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# Observing Intellectual Need and its Relationship with Undergraduate Students' Learning of Calculus

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**Abstract**
The concept of intellectual need, which proposes that learning is the result of students wrestling with a problem that is unsolvable by their current knowledge, has been used in instructional design for many years. However, prior research has not described a way to empirically determine whether, and to what extent, students experience intellectual need. In this paper, we present a methodology for identifying students' intellectual need and report the results of a study that investigated students' reactions to intellectual need-provoking tasks in first-semester calculus classes. We also explore the relationship between intellectual need, affective need, and students' learning. Although the overall percentage of students who reported experiencing an intellectual need was low, we found positive associations between intellectual need and learning.

**Keywords** Intellectual need · Calculus · Instructional videos · Flipped pedagogy

Problem solving has long been viewed as both an essential source and product of mathematical learning. However, as Schoenfeld (e.g., 1992) observed, instructors tend to engage students in routine exercises more than complex problems. Strategies to increase students' engagement and participation in the learning process are infrequently used in U.S. university mathematics and science classes (Walczyk & Ramsey, 2003; Ellis et al., 2014; Sonnet et al., 2014). Students' opportunities to rea-

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son about novel problems is further hindered by the predominantly procedural nature of mathematics textbook exercises (Lithner, 2004; Fuller et al., 2011) characterized much of students' activity as "problem-free" and noted that, even when the task may superficially appear to be a complex problem, students often "do not have a clear mental image of the problem that is being solved, or indeed an understanding that any intellectual problem is being solved" (p. 83). In contrast, "problem-laden" activity is grounded in and sustained by students' experience of *intellectual need* (Fuller et al., 2011).

The theory of intellectual need has been widely applied to analyze classroom instruction (e.g., Rabin et al., 2013; Zazkis & Kontorovich, 2016); guide professional development (e.g., Meyer, 2015); design effective "openings" and "hooks" for lessons (e.g., Abrahamson et al., 2011; Leatham et al., 2015); and create instructional tasks for use in both classroom (e.g., Koichu 2012; Caglayan, 2015; Foster & de Villers, 2015) and clinical contexts (e.g., Harel 2013b). Although these efforts share a common goal of necessitating particular mathematical skills and understandings, researchers' discernment of intellectual need has relied on their interpretations of indirect evidence, rather than empirically identifying students' experiences of intellectual need and relating these experiences to their mathematical activity and learning.

The goal of this paper is to introduce a methodology for explicitly identifying students' experiences of intellectual need through their engagement with a series of instructional videos and mathematical tasks, and to statistically examine factors that might be associated with these experiences. In addition, we employ quantitative methods to investigate the implications of students' experiences of intellectual need for their learning.

## Literature Review

Students' engagement with problematic situations has long been considered essential to support their mathematical learning, even as far back as Dewey's (1938) theory of inquiry. In Brousseau's Theory of Didactical Situations, mathematical problems are an essential aspect of didactical situations, which "constitute means to challenge the pupils' initial conceptions and to initiate their evolution" (Brousseau, 1997, p. 260). Chevallard's Anthropological Theory of the Didactic similarly postulates that any praxeology (i.e., human activity (*praxis*) that necessarily entails an implicit or explicit rationalization (*logos*)) is motivated by one's desire to resolve a problem or difficulty experienced within an institutional context (Bosch & Gascón, 2014). In accordance with this theoretical tradition of impasse-driven catalysts for cognitive development, Harel & Tall (1991) first introduced the concept of intellectual need, proposing that "if students do not see the rationale for an idea … the idea would seem to them as being evoked arbitrarily; it does not become a concept of the students" (p. 41). This notion was later formalized by Harel (1998) as the *necessity principle*: "For students to learn what we intend to teach them, they must have a need for it, where 'need' refers to intellectual need" (p. 501). From this perspective, a student experiences intellectual need when they enter a state of disequilibrium and feel a sense of curiosity that compels them to develop tools and techniques to resolve their cognitive perturbation. Although the concept of intellectual need describes a cognitive state, it is most often—and perhaps most effectively—provoked in social contexts.[1]

The concept of intellectual need has been widely applied to design and analyze classroom instruction. Much of this research has centered on features of mathematical tasks themselves rather than students' interpretations of and experiences engaging with them. Some researchers (e.g., Harel 2013a, 2017; Burger & Markin, 2016) have contrasted the features of tasks intended to provoke intellectual need from those unlikely to engender it. Others have described the various ways instructors or curriculum developers have designed tasks to incorporate intellectual need (e.g., Zazkis & Kontorovich 2016). Some studies (e.g., Foster & de Villers 2016; Fuller et al., 2011; Rabin et al., 2013) have described categories of activity or classroom episodes in which students appeared to *not* experience intellectual need, and highlighted the students' resistance to standard interpretations of mathematical concepts. Still others (e.g., Harel 2013b) have described the perturbations and resolutions associated with the historical development of various mathematical concepts, with the expectation that such developments might elucidate aspects of their psychogenesis within individuals. Evaluating the relationship between intellectual need and the obstacles to concept formation requires researchers to develop valid and reliable methods for empirically identifying students' experiences of intellectual need.

There have been relatively few instances where researchers have attempted to empirically examine students' experiences of intellectual need. In some cases (e.g., Harel 2010; Koichu, 2012), the researchers analyzed whole-class discussions and activities and inferred that students had experienced intellectual need by their collective reaction to what the researcher interpreted as an obstacle. For example, Harel (2010) identified moments when students' initial solution strategies failed, witnessed their subsequent adoption of a new strategy, and inferred that the students had experienced intellectual need. This method results in attributing a cognitive state of intellectual need to a group of people rather than to individuals. Leatham et al., (2015) suggested leveraging in-the-moment student thinking to the generation of intellectual need by using student's puzzlement and curiosity as the basis for further exploration. However, Leatham et al.'s (2015) discussion focused on the researchers' perspective of what likely would have generated intellectual need, rather than a systematic examination of individual students' interpretations, appraisals, and experiences. In other cases (e.g., Caglayan 2015), researchers interpreted students' enactment of a solution strategy as indicating a prior experience of intellectual need. These types of methods involve drawing conclusions about a student's psychological state (i.e., an experience of *need* or cognitive perturbation) based on indirect evidence (i.e., changes in mathematical activity or behavior). Although Harel's necessity principle (2008a) offers a plausible explanation of the students' psychological states during these episodes, the identification of students' experiences of intellectual need have typically been left to the researcher's interpretation of indirect and, sometimes, collective, rather than individual, evidence.

[1] The theoretical premises that contextualize the concept of intellectual need within Harel's (2008a) DNR instructional framework emphasize the social mediation of individual conceptual development.

## Theoretical Framework

Intellectual need is grounded in Piaget's (1985) notion of equilibration and is situated within Harel's framework, *DNR-based instruction in mathematics* (Harel, 2008a). Harel's framework "stipulates conditions for achieving critical goals such as provoking students' intellectual need to learn mathematics, helping them acquire mathematical ideas and practices, and assuring that they internalize, organize, and retain the mathematics they learn" (Harel et al., 2017, p. 267). The DNR framework consists of three categories of constructs: *premises*, *concepts*, and *claims*. DNR premises are explicit assumptions upon which the DNR concepts and claims are based. DNR concepts consist of constructs—consequent to the DNR premises—concerning the cognitive phenomena of teaching and learning mathematics. DNR claims are instructional principles that derive from DNR premises and concepts. Although the concept of intellectual need is situated within the instructional principles domain of the DNR framework, understanding this construct requires a review of the DNR premises that informed Harel's conceptualization of it.

Of the eight premises of the DNR framework, the *Knowing*, *Knowing-Knowledge Linkage*, and *Epistemophilia* premises are particularly relevant to intellectual need. The Knowing Premise asserts, "Knowing is a developmental process that proceeds through a continual tension between assimilation and accommodation, directed toward a (temporary) equilibrium" (Harel, 2008b, p. 894). The Knowing-Knowledge Linkage Premise states, "Any piece of knowledge humans know is an outcome of their resolution of a problematic situation" (Harel, 2008b, p. 894). The Epistemophilia Premise asserts,

> Humans—all humans—possess the capacity to develop a desire to be puzzled and to learn to carry out mental acts to solve the puzzles they create. Individual differences in this capacity, though present, do not reflect innate capacities that cannot be modified through adequate experience (Harel, 2008b, p. 894).

