

Throughout his noticing interview, Gabe also reflected on Valerie's reluctance to present. In describing this reluctance, Gabe was attending to Valerie's engagement and how certain behaviors signaled her vulnerability, hesitation and resistance. We can see this in Excerpt 2 of the transcript.

Figure 4. 7

Gabe's Noticing Interview Transcript Excerpt 2

7. G: Um, I I wonder (pause) I think she and she uses (pause) part of it is that she uses other students a lot of times to communicate, even even in Spanish sometimes.

8. M: With you?

9. G: Yes, yeah. And so, I wonder how much it is not having you know any, any kind of shield or any kind of, um, anything between her and the whole class. Um, a lot of times when she responds she has somebody there, you know, giving her like little little cues or little nudges. I think even even in Spanish, her math vocabulary isn't strong. Um.

10. G: Um, and again, a lot of this is an assumption, just based on demeanor and um how she approaches things. She usually seems to have, I think she has a lot of social confidence.

11. G: Um, (long pause) and, I I mean the presentation in general was very similar to some of the ones in English with a lot of reading from the board. Um, ah, you know ah um (long pause) yeah. I did note I mean when I first noticed some of that nervousness to come I thought that giving her another presentation and just again having that time to switch one order, having that time to mentally prepare um I was hoping would be alright. Yeah, so and and I wouldn't even say that's the (laughs) most resistance I've had for doing an oral presentation in the past, I've definitely had more.

In attending to engagement from Valerie that signaled that she was not self-advocating and that indicated her resistance to present, Gabe became more attuned to Valerie's vulnerability in presenting in Spanish to the class. This is evident in Excerpt 2, line 7 of the transcript where Gabe stated that Valerie might not be used to speaking directly with the class.

In transcript Excerpt 2, Gabe describes how Valerie creates support for herself in the mathematics classroom. In describing these supports, Gabe attended to Valerie's channels of communication. This can be seen in line 7 of his noticing interview, Gabe attended to Valerie's typical channels of communication in that he hypothesized that she used other students as shields in communicating with himself and the whole class in English and Spanish. In multimodal transcript (timestamp 35:25), we also see how Valerie drew upon her classmates for support when she asks Julia for help pronouncing West Virginia.

In line 10 of the transcript above, Gabe describes how he has attended to Valerie's social status over time. His attention to Valerie's lack of confidence in this instance is made evident in the multimodal transcript (timestamp 29:35) when he switches the presentation order and line 11 of the transcript when he explains why he switched the order of the presentation to give Valerie time to mentally prepare to present.

Additionally, Gabe's attention to Valerie's vulnerability can be seen in multimodal transcript (timestamp 29:35) when Gabe switches the presentation order. This attention can also be seen in line 11 of the transcript where Gabe explains that in having Julia, Valerie's friend, advocate, and Spanish speaking partner, go first, he hoped Valerie would be more comfortable presenting.

Gabe's noticing practices supported this co-constructed space for translanguaging between himself, Julia, and Valerie. In attending to aspects of Valerie's typical versus in the moment engagement, Gabe became more attuned to her vulnerability in presenting in Spanish to the class.

In this section, I have discussed Gabe's in the moment noticing practices that supported him in co-constructing a space for translanguaging with the students around Valerie's Spanish presentation. Gabe paid very close attention to signals from Valerie's body language about her experience during the episode. He was expecting her to advocate for herself in giving the presentation and when this did not happen, and Valerie was hesitant to present, Gabe started to pay more attention to support for her.

In the following section, I present the next vignette, in this case a whole class discussion, where students transitioned to small groupwork.

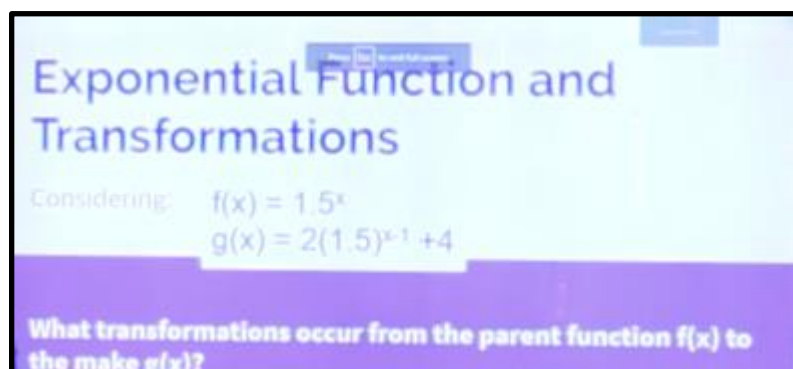
Vignette 2: Whole Class Discussion and Transition to Small Groupwork

The second example occurred in Spring 2020. In this episode, students demonstrated their ideas around transformations and exponential functions in the whole class, then transitioned to small groupwork to explore the relationship between the two mathematical concepts. My analysis revealed that this episode constituted a space for translanguaging that was supported by particular noticing and instructional moves (See Table 4.2).

The purpose of this classroom activity was for students to discover the connection between parent functions and exponential functions. Here is the prompt for the activity below:

Figure 4. 8

Vignette 2, Discussion Prompt



Throughout the class discussion, students shared their knowledge around transformations such as translations, dilations, and rotations. Immediately after the discussion, students worked in small groups around a task that Gabe assigned to graph an exponential function and its parent function and to annotate the patterns between the two functions.

I illustrate, in this vignette, how Gabe included a range of students in the discussion by engaging in a variety of teaching practices that drew out student sense-making and positioned students as classroom contributors. As Hansen-Thomas (2009) indicated, teachers using a variety of mathematical discourse practices such as choral responses, modeling standard forms, and encouraging mathematical language in small and large groups foster and broaden student participation, particularly among emerging bilingual students. In using a variety of practices to include more emerging bilingual students, teachers are taking into account students' evolving mathematical identities which are shaped by whether they have enough opportunities to engage in mathematical discourse (Takechi, 2015; Turner, Dominguez, Empson, & Maldonado, 2013;

Turner, Dominguez, Maldonado, & Empson, 2013). Although this class discussion occurred in English, I argue that the space for translanguaging was co-constructed during the whole class and continued into the transition to small groupwork.

The vignette opens with the whole class discussing their recollections of transformations. Students sitting in groups and Gabe is at the front of the room, readying his slides. When it is time to begin this part of the lesson, Gabe draws the class into a discussion around what they remember about transformations. Below I provide excerpts of the multimodal transcript around this initial whole class debrief and analyze how the space for translanguaging was co-constructed.

Figure 4. 9

Multimodal Transcript Part 1

Time	Gestural, Proximity, Gaze, Movement	Verbal
12:15	Gabe faces the class as he walks towards them and stops when he is about level with Rafa and Miranda's table and rotates his hands. Gabe ignores Victor and is looking towards Jonathon. Gabe looks towards Antonio, putting up a third finger. Gabe puts up another counting finger.	Gabe: What do you remember about transformations? Jonathon: Translations, rotations... while Victor says: They were annoying. Gabe: Translations (counts 1), rotations (counts 2)... Antonio: Reflections. Victor: Dilations. Gabe: Dilations.

The episode began when Gabe pulled up his lesson powerpoint and elicited the whole classes' ideas about transformations. In other words, Gabe drew the class into a discussion and positioned

the class as competent to build upon and make connections between the mathematics they already knew, transformations, and something new, exponentials. A number of students immediately began to participate in sharing what they knew about transformations. In response to Gabe's question, one student, Jonathon answered: "Translations, rotations..." while at the same time Victor said, "They were annoying." Gabe ignored Victor's dismissal of transformations and focused on Jonathon's response to launch the whole class discussion.

As Gabe revoiced Jonathon's listing of transformations, he counted along with his fingers, emphasizing each listed transformation. In other words, Gabe engaged in a revoicing to position move, or an explicit verbal, gestural, and other non-verbal positioning move by a teacher that places the original speaker in relation to other people, the task, etc. (de Araujo et al., 2018; Enyedy, Rubel, Castellón, Mukhopadhyay, Esmonde, & Secada, 2008). In revoicing to position, Gabe was making Jonathon's contribution explicit to the class. Antonio follows this interaction by offering another transformation, which Gabe again counted on his finger. Interestingly, Victor's next form of input was not ignored this time as Gabe treated it as another mathematical contribution. In this exchange, Gabe opened a space for student sense-making, and emphasized their sense-making through gesture.

The vignette continues with Gabe asking the class what each of the transformations look like.

Figure 4. 10*Vignette 2, Multimodal Transcript Part 2*

Time	Gestural, Proximity, Gaze, Movement	Verbal
12:31	Gabe is gesturing, palm outward to the class (like a wave). Gabe slides his hands left, right, up, and down. Gabe is nodding and waving both hands back and forth.	Gabe: Okay, what did a translation look like? Jonathon: They slide, so the graph doesn't change shape. Gabe: Sliding left, right, sliding up and down. Gabe: What did a reflection look like? Two students answer at the same time: Crosses the y-axis (Victor), Switches over one of the <u>axis</u> . (Jonathon).
12:51	His hands are gesturing outward again. Gabe brings his hands together back and forth.	Gabe repeats, Switching over axes. A mirror image, I think I heard, a mirror image. Um, what <u>about a dilation</u> ? Victor says, "Up or down, left or right." Gabe says, that's a translation," pointing at Victor. "Ah, shit" says Victor. Gabe: Shrinking, I think I heard. Shrinking, I like that as a word. Stretching. Stretching.

In this transcript, we see students responding to Gabe with their ideas. For example, in (timestamp 12:31), Jonathan says: "they slide." In response, Gabe slides his hands around in different directions. Again, when students explain the reflection across the y-axis, Gabe once again waves his hands back and forth.

Throughout these excerpts, Gabe was emphasizing gestures as a way to communicate mathematically, thus supporting a space for translanguaging. Studies show that pedagogical strategies of pairing gestures with revoicing can be especially beneficial for students, especially emerging bilingual students in that they can further ground mathematical discussions and narrate the meaning of concepts (Enyedy et al., 2008; Moschkovich, 1999; Shein, 2012). Especially in

reciprocating students' gestures, teachers are assigning competence to what can be considered student invented strategies (Shein, 2012). Furthermore, in students seeing each other's gestures validated as meaningful contributions, they learn broader ways to participate in the classroom. In broadening student modes of participation, teachers facilitate more opportunities to understand students', especially emerging bilingual students', mathematical thinking.

For example, Gabe asserted in his noticing interview after this videoed observation that what was not visible in the camera angle during this episode was that an exited bilingual student was making gestures when Gabe asked what a reflection looked like. The gestures that the student used were mimicked by Gabe, thus reinforcing the concept with gestures and revoicing her input.

Interestingly, Gabe offered input in the form of a couple of revoicing to position instances that were not audible in video from students such as "A mirror image, I think I heard, a mirror image," and "Shrinking, I think I heard. Shrinking, I like that as a word. Stretching," all the while reinforcing the definitions with gestures. Instead of simply telling students that dilations caused a shrink or stretch, Gabe pretended that he heard these ideas come from a student. In doing so, he did not position himself as the expert. This tactic seemed successful in that students continued to offer input and make connections between parent functions and exponential functions.

Once the class and Gabe finish their debrief around transformations, Gabe gave the next assignment to the class to work on in small groups. The episode continues as Gabe tasks the groups with exploring the relationship between parent functions and exponential functions.

Figure 4. 11*Vignette 2, Multimodal Transcript Part 3*

Time	Gestural, Proximity, Gaze, Movement	Verbal
13:10	Gabe gestures towards the two functions on the board and points towards his board to his right where his function is already hanging up. Gabe walks back towards the board, pointing at the functions and turning back towards the class. Gabe points with his thumb back at the board.	Gabe: We are going to look at exponentials with a transformations lens. Gabe: So, if I start here with the parent, the parent's 1.5 to the "x" power, here's the equation I chose. Gabe: What kind of transformations do you think are happening, (pause) based on these numbers that are happening from there? Gabe: Talk as a group. Kay, there's three. Talk as a group, see if you can determine all three. Talk as a group and determine if you can make out what they are based on what I've got here. Jonathon is talking about dilations, Valerie: Pienso que and another student talking about transformations. A student asks Gabe something, and Gabe says: Talk to your group first.

The episode concludes as the groups began graphing their exponential functions and parent functions. As group discussions began, Jonathon was talking about dilations, and Valerie said, "Pienso que" to her group. Likewise, Antonio was talking about transformations. Since Valerie immediately engaged in mathematical sensemaking in Spanish, it is implied that she felt comfortable doing so. Consequently, I argue that the space for translanguaging continued as students transitioned to small groupwork.

In this section, I described how the second vignette represented a space for translanguaging. Students were comfortable engaging across their linguistic repertoires in using

English, Spanish, and gestures in better voicing and communicating their ideas (Hansen-Thomas, 2009). Furthermore, Gabe emphasized the use of gestures to elicit and revoice student input. In this move, he was positioning all his students, including the emerging bilingual ones, as competent mathematics learners. Finally, students were comfortable engaging across English and Spanish during the small groupwork that occurred directly after the whole class discussion.

In the next section, I make apparent the instructional moves that supported this space. I illustrate these instructional moves through excerpts of the transcript below.

Vignette 2: Gabe's Instructional Practices

Several instructional practices supported the space for translanguaging that Gabe co-constructed with the class. These instructional practices involved Gabe: 1) using questioning strategies, 2) modeling the use of gestures, and 3) pretending to hear students' input to introduce new concepts and terminology. I illustrate these instructional practices below.

Gabe used a variety of questioning strategies to help students connect what they knew about transformations to exponentials. Instead of simply telling students about transformations, as seen in multimodal transcript (timestamp 12:15 and 12:31), Gabe elicited students' previous knowledge about them. In her noticing interview of Gabe's practice, Jeannette also attended to Gabe's questioning strategies.

Figure 4. 12

Vignette 2, Noticing Interview Transcript Excerpt 1

I was paying attention to the kinds of questions that he was asking, the response that he was getting from the kids. Just kind of pulling up their memory so that they could use that to move forward with something else. He didn't call on any kids, he just sort of waited for them to give answers. And then, um, again, once he got it like clear what he wanted, then he just had them working in their groups and he very clearly said, you guys need to talk about it in your groups.

In the transcript, above, Jeannette noticed that Gabe directed students to further discuss their thinking around the connection between parent exponential functions and transformations instead of describing the connection for them. In this way, Gabe was positioning the students as the experts in drawing upon what they knew about transformations. Gabe's questioning strategies appeared to be successful since students continued to explore connections between parent functions and exponential functions.

A second instructional practice Gabe used was to communicate through gestures. He made gestures as students were explaining complicated mathematical transformations, as well as mimicked students' gestures. In particular, when Gabe copied the gestures of the emerging bilingual student who was off camera, I argue that Gabe was translanguaging through his instructional practice. Mimicking the student's gesture also ruptured dominant classroom norms in that the teacher was re-positioning the student as the expert in the room. As Gabe asserted, reciprocating that student's body movements helped make a connection to what a transformation was by making that connection very "conducive to body movement."

Finally, Gabe also pretended to hear a student say something about transformations, instead of stating the information himself. We see this in the multimodal transcript (timestamp 12:51) where Gabe tells the class "A mirror image, I think I heard," and "Shrinking, I think I heard. Shrinking, I like that as a word. Stretching. Stretching." Instead of telling the class that a reflection is a mirror image and a dilation is a shrink or stretch, Gabe acted like a student said it.

In this section, I have explained how Gabe's instructional moves supported a space for translanguaging. Instead of positioning himself as the expert, Gabe positioned students as such by eliciting their previous knowledge and modeling the use of gestures. Additionally, Gabe

pretended to hear student input instead of simply telling students the answer. In the next section, I outline the noticing practices that supported these instructional practices.