Together, the DNR premises are assertions that derive from a variety of theoretical orientations including Aristotelian epistemology (e.g., Patterson 2002), situated cognition (Wenger, 1998), Vygotskian social constructivism (Vygotsky, 1978), Piaget's genetic epistemology (Piaget, 1971), and radical constructivism (von Glasersfeld, 1995).

Harel (2013b) described intellectual need as the perceived need to resolve "a perturbational state resulting from an individual's encounter with a situation that is incompatible with, or presents a problem that is unsolvable by, his or her current knowledge" (p. 122). This perturbation is rooted in the individual's experience within the discipline—in this case, mathematics—and is based on the learner's epistemological justification for the mathematical concept. Here, an epistemological justification is "the learner's discernment of how and why a particular piece of knowledge came to be" (Harel, 2013a, p. 8).

Intellectual need is distinct from what Harel (2008a; 2010b) described as *psychological need* and, later (e.g., Harel 2013a; 2013b) *affective need*, as well as related affective factors. Affective need is the motivation a student experiences to initially engage in the process of solving a problem (Harel, 2013b) and could involve a student's personal interest in a given topic, their perceived obligation to participate in school and respond in particular ways to teachers, to increase social or economic status, or to advance societal goals (Harel, 2013b).

Along with affective need, Morton (2010) noted that the emotions that students experience as they engage in problem-solving activity influence their cognitive engagement in the task. Tallman & Uscanga (2020) similarly argued that the conscious experience of an emotion affords or constrains subsequent cognitive activity. Pekrun (2000; 2006) proposed that individuals' experiences of emotions in achievement settings are influenced by their perception of their control over the activity and their perception of the importance of the tasks and potential outcomes. Several researchers (e.g., Pekrun & Linnenbrink-Garcia, 2012; Pekrun & Stephens 2012) have highlighted the significant epistemological role of one category of emotions—*epistemic emotions*, which are "emotions that arise when the object of their focus is on knowledge and knowing" (Muis et al., 2015, p. 173). Epistemic emotions include surprise, curiosity, and confusion (e.g., Fayn et al., 2019; Vogl et al., 2019). Nerantzaki (2021) noted that epistemic emotions are usually experienced as a result of "knowledge states involving discrepancy, incongruity, or conflict between cognitive schemas or incoming information" (p. 2). Thus, these emotions are likely to be closely related to students' experiences of disequilibrium and, consequently, intellectual need.). While surprise and curiosity tend to support students' participation and learning (e.g., Arguel et al., 2019), confusion can either support or hinder student engagement (e.g., D'Mello et al., 2014; Lodge et al., 2018).

Crucially, the primary affordance of affective need and epistemic emotions is that they respectively *stimulate* and *sustain* students' mathematical activity, whereas intellectual need has the potential to enhance the *nature* and *quality* of that activity, including the meanings students are positioned to construct as a result of acting to satisfy their intellectual need. To a greater degree than affective need, intellectual need has *epistemological significance*; engendering it places targeted mathematical meanings within a student's zone of potential construction.

## Research Questions

The goal of our study is to statistically explore various factors that might be related to students' experience of intellectual need (as operationalized by students' self-reported curiosity and wonderment). We also seek to identify relationships between students' self-reported intellectual need and learning from associated instructional video sets. Thus, our research questions are split into two groups that respectively reflect these two objectives:

Factors that relate to experiences of intellectual need:

1. What is the extent of the variation of students' intellectual need between sets of instructional videos? Are some categories of video sets associated with higher rates of students' intellectual need?

2. Are different instructors (and, implicitly, the ways they incorporate the tasks into their instruction) associated with different rates of their students' intellectual need?
3. Is there a relationship between affective need and intellectual need?
4. Does a student's background knowledge on a particular topic predict whether they experience intellectual need?
5. Does trying a task designed to provoke intellectual need and/or watching a video of students discussing the intellectual need-provoking task predict whether they experience intellectual need?

Relationship between intellectual need and learning:

1. Is there a relationship between students' intellectual need and their learning from a set of instructional videos?
2. Is there a relationship between students' affective need and their learning from a set of instructional videos?
3. Is there a relationship between experiencing an intellectual need-provoking task, viewing a video of students discussing the intellectual need-provoking task, and learning from the instructional videos?

## Methodology

Addressing our research questions required developing methods to accomplish three key objectives: (1) provoking students' intellectual need on an individual, rather than group, level; (2) identifying individual students' experiences of intellectual and affective need; and (3) measuring students' learning in connection with the *intellectual need-provoking* (IN-P) tasks. We leveraged the use of an online learning to implement these methods, since it would be difficult to accomplish (1) and (2) in the context of classroom instruction and provided a straightforward way to implement (3).

### Provoking Intellectual Need: Task Construction

To discern students' experiences of intellectual need, we needed to engage participants in tasks with the potential to provoke that need. For each of the 30 mathematical topics we investigated (a full list is included in the Appendix)—in our case, in first-semester college calculus—we conducted a conceptual analysis (Thompson, 2008) of the targeted concepts and skills, the curriculum in which the concepts were embedded, and the research literature about the likely background knowledge of the students who would be enrolled in the class. The first, third, and fourth authors were then part of a project (Weinberg et al., 2022) to collaboratively design IN-P tasks for which the targeted mathematical concept or skill was required. We intended these tasks to provide students with an opportunity to experience a perturbational state and endeavored to situate the problem within a context that might engage students' interests.

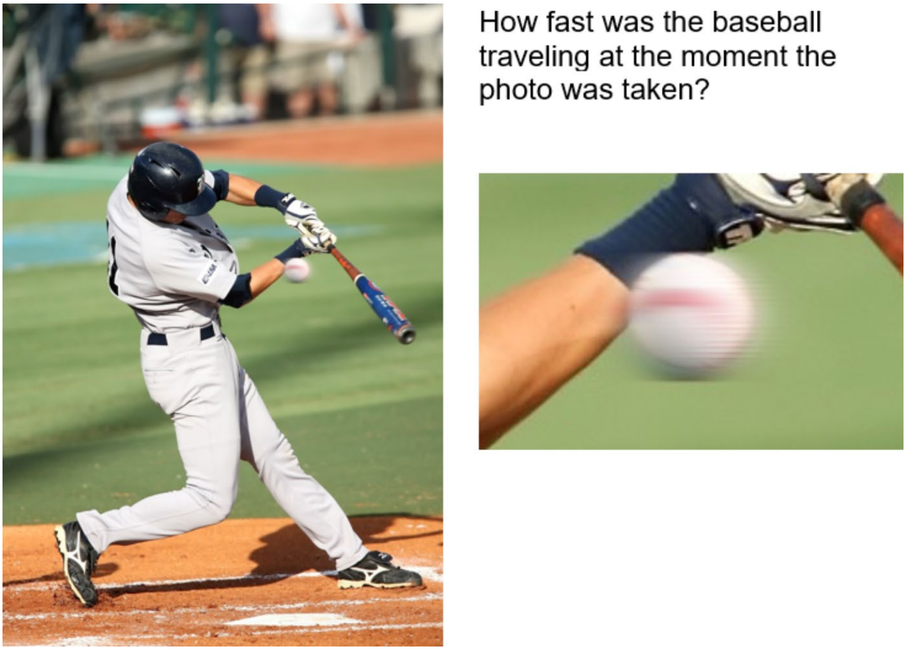



**Fig. 1** IN-P Task for Approximating Instantaneous Rate of Change

As an example, the IN-P task for the concept of instantaneous rate of change presented an image of a batter swinging at a baseball (as shown in Fig. 1) and prompted students to measure the speed of the ball at the instant the photograph was taken. The speed of a baseball is measured often during professional games, and we expected many students in our study to be familiar with this scenario. Although the students would intuitively know that the baseball was moving when the photo was taken, they likely would not immediately associate an interval of time (determined by the camera's shutter speed) with the picture. We anticipated that students who do not intuitively interpret photographs as depicting intervals of elapsed time would struggle to provide a reasonable value for the speed of the baseball, especially considering that their recent instructional experiences had focused on average rates of change, the quantification of which requires a multiplicative comparison of discernible changes in the measures of two covarying quantities. We surmised that this apparent incongruity between speed at an instant of time and speed over an interval of time might perturb students and evoke an intellectual need to construct another method for computing speed.