Vignette 2: Gabe's Noticing Practices

In this section, I identify Gabe's noticing practices which I argue facilitated the co-construction of a space for translanguaging in this episode. My analysis revealed that Gabe was attending to 1) ways to create opportunities for students to connect prior understandings to new mathematical concepts, 2) students' use of gesture as a form of mathematical sensemaking, and 3) opportunities to model student gestures to broaden mathematical sensemaking. I describe each of these noticing practices below.

Table 4. 2

Summary of Gabe's Noticing & Instructional Practices

Vignette	Instructional Moves	Noticing Practices
2	1) Using questioning strategies	1) Ways to create opportunities for students to connect prior understandings to new mathematical concepts
	2) Modeling the use of gestures	2) Students use of gesture as a form of mathematical sensemaking
	3) Pretending to hear students' input to introduce new language.	3) Opportunities to model student gestures to broaden mathematical sensemaking

First, Gabe attempted to structure this activity such that students were connecting their existing mathematical understandings to new content. Thus, he was attending to ways to support students in making these connections both through the design of the activity, as well as the way it played out. We see this in the following excerpt from the noticing interview that was conducted after this class session.

Figure 4. 13*Vignette 2, Noticing Interview Excerpt 1*

Gabe:

Uh, during that, uh, is very, I mean, last semester, last year, transformations were an emphasis, so kind of wanted to do something quick and I think, um, you know, using my hands to just review vocab I think too. And they also had graphs in front of them to see like, what does it look like on a graph to translate. They had that stuff in front of them. And so, um, that was like two minutes and just wanted to kind of have them just connect what we were doing currently to something they'd done in the past. And so, um, [inaudible] yeah. I don't know. I don't know if I would take more time than that. Um, and hope again, hopefully it's like with the group, at least one person in the group is connecting and then able to kind of spread that to group members that might not have been giving the best of attention, um, through there. So.

In this section of the transcript, Gabe attended to an opportunity to connect students' previous knowledge to what they were currently learning. This can be seen in the transcript above where Gabe says, "And so, um, that was like two minutes and just wanted to kind of have them just connect what we were doing currently to something they'd done in the past." Gabe's attention to these connections are clearly visible in the multimodal transcript of the vignette above in (timestamp 12:15), where he was "counting" the different types of transformations that students offered, and when he described hearing mathematical ideas from the students.

In this whole class discussion, Gabe asked questions to elicit student knowledge and review different types of transformations. As Gabe asserted in his noticing interview after class, he hoped that attending to students' gestures as a form of mathematical sensemaking would help students further make a connection to what a transformation was.

Figure 4. 14*Noticing Interview Excerpt 2*

Gabe:

So what's a reflection? And then, you know, saw her doing that with her hands, kind of mimic that as well. Um, and reciprocated her motions. So, um, yeah, I think that's, that's kind of what I was hoping to kind of see for some students is making a connection to what a transformation was. I think that's really conducive to body movement. Um, yeah.

In this section of the transcript, Gabe described how he noticed a bilingual student, Sonya, using gestures as a form of communication. He also elaborated that using gestures to describe transformations was “really conducive to body movement.” Gabe’s noticing gives us more context for his gestures in the multimodal transcript (timestamp 12:31), where he is nodding and waving his hands back and forth when he asks the class what a reflection looks like.

Additionally, in this whole class discussion, Gabe was attending to the ways that students were making sense physically of the mathematical ideas, and ways to support their explanations through his gestures. Not only did Gabe take up opportunities to model student ideas with gestures, but he was also attending to the gestures of particular students, in this case, Sonya. He viewed this as an opportunity to “spread” the ideas that were emerging among the groups of students who were participating to other students who might have been less engaged.

In this section, I have discussed Gabe’s in the moment noticing practices that supported his facilitation of whole class discussion and small groupwork. These noticing practices involved attention to opportunities for students to connect prior knowledge with the new ideas and to the use of gestures as a way to communicate and broaden mathematical sensemaking.

Vignette 3: Code Switching During Small Groupwork

The final vignette from Gabe's classroom comes from a videoed observation in the Spring of 2020 where I observed a pair of students codeswitching as a form of translanguaging while problem solving during small groupwork. After reviewing the video, this particular interaction caught my attention because the two students were problem solving across English and Spanish. Interestingly, the emerging bilingual student in this interaction, Raquel, had claimed previously she did not use Spanish to do mathematics in class.

The aim of this portion of the lesson was for students to review their understanding of quadratic equations by playing a review game in groups similar to Jeopardy. Students could win points towards their test if they solved the problem correctly. Prior to showing each problem, Gabe would present the type of quadratics problem that students were going to solve (e.g. solving for x under the radical, graphing quadratics, etc.). Based on how confident students felt about that type of problem, they would bet points towards their total score. The particular pair of students in this clip were already grouped with two other students but predominantly worked together in solving the problem of focus in this vignette. Their group bet six hundred points, a high amount compared to other groups, that they could solve for the value of ' x ' under a radical. The problem that students were assigned was $\sqrt{x + 4} = x$.

In this group, the two students of focus, an emerging bilingual student, Raquel, and an exited bilingual student, Julia, engaged in solving a quadratics problem in English and Spanish. According to Esmonde (2009), collaborative groupwork in mathematics classrooms, where students are each contributing towards joint problem solving, can help students explain not only their answers, but their strategies. Specific for students from linguistically nondominant

backgrounds, thoughtful and equitable grouping on the teacher's part can help draw students into joint problem-solving interactions that affirm their evolving identities as competent in the mathematics classroom (Esmonde, 2009; Hansen-Thomas, 2009).

The excerpt begins with Raquel and Julia sitting together within their group. When Gabe unveils the next problem, Raquel excitedly turns back to her group and begins working on the problem with Julia. Below, I provide portions of the multimodal transcript around Julia and Raquel's interaction and show how this interaction constitutes a space for translanguaging.

Figure 4. 15

Vignette 3, Multimodal Transcript Part 1

Time	Gesture, Proximity, Gaze, Movement	Verbal
28:42	Raquel is facing the board while Gabe unveils the next problem. Raquel excitedly turns back to her group. Julia begins writing down the problem with her expo marker on her desk. Raquel's arms are stretched out on Julia's desk and she is pointing at Julia's work the entire time Julia is writing. Julia continues working out the problem then turns to look at Raquel. Julia is smiling. The two girls are still working.	Raquel: We know we know we know we know. Raquel: Okay, -4. Raquel says something to Julia to where Julia agrees: <i>Sí</i>. Julia: <i>¿Cuánto es?</i> Raquel: 16, no wait, is that positive 16? Julia: Yeah.

The space for translanguaging emerges when Gabe puts the problem on the document camera and Julia and Raquel begin working together to solve for 'x' under the radical. Raquel turned to Julia excitedly, "We know we know we know we know we know." Raquel's excitement seemed to set

the tone for the duo working out the problem in that Julia immediately started writing it on her desk with an Expo marker. This suggested that Julia picked up on her partners' enthusiasm and positioned herself as competent in beginning the problem. In the above excerpt (timestamp 28:42), Julia made a bid to co-construct a translanguaging space in codeswitching between English and Spanish to make sense of the mathematics. Again, the fact students felt comfortable codeswitching means that there was likely a class norm of problem solving across languages.

While Julia writes down the problem (timestamp 28:42), Raquel guides her through the problem-solving steps. As Julia and Raquel problem solved, neither positioned the other as “better” at mathematics, despite the fact that Raquel, with fewer English skills, would often be positioned as less competent within dominant classroom social hierarchies. When Raquel and Julia finish the problem, Julia is smiling. This indicated some level of satisfaction with her problem solving before giving the marker to Raquel. In the next excerpt, the space for translanguaging continues as Julia and Raquel check over their problem.

Figure 4. 16*Vignette 3, Multimodal Transcript Part 2*

Time	Gesture, Proximity, Gaze, Movement	Verbal
29:27		Julia: Wanna check it, just in case?
	Raquel takes the marker and points at some of Julia's work.	Raquel: You, you, <i>qué hace?</i>
	Julia is pointing thoughtfully at her work and takes back the marker and specifies her point in writing before handing the marker back to Raquel.	
		Julia: -3.
	Raquel begins to write.	Julia: You do it.
	Julia reads her work to her in English.	Raquel: ¿ <i>Qué?</i>
		Julia: Square root of x squared, plus four...
	Raquel is writing and indicates she is following along.	Raquel: Uh huh.
	Raquel continues working out the problem.	Julia: Equals x.

In this episode (timestamp 29:27), Julia asks Raquel to check over her work, “Wanna check it, just in case?” In this move, Julia positioned Raquel as competent in that not only had Raquel guided Julia through the problem-solving process, she was also qualified to check over it. This also indicates that the problem-solving process is collaborative. In response, Raquel positioned Julia as having made sense of the problem when she asked, “¿You, you, que hace?” Thus, the two students continued co-constructing their space for translanguaging in positioning themselves as competent in mathematical sensemaking across languages. Julia explains what she did in her head to Raquel before handing the marker back to her. Again, Julia held Raquel accountable for checking her work. The episode continues as Julia reads her work to Raquel and Raquel checks

over the problem. Since both students engaged in using Spanish throughout both excerpts, it was implied that using both languages was productive for both students.

Interestingly, during her student interview (Interview, 1/22/2020), Raquel named her teammates as something that helped encourage her to participate in math class. Raquel explained that they encouraged her to share her ideas, which was especially helpful to her since she was shy. Raquel asserted in her interview that she did not really use Spanish in math class. She clarified that she used it for translating or to understand an unfamiliar word in English. With respect to this clip, however, Raquel moved fluidly between languages. Additionally, in the interview, Raquel explained that sometimes it was very difficult for her to do mathematics independent of her teacher. In this multimodal transcript above, not only was Raquel doing mathematics independent of her teacher, she was an active participant involved in mathematical sensemaking.

In the next section, I highlight Gabe's noticing and instructional practices over time that I argue supported the translanguaging space that Julia and Raquel co-constructed. Gabe and I did not have a noticing interview about the problem-solving interaction between Julia and Raquel since it had not caught my attention at the time. Since Gabe was not part of the group's interaction, I examined instructional practices in Gabe's noticing interviews that I argue relate to the co-construction of this space by the two students.

Vignette 3: Gabe's Instructional Practices over Time

Gabe engaged in several instructional practices that I found might have supported the space for translanguaging that Raquel and Julia co-constructed. These instructional practices included: 1) facilitating student discussions across languages and 2) leveraging language as a resource in groupwork.

Early in the year, Gabe set a norm for groupwork discussions to occur across languages. During his baseline interview (Interview, 3/7/2018) Gabe described his classroom practices and norms facilitating student discussions. He explained that he emphasized the importance of expressing mathematical ideas and concepts in students' language of choice. Gabe argued that mathematics could be a middle ground for English Language Learners in schools, especially as the subject is an interpretation of symbols and numbers. He claimed that for many of his English Language Learner students, simply having text in Spanish did not help them. They needed to have discussions in Spanish to connect their interpretations of numbers and symbols.

Additionally, Gabe explained a norm he attempted to uphold for grouping students (Interview, 12/20/2019). During a noticing interview after this particular class, Gabe reflected on his goals in grouping the teams for this review activity and for the class (Interview, 12/20/2019). Gabe's hope with these groupings were for students to look to each other for justifications and reasoning. Additionally, Gabe asserted that since these students had already been together for a year, they were familiar with each other's strengths during groupwork.

Furthermore, Gabe drew upon students' strengths in languages when constructing groups. Gabe specified that by intentionally grouping students who spoke English and Spanish, he leveraged both languages through student voice since students could generally explain the mathematics better to each other than he could (Interview, 12/20/2019). Later, in an interview on 1/17/2020, Gabe gave the example of a specific emerging bilingual student's strengths, Mateo. According to Gabe, Mateo's strengths were interpreting concepts and expressing them in a way that other students could understand (i.e. he called Mateo a bridge). In the transcript below, Gabe elaborated on his instructional choice to group another emerging bilingual student, Elena with Mateo.

Um, Elena needs to build confidence a lot too. Um, I think she's very, she's a good kind of captain of a group and I think she does, she cares a lot about her group, uh, or she cares a lot about her grades and, um, success. And, uh, I don't know if she's been set up a lot of times to have success and so hopefully kind of in her position to, she's the captain of that group. Um, and so in a position to kind of leverage other people's strengths and, and um, I think she gets along with people, which is absolutely a skill, but, um, but she does, she needs math support. Like she needs people to kind of like re-explain things. Um, part of that could be language. Um, and so that's where I hope Mateo comes in again as kind of the communicator who's, uh, speaks English and Spanish well (Interview, 1/17/2020).

Gabe elaborated on how he leveraged the strengths of Mateo and another emerging bilingual student in his class, Elena. Gabe attributed some of Elena's need to build confidence in mathematics with language, yet recognized that she was a good group captain since she really cared about her team members and tried to be supportive of them. Mateo was likely able to explain ideas and concepts to her in Spanish and English. Ultimately, by setting a classroom norm for students to leverage each other's strengths during group work, Gabe positioned his students to first lean on each other, not him.

During their joint student interview (Interview, 01/22/20), Elena and Mateo agreed that the people in their class encouraged them to participate. Since the same people were in their classes the past year and a half, they both felt a familiarity with their classmates and teacher, thus affirming Gabe's perception that students were familiar with each other. They also described, however, that they were both afraid of being wrong and were intimidated by "smart people." Their apprehension with grouping further confirmed Gabe's reasoning for his careful attention to it.

Over time, Gabe established this classroom norm of student discussions across languages and leveraging language as a resource in groupwork. In the following section, I discuss Gabe's noticing practices that informed the instructional moves outlined above.

Vignette 3: Gabe's Noticing Practices Over Time

In this section, I identify Gabe's noticing practices that supported this space for translanguaging. My analysis revealed that throughout his interviews, Gabe attended to 1) students' perceptions of the classroom which informed his instructional practice around grouping emerging bilingual students and 2) grouping emerging bilingual students such that they could leverage language. These noticing practices support the way that students' linguistic strengths play out in groupwork. I examine each of these noticing practices below.

Table 4. 3

Summary of Vignette 3 by Gabe's Instructional & Noticing Practices

Vignette		Instructional Moves	Noticing Practices
3	Raquel & Julia codeswitching during groupwork.	1) Facilitating student discussions across languages	1) Student' classroom perceptions which informed his instructional practice around grouping emerging bilingual students.
		2) Leveraging language as a resource in groupwork	2) Grouping emerging bilingual students such that they could leverage language.

In one of his interviews, Gabe explained how he grouped students as a result of student feedback based on a classroom perception survey. For example, Gabe attended to student perceptions that their groupings were not fair since he was grouping students based on perceived mathematical ability. Below is the transcript excerpt of Gabe's noticing interview on 12/20/2019 around grouping.

Figure 4. 17*Noticing Interview Transcript Excerpt 2*

Yeah, I think, um, I think they've noticed in, in groupings that I have, they've noticed before that I've tried to get a mix of, um, math ability when I set up groups. Um, you know, that cause it'll, it'll often happen after a unit test and I give the test back at the same time. And so, um, not this year, but last year like they commented that they did kind of notice that, um, that groups are being set up in ways that, uh, were homogenous, heterogeneous, um, as far as math ability and, um, I haven't had the highest stakes previously, so I don't know if that's something to compare.