### Provoking Individual Intellectual Need: Student Problem-Solving Videos

As described above, previous studies of students' intellectual need have involved group-level observations. Furthermore, students' engagement with IN-P tasks in prior research have typically taken place in settings where they could discuss the tasks with classmates. Although we intended our IN-P tasks to help students enter a perturbational state for the targeted concepts, we hypothesized that working on a task individually might not lead a student to experience disequilibrium. In particular, it might be possible for students to not recognize the limitations of their initial way of thinking. Researchers have demonstrated that facilitating students' awareness of the complexity of the situation can improve their constructive reasoning (e.g., Sinha et al., 2021). Thus, to engender and identify intellectual need on the individual level, we built on ideas about vicarious participation in mathematical exploration (e.g., Lobato et al., 2019) and designed a student problem-solving video to accompany each IN-P task. In each video, a pair of actors posed as calculus students attempting to solve the IN-P task. These videos were loosely scripted so that the actors demonstrated a variety of plausible ways of thinking that reflected common challenges encountered by students learning the concept. Each student problem-solving video concluded without the IN-P task being resolved. These videos were designed to take the place of group discussion and exploration and for students to be persuaded by—or identify with—the interpretations, conceptions, and ways of thinking demonstrated by the actors, and then to be challenged by the confusions, counterexamples, or refutations tentatively expressed by the actors. We expected the resulting disequilibrium to engender students' intellectual need by focusing their attention on the conceptual obstacles to learning the targeted concept or skill and, thus, provide a foundation for the learner's epistemological justification for the mathematical concept.

We illustrate these design principles by describing the student problem-solving video for instantaneous rate of change (using the baseball task described above). This is a concept for which students face significant challenges (e.g., Carlson et al., 2002; Confrey & Smith, 1994; Orton, 1983; Thompson, 1994; Zandieh, 2000). For example, Carlson et al., (2002) and Thompson & Carlson (2017) documented the importance of constructing an image of variation where the amount of change of one variable is coordinated with changes in another variable to conceive of average rate; then an instantaneous rate at a moment may be viewed as an average rate over an interval so small that changes in the values of these variables are essentially proportional. In the student problem-solving video, the actors expressed that although the ball was in motion while the photo was taken, the photo does not depict the ball moving because "there is no time happening," and they would need a change in time to quantify the ball's speed. After a brief discussion, the actors identified the blur of the baseball in the photo as illustrating a change in time, but pointed out that this temporal duration didn't correspond to a single moment, or instant, in time. The actors were ultimately unable to compute a value for the requested instantaneous rate, and the video concluded with the narrator summarizing the actors' observations and confusions.

## Identifying Intellectual and Affective Need

Intellectual need is an individual, conscious experience with the potential to influence students' mathematical thinking. The educational value of a student experiencing intellectual need results from the strategic decisions they are positioned to engage in to resolve an impasse they construct. Thus, our goal was to identify students' experiences of intellectual need at the individual level and to ground this identification in students' perceptions of their experiences. There has been little discussion in the research literature of what might constitute evidence for students' experiences of intellectual need, and prior research methods have either relied on indirect indicators of intellectual need (e.g., Caglayan 2015) or ascribed intellectual need to groups rather than individuals (e.g., Harel 2010; Koichu, 2012).

To operationalize an empirical identification of students' experiences of intellectual need, we leveraged Pekrun et al.'s (2016) descriptions of epistemically-related emotions to identify emotions that would reflect internal cognitive states of disequilibrium. Of the seven emotions Pekrun et al. distinguished (surprise, curiosity, enjoyment, confusion, anxiety, frustration, and boredom) curiosity appeared to be most closely aligned with intellectual need. Curiosity is "the complex feeling and cognition accompanying the desire to learn what is unknown (Kang et al., 2009, p. 963); it is experienced when a student recognizes a gap in their knowledge (Metcalfe et al., 2020) and can be thought of as an "appetite" for new or missing information (Shin & Kim, 2019).[2] Thus, we operationalized intellectual need in terms of "wonderment" and "curiosity." That is, we asked students the following question after they had worked on the IN-P task and/or watched the student problem-solving video:

When you were working on this task, were there any parts where you genuinely were curious or were left wondering about something? If so, please state them in the box below; if not, please leave the box empty.

Student interest—grounded in affective need—can prompt their engagement with the problem (Hidi & Renninger, 2006). Thus, we felt that it was important to distinguish epistemic emotions from the student's interest in the underlying context—that is, an aspect of their affective need for engaging in the task. Since the focus of our research is on intellectual need, our measure of affective need was mainly being used to empirically test that our measure of intellectual need described something different than affective need. A common strategy to construct a measure of interest is to explicitly ask students about their interest (e.g., Fedesco et al., 2019; Thompson et al., 2015; Turner & Silva, 2006). Thus, immediately before the curious/wondering question, we asked:

> The task you just worked on dealt with the context of [context—e.g., "the speed of a baseball"]. In your honest opinion, how interesting/enjoyable was this context?

Throughout the results, when we refer to a student experiencing an intellectual need, we mean that they responded "yes" to the curious/wondering question; when we refer to a student experiencing an affective need, we mean that they responded either "somewhat interesting" or "very interesting" to the interesting/enjoyable question.

[2] The epistemic emotion of confusion also has potential connections to intellectual need. However, some researchers (e.g., D'Mello & Graesser (2012)) have pointed out that students often experience confusion negatively, leading them to reduce their intellectual engagement in the problem-solving activity, rather than reacting to disequilibrium in the way envisioned in the DNR framework.

### Measuring Learning

To identify the relationship between intellectual need and learning, we sought to measure student learning as a direct result of the instruction. Thus, we needed to measure each student's background knowledge prior to and immediately after instruction. To do this frequently—and to do so for a large enough sample of students to support our statistical analysis—we conducted our study in the context of an online learning environment.

For each topic (listed in the Appendix), we created 1–4 questions that were designed to assess students' understanding. The questions were administered to the students in a multiple-choice format prior to attempting the IN-P task or experiencing any instruction. Then, after showing the instructional videos, we presented students with a set of questions that corresponded to the pre-video questions but substituted different numbers, functions, or contexts. Post-video questions that involved computations were presented in a free-response format that was automatically evaluated by a computer algebra system. The students were given an unlimited number of attempts to solve the post-instruction questions, and the system that hosted the questions indicated whether each of their responses were correct. We evaluated the correctness of an answer to a multiple-choice question based on whether the initial response was correct, and on the first two attempts for free-response questions.

## Materials, Participants & Methods

### Materials

The first, third, and fifth authors identified 30 topics (listed in the Appendix) typically taught in first-semester calculus. To conduct the study in an online learning environment, we created a set of 1–3 instructional videos for each target topic; each set of videos (a full list of topics is included in the Appendix) included a solution to one of the IN-P tasks and additional explanation of the underlying concept. These videos, which are available online at https://calcvids.org, were designed using Mayer's (2014a) recommendations for best practices in multimedia design and grounded in a conceptual analysis that focused on supporting students' quantitative and covariational reasoning (e.g., Thompson 1994; Carlson et al., 2002). The sequence of videos began with the development of the concept of constant rate of change as a proportional relationship between varying amounts of change in the measures of two continuously covarying quantities, and used this idea consistently to develop the ideas of average and instantaneous rates of change, accumulation, and the Fundamental Theorem of Calculus. The topics mirrored those commonly included in introductory Calculus textbooks, with the exception of limits, which were integrated into several videos rather than being presented as a stand-alone topic. In this way, we designed the materials to introduce and support quantitative reasoning as a way of thinking[3]

[3] Regarding the relationship between quantitative reasoning and mathematical ways of thinking, Tallman & Frank (2020) explained: "The inclination to conceptualize situations in terms of quantities and quantita-

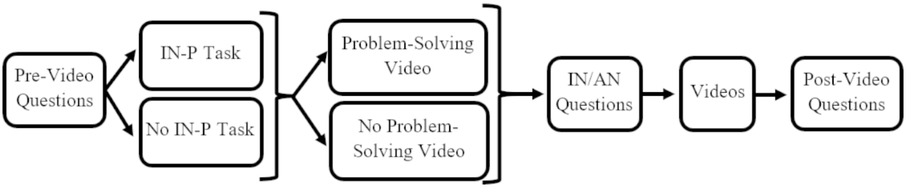



**Fig. 2** Experimental Design

that would engender particular ways of understanding specific calculus concepts (i.e., leveraging the duality and repeated reasoning principles in Harel's (2010) DNR framework).

For each topic, we created a collection of problems to be solved prior to and after watching the instructional videos, an IN-P task, and a student problem-solving video. The materials for each topic—a *video set*—were hosted on Ximera (ximera.osu.edu) (a system that hosted the sequence of pages that displayed the questions and videos and evaluated the students' responses).

The experimental design is shown in Fig. 2[4]. For each video set, each student was randomly assigned to be presented with the IN-P task or not and to see the student problem-solving video or not. In cases where students saw neither the task nor the video, they were not asked about affective need, and the intellectual need questions were phrased to ask about the pre-video questions the students had worked on.

## Participants

The participants in the study were 2,733 students who were enrolled in first-semester calculus classes at one of 18 universities during the Fall 2019 and Spring 2020 semesters. The universities included both public and private institutions, ranging in size from roughly 3,000 to over 35,000 students, from all regions of the United States and one institution in Indonesia. The institutions included small, private liberal arts colleges, regional colleges and universities, and large, research-focused universities. Individual class sizes ranged from 20 to over 200 students.

There were 28 participating instructors over the two semesters. Prior to using the instructional materials, each instructor participated in a professional development session in which they learned about the design of the videos, the idea of intellectual need, and the design and purpose of the student problem-solving videos.

At one institution (a large, public university in the Mid-South United States) calculus classes were organized into "sections" of approximately 20–30 students each, with a different instructor teaching each section; there were a total of 10 instructors teaching these coordinated sections over the two semesters (some instructors taught

tive relationships is one that can be productively applied to make sense of several mathematical ideas. An individual who maintains an orientation across a variety of mathematical domains to identify measurable attributes of objects and to define relationships between them possesses a cognitive characteristic of the mental act of interpreting and analyzing that can be called *reasoning quantitatively"* (p. 73).

[4] The full design involved a third experimental condition to investigate a different set of research questions. This full design, which is shown in the Appendix, more clearly illustrates the percentage of students who were not asked the AN questions.

two sections in a semester, and most instructors taught sections in both the fall and spring semesters). One member of the research team served as the coordinator of these sections by assigning a textbook, providing a schedule of topics, and assigning common homework and exams. The coordinator required the instructors to assign a common subset of the video sets to their students, but beyond this there were no additional requirements for instructors to incorporate the videos into their classes.

The 18 other instructors were voluntarily participating in the project research; within this group, two instructors were from the same institution and the 16 others were all from different institutions. These instructors followed the instructional requirements of their own institution. Each instructor selected a subset of the video sets to assign to their students, but the research team did not otherwise specify how the instructors should incorporate the videos into their classes.

At the beginning of the semester, the students completed a survey in which they were given the opportunity to allow the research team to include their responses in the data corpus. Students were able to access all the video sets and we hypothesized that some ambitious students might access video sets independently. If an instructor had relatively few students submit work for the video sets, or relatively few students from one class complete an individual video set, then this could introduce unintended bias. To eliminate these instances (i.e., one student watching a video set) from the data, we removed data from instructors who had only a small number of participating students across the semester (specifically, when an instructor had fewer than 100 total submitted video sets) which resulted in dropping 7 instructors with a total of 128 video set instances. We also deleted cases where fewer than 25% of the students completed a particular data set from a particular instructor. This dropped a total of 988 video set instances. An additional 4293 video set instances were dropped due to missing instructor data.

### Categorizing Video Sets

Given the consistency in the ways of thinking and understanding the video sets were designed to elicit, the tasks and methods of presentation among the videos had numerous similarities with each other—for example, the video set about average rate of change had similar tasks and underlying conceptions as the video about constant rate of change. To account for this similarity in our statistical exploration, we identified each video set as falling into one of the following six categories:

- Quantitative/proportional reasoning (4 video sets).
- Graphical/(co)variational reasoning (4 video sets).
- Computation (15 video sets).
- Interpreting mathematical expressions (1 video set).
- Definitions (2 video sets).
- Applications of derivatives (4 video sets).

As we generated models in our statistical analysis of the data, the relatively small number of video sets in some of these categories resulted in not all the distinctions between categories being significant. Thus, in our final model, we only distinguish between the 15 video sets that focused on computation and the 15 others.

### Statistical Design

The class of statistical models we used for our analysis are called Generalized Hierarchical Linear Models (Raudenbush & Bryk, 2002), sometimes referred to as mixed models. Mixed models are an extension of ordinary least-squares (OLS) regression or generalized linear regression models that can be used with data that do not satisfy the assumptions of OLS regression. Mixed models are appropriate for our data because we have nested and cross-nested cases: video sets are nested in students, students are nested in instructors, and both students and instructors are cross-nested with video sets. The nested and cross-nested data structure violates the OLS regression assumption that each case is independent of the other cases. For example, two video sets done by the same student are expected to be more similar than video sets done by two different students.

We use models with two outcomes to answer our research questions: one with intellectual need as the outcome, and one with score on the post-video questions as an outcome. Both outcomes are measured with a binary variable. In the case of intellectual need, a 1 signifies that a student responded "yes" to the question, "Were there any parts where you genuinely were curious or were left wondering about something?", and 0 signifies that they responded "no" to the same question. For the achievement models, the outcome is a binary variable with 1 representing any growth in the percentage correct from the pre-video to the post-video questions, and a 0 representing no growth (no change or negative change). Because most of the video sets had a very small number of pre- and post-video questions, we felt that a binary outcome would be more appropriate than using the percentage, gain score, normalized change scores (Marx & Cummings, 2007), or other outcome measure. Since the outcomes of both models are binary, the lowest level regression model (video set) was a logistic regression model, with all higher levels considered to be linear regression models with the basic variance assumptions as described by Raudenbush & Bryk (2002).

For each outcome we ran two models: unconditional and conditional. Unconditional models have no predictor variables but show the extent of variation at each level of the model. We use this model to describe the extent the outcome varies between students, between teachers, and between topics. The conditional model includes predictor variables and is used to test the relationship between the predictor variables and the outcome variable.

Although mixed models are neither OLS regression models nor GLM models, the coefficients of the models can be interpreted in the same way. Variables in the model are tested to see if there is a statistically significant association with the outcome variable. Because the lowest-level model is a logistic regression model the coefficients are given in logit units, or log-odds. The size of the effects in log-odds are not easily interpreted due to the non-linear nature of the logistic function. However, positive coefficients indicate an effect that increases the probability of a student indicating an intellectual need (IN model) or showing evidence of learning (Learning model). Also, the larger the coefficient the larger effect the variable has on the outcome.