In this section of transcript, we see that Gabe attended to student perceptions of how he would group them after they received tests back from him. In grouping students homogeneously and heterogeneously, Gabe elaborated that students perceived they were being grouped by mathematical performance, whether they had the same or mixed mathematical ability. As a result of student perceptions, Gabe decided to group students differently.

In many of his noticing interviews, Gabe also described trying to ascertain students' linguistic strengths. He would pay attention to, for example, the language practices of individual students, when they used different resources (e.g. during math versus social talk), and their reading and writing skills across languages. Noticing students' linguistic strengths was connected to the noticing above, in terms of structuring groupwork. We saw this above when Gabe describes his attention to the strengths that Mateo and Elena brought to their group (Figure 4.18).

In this section, I have discussed Gabe's noticing practices over time that supported his facilitation of whole class discussion and small groupwork. Over time, Gabe attended to his students' perceptions about how he grouped them as a result of their performance on tests. For Gabe, attending his students' perceptions about grouping further highlighted the importance of

carefully attending to different grouping strategies around students' language practices. In the next section, I summarize all three spaces for translanguageing and make explicit the noticing practices and instructional practices that supported them in the moment and over time.

Summary

The three vignettes of spaces for translanguageing that were constructed in Gabe's classroom illustrate how Gabe attended to different aspects of the classroom and organized his instruction accordingly. In the first example, Valerie presented her mathematical work in the language of her choice. For Gabe, his participation in this vignette was used to continuously leverage Valerie's expertise in the language of her choice, thus further developing her agency in communicating her own ideas.

Gabe's noticing informed classroom practices that allowed students to present their work in the language of their choice. In the vignette, Gabe was attending to Valerie's energy in that she was uncomfortable to risk presenting and how she was communicating with another student instead of him. These noticing practices informed Gabe's instructional modifications.

The next space for translanguageing involved students and teacher engaged in reciprocal translanguageing where Gabe positioned students as experts during whole class and small group activity. In the moment, Gabe was attending to student gestures and as a result, modeled using more gestures as they aligned with transformations. Throughout his interviews, Gabe explained his attention to application of language, talk structures, and student groupings during whole class check-ins. These noticing practices resulted in Gabe making instructional moves such as how he positioned students, guided them towards mathematical language, and used gestures.

In the space for translanguageing where students were codeswitching in small groups, Julia and Raquel seemed to comfortably problem solve across languages and leaned on each

other instead of the teacher or other students. This space was supported by a lot of norm building that Gabe had done prior to the interaction. For example, in attending to individual students' strengths, Gabe was able to intentionally and successfully group students. These groupings were for the most part successful in that many students affirmed that their teammates were valuable to their learning and sensemaking of mathematics. By noticing that students are comfortable expressing their mathematical ideas in multiple languages during class, Gabe set a norm for students to engage in groupwork in the language of their choice.

In Chapter 5, I turn to my analysis of spaces of translanguaging in Jeannette's classroom.

Chapter 5. Findings: Jeannette's Classroom

In this chapter, I present three vignettes in the context of Jeannette's mathematics classroom. Additionally, I describe the different instructional moves and noticing practices that supported the potential, in some cases, for spaces for translanguageing. I argue that Jeannette's noticing practices made central student buy-in around mathematics as it related to their experiences, language and culture. This focus also informed her translanguageing practice that helped students develop mathematical language.

Again, I show three excerpts of classroom activity to demonstrate various spaces where teachers and students co-constructed opportunities for mathematical sensemaking. These spaces for mathematical sensemaking were opened and constrained through Jeannette's attention to balancing the development of mathematical language and mathematical goals for her students.

Vignette 1: Miguel's Turn

The first vignette that demonstrated evidence of a space for translanguageing in Jeannette's class comes from Fall 2018. Jeannette was reviewing the second question in a warm up task that involved solving multi-step equations with the class. Here is the warm up problem:

Figure 5. 1

Solving Equations Warm Up

Fluency	Wed Oct 3 or	Thurs Oct 4	Objective: We will be able to accurately solve equations with several steps. F-IF.1, F-IF.2
Solve for x:			
1.		2.	
$9 - 3(x - 6) = 12$		$3(x + 4) = 15 - 2x - 13$	

Jeanette invited the students to walk through the steps to solving the problem with her.

This was not a high cognitive demand warm up activity, but Jeanette was asking students to explain why certain steps made sense. According to Jeannette, what made it higher in cognitive demand in this instance was that one of her emerging bilingual students explained in English for the first time (Interview, 10/4/2018). In asking the class to guide her through the review problem, Jeannette was also eliciting students' prior knowledge (Hansen-Thomas 2009) and encouraging the use of mathematical language. As previously discussed, eliciting students' contributions, especially those of emerging bilingual students, has implications for affirming students' evolving identities in mathematics class (de Araujo, Roberts, Willey, & Zahner, 2018; Takechi, 2015; Turner, Dominguez, Empson, & Maldonado, 2013; Turner, Dominguez, Maldonado, & Empson, 2013). In other words, Jeannette was positioning her class as having the authority to engage in mathematical problem solving, thus setting the stage for validation of the mathematical contributions they were about to make and potentially broadening their future participation.

The ninth-grade student of focus in this example, Miguel, had just moved to the United States from Mexico the year before this interaction was documented and was raised in a Spanish-dominant home. The High School fostered biliteracy courses at the time this vignette occurred and was transitioning towards a dual immersion program, thus Miguel's schooling occurred in both English and Spanish. However, his mathematics class was conducted primarily in English. Despite this, from my observations and discussions with Jeanette, Miguel only spoke Spanish to his classmates and to Jeannette, whether or not they responded in English or Spanish.

The vignette begins with Jeannette reviewing the warm-up at the front of the room on the Smartboard. Miguel is sitting with his classmates in groups of four facing the board. Miguel is seated with three English-Spanish bilingual classmates. In this example, Miguel volunteers to explain his answer and surprises Jeannette by explaining his problem solving in English for the

first time. The fact that Miguel chose to speak in English and the class did not register this as unusual speaks to the notion of the classroom as a space for translanguaging.

In the section below, I present excerpts of Miguel's contribution and illustrate how this example constituted a space for translanguaging.

Figure 5. 2

Vignette 1, Multimodal Transcript Part 1

Time	Gesture, Proximity, Gaze, & Movement	Verbal
10:25	<i>Jeannette writes down 5x, she looks at the board while she writes. Prior to the other student talking, Miguel had his hand raised to answer. Jeannette did not call on him so he puts his hand down while the other student is talking.</i> <i>Miguel is still watching the board.</i> The same other student responds and Jeannette, while watching him, nods, and cancels out the 5x.	Jeannette: Tell me what to do. Student: Bring down the 5x. Jeannette: Why is that 5x by itself now? Student: It's going to cancel. Jeannette: We're going to zero it out, right there.

This warm up debrief has started already when this excerpt opens. In this transcript, we see Jeannette inviting the class to “tell her what to do.” Miguel and his table mate both volunteer to answer. Jeannette chooses Miguel's table mate to answer first, and he walks her through the next step in the problem. While his table mate was guiding Jeannette through the problem, Miguel was attentively watching the board, thus signaling his engagement with the problem and group shareout.

In the next excerpt, Miguel volunteers to help guide Jeannette through the problem again.

Figure 5. 3*Vignette 1, Multimodal Transcript Part 2*

Time	Gesture, Proximity, Gaze, & Movement	Verbal
10:40	Again, Miguel raises his hand (his hand is not raised fully in front of him). Jeannette sees him and reaches for a clothespin from her bucket. Jeannette looks back at the board then down at her clothespin and gives a little smile. She points at the board.	
	Jeannette is nodding and writing.	Jeannette: Miguel, what are we going to do now?
		(1 second pause) Miguel: Eh, divide.
		Jeannette: Divide by what?
	Jeannette is writing the steps.	(1 second pause) Miguel: Five.
		Jeannette: Kay. And then what?
	Jeannette smiles at Miguel.	(1 second pause) Miguel responds slowly: And then x is alone.
	Jeannette follows along and is writing out the next step, -10 divided by 5.	Jeannette: Why is x alone?
		(1 second pause) Miguel: Because 5 divided by five is x.
	Jeannette nods and looks back to the whole class.	Miguel: 2 negative.
		Jeannette responds: Okay.
	Miguel goes back to looking at his work on his desk.	Jeannette: I'm not going to box that up because I want to check it real quick.

When Jeannette asks for another volunteer, again, Miguel raises his hand (timestamp 10:40). This suggested he was making a bid for a mathematical contribution. Although Jeannette sees him raising his hand, interestingly, she does not simply call on him. She maintained her classroom norm of reaching into her bucket of clothespins which had the names of the students in her class to call on students to share their ideas by. Jeannette conveniently “chose” Miguel. This move suggested that she did not want to draw extra attention to him. Miguel’s persistent bid to share steps to the problem, and Jeannette’s selection of Miguel as the second presenter were precursors to a space for translanguageing. Recall that up until this point, Miguel always shared his ideas in Spanish in this class. By inviting Miguel to share his answer, then, Jeannette was also

inviting him to express his problem-solving steps in whatever language he chose.

Notably, Miguel responds in English, “Eh, divide.” As noted in the introduction, this was the first time Miguel had ever given a mathematical answer to Jeannette in English. As they work through the problem, Jeannette does not draw extra attention to Miguel’s contribution, and instead nods matter of factly and continues to write on the board.

Although Jeannette’s class was generally fast paced and, on a schedule, in this interaction, Jeannette slowed it down and gave Miguel “wait time” with scaffolded questions. The one second pauses between Jeannette’s questions and Miguel’s answers were examples of Jeannette providing wait time. More specifically, throughout their interlude (beginning in timestamp 10:40), Jeannette did not invite the whole class to participate in their interaction and took time to ask him scaffolded questions, thus helping him build upon his explanation in English. In doing so, Jeannette dismissed a fast-paced review so Miguel could pace his explanation into English. In slowing down the pace of the class to include Miguel’s explanation, Jeannette supported him in expanding on how he normally participated in his mathematics class to include English. Also important to note is that not only did Miguel appear comfortable testing his English in this instance, the class did not seem to attribute any significance to him answering in English for the first time. From an outsider’s perspective, one would never guess that anything was out of the ordinary.

Additionally, Jeannette did not revoice any of Miguel’s responses or redirect his contribution to more conventional mathematical terms (Hansen-Thomas, 2009). For example, Miguel responds (timestamp 10:50), “And then x is alone.” A more conventional response a student might give would have been “to get ‘ x ’ by itself.” Instead of revoicing Miguel’s wording to “get ‘ x ’ by itself”, Jeannette positioned his choice of language as competent by questioning him

“Why is ‘x’ alone?” Miguel elaborated because they had divided by 5 and the answer was two negative. Again, instead of revoicing his answer, Jeannette agreed with him.

In this vignette, I argue that a space for translanguaging was co-constructed in the interactions between Jeannette, Miguel, and the class. In volunteering to answer and explain his answer in English for the first time, Miguel made a bid to co-construct a space for translanguaging. Jeannette continued co-constructing this space by asking Miguel scaffolded questions and offering him encouragement throughout his answers. In giving Miguel time to explain in the language of his choice, Jeannette ruptured a dominant classroom norm of fast-paced mathematics. Furthermore, neither Jeannette nor Miguel drew attention to the fact that this was Miguel’s first time not only speaking English in class, but also that it was his first time providing answers in English in front of the whole class. Throughout this interaction, Miguel appeared comfortable explaining in English. By maintaining the problem-solving session with Miguel alone, Jeannette positioned Miguel as having the authority to finish the explanation of the problem on his own. This rupture was further supported in that the class did not indicate they were surprised or uncomfortable while Miguel explained his answer.

In the section that follows, I analyze the instructional practices that supported the space for translanguaging that Jeannette and Miguel co-constructed. I detail these instructional practices through an excerpt of the noticing interview with Jeanette that followed this classroom observation.

Vignette 1: Jeannette’s Instructional Practices

In this section, I present Jeannette’s instructional practices that supported Miguel’s contribution of problem-solving steps in English. These instructional practices were as follows:

1) acting like nothing was out of the ordinary to the rest of the class when Miguel explained his

answer in English and 2) appearing to maintain classroom norms while creating a specific type of opportunity for Miguel. To illustrate Jeannette's instructional practices, I refer to the multimodal transcript above and to an excerpt of Jeannette's noticing interview transcript.

Throughout the time Miguel was explaining his problem solving in English, Jeannette acted like nothing was out of the ordinary. For example, when Jeannette asks Miguel what they are going to do next and Miguel responds in English (timestamp 10:40), Jeannette simply nods and follows up with additional questions. In timestamp 10:50, when Jeannette questions him for the next step, Miguel clarifies with "and then x is alone," instead of the more common phrase, to "get x by itself" or "isolate the variable." Jeannette does not revoice Miguel's choice of words and incorporates his language into her own by posing the question, "Why is x alone?" After Miguel finishes explaining, he adds that the answer is two negative, to which Jeannette agrees instead of correcting his answer to negative two.

Jeannette also improvised a specific type of participation structure for Miguel in order to support the co-construction of a space for translanguaging with him. Jeannette clarified in her interview that she intentionally called on Miguel even though it was not really his name on the clothespin. In describing how she invited Miguel to share, Jeannette referred to her clothespin bucket as a tool to intentionally (or randomly) choose students to participate in class.

I have a bucket and all of their names are in the bucket on clothespins. Um, so it's completely random. Now in this class I have, um, all of the kids in the bucket. But like for today there was, there was a kid who was dying to give an answer and um, and I hadn't called his name yet. And so when I picked a different kid's name that I know is just fine, I lied and said it was the other kid that was super excited to try to, um, give an answer. I can't remember which kid when, but so sometimes I'll do that. Sometimes I will see a kid's name that I know is having a bad day and I'll just say, Oh that kid's absent and I'll pretend like I didn't pick that. You know what I mean? So I kind of modify it. So it's mainly um, random but with a little bit of intention on my part. Um, so that the kids who are quiet and, um, try to be invisible as much as possible, I think can't stay invisible so that it kind of draw them out a little bit and let them know I see them. Um, that's why I

use that method. I had picked Mario over here. Mario is a senior, he's just taken this class cause he needs another math class and he's fine. He doesn't need to speak out, but Miguel needed to (Interview, 10/4/2018).

Jeannette also scaffolded Miguel's mathematical contribution by asking him questions that required one-word responses. Additionally, she slows down the interaction by asking lower inference questions so that he explains in simpler terms. As Jeannette explains in the interview excerpt below, she generally used this strategy of asking lower inference questions before making a switch to a higher order question in English with all her English Language Learner students in order to give them extra time to respond.

Yeah. Yeah. And so I do that with him actually regularly. I do that with all of my Spanish language students. Um, I try to remember to do it regularly because I want to give them the chance to answer it in English. Generally. They'll stare at me for a minute and then I'll say, you know, 'En español está bien.' And then they'll give me the full explanation in Spanish. But I always wait for a second to see if they're gonna try. Not even more than a second. You know, I give some wait time to see if they're going to try to do it in English. And sometimes they do. And then sometimes they'll just automatically go if they're going to try to go with it. So, yeah, that was not, um, not completely unusual for some of my Spanish speakers, but the first time that happened with Miguel. Yeah. Even in small group (Interview, 10/04/2018).

Within this co-constructed space, it is important to note the response by the rest of the class. The class did not indicate that they were surprised with Miguel's explanation being in English, nor did anyone try to correct his explanation.

In this section, I have discussed the instructional practices related to this space. Below, I discuss the noticing practices these instructional practices.

Vignette 1: Jeannette's Noticing Practices

The noticing practices that emerged from my analysis of Jeannette's noticing interview included attending to: 1) whether the classroom appeared "normal" to students, 2) how her body language invites (or not) students to participate in a class discussion, 3) students' level of

comfort expressing mathematical ideas in English, 4) mathematical errors, and 5) which students need to have an opportunity to share mathematical ideas. To demonstrate Jeannette's noticing practices, I refer to the transcript of Jeannette's noticing interview.

Table 5. 1

Summary of Vignette 1 by Noticing & Instructional Practices

Vignette	Instructional Moves	Noticing Practices
1 Miguel explains in English for the first time.	1) Acting like nothing was out of the ordinary to the rest of the class. 2) Appearing to maintain classroom norms while creating a specific type of opportunity for Miguel.	1) Whether the classroom appeared "normal" to students. 2) How her body language invites (or not) students to participate in a class discussion. 3) Students' level of comfort expressing mathematical ideas in English. 4) Mathematical errors 5) Which students need to have an opportunity to share mathematical ideas?

First, Jeannette described her own excitement during her interaction with Miguel since it was the first time she had heard Miguel give an explanation in English. Jeannette voiced that she attended to her reaction to Miguel's responses. We see Jeannette make this explicit in the transcript excerpt below of her noticing interview.

Um, so as for Miguel's interaction with me. I um, was trying not to seem like I was too excited about him speaking because yeah. And I was trying not to because I didn't want to scare him off of continuing. I just wanted it to feel like normal. Like this is what we do every day so that he would just continue with what he was doing. Yeah. So I think I also stopped looking around the room and trying to include other people with my eyes or my, you know, whatever. Because I was so excited about drawing out what I was drawing out from Miguel (Interview, 10/04/2018).

In this portion of the transcript, Jeannette describes that she did not want to scare Miguel from continuing his explanation, so she acted like nothing was out of the ordinary. Jeannette also noted that she intentionally moved her eyes away from other students in the class so she could

focus on her interaction with Miguel. This is interesting since Jeannette was explaining that she needed to look at Miguel to better center his explanation and help him build upon his contribution. In both cases, Jeannette's noticing is trained on creating a familiar and safe space for Miguel.

In recognizing that Miguel was giving an explanation in English for the first time, Jeannette also noticed his fluency in expressing his mathematical ideas in English. We see this in the excerpt of the transcript below.

In that moment, what was standing out to me was how comfortable he was with using English in giving his answer. Um, he wasn't, um, stopping and then starting, he wasn't thinking for a minute about how to say it. He was just giving his explanation in English. And that's what I was noticing in that moment (Interview, 10/4/2018).

Above, Jeanette is paying attention to more than the accuracy of the words in English. She is also noticing stops and starts, and the time it takes for Miguel to respond. Jeannette's noticing around Miguel's fluency was important because she recognized the importance of supporting him continuing to explain his answer.

Jeannette specified that she acted like nothing was out of the ordinary in nodding and giving encouragement while noticing if she would have to correct Miguel's conceptual understanding.

So I guess I did a lot of nodding and writing down and saying yes, I'm just saying the correct math in it, in the correct. Using the correct terminology. So he was just spot on. Um, I did worry in the middle of it that there would be something that I would have to correct. Um, I mean there are certain things where you can let it slide and then certain things where if you let it that makes the math more difficult later. Um, so I was really trying to make sure or just hopeful, really hopeful that as he went along that it was going to be something that I could agree with. I didn't want to correct him on any, I was hoping I would not have to and I didn't have to correct him. He was doing, it was spot on. (Interview, 10/4/2018).

In the excerpt above, Jeannette was attending to mathematical mistakes that needed correction

versus those she could let slide. Jeannette noticed the flow and correctness of Miguel's explanation in English. She had been worried she would have to intervene to correct him, which might have undermined the space they were co-creating.

Jeannette's noticing practices supported students needing to have an opportunity to share ideas. Prior to Jeannette calling on Miguel, Miguel had raised his hand to share his ideas around the same problem (see multimodal transcript timestamp 10:25). In attending to his self-advocacy in raising his hand twice, Jeannette noticed that Miguel really wanted to explain his answer. She also noticed the tension between cutting off another student so that Miguel could contribute and supporting Miguel's enthusiasm. In this case, the clothespin was a way to improvise a solution.

In this section, I have discussed the noticing practices that supported this co-constructed space for translanguaging. These noticing practices involved Jeannette attending to ways to appear to maintain business as usual while creating a safer space for Miguel to express his mathematical ideas in English. In the following section, I present the next vignette, which, in this case, supports a potential space for translanguaging.

Vignette 2: Youth Culture

The second vignette from Jeannette's classroom involves her adaptation of an activity from the CPM curriculum to become more relevant to youth culture. This vignette focuses on the instructional moves that Jeannette made to support a potential space for translanguaging, instead of the co-construction of one. The lack of evidence for the co-construction of a space for translanguaging in this vignette is due to my inability to access the original video of this activity due to unforeseen circumstances.⁵ Instead, I illustrate the vignette through a summary of the

⁵ Due to the Colorado stay-at-home order during CO-VID 19 during the Spring 2020 semester, I did not have the opportunity to access and analyze the video demonstrating a space for translanguaging.

class activity and task, neither of which have details about students' engagement.

In 2018, Jeannette gave the class the following problem as an introduction for setting up systems of equations.

Figure 5. 4

Original Mathematical Task to Introduce Systems of Equations

3. Model with mathematics. Bags of maple granola cost \$2 more than bags of apple granola. The owner of a restaurant ordered 6 bags of maple granola and 5 bags of apple granola. The total cost of the order was \$56.

a. Let m represent the cost of a bag of maple granola. Write an expression for the cost of 6 bags of maple granola.
 $6m$

b. Use the variable m to write an expression for the cost of a bag of apple granola.
 $m - 2$

c. Write an expression for the cost of 5 bags of apple granola.
 $5(m - 2)$

d. Write an equation to show that the cost of 6 bags of maple granola and 5 bags of apple granola was \$56.
 $6m + 5(m - 2) = 56$

e. Solve your equation to find the cost per bag of each type of granola.

My Notes

$m = \text{maple granola}$
 $m - 2 = \text{apple granola}$

In 2019, Jeanette recalled that this introductory problem was not interesting to the students.

Thus, she opted to adapt it in the following way:

Figure 5. 5

Adapted Mathematical Task to Introduce Systems of Equations

Intro to Systems of Equations

Name _____

Date _____ Per _____

Cardi B and Lil Pump are scheduling concerts in Denver this summer.

Cardi B is going to be headlining at the Fillmore Auditorium. The use of the Fillmore will cost her \$1830, and her tickets are selling for \$51 each.

Lil Pump is going to be performing at the Gothic Theatre. It will cost \$6480 for use of the venue, and his tickets are selling for \$27 each.

A. Find the number of tickets each artist needs to sell in order to **break even**.

B. Find the number of tickets they could sell to make the same profit.



I interviewed Jeannette about the problem she created for the next day after the introductory task based on the modified version.

Figure 5. 6

Adapted Mathematical Task for Day 2 Systems of Equations

More Systems of Equations	Name _____
Standard Form Equation	Date _____ Per _____
1. Cardi B and Lil Pump are coming in concert to Denver this summer. Cardi B is going to be performing at the Fillmore Auditorium. Tickets have sold for \$51.00 each. Lil Pump is going to be playing at the Gothic Theatre. His tickets have sold for \$27.00 each. They have the same management company, XL Mgt. XL found that the total number of tickets they sold was 2000. They made a gross total of \$82800. They need to find out how many of tickets Cardi B sold, and how many tickets Lil Pump sold. A. Define your variables: B. Write an equation that shows that the total number of tickets sold for both concerts is 2000: C. Write an equation that shows that the total amount of money they made off the sale of tickets is \$82800: D. Find the number of tickets sold by each artist:	

I argue that the modification of the problem had the potential to support a space for translanguaging because it was more culturally relevant to the students. It was also a more challenging task.

During her noticing interview (Interview, 03/07/2019), Jeannette explained that the unit on solving systems of equations could be dry and unengaging for students. When she initially introduced the systems of equations unit involving the original problem with granola the previous school year (see Figure 5.4), she recognized that the focus of the task on restaurant owners and granola had been designed to engage in a White, middle class, and adult Discourse (Gee, 1996). In other words, this original problem drew upon a distinctive way of not only speaking, but of being and acting in a socially recognizable way (i.e. White, middle class, and adult). Thus, Jeannette decided to adapt the mathematical task to reflect a youth culture Discourse. In having the opportunity to participate in a culturally relevant youth Discourse (Gee, 1996), I argue that students were more fully able to access their linguistic repertoire, thus engaging more with the problem. In this adapted activity (see Figure 5.6), students were applying their knowledge of systems of equations to explore concert profits.

Below, I summarize how the whole class engaged in this adapted mathematical task.

The episode began when Jeannette reintroduced the problem and asked students to “remember” who was coming to Denver over the summer (see Figure 5.6). Jeannette reviewed how to find the number of tickets sold using Cardi B’s equation then asked the class to work independently to find the number of tickets sold for the other rapper, Lil Pump. In addition to finding the ticket sales for both rappers, Jeannette asked students to analyze the systems of equations for both Cardi B and Lil Pump’s equations using the mathematical online graphing tool, Desmos, and to interpret the y-intercepts, slope, and points of intersection to better compare and understand each rapper’s ticket sales at local concert venues to the area. She let them know that they should not use any numbers in their explanation. While students were talking in their groups, Jeannette circulated around the room and checked in with students about their intuitions.

Students ended the lesson by writing the equation with the assigned profit value on their worksheet, then finding the number of tickets sold for that profit value.

I argue, here, that by making the task more culturally relevant to students and by turning the mathematics over to them to work on in groups, a space for translanguaging was likely to emerge. In the section that follows, I detail the task modification rationale.

Vignette 2: Jeannette's Task Modification

The year prior to assigning the adapted task, Jeannette hypothesized that students were disinterested in the original problem because of its context. The last time she taught systems of equations, she used the given activity from the curriculum. This activity involved students solving the system to find the weight of bagged granola. She describes how students engaged with the original problem which we see in the transcript below.

Figure 5. 7

Vignette 2, Transcript Excerpt 1

1. **Jeannette:** So the unit that we're supposed to be doing is solving systems of equations. It's not easy and it can also be very dry. And if you start with naked numbers and say this is how you do it, you lose everybody. Everybody but that one kid that just loves to solve problems. Maybe five kids. Um, so I wanted to start with something engaging. So I used the book last year.
2. **Jeannette:** The problem from the book literally introduces this with a granola problem.
3. **Monica:** Like the bar?
4. **Jeannette:** No. Like a bag of granola. So this bag of granola costs you know six dollars per pound. This bag costs four seventy per pound. It's such a boring problem. I actually had to go buy bags of granola just so the kids could see what the heck I was talking about cause most of them their frame of reference is a granola bar. When I started

introducing the problem they were like, you know, they just didn't know what I was talking about literally.

In *line 1*, Jeannette explains that problems involving systems of equations are challenging, especially when there is no context provided. She goes on to explain in *line 4* that she used the original problem with bags of granola when she first taught systems of equations using this curriculum. Since students did not know what a bag of granola was when she introduced the problem, (*line 4*), Jeannette brought a few bags of granola in for the class. In other words, she modified the task based on aspects of it that she felt would be more relatable for students, thus making it more meaningful. Creating opportunities to engage in meaningful contexts for mathematics is an important aspect to plan for in translanguaging (Garcia & Sylvan, 2011; Garcia & Wei, 2014; Hornberger & Link, 2012).

She noticed that the task still wasn't very meaningful to students and decided to solicit students' ideas about meaningful subject-matter, in this case, Hip Hop rappers. Jeannette learned that her students were interested in the rappers Cardi B and Lil Pump and modified the task to incorporate these celebrities with mathematical contexts that were meaningful to students. This process is detailed in the transcript excerpt below.

Figure 5. 8

Vignette 2, Transcript Excerpt 2

5. Jeannette: Mhm. That day I bought bags of granola that was a little more entertaining but really? Granola? So, I thought, well, what was something that I could do that would be more interesting? And then I just asked the kids for names so that they would have

name recognition and it wouldn't be coming from me. If you're going to ask for rappers coming from me they are not going to be ones that they've heard of (laughs).

6. **Monica:** What are you noticing now that you are using the Cardi B problem?
7. **Jeannette:** Versus granola? Two million times more interesting (laughs). That might be low. The granola problem, honestly and truly they were like can we have some granola? That was the most interesting thing to them. They wanted to try the granola (laughs). But the problem itself, they didn't care.

In *line 5*, Jeannette explains how she solicited students for rappers they liked. In learning about meaningful contexts to students in the form of musical artists, Jeannette positioned herself as a co-learner with them as students learned how to solve systems of equations, thus inviting a potential culture of translanguaging (Garcia & Kleyn; Phakeng & Moscovitch, 2013; Staat, 2009) into the classroom. In other words, by positioning herself as a co-learner with students, Jeannette created an opportunity to leverage student Discourse in the mathematics classroom.

Jeannette describes students' engagement in *line 7* once she modified the task. She hypothesized that bringing these rappers and their lives into the students' mathematical work increased classroom engagement. By adapting the mathematical task, Jeannette invited a culture of translanguaging, thus opening a potential space for translanguaging where she could highlight connections between mathematics and young people's interests (Staat, 2009).

In the next section, I illustrate Jeannette's noticing practices that supported this task redesign.

Vignette 2: Jeannette's Noticing Practices

In this section, I identify the noticing practices that supported Jeannette's decision to modify the task and to solicit students' ideas for it. My analysis revealed that Jeannette was

attending to: 1) context of the mathematical task, 2) student energy in the classroom, 3), youth culture and what is interesting to young people, and 4) Whiteness in the presentation of the original task. I illustrate these noticing practices through referring to the excerpts above and below of Jeannette’s noticing interview.

Table 5. 2

Summary of Vignettes by Jeannette’s Noticing & Instructional Practices

Vignette	Instructional Moves	Noticing Practices
2 Jeannette adapted a task to one that drew upon youth culture.	1) Redesigning the problem.	1) Context of a mathematical task. 2) Student energy. 3) Youth culture and what is interesting to students. 4) Whiteness in the presentation of the original task.

One noticing practice that emerged from the interview was Jeannette’s attention to the context of the mathematical task. As we saw in Excerpt 1, Jeannette noticed that her previous students were confused about granola bars versus bags of granola. While changing a task context to be relevant to students may not necessarily be sufficient for supporting spaces for translanguaging, it may be an important component.

Throughout her noticing interview, Jeannette also reflected on the nature of students’ engagement with the original versus new problem. Here, Jeannette describes how the students were more interested in eating the granola than engaging in the problem (*Excerpt 2*) and how their engagement in the mathematics changed with the new context.

In attending to student disinterest in the problem, Jeannette also became attuned to an opportunity to draw upon youth culture. In this case instead of trying to figure this out herself, Jeannette recognizes that the kids are the experts in this area, “And then I just asked the kids for

names so that they would have name recognition and it wouldn't be coming from me” (*Vignette 2, Transcript Excerpt 2*).

With respect to the lack of cultural relevance of the task, Jeannette noticed that the original task reflected an adult, middle class, White-centric culture since “other groups of people do not buy bags of granola on a regular basis” (Interview, 3/7/2019). In describing this disconnectedness of the task to youth culture, Jeannette was attending to Whiteness.