Table1 displays a list of the variables we use in the models. We measured several other variables and tried many two-way interaction terms in the models, but removed those that were not statistically significant. The models were drawn from a data set with approximately 26,000 instances of students completing video sets, approximately 1,550 students, 25 instructors, 14 institutions, on 30 video sets. We were not able to use all data for the conditional models due to multiple attempts by the same student and missing data. For the IN model there were 26,056 completed video sets, 1,559 students, 30 video sets (topics) and 25 instructors. The Learning model used 15,588 completed video sets, 1,524 students, 29 video sets (topics) and 25 instructors. The smaller data set for the Learning model is largely explained by a greater rate of missing data with the pre/post scores and many students that had perfect scores on pre- and post-tests, which were not usable for our conditional model. Also, one video set was not used in the analysis because of poorly framed problems on the post-test that did not accurately capture students' understanding. It was possible for a student to begin working on a video set multiple times. In such cases (approximately 4.3% of instances), we believed that the student's last interaction with the set was most likely to include the student's completion of all components in the experimental design, and, thus, used their last interaction in the models.

To capture effects of survey fatigue, we included as a variable the ordinal number associated with each video set, as listed in the Appendix. Although instructors were not required to use all the video sets or to use them in the presented order, our ordering matches what is commonly found in introductory Calculus textbooks, and this ordering generally matched the point in the semester in which the video set was assigned.

Due to the full experimental design (shown in the Appendix), roughly 1/3 of the students were neither shown the IN-P task nor the student problem-solving video. Therefore, models with the affective need measure were done with only about 2/3 of the data, but results were similar across models with the full data and with the reduced dataset with affective need included.

## Results

We begin by describing the results for the unconditional models for intellectual need and achievement growth. Then, we repeat these analyses using conditional models. Throughout the results, we use the term "experience intellectual need" to refer to the students responding "yes" to the question, "Were there any parts where you genuinely were curious or were left wondering about something?" We use the label "typical student," to represent students who are represented by the intercept of the statistical model. Typical students become the reference group for making comparisons (e.g., comparing "typical students" and "students that reported intellectual need"). Which students are "typical" depends on the formulation of the model and can be determined by thinking about students that have a value of zero on all variables. Continuous variables have been mean-centered, so a value of zero means an average value on that variable. "Typical instructors" or "typical lessons" represent instructors or lessons with the mean in the outcome of the model. For example, typical instructors

**Table 1** Definitions and Descriptions of Variables

| Variable Name and Symbol | Description |
|---|---|
| **Outcome Variables** | |
| Intellectual Need (IN) (also a predictor for L) | A binary variable with 1 indicating students reporting an intellectual need and 0 otherwise. Mean=0.12; SD=0.32 |
| Learning (L) | A binary variable with 1 indicating students improved their score from pre- to post-video and 0 otherwise. Mean=0.44; SD=0.49 |
| **Predictor Variables** | |
| Affective Need (AN) | A binary variable with 1 indicating that students find the context “somewhat” or “very” interesting, 0 otherwise. Mean=0.44; SD=0.49 |
| Intellectual Need- Provoking Task (IT) | A binary variable with 1 indicating students were presented an IN-P task and 0 otherwise. |
| Computational Topic (CT) | A binary variable with 1 indicating that the video set topic was largely computational, 0 otherwise. 15 of the video sets fell into this category |
| Pre-Video Score (Pre) | A variable that represents the score (from 0 to 100) on the pre-video test. Mean=36, SD=32. For use in the models this variable has been standardized to a mean of 0 and an SD of 1 |
| Lesson Order (LO) | The number of each lesson during the semester (5 indicates the 5th lesson). This variable was centered at lesson 15, the middle of the semester. |
| Problem-Solving Video (PSV) | A binary variable with 1 indicating that students were shown the student problem-solving video, 0 otherwise |
| Coordinated Institution (SI) | A binary variable with 1 indicating an instructor was at the institution with multiple, coordinated calculus sections, 0 otherwise. |

in the models with intellectual need as the outcome, are instructors (or a hypothetical instructor) whose students report experiencing intellectual need at the average rate of all instructors.

## Intellectual Need Models

### Unconditional Model

To answer the first set of research questions, we ran unconditional mixed models to understand how much variation there is at the student, instructor, and video set levels. The results of the unconditional model are displayed in Table2, which lists the intercept, variance, and standard deviation (square root of the variance) of each level. To make the results easier to interpret, we have included conversions of log-odds to percentages in brackets. The intercept represents typical students working on a typical video set from a typical instructor. Thus, students would experience

**Table 2** Intercept and Random Effects of the Intellectual Need Unconditional Model

| | IN Outcome |
|---|---|
| Intercept | -2.99 [4.8%] |
| Level | Variance (SD) [-1 SD, +1 SD] |
| Student | 2.64 (1.63) [1.0%, 20.4%] |
| Instructor | 0.52 (0.72) [2.4%, 9.4%] |
| Video Set | 0.23 (0.48) [3.0%, 7.5%] |

an intellectual need in response to the task and/or student problem-solving video in 4.8% of the typical video sets. The two percentages next to the variance and standard deviation show percentage estimates of one standard deviation up and down from the intercept for each level. Each interval assumes that the values in the other levels are for a typical case (student, instructor, or lesson), which is the intercept value. For example, on average, students reported that they experienced an intellectual need in 4.8% of the video sets, but different students had different rates at which they reported experiencing an intellectual need. The interval [1.0%, 20.4%] means that students 1 SD less than the mean only reported experiencing an intellectual need on 1.0% of the video sets and students 1 SD above the mean reported it at 20.4%. Students, of course, could report much higher (lower) if they are in a high (low) lesson or a high (low) class, or further than 1 SD away from the mean. The intervals are not symmetric around the mean because the conversion from log-odds to percentages is a non-linear transformation. A move towards 50% will be a larger jump than a move away from 50%.

**Variation of intellectual need by instructor.** Although there was less variability than between students within a classroom, there was still a reasonable amount of variation across instructors in the data set. The typical student (one that experienced intellectual need at the average rate) in the typical lesson (one where students experienced intellectual need at an average rate of all lessons) reported experiencing an intellectual need in 2.4% of video sets in some teachers' classes (down 1 SD) but 9.4% in other teachers' classes (up 1 SD)—roughly four times the rate in some classes compared to others. The interval for two standard deviations from the mean was 0.1–17.5%. This means that at the low end, some teachers rarely had average students (in a typical lesson) who reported experiencing an intellectual need, while at the other end, some teachers had average students reporting an intellectual need on over a sixth of the video sets.

Because most institutions in our data set were represented by a single instructor, it is difficult to tease apart variation due to instructor and variation due to institution, curriculum, or pedagogy. To calculate a more accurate estimate of the variation due to instructors, we ran an unconditional model using only the data of the institution that had multiple coordinated sections of calculus—ostensibly, controlling for institutional-related student factors and curriculum. Although not quite as large as in the original estimate, there was still a reasonable amount of variation. The standard deviation at the instructor level was 0.63, which is relatively close to the 0.72 standard deviation with all instructors included. This result is evidence that the instructor variation in our model was due mainly to differences in instructors, rather than other institutional factors.

**Table 3** Conditional Model with Intellectual Need as the Outcome

| Fixed Effects | IN Outcome | Marginal Effect in Percentage[a] |
|---|---|---|
| Intercept | -2.12*** | 10.7% |
| Intellectual Need-Provoking Task (IT) | -0.294*** | -2.5% |
| Affective Need (AN)そ | 0.382** | 4.3% |
| Pre-Video Score (Pre) | 0.768** | 9.9% |
| Lesson Order (LO) | -0.044*** | -0.40% |
| Problem Solving Video (PSV) | -0.553*** | -4.2% |
| Coordinated Institution (SI) | -1.35*** | -7.7% |
| Intellectual Need-Provoking Task*Problem Solving Video (IT*PSV) | 0.532*** | 6.3% |

Note: Asterisks indicate statistical significance on the following scale: * indicates $p<.05$; ** indicates $p<.01$; *** indicates $p<.001$

[a] Because of the study design, only about 2/3 of students were introduced to a context during the online lessons and thus only 2/3 were asked the question that measures Affective Need. The coefficient for Affective Need was generated using a subset of the data with the same variables entered into the model, when feasible. Because there are no students in the non-treatment, non-video option in the restricted data, we cannot introduce the IT*V interaction term into the IN-Outcome model. Thus, the Affective Need coefficients should be interpreted with caution, although the direction of the effect is the same as in the raw data and the size of the effects are similar to the raw data

**Variation of intellectual need by video set.** The variation across video sets was less than between teachers or between students in a class, but for a typical student in a typical teacher's class, most lessons (about two-thirds) elicited intellectual need of 3.0–7.5% with extreme values (up and down two standard deviations) of 1.9% and 11.6%. These are low values in absolute terms but show some lessons were over 6 times more likely to generate an intellectual need than others.