Well, in that specific example it is very White-centric. Very. To an extreme almost. Very middle-class White-centric. Um, I don't think other groups generally buy bags of granola on a regular basis enough to...but it also is something if even if you're middle-class and you're Anglo and you eat granola every morning, you kind of don't care about buying it if you're a kid no matter what. Right? It's just...I don't think there was a thought put into just youth culture. It's very adult. Very very. And that goes across the board outside of race and color and socioeconomic status. And actually most kids just don't even go grocery shopping. So, I mean, yeah. I mean not on their own (Interview, 3/7/2019).

In this portion of the transcript, Jeannette explains that the problem was not very relatable even for White students because buying bags of granola is very adult. She determined that youth culture was not considered when the original problem was written. Jeannette attended to how youth culture and disrupting Whiteness were crucial elements to consider in the redesign of the lesson. To further prove her point, she argued that youth do not typically go grocery shopping, at least not on their own!

I argue that Jeannette's noticing practices supported a potential space for translanguaging by redesigning the original task. In attending to the disengagement of the class with the original problem, Jeannette became attuned to an opportunity to leverage youth culture. In doing so, Jeannette positioned herself as a co-learner in the classroom, thus inviting a *culture of translanguaging* (Staat, 2009) when she asked for student input towards the task redesign. At the same time, Jeannette ruptured a norm of drawing upon mathematics tasks centering Whiteness.

In the next section, I illustrate how Jeannette's instruction did not support a space for translanguaging to emerge in the moment but may have encouraged future ones.

Vignette 3: What was the tilde for?

The third vignette from Spring 2019 focused on the instructional moves that Jeannette made to support a potential space for translanguaging. In this case, she drew upon students' linguistic funds of knowledge to make sense of a mathematical symbol. Jeannette made instructional moves consistently over the two years with each of her Math 1 classes (Interview, 5/2/2019). I argue that this particular vignette illustrates how a potential space for translanguaging became narrowed during the lesson, in part, because students did not have the opportunity to engage in mathematical discourse, in either language. However, I also contend that this instructional move could have supported the emergence of spaces for translanguaging in future class meetings.

In this vignette, Jeannette's aim was to make a connection with the whole class between the mathematical similarity symbol ($\sim$) and the Spanish tilde in writing similarity statements between geometric figures (Interview, 5/2/2019).

The vignette begins with Jeannette at the front of the room and students sitting in groups facing the front. Jeannette is reviewing similarity theorems with students and teaching the class how to write a similarity statement before they began writing geometric proofs. Prior to reviewing the similarity theorems, Jeannette realized she had not taught the class similarity statements. Below is the transcript in which Jeannette teaches similarity statements after the class lets her know that they do not know how to write them.

Figure 5. 9*Vignette 3, Transcript Excerpt 1*

1. **Jeannette:** Oh, then let's do this now. Yeah. So, so underneath where you wrote from the similarity to this similarity statement sorry, I do different things for every class, so I never know. Okay. So we're going to say triangle MON. Um, and then I did not write a congruency symbol. What I wrote instead was a similarity symbol and that symbol, that just that one squiggle, means is similar to. So, triangle MON is similar to. And, that's just like a little tilde. Okay? And I could have done any, any way I wanted. I did MON cause it spells mon. Which you know, whatever. But anyway, now I actually have to have everything with corresponding angles in the same place. So look at that first triangle, what angle corresponds to angle M in the other triangle?
2. **Student:** P.
3. **Jeannette:** Okay. So we're going to put the P and then what corresponds to O, right?
4. **Student:** R.
5. **Jeannette:** R and then the last one is T. Okay. So when you write the similarity statement, it's very similar to writing a congruency statement. Uh, it's just, you don't write the congruence symbol, you just write this similar, same kind.

In the transcript, Jeannette relates the similarity statement to the Spanish tilde. After Jeannette and her students write the similarity statement for the triangles, Jeannette says, “What I wrote instead was a similarity symbol and that symbol, that just that one squiggle, means is similar to. So, triangle MON is similar to. And, that's just like a little tilde” (line 1). Her translanguaging practice of connecting the Spanish tilde with a mathematics sign for similarity had the potential to support a space for translanguaging since she was drawing upon students’ linguistic funds of knowledge to make mathematical connections. However, as the class progressed, Jeannette did not continue to draw upon students’ linguistic funds of knowledge to make the connection between the tilde and congruency statement (line 1). Additionally, Jeannette’s lecture on proving congruency and similarity continued for approximately twenty minutes. During this time, she posed eleven lower-level questions to students in the form of I-R-E (initiation-response-evaluation). Thus, students were not able to engage in mathematical

sensemaking, and a space for translanguaging did not emerge.

Below, I explain more about Jeannette's instructional practice of drawing on students' biliteracy practice.

Vignette 3: Instructional Practice

In this vignette, Jeannette is drawing upon students' biliteracy practices to encourage students to make a possible connection between mathematics and Spanish syntax. In this case, the instructional practice involved connecting the Spanish tilde to the mathematical concept of congruence.

Jeannette outlines this instructional move below in her noticing interview from the 2017-2018 school year before this vignette occurred.

Figure 5. 10

Vignette 3, Noticing Interview Transcript Excerpt 2

I wanted to name it [tilde] because the kids know they use tildes...a lot. Okay. So our school, um, has a large, large number of kids who are biliterate. Okay. So a large chunk of our kids, you know, speak Spanish as their first language. That's not different from a lot of schools, but a lot of them take Spanish literacy classes. So they read, write, and understand the syntax and grammar and usage and all of the things that they do in their language arts class. They do the same thing in Spanish. So I wanted to connect it to a word that they probably knew or would know when they do start taking Spanish classes (Interview, 5/10/2018).

Here, Jeannette describes how she drew upon students' (possible) experiences in their Spanish literacy classes to connect mathematical symbols and notation to Spanish syntax. Additionally, she explains how she can "rely on" students knowing what a tilde is.

Figure 5. 11*Vignette 3, Transcript Excerpt 3*

Not all of them are taking [Spanish language arts courses], but most of them are. And it does include the kids who are not Spanish dominant and eventually we're going to move towards being dual language. Um, we're just in the process of moving towards that. But I can rely on the majority of my kids knowing what a tilde is, you know, it's kinda nice (Interview, 5/2/2019).

Jeannette explained that previous to the past few years, she was not able to make the tilde connection since the school did not offer Spanish language arts and other biliteracy classes at the time and she could not count on students knowing how to read and write in Spanish. Now that there was a heavy emphasis on biliteracy courses and the school was transitioning towards a dual immersion program where students were in Spanish language arts classes, she could count on them knowing what a tilde was.

In the next section, I outline the noticing practices that supported this connection to students' biliteracy.

Vignette 3: Jeannette's Noticing Practices

Jeannette's noticing practices in this instance were geared towards biliteracy. These included attending to 1) opportunities to make connections between mathematical discourse and Spanish syntax, and 2) noticing distinctions between types of understanding among emerging bilingual students. I describe each of these noticing practices below.

Table 5.3*Summary of Vignettes by Jeannette's Noticing & Instructional Practices*

Vignette	Instructional Moves	Noticing Practices
3	1) Drawing upon students' biliteracy.	1) Noticing opportunities to make connections between mathematical discourse and Spanish syntax. 2) Noticing distinctions between types of emerging bilingual students' understandings.

First, it is clear that Jeanette attended to opportunities to make connections between mathematical discourse and Spanish syntax (*Excerpt 1*). In this case, the connection revolved around a mathematical sign ($\sim$), which looks like the Spanish tilde so Jeannette is helping students become familiar with a mathematical form that is also in the Spanish language. Jeannette explained that she knew she could make this connection since students were taking biliteracy courses (*Transcript Excerpt 2*).

Second, Jeannette was attending to distinctions between emerging bilingual student and bilingual student understandings and ways in which mathematical discourse was not accessible to them. She described how she used to teach mathematical similarity differently in previous years since she could not count on student biliteracy practices. We see this in the excerpt below.

Figure 5. 12*Noticing Interview Transcript Excerpt 4*

Well, I think you picked this because I chose to give to equate it to a tilde symbol? And um, and yeah, and I do that every time we have to use that. Like I'll say congruency symbol is just like an equal sign but you put a tilde over it and I'll just, because it's convenient, like, um, the nice thing and it didn't used to work as well. Um, because my students, even though they spoke Spanish, they didn't read and write in Spanish. Some did and some didn't. But more did than did because they had been pushed away from them English right. And now because we're a completely bilingual school, most of the kids, especially the Spanish speakers, but most of the kids were taking Spanish literacy classes so they all know what a tilde is (Interview, 5/2/2019).

Here, Jeannette attended to differences between the ability to speak, read and write in Spanish among bilingual students. She explained that at the time of this vignette occurring, what made their school population unique was that many of their students took Spanish literacy classes and understood the syntax and grammar of the Spanish language. Jeannette claimed that drawing upon student biliteracy was relevant even if students had not already started taking Spanish literacy classes since they would in the future. Additionally, Jeannette explained that this practice was relevant for all students since they all took Spanish language arts classes.

This vignette illustrates how Jeannette leveraged students' hybrid language practices, which is important since she drew a connection between students' linguistic funds of knowledge to potentially build upon their mathematical learning. Drawing upon students' linguistic funds of knowledge is powerful in that it helps move emerging bilingual students into empowering spaces where they can create and sustain academic, bicultural identities (Gort & Sembiante, 2015; Palmer & Martinez, 2013). While the space for translanguaging was narrowed when students were not able to make meaning of the mathematical signs and ideas themselves, the connection still signaled a way to do mathematics in this classroom.

Above, I discussed the noticing practices that Jeannette engaged in over time to connect Spanish syntax with mathematical similarity. These noticing practices involved Jeannette attending to differences in students' biliteracy practices and opportunities to engage in mathematical discourse.

Summary

The three given examples I have provided from Jeannette's classroom involved either supporting spaces for translanguaging or the potential spaces for them. In the space where Miguel explained his problem solving in English for the first time in math class, dominant classroom norms were ruptured in that the teacher really took time to understand Miguel's thinking at the expense of losing the class's attention. In that moment, Jeannette was attending to the flow of English and the correctness of what Miguel was saying.

This space for translanguaging was supported by her noticing practices and instructional moves over time. For example, in attending to emerging bilingual students' pacing into English, student energy and self-advocacy, Jeannette was able to ask carefully scaffolded questions that helped students express their thinking. By noticing when students were comfortable expressing their mathematical ideas, Jeannette set a norm for students to engage in problem solving in the language of their choice.

The potential space for translanguaging occurred when Jeannette redesigned a mathematical task to encourage more student buy-in. When Jeannette taught the same lesson before with bags of granola weight, she noticed the disengagement of her students and related this to the context of the problem in middle-class, adult, white culture. In redesigning the problem, Jeannette noticed much more student buy-in and engagement.

The third vignette exemplifies how spaces for translanguaging can be constrained despite instructional moves that draw upon student biliteracy practices. Spaces for translanguaging are characterized by students' mathematical sensemaking, which did not emerge in this case. However, the analysis revealed the intricacies of Jeannette's attention to Spanish language practices and the implications of these for her instruction.

Chapter 6. Cross Case Analysis

Although Gabe and Jeannette both supported spaces for translanguaging in their mathematics classrooms, the implementation of those supports looked different across the classroom of each teacher. In this chapter, I present themes across each teachers' pedagogy and their instructional moves that supported these spaces. Furthermore, I discuss themes in each teacher's noticing practices. Finally, I summarize the culture of each teacher's classroom.

To identify the practices that were emphasized in each teacher's classroom, I used MAXQDA Code Matrix Browser in addition to MAX Maps to create visual summaries of the instructional practices and noticing practices codes that appeared most often to help me organize my analysis. Additionally, using these tools for visual analysis was helpful in that they helped me identify the unique practices of each teacher that contributed towards their classroom culture.

Gabe's Instructional Practices

Across my observations in Gabe's classroom, I observed Gabe engaged in several instructional patterns that persisted over time and were unique to his teaching style. The patterns of Gabe's instruction informed his noticing practices and vice versa. In the following sections I describe the type of instruction that was central to Gabe's teaching practice both overall and specific to emerging bilingual students.

Below is a table showing which instructional practices appeared most often between my classroom fieldnotes and Gabe's noticing interviews. Below I describe each of these practices in greater detail.

Table 6. 1*Gabe's Most Frequent Instructional Practices*

Instructional Practice	Order of Most Frequent Codes
Grouping	1
Teacher Biliteracy Practices	2
Teacher Questioning Strategies	3
Emerging Bilingual Student Grouping	4

Groupwork. The most prevalent practice that emerged from my analysis of Gabe's classroom was his grouping strategies. In each of Gabe's classes that I observed, students were always sitting in groups. Additionally, in each of Gabe's noticing interviews, he reflected on his instructional practice of grouping students, many times without prompting. Gabe explained that his classroom norm in creating groups was to consider students' strengths (Interview, 12/20/2019). Along with recognizing students' strengths, Gabe described his goal for the students in this particular class to recognize and leverage each other's strengths in groupwork since they had been together in the same class for going on two years. Importantly, Gabe described how he leveraged language as a resource in creating groups consisting of emerging bilingual students and bilingual students so they could problem solve and engage in mathematical discussions in the language of their choice.

Eliciting students' mathematical reasoning. Gabe's instructional practices and the focus of his practice from his baseline interview also centered on eliciting students' mathematical reasoning. As much as possible, he worked to position new mathematical ideas as coming from

students, instead of himself. Even if students were learning new concepts, Gabe would connect these concepts to students' previous knowledge with instructional moves such as rephrasing or clarifying what students were saying (Interview, 01/17/2020). In her noticing interview of Gabe's practice (Interview, 1/24/2020), Jeannette pointed out how Gabe gave instructions to students and did not just give them the answer. Additionally, Jeannette noticed that students did not look surprised or confused by Gabe's directions, thus she assumed that students were used to directions and activities like these.

Gabe employed this instructional practice in particular ways to support his emerging bilingual students. One of the ways he encouraged students' mathematical sensemaking was through using gestures with his students and mirroring gestures from them. Encouraging students' use of gesture supported bilingual and emerging bilingual students (and all his students) in that it expanded upon ways they could contribute their mathematical ideas.

A second approach to encouraging public reasoning was to position the contributions of emerging bilingual students as competent. For example, Gabe encouraged a classroom norm for students to engage in mathematical sensemaking with each other and with him in the language of their choice. Moreover, when students did engage in mathematical sensemaking across languages, Gabe and the rest of the class did not treat the use of multiple languages as out of the ordinary.

Setting classroom norms around biliteracy. During his baseline interview, Gabe described his classroom practices around language and biliteracy (Interview, 03/15/2018). He explained that he emphasized the importance of students expressing mathematical ideas and concepts in their language of choice. Students should have a space to experiment with ideas no matter the language. Gabe set a norm for students to share their mathematical ideas in the

language of their choice through his grouping strategies as described above. During his baseline interview, Gabe explained that he wanted students to feel comfortable taking risks in the classroom and expressing new ideas. He emphasized that the risk should be in experimenting with mathematics, rather than language. More specifically, encouraging student biliteracy practices supported students taking risks with mathematics in that students could feel free to explore mathematical concepts and ideas, no matter the language.

Gabe reflected during an interview how his biliteracy teaching practices had evolved over the years. For example, he now gave his emerging bilingual students combined English and Spanish versions of their tests (Interview, 12/20/19). Previously, Gabe had given his emerging bilingual students separate drafts in English and Spanish so they could reference both at the same time (Interview, 12/20/2019). Gabe explained that combining languages was an important practice because he was no longer singling out his emerging bilingual students as non-English dominant speakers, thus disrupting English-dominant classroom norms. This also supported students in being able to draw upon both languages during mathematical sensemaking.

In this section, I discussed the instructional practices that were most prevalent in Gabe's classroom. In the next section, I discuss his positionality and disposition towards equitable instruction.