## Conditional Model

The results of the Intellectual Need conditional mixed model are displayed in Table3. Each variable that was not significant was dropped from the model unless it was a main effect in an interaction term. The coefficients of the model are given in log-odds. To aid in interpretation a percentage is listed next to each coefficient; these are marginal percentages given a unit increase in the variable from the model intercept, with all other variables equal to zero. The percentages are not additive like in a linear regression model, and the effect of a variable could be larger or smaller than the listed percentage depending on the values of other variables.

**Intercept.** The intercept in the conditional model (log-odds of -3.064 and percentage of 10.7%) was higher than in the unconditional model (-2.12, 4.5%). Although the 4.5% represents an overall rate at which all students across all institutions, instructors, and video sets reported an intellectual need, the intercept in the conditional model indicates the average rate at which students not at the coordinated institution

**Table 4** Marginal Effects Illustrating the Problem-Solving Video and IN-P Task Interaction Term

| | No Problem-Solving Video (PSV) | Problem Solving Video (PSV) |
|---|---|---|
| No IN-P Task (IT) | 10.7% | 6.5% |
| IN-P task (IT) | 8.2% | 8.1% |

reported having an intellectual need on a video set during the middle of the semester, with a mean score on the pre-video questions, and not receiving the IN-P task or the student problem-solving video.

**Coordinated Institution.** The students in the coordinated institution reported having an intellectual need at a rate 7.7% points less than the 10.7% percent intercept (holding other variables at zero). The average rate, then, for the students in the coordinated institution was about 4%. This difference may be explained by the fact that all calculus instructors at the coordinated institution were encouraged to use the video sets as part of their calculus course, whereas the other instructors were self-selecting to use the video sets. Alternatively, it is possible that students in particular fields of study were less likely to report experiencing intellectual need, and that these majors constituted a larger proportion at the coordinated institution than at other institutions.

**Relationship between Task, Problem-Solving Video, and Intellectual Need.** The IN-P task (IT), -0.294, and watching the Problem-Solving Video (PSV). -0.553 had negative marginal effects of -2.5% and −4.2%, respectively. That is, students who saw the IN-P task were 2.5% points less likely to report experiencing an intellectual need, dropping from 10.7 to 8.1%. Students who watched the problem-solving video were also less likely to report experiencing an intellectual need. Keeping other variables at zero, the marginal effect of watching the problem-solving video dropped the average rate from 10.7 to 6.5%. However, the effects were not additive because there was a significant positive interaction term between the two variables. This means that students who both saw the IN-P task and watched the student problem-solving video were more likely to report experiencing an intellectual need than would be expected by the combined effects of each. One meaning of the interaction term is that the effect of doing the IN-P task depends on if they watched the video and vice-versa.

To make sense of these interacting effects, we held one variable constant and considered the effect of the other. Watching the problem-solving video had a large negative effect if not done with the IN-P task but made little difference if watched in conjunction with the IN-P task (the coefficients for PSV and the IT*PSV are about the same size and in different directions—0.553 versus 0.532). Alternatively, students who watched the problem-solving video actually increased their rate from 6.5% to about 8.1% if they watched the IN-P video as well. Table4 shows these effects in marginal percentages.

**Relationship between Background Knowledge and Intellectual Need.** Students with above average pre-video scores (1 SD above average), were almost twice as likely to report experiencing intellectual need than students represented by the intercept value, with average rates of 20.6–10.7% respectively (keeping other variables at zero). Students scoring one standard deviation below average on the pre-video math problems dropped the average rate of reporting an intellectual need to 5.3%. This

**Table 5** Intercept and Random Effects of the Unconditional Learning Model

| | Learning Outcome |
|---|---|
| Intercept | -0.306 [42.4%] |
| Level | Variance (SD) [-1 SD, +1 SD] |
| Student | 0.144 (0.379) [33.5%, 51.8%] |
| Instructor | 0.004 (0.065) [40.8%, 44.0%] |
| Video Set | 0.443 (0.666) [27.4%, 58.9%] |

finding suggests a strong connection between closely related content knowledge and the probability of experiencing intellectual need.

**Relationship between Affective and Intellectual Need**. There was a significant positive association between affective need (AN) and intellectual need. Typical students (represented by the intercept) were about 40% more likely to report experiencing intellectual need if they reported experiencing affective need, with an average rate of 15.0% vs. 10.7%.

**Effects of Lesson Order on Intellectual Need.** The rate at which students reported having an intellectual need decreased, on average, across the semester. This is illustrated with the negative coefficient of the Lesson Order variable: -0.044. This variable was centered at 15, about the middle of the semester, so the intercept in the model represented the average rate at which students reported having an intellectual need at the middle of the semester. Lessons near the beginning of the semester (around lesson 3) had a rate of 16.9%, while lessons near the end of the semester (3 from the end) had an average rate of 6.9%.

## Learning Models

### Unconditional Model

The results of the unconditional model with learning as the outcome are displayed in Table5. The unconditional model showed that typical students with a typical instructor would improve from their pre-video to post-video score on 42.4% of typical video sets. The story for the Learning model is somewhat different than that of the IN model. There was considerable variation across video sets and students in the Learning model, but the variation at the instructor level was not significantly different from zero, which implies that the effectiveness of the video sets in helping students learn is independent of instructor and institution. This implies that each video set tended to be consistently difficult (or not) for most students, but that there was variation among the sets. This is different than all video sets being of similar difficulty, but some students consistently finding them easy and others consistently finding them challenging—although variation between students was still significant and meaningful, just not as large as the variation between video sets.

### Conditional Model

The results of the conditional model that tests association with students' learning are displayed in Table6. As before, the outcome variable in this model was a binary

**Table 6** Conditional Model with Learning as the Outcome

| Fixed Effects | Estimate | Marginal Effect in Percentage[a] |
|---|---|---|
| Intercept | -0.853*** | 29.9% |
| Intellectual Need (IN) | 0.311** | 6.9% |
| Intellectual Need-Provoking Task (IT) | 0.118** | 2.5% |
| Affective Need (AN)† | 0.119** | 2.5% |
| Computational Topic (CT) | 0.554 | 12.7% |
| Pre-Video Score (Pre) | -1.18*** | -18.3% |
| IN*CT | -0.356* | -6.9% |

Note: Asterisks indicate statistical significance on the following scale: * indicates p<.05; ** indicates p<.01; *** indicates p<.001

[a] These percentages represent the effect of increasing the variable from 0 to 1, for dichotomous variables, and from the mean to one standard deviation above the mean, for continuous variables. All continuous variables were mean centered. Due to the non-linear model at level one, these percentages cannot be added together to find compounding effects

variable with a 1 indicating any growth in percentage from pre-video to post-video questions. The level-one model, like in the previous models, was a logistic regression model, so coefficients are in log-odds units. To aid in interpretation we have included the marginal effect as a percentage based on movement from the intercept, keeping all other variables at zero (continuous variables were mean centered).

**Intercept.** The intercept is interpreted as the average rate at which certain students learned from the video sets as measured by a growth in percentage from pre-video to post-video. Students represented by the intercept are students with an average pre-video score on a non-computational topic that reported no intellectual or affective need and did not see the intellectual task. The average rate for learning from a video set was 29.9%.

**Relationship between Intellectual Need, Video Set Topic, and Learning.** There was a significant positive association between experiencing intellectual need and learning. Typical students (represented by the intercept) showed evidence of learning on approximately 6.9% more lessons if they experienced an intellectual need, an increase of about 23% (from the average/intercept of 29.9% up to 36.8% of lessons) over students that did not experience an intellectual need. However, there was a significant interaction term with the IN variable and topics coded as computational, so reporting an intellectual need had a different association with learning for computational topics than for non-computational topics. In non-computational topics, reporting an intellectual need increased the probability of showing evidence of learning by 6.9% (for a typical student), but for computational topics the intellectual need raised the probability by 11.6% (for a typical student).