Gabe's positionality and disposition towards equitable instruction

Every teacher brings a unique disposition towards teaching that is shaped, in part, by their positionality (Wager, 2014). My analysis of Gabe's disposition and positionality draws primarily from his baseline interview (Interview, 3/15/2018).

During his baseline interview (Interview, 3/15/2020), Gabe explained that as a white male, he never had to think about his own race or gender growing up. He explained that as a

student, he thought he was being recognized for what he was accomplishing as a form of merit. Gabe explained that later in life as a former college football player and football coach, he became more aware of systemic injustices that several men of color on his teams faced. In his interview he elaborated further to explain that he recognized some of his accomplishments were in part, due to systemic privileges that gave him a step up in life. Understanding these privileges informed his teaching, and the way he attended to the classroom. Although he explained that he understood that students from different backgrounds could not have his same experience as a White man living in the dominant White culture, he could help empower them to participate within that culture. According to Gabe, participating within that dominant culture could help empower students with new opportunities, thus changing the status quo and rupturing societal norms.

Gabe also had a particular perspective on equitable mathematics instruction (Interview, 3/15/2020). To Gabe, equitable instruction had a lot to do with accessible mathematics. Gabe explained that to teach mathematics in a way that was accessible to students, he looked for connections in mathematics to students' everyday lives. In his words, "Instead of the what am I trying to do in an activity, I can bring in the why...getting to the what the purpose is, what matters for students or look for connections [to their lives]" (Interview, 3/15/2020). For example, one of Gabe's teaching goals was for students to be able to leverage mathematics to become informed consumers towards decision making instead of rote consumers for standardized testing.

Gabe's positionality and disposition were intimately bound up in his noticing practices. Below, I outline the noticing practices that were pivotal to his instructional moves.

Noticing practices that were pivotal to Gabe's instruction

Throughout Gabe's noticing interviews, he attended to several aspects of the classroom that reflected themes in his noticing. In the following sections, I describe the type of noticing that was most influential to Gabe's teaching practice both overall and for emerging bilingual students.

Below is a table of Gabe's five most common noticing codes (Figure 7.2). I discuss the percentages of the most common code frequencies to summarize themes in Gabe's noticing.

Table 6. 2

Gabe's Most Frequent Noticing Practices

Teacher attention to:	% of Top 5 Codes
Student participation in classroom interactions	44%
Positioning	25%
Developing mathematical language	15%
Students' confidence	12%
Tensions around standardized testing	6%

Noticing Student Participation in Classroom Interactions. Throughout his noticing interviews, Gabe attended to how students were participating in the classroom, both traditionally and untraditionally. Of the five main teacher noticing codes that came up most often for Gabe, codes related to his attention to classroom interactions accounted for 44% of his main noticing codes. For Gabe, this attention to classroom interactions most often took the form of noticing how students were participating in groupwork. For example, Gabe attended to how students were positioning each other (e.g., who was the expert, who was quieter) during group assignments

(Interview, 2/25/2019). He paid attention to the strengths of each student and what this meant for the organization of groups. Specific to emerging bilingual students, Gabe attended to which classmates emerging bilingual students worked well with and how they interacted in problem solving across languages (Interview, 12/20/2020).

Positioning. Another frequent noticing practice for Gabe was consistently attending to opportunities to position students as experts. This included attending to opportunities to affirm and build on what students knew in order to help them develop mathematical vocabulary. In this section, I summarize Gabe's attention to positioning (25% of main codes) and opportunities to develop mathematical language (15% of main codes) together since Gabe's noticing practices around those codes were related. For example, in one of Gabe's noticing interviews, he attended to how much new mathematical vocabulary he was introducing to students at one time, which he viewed as a missed opportunity to pause and allow students to make sense of the information being used (Interview, 03/15/18). Additionally, Gabe elaborated how he missed the opportunity to build in time for practicing and applying new terms since he noticed the class was silent when he was introducing vocabulary. Moreover, in this interview, Gabe was referring to how he tacitly positioned himself as the mathematical language expert in this lesson since he noticed he did not include all students based on the academic language he was using. This situation speaks to a broader aspect of positioning that was of importance to him, which was how he attended to the positioning of student ideas. More specifically, Gabe grappled with how to include more student ideas in the classroom (Interview, 03/15/18). He acknowledged that the way he grouped students did not necessarily facilitate their talk about their ideas and creative work. Thus, he was paying close attention to classroom "silences" and what they meant. In this example, Gabe also attended to components of the lesson related to positioning and mathematical language that he wished had

gone better such as his choice and application of vocabulary with the whole class, student check ins, and groupwork.

Students' Confidence. Gabe also consistently attended to students' confidence (12% of the main codes) while students were sharing their ideas either during whole class or groupwork (Interview, 1/17/2020). We see this in Chapter 4, when he attended to students' confidence sharing their mathematical ideas during classroom activities. This was particularly true in attending to the confidence of his emerging bilingual students not only in mathematics (01/17/2020) but also in their social interactions (11/28/2018) and how each of these played out in groupwork.

Tensions from standardized testing. Throughout his noticing interviews, Gabe also attended to tensions (6% of main codes) which included classroom influences from standardized testing. Gabe noticed opportunities to teach students how to take multiple choice tests since he believed that knowing how to take a test is a skillset required to be successful within the dominant system of schooling (5/15/2019). Additionally, Gabe attended to the tensions he experienced in prioritizing teaching to standardized tests. Specific to emerging bilingual students, Gabe attended to the tension he felt with providing resources in English and Spanish to emerging bilingual students when they would have to take standardized tests in English (Interview, 11/28/2018).

Vulnerability. Although this did not come out in the MAXQDA analysis, the Co-ATTEND Team noticed that Gabe often seemed to be aware of situations in which his vulnerability could enhance relationships and build trust with individuals, both with students and members of the Co-ATTEND team. For example, Gabe reflected on how attending to his own vulnerability speaking Spanish helped him further appreciate the risk it took for emerging

bilingual students to self-advocate for what they needed in the classroom (Interview, 12/12/2019). Furthermore, Gabe attended to how his speaking Spanish helped his emerging bilingual students realize that he was willing to position himself as vulnerable with them (Interview, 12/12/2019). In a paper he was authoring as part of the Co-ATTEND team, he described how he began to key in on the bravery and vulnerability that his students (particularly young men of color) were demonstrating when sharing their mathematical ideas.

Gabe's noticing practices were inextricably linked to the culture that emerged in his classroom. I summarize his classroom culture below.

Summary

Gabe's classroom culture was one in which students took ownership of their ideas and expressed them in ways that challenged traditional ways of contributing in class. Gabe developed classroom norms around student ownership by encouraging student exploration of mathematical ideas (e.g. inquiry-based learning) during group work. Exploring mathematical ideas involved students having to rely upon each other instead of Gabe simply telling them the answer. Additionally, students expressed their mathematical ideas in untraditional ways since they were typically responsible for sharing with the whole class and other groups before Gabe gave his input. By encouraging his students to express their mathematical ideas in untraditional ways, Gabe empowered students to broaden their way of participating in mathematics.

I observed major differences between the culture that emerged in Gabe's classroom and the one that emerged in Jeannette's. These differences played out in terms of the instructional practices and noticing, which were tied in part to classroom goals and teacher positionality. Below, I discuss the practices that contributed towards Jeannette's classroom culture.

Jeannette's Instructional Practices

Across my observations in Jeannette's classroom, I observed her engage in several instructional patterns that persisted over time and were unique to her teaching style. In the following sections I describe the type of instruction that was central to Jeannette's teaching practice both overall and specific to emerging bilingual students. I also draw comparisons between Gabe and Jeannette's practices to highlight the differences that I observed.

Below is a table of my analysis (Figure 7.3). Interestingly, the same instructional codes that were most frequent for Gabe, were the most frequent for Jeannette. However, the enactment of those instructional practices looked very different in each teacher's classroom. The most prevalent instructional practice for Jeannette was engaging in biliteracy practices; grouping and questioning had similar numbers of codes. Below I describe each of these practices in greater detail.

Table 6. 3

Jeannette's Most Frequent Instructional Practices

Instructional Practice	Order of Most Frequent Codes
Teacher Biliteracy Practices	1
Teacher Questioning Strategies	2
Grouping	3

Setting classroom norms around biliteracy. During her baseline interview (4/13/2018), Jeannette explained that she saw part of her job as a teacher to help students develop mathematical language. Jeannette explained that she typically taught mostly bilingual and emerging bilingual Latinx students, so this was usually on her mind. Jeannette explained that she

could never assume there was not an emerging bilingual student in her classroom. Even when students do not have marked accents in English, she explained, some may have just moved from Mexico or another foreign country. Furthermore, Jeannette explained that even if a student was labelled as “proficient” in English, they could still get confused in that some words sounded like a word in English but then meant something different in math class. To help students distinguish between languages, for example, Jeannette connected the use of cognates and grammatical structures between English mathematical terms and Spanish terms with students (Interview, 5/2/2019). Although Jeannette made sure to explain terms and be very clear about them, she also explained that she could not assume students in her class were biliterate. As an extra precaution, Jeannette explained that she took extra measures with students to ensure they understood new terms through approaches such as acting things out (e.g. pointing) or having them highlight new mathematical text that was related to new concepts. This was similar to Gabe’s practice in that he also had students highlight unfamiliar words or used gestures with students.

Furthermore, throughout most of her interviews, Jeannette would provide explanations about how she attempted to draw connections between Spanish and English in her instructional practices. For example, Jeannette gave her students combined English and Spanish texts. While Gabe gave students tests in both languages, everything Jeannette gave her students was combined with English and Spanish text, including students’ notes and warm up problems. As shown in Chapter 5, other biliteracy practices that Jeannette engaged in were leveraging students’ biliteracy practices and redesigning tasks, so they were more relevant to students.

Jeannette also explained that she intentionally spoke Spanish during mathematics class to set a norm for students that it was alright to use it (Personal Communication, 4/10/2020). To encourage students to converse in the language in which they were most comfortable, Jeannette

explained she would join students' conversations during groupwork in Spanish or during activities with the whole class where she would respond or pose a question in Spanish to students. These conversations could be about mathematics or not. Furthermore, Jeannette was also intentional about responding to students in Spanish even if they asked her a question in English. Jeannette's conversational Spanish practice differed from Gabe's in that overall, she was more proficient in it and engaged more in Spanish with students. Gabe explained that using Spanish enabled students to see that he was willing to make himself vulnerable, thus further developing his relationship with emerging bilingual students (Interview, 12/12/2019). On the other hand, Jeannette used Spanish to set a classroom norm so that the class could speak it not only in conversation, but also when engaging in mathematical sensemaking.

Questioning. Jeannette also integrated biliteracy practices in her questioning strategies when accounting for the emerging bilingual students in her classroom. Consistent with her assertion about the importance of students acting things out as a language practice (Interview, 4/13/2018), Jeannette had students physically point to written questions in order to make sure they were answering them. Additionally, Jeannette made a point to consistently instruct students to reread word problems to make sure they understood the question. As discussed in Chapter 5, if emerging students were hesitant to answer her scaffolded questions in English, she would codeswitch to let them know they could share their ideas in Spanish. Jeannette's questioning strategies were different from Gabe's in that her overall focus with questioning did not seem to be as much on eliciting student sense-making; rather, it seemed to be to ensure students were interpreting and expressing mathematics correctly.

Groupwork. Jeannette also focused on groupwork in her class but in a different manner than Gabe. Unlike Gabe, who reflected on his grouping strategies in almost every interview,

Jeannette said less about hers. She did utilize groupwork to facilitate whole group and small group check-ins where students talked about unfamiliar terms or to complete a mathematical task. Thus, Jeannette did not assign as many different types of structured groupwork activities as Gabe did in his class. However, Jeannette's class was always seated in groups at different tables, indicating that her expectation was for students to be able to work together.

In this section, I discussed the instructional practices that were most frequent in Jeannette's classroom. In the next section, I discuss Jeannette's positionality and disposition towards equitable instruction.

Jeannette's positionality and disposition towards equitable instruction

My analysis of Jeannette's positionality and disposition towards equitable instruction draws mostly from her baseline interview (4/13/2018). Additionally, much of what Jeannette emphasized in her baseline interview was evident in her noticing interviews, and these informed my analysis as well.

In Jeannette's baseline interview, she reflected on how she identified racially as Latina (Interview, 4/13/2020). Her identity informed her teaching of predominantly Latinx students. Jeannette explained how growing up she would never see anyone like herself in anything outside of her own family (e.g. math problems, movie credits, etc.). For Jeannette, it was very important to emphasize to her students that people like them could achieve. This strong conviction was connected to the ways that Jeannette encouraged student buy-in in mathematics and helped them see themselves in problems (Interview, 3/7/2019). Additionally, she argued that this buy-in did not have to necessarily be culturally relevant. Instead, her goal was to bring things to life for her students through experiences and to empower them to be successful (Interview, 4/13/2018).

Jeannette explained her idea of equity as “delivering the thing that everyone needs” (Interview, 4/13/2018). By this, she meant that equity is a form of success, which is different for every person. In helping students be successful, it was Jeannette’s hope to not only empower students, but to help give them access to resources that granted them an appreciation for the richness and beauty of mathematics. She explained, however, that this did not necessarily happen all the time since she did not think she was equitable all the time. She elaborated that it might not seem to an outsider walking into her classroom that her classroom was equitable, but it was a constant work in progress. This process of creating equity, for Jeannette, occurred primarily through developing relationships with her students. Jeannette explained that her goal was to know at least one thing about each student and for her students to know that she cared about them. These relationships with students helped form the backbone of her classroom culture, which was that students should not walk into her classroom feeling like they were there just to do mathematics.

Jeannette’s positionality and disposition were intimately bound up in her noticing practices. In the section below, I explain the noticing practices that informed her instructional moves.

Jeannette’s Noticing Practices

Throughout Jeannette’s noticing interviews, she attended to several aspects of the classroom that reflected themes in her noticing. Below, I describe the type of noticing that was especially prominent in Jeannette’s teaching practice both overall and for emerging bilingual students.

Below is the table of Jeannette’s five most common noticing codes (Figure 7.4). Like the table in Gabe’s noticing practices section, I discuss the frequency percentages with respect to the

five most common main codes to discuss themes in Jeannette's noticing.

Table 6. 4

Jeannette's Most Frequent Noticing Practices

Teacher attention to:	% of Top 5 Codes
Developing Language	31%
Student Energy	20%
Engagement with mathematics	18%
Positioning	16%
What was not noticed	15%

Opportunities to Develop Language. Throughout her noticing interviews, Jeannette attended to opportunities to develop mathematical language in both English and Spanish. Codes related to this accounted for 31% of Jeannette's main noticing codes. For Jeannette, this attention to opportunities to develop language most often took the form of noticing opportunities such as introducing new vocabulary via visuals, acting things out, etc. (Interview, 4/4/2019). Specific to her emerging bilingual students, Jeannette noticed how they were taking up their biliteracy classes outside of mathematics class, thus she attended to connections between Spanish grammar and syntax, and mathematical notation and concepts. She also attended to the biliteracy experiences of her students with the understanding that not all of her emerging bilingual students and bilingual students had had the same literacy experiences with their second language. Attending to opportunities to develop mathematical language with students' biliteracy experiences was more of a focus for Jeannette whereas for Gabe, his noticing around language was more so towards opportunities for students to explore mathematical vocabulary in groupwork.