**Relationship between Affective Need and Learning.** There was a significant positive association between affective need and learning, but it was only about a third of the size of self-reported intellectual need on learning—2.5% versus 6.9%.

**Relationship between the Intellectual Need Provoking Task and Learning.** There was a significant positive effect for students who were shown the IN-P task.

Typical students seeing the IN-P task showed evidence of learning on 2.5% more lessons than students that did not see the IN-P task.

## Discussion

### Summary and Discussion of Results

In this first subsection, we provide and discuss answers to our research questions. We first discuss the factors that influence provocation of intellectual need (research questions 1–5). Then, we discuss the relationship between intellectual need and learning (research questions 6–8).

### Factors that Relate to Intellectual Need

**Extent of Variation of Students' Intellectual Need.** Overall, there was a relatively low rate of students reporting an experience of intellectual need: in a typical instructor's class and for a typical video set, only 4.8% of students reported experiencing curiosity or wonderment. We consider this to be a surprisingly low rate given the directed efforts to provoke intellectual need. One potential explanation for this low rate could be students' affective response in which they experience disequilibrium in terms of confusion rather than curiosity (e.g., Vogl et al., 2019). In particular, if students' view of their role in a mathematics class does not include engaging in situations that are complex, novel, and difficult to understand, they might tend to experience confusion or frustration (Fayn et al., 2019), which can reduce their engagement in problem-solving (e.g., Arguel et al., 2019). Another explanation is that, as Weinberg & Jones (2022) suggested, provoking intellectual need requires considerable planning and intervention by an instructor. Students' experience of intellectual need in this study was potentially limited by aspects of the experimental design, namely that the intellectual need-provoking tasks and videos were presented online without the presence of an instructor to mediate the use of the tasks/videos or to facilitate the creation and resolution of the students' disequilibrium.

Students were much more likely to experience intellectual need in response to some video sets than others. This means that some mathematical topics, tasks, or problem-solving videos were more effective at helping students experience a state of disequilibrium. There wasn't a relationship between the topic being computational and students experiencing intellectual need, so the relationship between video set content and intellectual need warrants further investigation, perhaps with consideration of the specific category of intellectual need being supported (e.g., *certainty*, *causality*, *computation*, *communication*, or *structure*).

**Association Between Instructors and Intellectual Need.** Instructors—and, implicitly, the ways they incorporate the video sets into their instruction—are associated with different rates of students experiencing intellectual need. The between-instructor variation at the institution with multiple coordinated sections of calculus is evidence that the instructor variation in our model is due mainly to differences in instructors rather than other institutional factors. Not only was the between-instructor variation dramatic, but this variation at the institution with multiple coordinated sections of calculus was nearly as large as the variation between institutions. All instructors at the coordinated institution were required to use the video sets as part of their instruction, whereas the instructors at other institutions were self-selecting to use the video sets. Thus, the small difference in rate between the coordinated institution and other institutions could possibly be explained by the way the video sets were used by the self-selecting instructors at other institutions or the demographics of students at the other institutions. With this self-selection in mind, the rate at the coordinated institution may be more typical of the rate at which students report an intellectual need in the treatment in our study. Our results add one more outcome that varies significantly across instructors such as effectiveness (e.g., Rivkin et al., 2005) and instructional quality (e.g., Bergsten 2007).

The variation between instructors is particularly surprising because the students' interaction with the video sets occurred outside of regular class meetings, and we would expect their reaction to the tasks and problem-solving videos to be independent of the pedagogy that was employed during class meetings. This result suggests that there is a complex relationship between pedagogy, curriculum, and students' interaction with the out-of-class learning materials, and these relationships warrant further empirical investigation. It is plausible, for example, that instructors' efforts to influence students' beliefs about the nature of mathematics, their image of the aptitudes that define mathematical proficiency, and their identity in relation to mathematics might affect students' expectations of the role the video sets might play in learning. Students whose instructors are purposeful about supporting their beliefs, mindset, and identity in these ways might be more inclined to appraise intellectual need-provoking tasks as a necessary catalyst to their learning, rather than a superfluous prelude to the instructor's presentation of rules and demonstration of procedures, which are often prioritized by students in pursuit of performance goals. The complex relationship between students' experience of intellectual need and characteristics of the classroom community of practice–including the social and sociomathematical norms continually negotiated by students and instructor–is reflected in the many DNR premises and concepts intricately related to the necessity principle. Our findings demonstrate a need for researchers to explore the relationship between, for example, students' mathematical ways of thinking, their subjective interpretation of instructional experiences, and their orientations to interacting with intellectual-need provoking stimuli.

**Association Between Affective and Intellectual Need.** There was a significant relationship between students' experiencing intellectual and affective need. One explanation for this result is that there is a significant cognitive or emotional overlap between the factors that engender the two types of need. Thus, it may be important to consider both students' affective responses to the problems (see, e.g., Arguel et al., 2019) and the problem context when constructing an IN-P task (e.g., Mayer 2014b). However, in the conditional learning model (discussed later) the measure of intellectual need and affective need were both significant, providing strong evidence that our measures for these two variables were not measuring the same construct.

**Association Between Background Knowledge and Intellectual Need.** Students who had more extensive background knowledge for a task—as measured by their performance on the pre-video questions—were more likely to experience an intellectual need than other students. One explanation for this result is that students need a certain level of knowledge about the background mathematical concepts to engage in the IN-P task in the intended way. Alternatively, students might need the background knowledge to *identify* their experience as one of intellectual need. Similar to previous theoretical recommendations (e.g., Harel 2008a; Weinberg & Jones, 2020; 2022), and consistent with Harel's (2008a) subjectivity and interdependency premises, both explanations suggest that IN-P tasks need to be carefully tailored to particular student knowledge and characteristics in order to provoke intellectual need.

**Effects of IN-P Task and Video on Intellectual Need.** Beyond the students' own background knowledge and other characteristics, it appears that the ways we structure the video-watching process can impact the students' experience of intellectual need. Students who (only) tried the IN-P task or (only) watched the student problem-solving video were less likely to experience intellectual need. However, for students who watched the problem-solving video, those who also tried IN-P task were more likely to experience intellectual need. This finding could be understood in terms of students' epistemic emotions, in which solely watching the student problem-solving video left students feeling more confused or frustrated than curious, but the combination of the task and the video facilitated students' awareness of the complexity of the situation (e.g., Lodge et al., 2018; Sinha et al., 2021; Vogl et al., 2020). This result suggests that merely provoking intellectual need is not a straightforward process, and that it would be useful for educators to have a framework to support the design and implementation of IN-P tasks (Weinberg & Jones, 2020; 2022).

We observed a significant effect of lesson order on whether students indicated that they experienced an intellectual need. One possible explanation for this result is that students experienced increasing fatigue, stress, or indifference to the calculus concepts over the course of the semester, and these factors influenced their potential for experiencing intellectual need. Alternatively, students could have reported intellectual need at a lower rate due to survey fatigue. Students who indicated that they experienced an intellectual need were then asked to provide an explanation of what they were curious about. This response required more time and effort on the part of the students. We expect students picked up on this and became more reluctant to report having an intellectual need as the semester progressed.

### Factors that Relate to Learning

**Relationship with Intellectual Need.** There is a significant, positive association between a student experiencing intellectual need and demonstrating learning from the instructional videos (as measured by their change from the pre- to the post-video questions). This result aligns well with the theory of intellectual need, which posits this relationship between need and learning (e.g., Harel 2010). It is noteworthy that our measures of learning and of intellectual and affective need are fairly simple, but even with these simple measures we found significant effects. Our measures of learning—a small collection of computational problems and, for conceptual questions, a multiple-choice format—were basic and only administered immediately after the conclusion of the instructional videos. Similarly, both intellectual and affective need were captured through a single yes/no question that was administered at a single point in time. Errors in measuring these constructs would make it more difficult to detect effects in statistical models. Further research will lead to better measures and more accurate understanding of the relationships between these (or more refined) constructs.