Student Energy. Another frequent noticing practice for Jeannette was consistently attending to student energy (20% of main codes). This included Jeannette attending to students' body language as a reflection of student feelings and level of comfort either with the mathematics or in using Spanish to problem solve. I also describe how attention to student energy was related to attention to students' mathematical work (18% of main codes) since her noticing practices around those codes were related. For example, in one of Jeannette's noticing interviews, she noticed that a student was very nervous to share her problem solving in front of the class based on the fact that she was facing the board as she talked and never turned around. Jeannette noticed that this student generally seemed unsure of herself in mathematics class so she did not ask her to turn around, nor did she ask clarifying questions (Interview, 5/2/2019). More broadly, this situation speaks to how Jeannette attended to the feelings of individual students based on her read of their body language. In addition to attending to individual student energy, Jeannette also attended to energy from the whole class with respect to student buy-in. As discussed in Chapter 5, Jeannette noticed when her whole class seemed disinterested or had not bought into their mathematical work, and redesigned tasks and activities to better reflect student interests. In noticing opportunities to redesign tasks, Jeannette's noticing was different from Gabe's. Gabe generally attended to opportunities to relate mathematics to its applicability later on in life, rather than students' interests in the moment.

Jeannette also frequently attended to how students were grappling with the mathematics (18% of main codes) they were learning. Specifically, Jeannette attended to how students were demonstrating their understanding of mathematics and what kinds of connections she could make with them. For example, in one of her noticing interviews Jeannette described how she noticed that every group knew the answer to a mathematical problem, but no one was saying anything

when prompted (Interview, 3/7/2019). Interestingly, Jeannette attended to her own uncertainty as a result of her students' silence. She acknowledged that not being sure about why they were silent led her to randomly choose a person to respond to her questioning which was not her normal instructional practice (Interview, 3/7/2019). Specific to emerging bilingual students, Jeannette attended to the importance of calling on students that needed to share their ideas. For example, in another interview, Jeannette explained that she noticed that a particular student who did not have a lot of mathematical confidence "needed to shine" (Interview, 5/2/2019) and wanted to share his answer. As a result, she intentionally did not call on one of her emerging bilingual students, Miguel, who was also very eager to share.

Positioning. Jeannette also frequently attended to how she positioned students and her own physical positioning in the room (16% of main codes). Like Gabe, Jeannette attended to how students in her classroom participated traditionally and untraditionally. For example, both teachers attended to how students participated traditionally by noticing which students were contributing (e.g. students who were considered "smart" by their peers) and how they contributed during whole and small group work (e.g. telling their group what to do or sharing their ideas in ways that helped classmates build on their ideas). However, a difference in Gabe and Jeannette's noticing practices around positioning was that Jeannette frequently attended to her physical positioning in the room and how that influenced students. For example, in one of her noticing interviews (4/4/2019), Jeannette attended to how she stood in front of the board for a long time during one particular class. As she reflected, Jeannette noticed a tension between taking the time to circulate around the room and making sure she had enough time to teach students the material they needed to know. In another example, Jeannette was noticing when students were shy to present at the board and thus would intentionally walk to the back of the

room. She noticed that when she walked away, those students would have to look across the room at her and as a result, seem to be looking at the class. As discussed in Chapter 5, specific to emerging bilingual students, Jeannette attended to opportunities to position student biliteracy practices and student interests as resources in that she would adapt her lesson and tasks to accommodate them.

What was not noticed. Another approach to noticing in Jeanette's practice that was distinct from Gabe's was Jeannette attention to aspects of the classroom activity that she did not notice while teaching (15% of codes). Jeannette often considered these missed opportunities, whether they had to do with discussions she missed (Interview, 5/10/2018) or not noticing when students needed extra help (or not) with terms in problems (Interview, 4/4/2019). For instance, in one particular interview, Jeannette attended to how the term metalloid was an unfamiliar word to students in a mathematics problem. She noticed that she drew the class into a discussion about what a metalloid was but missed the opportunity to let students know that they did not actually need to know that term to continue solving the problem. In addition to noticing missed opportunities, Jeannette also intentionally did not draw attention to instances such as students walking in late (Interview, 5/3/2018). As discussed in Chapter 5, specific to emerging bilingual students sharing their ideas in English or Spanish, Jeannette would attend to her own reaction to make sure nothing appeared out of the ordinary for students who did not normally share ideas in English (Interview, 10/4/2018).

Jeannette's unique noticing practices were tied to her instructional practices, which shaped the culture of her classroom. I summarize her classroom culture below.

Summary

Jeannette's mathematics teaching tended to be traditional in its pedagogical approach meaning she was more likely to use direct instruction. At the same time, her attention to getting to know her students made her classroom a meaningful experience for her students. By getting to know her students, Jeannette could leverage students' interests and language resources in the classroom. As a result of leveraging students' language practices, Jeannette modeled making connections between students' biliteracy resources and mathematical concepts while encouraging students to do the same. Furthermore, Jeannette developed norms around students engaging in conversations across languages by speaking both English and Spanish with her students during mathematics class. This classroom norm of engaging across languages helped further promote students' academic discussions in both English and Spanish.

In this chapter, I have outlined the similarities and differences between Gabe and Jeanette's instructional practices, noticing practices, positionalities, and ultimately, classroom cultures. In Chapter 7, I discuss the findings from Chapters 4, 5, and 6 and bring them into conversation with each other and the field of mathematics education research.

Chapter 7. Discussion

In this chapter, I summarize the main findings of this dissertation and connect these findings to the literature.

Shifting from Biliteracy towards Translanguaging

Spaces for translanguaging in the mathematics classroom involve co-constructed discursive moves. These co-constructed discursive moves involve teachers and students engaging in translanguaging practices which are constitutive of students' biliteracy practices. Although biliteracy practices can involve the integration of two languages within the classroom, traditionally they have been encouraged by dominant systems (e.g. dual immersion programs) to be kept separate. Translanguaging, on the other hand, challenges dominant language ideologies in that it involves students accessing their full linguistic repertoire across languages. With this consideration in mind, both Gabe and Jeannette's instructional practices shifted over the years in how they drew upon biliteracy practices towards translanguaging practices in their teaching. For example, Gabe noted that previous to the past year of this study (i.e. Phase 2), he gave separate English and Spanish written tests to his emerging bilingual students whereas his current practice was to combine them. For Jeannette, on the other hand, the role of the school context was a tool she used to support students' translanguaging. For example, Jeannette noted shifts in her teaching practice since changes in her school's biliteracy program supported her in currently being able to draw upon making mathematical connections in her classroom with Spanish. For both teachers, these shifts towards bringing both languages together in classroom spaces (Gort & Sembiente, 2015, Palmer & Martinez, 2013; Takeuchi, 2015), supported the likelihood of students engaging in co-constructed spaces for translanguaging.

Bids for Spaces for Translanguaging

Additionally, spaces for translanguage in the mathematics classroom involve co-constructed spaces for mathematical meaning making. These co-constructed spaces for meaning making involve either the teacher or students making a bid to translanguage which is taken up or not by the classroom participants. In Jeannette's classroom, she regularly engaged in these initial bids to translanguage by using Spanish as a translanguage practice. On the other hand, Gabe, planned to engage students in spaces for translanguage through utilizing his own translanguage practices and that of his students through his attention to grouping together bilingual and emerging bilingual students. For both teachers, their translanguage stance in bringing both languages for students to engage in mathematical meaning making (García, et al., 2017), supported the likelihood of students accepting bids to translanguage or not.

García, Ibarra Johnson, and Kleyn (2017) have introduced the concept of translanguage corriente to describe how students enter a particular type of translanguage practice in classrooms. Although spaces for translanguage have similarities to a translanguage corriente as conceptualized by García, et al. (2017), the two concepts are not the entirely the same. According to García et al. (2017), a translanguage corriente involves a flow of students engaging in bilingual practices where "...the translanguage corriente, produced and driven by the positive energy of student' bilingualism, flows throughout all classrooms. Metaphorically, we think about the translanguage corriente as a river current you can't always see or feel, but that is always present, always moving, and responsible for changes in the (classroom) landscape" (García et al., 2017, p.xi). In other words, García et al. (2017) mean that bilingual students naturally engage in bilingual practices in the classroom to some degree regardless of the authorized linguistic practices. In contrast, I have conceptualized a space for translanguage in the mathematics classroom as involving students choosing to take up bids from other classroom

participants. García et al. (2017) do not focus on the students' choices to engage in translanguaging practices during classroom interactions. At the secondary level especially, students are more likely to be accustomed to engaging in English dominant norms of schooling and may or may not accept the bid to translanguage. Students choosing to accept these translanguaging bids (or not) reflect their positioning of translanguaging as a competent practice in the mathematics classroom (Gresalfi et al., 2008; Jilk, 2016; Turner et al., 2013). Positioning translanguaging as a competent practice (or not) has ramifications for how students participate in mathematics. As a result of accepting (or not) each other's bids to enter a space for translanguaging, classroom participants' mathematical meaning making can be enhanced (García et al., 2017).

Supporting (and Narrowing) Spaces for Translanguaging

Furthermore, the results of this research demonstrate how spaces for translanguaging emerge through and can be narrowed by classroom discursive moves. Spaces for translanguaging emerged when Gabe and Jeannette supported students' opportunities for mathematical sensemaking across students' language repertoires. For both teachers, these broadened discursive opportunities involved formalized language performances (e.g. presentations) that integrated students' academic language and content to embody multiple ways for students to use language (Gort & Sembiente, 2015). For example, in the context of both teachers' classrooms, spaces were opened when emerging bilingual students were encouraged to contribute no matter which language they used in front of the whole class or during groupwork. In contrast, spaces for translanguaging could narrow if teachers did not provide integrated language resources or did not invite students to engage in mathematical sensemaking. Expanding upon students' opportunities

to contribute in both languages has implications for a deeper mathematical and conceptual understanding (de Araujo et al., 2018; LópezLeiva et al., 2013; Torres & Khisty, 2013).

Spaces for Translanguaging Serve Different Purposes

Spaces for translanguaging served different purposes based on the teachers' goals, positionalities, and translanguaging stances (García et al., 2017). As a reform-oriented mathematics teacher, Gabe was committed to reform mathematics teaching practices. Mathematical reform practices "...are characterized by an emphasis on problem solving, critical thinking through discovery learning, and communication and discussion in group situations" (Hansen-Thomas, 2009; NCTM, 2000). Specific to Gabe's mathematics classroom, reform mathematics teaching practices involved him emphasizing student strategies and building upon them (Drake & Sherin, 2006) in order to help students develop mathematical language. In his classroom, spaces for translanguaging served to leverage students' strengths in positioning them to take ownership of their learning. For instance, Gabe attended to opportunities to leverage language as a resource and consistently grouped emerging bilingual students with bilingual students to allow for them to potentially expand upon their mathematical discussions. On the other hand, Jeannette was concerned with making sure students experienced buy-in when solving mathematical problems. Her positionality, as a Latina, supported her attending closely to students' experiences, linguistic practices, and culture. Translanguaging in her classroom served to help students make connections between their funds of knowledge and mathematics. Similarly, Gutierrez (2002) argued that high school mathematics teachers who got to know their students and drew upon their cultural diversity and funds of knowledge in Calculus courses, provided them with diverse opportunities for mathematical discussions across languages.

Balancing Language and Mathematics Goals

Both teachers attended to their own tension regarding how to balance their focus on language and mathematical goals. The results of this dissertation indicate that this balance stems from the co-construction of discursive spaces where not only the teacher considers this balance but also acts upon it in order to invite students to contribute. For example, Gabe reflected that how he introduced and positioned new mathematical vocabulary had implications for how students contributed to the discussion around new mathematical language. For Gabe, positioning students' previous knowledge as a viable contribution towards new concepts (i.e. versus positioning himself as the teacher expert teaching new concepts), had different outcomes for students engaging in a co-constructed space. Additionally, Jeannette considered how much teacher focus on the positioning of mathematical language could subtract from students doing the mathematics. Too much focus on mathematical language could diminish students' belief in their own abilities to do mathematics. This finding is consistent with research studies showing how teachers balance mathematical content and invite students to contribute towards mathematical sensemaking has implications for students' identities as doers of mathematics (Abrue & Cline, 2003; Cobb, Gresalfi, & Hodge, 2009).

Noticing Practices to Support Translanguaging

My research illuminated aspects of teacher noticing that supported spaces for translanguaging that have received less attention in the field. These include teachers' attention to classroom positioning, to student energy, and to student participation over time. For both teachers, their noticing allowed them to engage in translanguaging shifts or moment to moment decisions that teachers make to change the lesson to allow for translanguaging to occur (García et al., 2017).

Although they had different approaches to developing mathematical language, overall both teachers attended to opportunities to position students' language uses as a strength and launchpoint for learning mathematics. Jilk (2016) and Louie (2017) similarly argued that attention to students' linguistic and communicative strengths was important as it helped position students to notice their own mathematical strengths, thus changing their patterns of participation to better learn mathematics.

Additionally, students feeling invited to participate is also reflective of how teachers attend both to positioning and student energy during classroom interactions (van Es, Hand, & Mercado, 2017). Both Gabe and Jeannette attended to student energy as reflected in students' body language; this attention to body language is also important to consider in how students are physically participating in spaces for translanguaging. This complements van Es, Hand, & Mercado's, (2017) finding that attending to student energy can be a form of understanding how the teacher constructs opportunities for students to take up space.

Important to note is that while the teacher might be attending to student body language and energy, this attention is not focused on classroom management, such as the SLANT strategy, which is a strategy primarily used in urban classroom settings where teachers gain student attention and compliance as a form of control (Kretchmar & Zeichner, 2016). Instead, I argue that the teachers here are engaged in a translanguaging stance (García et al., 2017), or one that assumes that a "classroom space must promote collaboration across content language, people and home, school and community" (p. xii). Attention to students' feelings is a focus of the stance, that takes into account the risks students take when engaging in new academic content. The way teachers attend to and position students' risk taking has implications for boosting students' self-esteem (or not), thus influencing whether students continue to be willing to engage in new ways

of engaging in mathematical sensemaking (Jilk, 2016).

The results of this dissertation indicate the importance of understanding spaces for translanguaging as they are supported by a cycle of teacher attention and instructional practices, not only in the moment, but over time. In addition to noticing and acting upon opportunities to leverage language in the moment, both teachers attended to how their students were participating in different ways over time (i.e. student roles, who was being positioned as the expert in a group, etc.). As a result of this noticing, both teachers created opportunities to empower more students to participate in classroom structures (i.e. groupwork, whole class discussion, etc.). This finding is consistent with research studies pointing to the importance of teachers attending to students' trajectories of participation in a classroom (Moschkovich, 2007; 2015), since these can potentially shape students' perceptions of theirs and classmates' mathematical abilities based on different cultural and linguistic backgrounds.

Chapter 8. Conclusion

In this dissertation case study, I explored the mathematics teacher noticing practices that supported teachers and students in co-constructing spaces for translanguaging in the context of two high school mathematics classrooms. In reviewing these spaces, I found that both teachers in this study shifted in their biliteracy teaching towards a culture of translanguaging, and for different purposes: one to support the mathematical inquiry of emerging bilingual students, and the other to support emerging bilingual students in feeling like they belong in a mathematics classroom. Noticing was shown to be intimately bound up in teachers' dispositions and goals, which meant that spaces for translanguaging looked different across each teacher's classroom. At the same time, both teachers attended to how students were positioned discursively (and positioned themselves), facets of students' participation, including energy and body language, and the challenge of balancing language and mathematics goals. Below, I discuss the limitations of this study as well as directions for future research.

Limitations

In this section, I outline the following limitations from this study. These limitations included time, permission, and member check constraints.