**Relationship with Affective Need.** As with intellectual need, there was also a significant, positive association between affective need and learning. This result suggests that students who experience an interest in the problem context are more likely to learn from the associated videos. This result falls in line with other empirical research that finds student interest is significantly associated with learning in K-12 schooling (Schiefele et al., 1992). However, empirical connections to student interest and learning in university mathematics have been harder to find (Hailikari et al., 2008). In addition, this result provides evidence for Harel's (2008a) proposal that intellectual and affective needs are related, but separate, components in the complex process of learning.

**Relationship Between IN-P Tasks and Problem-Solving Videos.** In contrast to the experience of intellectual need, students who tried the IN-P task were more likely to learn from the instructional videos. However, watching the student problem-solving video was not associated with learning, and the interaction between the IN-P task and the problem-solving video was also not associated with learning. One potential explanation for these findings is simply that more experience with the content is better, but this conclusion is not supported by the effect on the problem-solving video, which would provide more experience with the content but is not significantly associated with learning.

## Methodological Contributions

In our study, we developed methods to provoke and empirically identify students' experiences of intellectual need. This methodology included developing intellectual need-provoking tasks, videos related to each task to foster students' experience of intellectual need, and survey questions—administered at the point where we thought students might be experiencing disequilibrium—to enable students to report feelings of affective and intellectual need. In the survey, we operationalized intellectual need using the language of curiosity and wonderment and attempted to distinguish intellectual need from affective need.

Although the present study was conducted in an online learning environment, we hypothesize that the methodology could be adapted to in-person contexts. The pre- and post-instruction questions could be administered in-person. Rather than watching the student problem-solving video, students could engage in discussions about the IN-P task and the instructor could highlight the problematic conceptions that would have been identified in the video. From a research perspective, the only shortcoming would be the difficulty in collecting data from more than a handful of students.

There are several potential limitations of our methodology. First, our identification of students' experiences of intellectual need were based on their perceptions of our use of "curiosity" and "wonderment," but it is possible that students interpreted these terms in ways that were inconsistent with our intentions. Consequently, we believe that in-depth qualitative work to develop valid ways of identifying and measuring intellectual need is needed. These measures could help identify intellectual need after-the-fact, like in the survey in this study, as well as to identify intellectual need in "real time." The timing of the survey means that students could have been describing their reactions to the pre-video questions rather than the IN-P task or video. Students might have been reluctant to respond affirmatively to the intellectual need question because doing so would require them to write additional information; in particular, the decreasing rate of intellectual need across the semester (illustrated by the lesson order variable) suggests that survey fatigue likely influenced the students' responses. Finally, and perhaps most importantly, we do not know the extent to which intellectual need can be provoked by a single task, even when the task is accompanied with a student problem-solving video. Weinberg & Jones (2022) suggest, instructor interaction and intervention might be essential to moving students into the state of disequilibrium for which the intended concept or topic would provide a resolution, and our methods might have been insufficient to actually provoke genuine intellectual need.

### Generalizing to Other Learning Environments

The present study was conducted in an online learning environment, which raises questions about the extent to which our results could generalize to other learning environments (Bosch & Gascón, 2014). That is, if we imagine the same instruction occurring during class sessions, how might students' experiences of intellectual need and their learning from watching the videos change if the IN-P tasks were enacted during class sessions and what role might be played by the student problem-solving videos?

We have no reason to suspect that any aspect of the kinds of IN-P experiences we intended to engender were specific to or in any way enhanced by the virtual format. In some ways the constraints that arose from the online environment probably mean that our experimental context might have hindered students' experiences of intellectual need—or at least their willingness to report such experiences. Furthermore, we hypothesize that the online implementation of the tasks is probably not as strong as what instructors could have done in person. That is, the online environment is likely an inferior mode for engaging students in thinking about the ideas in the IN-P task, and students might be more likely to experience curiosity or wonderment under the direct guidance of their instructor, provided the instructor was skilled in facilitating their students' participation in collective mathematical activity or at incorporating the student problem-solving video into the class exploration.

## Conclusion

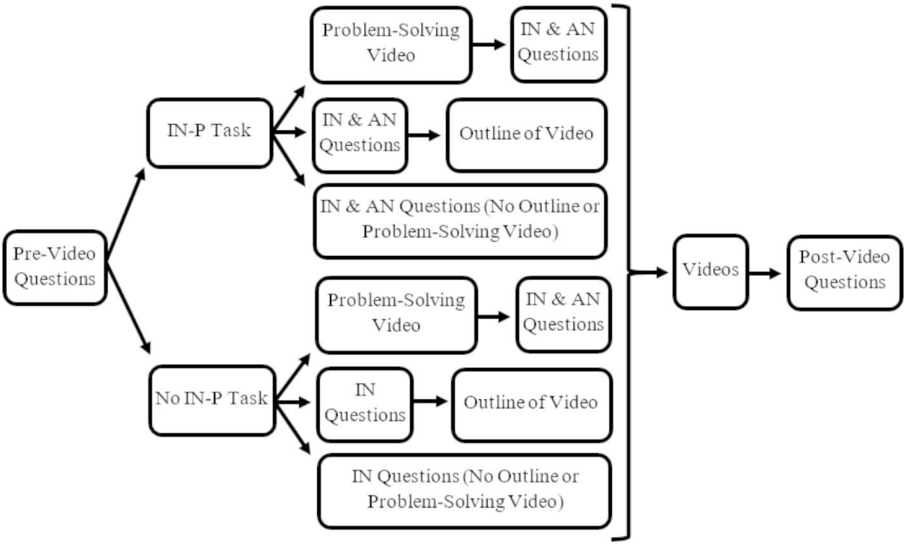
This study makes a significant methodological contribution to the design and evaluation of intellectual need-provoking tasks and materials. Our methods provide a step toward empirically identifying students' experiences of intellectual need and connecting those experiences to their learning. The combination of IN-P tasks and student problem-solving videos demonstrate some relationships with students' experiences of intellectual need and learning and provide a framework for further investigating the design of instructional tasks. The incorporation of the intellectual and affective need survey at the point in time when students are likely to experience disequilibrium similarly provides a starting point for continuing to investigate and understand intellectual need and the circumstances that provoke it.

Our results also shed light on some of the factors that might impact students' experiences of intellectual need and how these factors influence learning. The relationship between intellectual need, learning, the structure of the students' interaction with the video sets, the students' background knowledge, and the instructor's pedagogy is complex. Taken together, these results highlight the importance of continuing to study intellectual need and to create a framework for helping instructors design and implement intellectual need-provoking tasks.

## Appendix

**Calcvids Topics**.

1. Constant Rates of Change.
2. Graphing Constant Rate of Change.
3. Varying Rates of Change.
4. Graphing Varying Rates of Change.
5. Average Rates of Change.
6. Approximating Instantaneous Rates of Change.
7. Continuity.
8. Differentiability and Local Linearity.
9. Limit Definition of Derivative.
10. Using the Limit Definition of Derivative.
11. Interpreting Derivatives.
12. Slopes of Secant and Tangent Lines.
13. Graphing Derivatives.
14. Basic Derivative Rules.
15. The Product Rule.
16. The Quotient Rule.
17. The Chain Rule.
18. l'Hopital's Rule.
19. Mean Value Theorem.
20. Related Rates.
21. Implicit Differentiation.
22. Introduction to Optimization.
23. Optimization: Algebraic Modeling.
24. Introduction to Riemann Sums.
25. Riemann Sum Notation.
26. Definite Integrals.
27. Antiderivatives.
28. The Fundamental Theorem of Calculus, Part 1.
29. The Fundamental Theorem of Calculus, Part 2.
30. U-Substitution.

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## Declarations

**Conflict of interest** On behalf of all authors, the corresponding author states that there is no conflict of interest.

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