While I spent considerable time with these teachers while they taught, I was not present in the other classes they taught during the day. As such, I do not have data that supports the degree to which the teachers and students constructed spaces for translanguaging in their other classes. The findings from this dissertation are limited to the class periods I was in each teacher's class.

Additionally, due to the lack of student permission, I was not able to collect as much formal data from Jeannette's classroom as I did in Gabe's classroom during Phase 2. It is

possible that I would have had additional insights to add to my interpretations of the support for spaces for translanguaging in her classroom, especially since this past year was The High School's first year as an official dual immersion program. Furthermore, my access to Jeannette's existing video data was limited since the data was kept on the Co-ATTEND server and the Colorado stay at home order was in place.

Finally, this dissertation consists of my inferences of how the teachers supported spaces for translanguaging. Although I checked in with both Gabe and Jeannette throughout the school year, once I identified these spaces, I did not do member checks with them due to time constraints. I have illustrated that these spaces are co-constructed, thus, it would have been interesting to understand what the teachers would have added about the noticing and instructional practices I identified.

Directions for Future Research

This study has several implications for theory and practice around teacher noticing for spaces for translanguaging. Additionally, the findings around teacher noticing have implications for teacher education programs and classroom language policies. I discuss these implications in further detail below.

Current work on teacher noticing for equity focuses on teachers' attention to moment to moment classroom interactions (Erickson, 2011, Hand, 2012; Louie, 2017; Turner, Dominguez, Maldonado & Empson, 2013; Wager, 2014). This study has implications for the field of teacher noticing and translanguaging regarding how teachers learn to notice over time in ways that support spaces for translanguaging. Recall that noticing practices over time refer to practices that teachers engage in throughout the school year that support moment to moment classroom interactions. For example, by administering student perception surveys over the two years he had

this cohort of students, Gabe became more attuned to different student perceptions of their classroom experience. Jeannette, on the other hand, attended to students' biliteracy practices over time. Other sustained noticing practices I identified in this study included teacher attention to students' classroom perceptions of mathematical ability and grouping. Both teachers' sustained noticing practices resulted in them developing classroom norms that facilitated the greater likelihood that spaces for translanguaging would occur in their classrooms. Future research might focus more on how sustained teacher noticing supports the likelihood of spaces for translanguaging occurring.

Current teacher noticing studies point to the importance of examining how teachers notice based on their dispositions and stances, which include their goals (van Es, Hand, & Mercado, 2017), and ideologies (Louie, 2017). Henderson & Palmer (2015) explain that teachers' language ideologies influence how they interpret and enact language separation school policies. This study further reinforced that teacher dispositions are linked to their noticing practices. For example, this study found that the teachers attended to the vulnerability involved in linguistic risk taking, mathematical sensemaking, and student learning more broadly. This noticing led Gabe to model risk taking by making himself vulnerable in his attempts to speak Spanish. Another example of attending to student vulnerability was Jeannette intentionally leaving the class out of an interaction with an emerging bilingual student so he could feel more comfortable giving his mathematical answer in English for the first time. Thus, this study points to the need to attend to vulnerability as a classroom norm to support students and their sensemaking.

Additionally, with a focus on teacher and student vulnerability, this study has the potential to push on the more pervasive and troubling literature around student body language as

a tool for classroom management (Kretchmar & Zeichner, 2016). This study shows that teachers' attention to students' body language led to the likelihood of teachers creating more culturally responsive mathematical tasks and activities. For example, in attending to students' body language as a signal for student engagement, Jeannette became attuned to the Whiteness prevalent within mathematical tasks. As a result, she created tasks and activities that supported a classroom culture where students were encouraged to draw upon their linguistic funds of knowledge as they engaged in mathematical sensemaking, thus supporting the likelihood that spaces for translanguageing would occur.

Furthermore, combining the literatures around translanguageing and teacher noticing has implications for teacher education. More specifically, this research has implications for pre-service mathematics teachers learning to notice towards translanguageing and creating spaces for translanguageing in the classroom. Current research around teacher noticing specific to emerging bilingual students points to the importance of preservice teachers utilizing teacher noticing to redesign tasks and pair meaningful mathematical connections across students' communities (Fernandes, 2012; Aguirre et al., 2012). My study suggests the need to investigate teacher noticing of opportunities to facilitate mathematical discussions that include emerging bilingual students and grouping emerging bilingual students in ways that leverage their linguistic practices. For example, in attending to emerging students' patterns of interactions with classroom participants, Gabe facilitated groupwork and classroom discussions to support his emerging bilingual students. Jeannette, on the other hand, attended to student energy to inform how she asked scaffolded mathematical inquiry among her emerging bilingual students. These linguistic supports that Gabe and Jeannette demonstrated are not intuitive, especially for pre-service teachers at the beginning of their teaching careers. Learning about noticing practices that serve as

linguistic supports could support pre-service teachers in capturing student engagement in spaces for translanguaging. These linguistic supports could include gesture, opportunities to ask scaffolded questions, and opportunities to draw upon student biliteracy. For example, Gabe attended to the gestures of a particular emerging bilingual student and incorporated them into a classroom discussion, which served to validate her mathematical contribution while introducing a new way to represent the mathematical concept. In learning to attend to the diverse ways emerging bilingual students contribute mathematically, pre-service teachers can be sure to include their ideas in ways that affirm students' competence and identity (Gresalfi et al., 2008; Jilk, 2016; Turner et al., 2013) as doers of mathematics.

Directions for future research point to learning more about how teachers and students co-construct their use of English and other languages specific to their classroom, regardless of the school and district policies around language. This study affirms that when mathematics teachers attend to opportunities to co-construct spaces for translanguaging with emerging bilingual students, emerging bilingual students have more opportunities to engage in mathematical sensemaking, regardless of classroom language policies. Although Jeannette's school was a dual immersion program and her math class was supposed to be in English, Jeannette encouraged her students to draw upon their Spanish biliteracy resources from non-mathematics classes to make connections to mathematical ideas. If more studies are designed to investigate how teachers leverage students' bilingualism in the mathematics classroom, students' strengths can be supported in ways that support their achievement outside of monolingual norms of schooling.

This study also has implications for future research around policies at the school and classroom level. García et al. (2017) assert that many bilingual classrooms and school programs conceptualize languages as separate and enact language separation policies, which limits

students' English and Spanish language development. For example, students are never labeled as Spanish *and* English dominant, they are either one or the other (García et al., 2017). I argue that teachers and students can be considered language policy makers within their own classrooms (Henderson & Palmer, 2015; Palmer & Martinez, 2013). In further training teachers and pre-service teachers that classroom participants can be language policy makers, more students can be supported in mathematics across languages.

Conclusion

Mathematics teachers navigate complex classroom interactions. How they navigate this complexity is tied to their sociocultural backgrounds and classroom goals, which inform their dispositions towards noticing. Noticing opportunities to support spaces for translanguaging is a particular kind of challenge, in that it requires teacher attention to not only language hybridity, but also to whether or not the language practices offer a way for emerging bilingual students to be seen as valuable, and in this way break down dominant hierarchies around language and culture in and beyond the mathematics classroom. Although there are many studies foregrounding teacher noticing and translanguaging as separate bodies of research, there has been a gap surrounding teachers noticing for equity in the secondary mathematics classroom and implementing language practices.

In this dissertation, I have presented examples of how teacher noticing can lend itself towards supporting co-constructed spaces for translanguaging in classroom interactions. Mathematics teachers are not trained to really “see” students, apart from the mathematics they do. This means that the brilliance of students, particularly those from less-dominant linguistic and cultural backgrounds, may be invisible, hidden behind structures that prevent all of students' linguistic and personal resources from being viewed as gifts to their learning. In truly seeing

students and what they bring to their learning, we have the potential to re-humanize groups of young people that have been left out of the mathematics community.

Additionally, it would also have also been interesting to focus more on the discursive moves of the students (Gravemeijer & van Eerde, 2009) and how they positioned and supported each other (Turner et al., 2013) in these spaces.

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Appendix A

Baseline Interview Co-ATTEND Research Project (2017-2018)

Send the following prompt one week prior to the interview:

Next week, we have an interview with you to get a sense of your goals, expectations and commitments with respect to your teaching. We'd also like to hear your thoughts about the ways that mathematics teaching, and mathematics education, more broadly, relate to issues of equity/justice/fairness. We wanted to give you an opportunity to spend a little time thinking about this, and we have a pre-interview activity that we would like you to do. It is no problem if you don't have the time to do this before the interview. You are welcome to do it then, too.

The activity involves describing your ideas about equity through words and/or drawings. For example, you could make a concept map. To make a concept map, write equity in the center of a blank sheet of paper, and draw lines to other ideas, thoughts or emotions that you feel connected to it. You are also welcome to do something more freeform, and just draw out your ideas, either on paper or on a computer. Whatever works! We will ask you to talk to us about whatever you made at the interview.

Day of the baseline interview:

Introduction:

Thank you for taking time to meet today. You were highly recommended by various people in the district! We'd like to learn more about you, your teaching, and your goals for students.

1) How is your year going?

- a. What are your goals for your students?
- b. What do you want them to be able to do mathematically?
- c. What kind of people do you want them to become/ support them in becoming?
- d. How do you organize learning to support these goals?

- 2) What would your ideal classroom look like? (How would you structure learning?)
 - e. What is your classroom like now?
 - f. Are there constraints you face that make it challenging to reach your ideal? (Note: Steer them back to issues of equity and learning.)
 - g. How do you navigate these constraints?

- 3) Can you describe the drawing you made to us?
 - h. Do you have any stories about your experiences, or your students, that relate to your drawing?
 - i. Tell us more about how you think about these ideas in relation to your teaching?
 - j. Have your views on equity changed at all over time? Has your teaching changed?

- 4) We'd like to do another activity with you. It's called, my sociocultural self. We can do it together.
 - k. List in each circle the identities you carry with you/you ascribe to in the school setting and out of school. Once we take a few minutes to write our identities, we will talk about them.
 - l. How do your identities shape your teaching?

Questions specific to language:

- 5) How do you think about language in the mathematics classroom?
 - m. How do you think about mathematics as a language?
 - n. Why is language important in a math classroom?

6. Are there ways that you intentionally organize your pedagogy for mathematical literacy to emerge among your bilingual students?

Appendix B

Fieldnote Protocol

Written Field Note Template:

Date/ Time	
School/ Grade/ Course	
Participants	
Observer Name	
References (textbook info, urls, image source, etc.)	
Notes/ Comments	

Abstract:

Class Layout: (any changes)

Fieldnotes:

Time	Observation Notes	Researcher Notes

Reflections (what stood out to you the most):

Summary/Memo:**Questions:**

In addition to following the Co-ATTEND field note protocol (paying attention to equitable practices, groupings, positioning, etc.), I will pay attention to the following:

- Are different languages spoken in the classroom? At what point do different languages emerge in the classroom?
- What are the official rules surrounding language practices for the district and school?
- How often is Spanish used in class? Is Spanish sanctioned before, during and or after class?
- How are mathematical mistakes positioned in the classroom?
- What is the physical set up of the classroom? How are students grouped?
- How are student positioned in the classroom? Is mathematical expertise leveraged? How is math used as a relevant part of everyday life?
- How are languages used to help leverage students' experiences? How are students being humanized in the math classroom?

Appendix C

Noticing Interview Protocol

Introduction

Thank you for participating in this interview today. After observing your lesson, we identified some interesting moments in your teaching. What we'll do today is view those clips and then talk about what you were thinking during these moments. If there were other moments that stood out to you, we can also go back and look at those to learn from you why you found them noteworthy. Our discussion is going to be video and audiotaped and I may take notes as well. Do you have any questions before we get started?

Show Clip #X

1. What did you notice as you watched this clip?
2. What kinds of things were you paying attention to during this interaction? What was standing out to you? What about the students' mathematical work was standing to you?
3. Can you describe some of the choices you made in your interactions with [x student] and why you made them?
 - Why did you choose to pursue a student's question?
 - Why did you respond to a student's idea that way?
 - How did you decide which students to invite to participate?
 - How is that student's experience in your class/ at home/ etc... relevant to this interaction?
4. How was this instance related to translanguaging?
5. What was your role in supporting students' multiple linguistic resources? How do these practices support developing student mathematical sensemaking?

Note: The interview will follow this format for each video clip. After discussing these segments, the interviewer will ask:

Is there anything else you want to share about what was standing out to you during that interaction? Anything that stood out to you in this clip?

Appendix D

Student Interview Protocol

This is a 15-20 minute interview with students, 1 pair at a time:

1. How did you each solve this task? (5 minutes)
 - a. How was knowing Spanish helpful or not to your problem solving?
(maybe) On a task by task basis: (Look at task and see if cognates or grammatical structures that you could discuss.)
 - b. (maybe) How did you each use key words in this problem?
2. What are the different ways you each use language in the classroom? (10 minutes)
 - a. How do you see yourself as a math learner?
 - b. How do you think your teacher sees you as a math learner?
 - c. What kinds of things in this class encourage you to participate? What kinds of things in this class make you not want to participate?"
 - d. What are the different ways you each use Spanish in the classroom (per first question)

Extra Questions if time.

- What language(s) do you feel comfortable speaking in your math class? Why/why not? Tell me which languages you think your teacher prefers you to speak (does he have a preference)? Do you feel comfortable speaking Spanish? How about English?
- Does everyone speak Spanish in this math class? Do you think there is a language that is better to use in this class? Why/why not? What do your English-speaking classmates think of you using Spanish? What do they think of you as a math student? (Do you think there's any differences between the way Spanish speakers and English speakers are labeled or are appreciated in this class - by other kids? by the teacher?)

Appendix E

Mathematics Identity Survey

This survey was developed to examine the degree to which you identify with mathematics, meaning. For the purposes of this survey, mathematical identity is your vision regarding your ability to participate in mathematics successfully (Boaler & Greeno, 2000).

1. What is your grade level?
A. 9th C. 11th
B. 10th D. 12th

2. What is your gender:
A. male C. female
B. other D. prefer not to say

3. How many languages do you speak?
A. 1 B. 2
C. 3 D. 4 or more

4. What language(s) do you speak?

5. What language(s) do you speak at school?

6. What language(s) do you speak in your math class?

7. Which race do you identify with?
A. Hispanic / Latino B. White / Caucasian
C. African- American D. BlackAsian-American
E. Pacific Islander F. Native American
G. Other

	Please mark the degree to which you agree with the following statements.	Never	Sometimes	Almost Always	Always
1	I feel comfortable sharing my ideas in math class.				
2	I consider myself smart in math.				
3	If I don't understand something, I would rather keep it to myself.				
4	I have different kinds of opportunities to participate in math class.				
5	I am confident sharing my ideas in math class via groupwork.				
6	If I cannot solve a math problem quickly, then spending a lot of time on it probably will not help me.				
7	I express my mathematical ideas with visuals.				
8	I voluntarily share my ideas in front of the whole math class.				
9	I feel comfortable exploring new ideas on my own in math class.				
10	I am confident in my ability to learn math.				
11	I can only learn math when someone shows me how to do the problem.				

12	I try other math strategies besides the one my teacher teaches me.				
13	I let someone know if I do not understand a problem in math class.				
14	I do not share my ideas in front of the whole math class				
15	I am confident doing math even when my peers are doing math in other languages.				
16	I will only practice a math problem the way my teacher taught me.				
17	I express my mathematical ideas using gestures.				
18	It does not affect my future if I am not good at math.				
19	If I do not get a good grade in math for the school year, I will probably not get a good grade in later math classes.				
20	I am confident working individually in math class.				
21	If I am good at math, I will be successful.				
22	I do not let anyone know if I do not understand a problem in math class.				

