Effect of loop quantization prescriptions on the physics of non-singular gravitational collapse

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We review recent developments in the physical implications of two loop quantization strategies for the interior of a homogeneous dust cloud. The first is the loop quantization with holonomies and the triads while the second is with holonomies and the gauge covariant fluxes. Although both of the quantization schemes resolve the classical central singularity regardless of the initial conditions, they also lead to a distinct phenomenology. For the first loop quantization, we find that when the dust mass is larger than a threshold value, both black hole and white hole would form and their evolution is symmetric with respect to the bounce point, leading to black hole-white hole twins. In contrast, in the second quantization, the evolution of the outermost dust shell is asymmetric with respect to the bounce point, and as a result the black hole-white hole twins can never form. Even in the situation when both black hole and white hole can form, the mass of the latter is only $2/\pi$ of the mass of the former.

Keywords: Homogeneous dust collapse; loop quantum cosmology.

1. Introduction

The singularity problem in the gravitational collapse results from the breakdown of the classical theory of general relativity (GR) and can thus be resolved by quantum gravity. Understanding the role of quantum gravity in determining the final state of the collapsing stars can help provide insights on the fundamental questions related with cosmic censorship conjecture and black hole evaporation. Because of the quantization ambiguities in quantum theories, it is important to understand if the physical predictions are robust against various quantization prescriptions. In the following, we address this question for a collapsing dust cloud whose interior spacetime is described by the Lemaître-Tolman-Bondi(LTB) metric and study two quantization prescriptions in a loop quantum gravity (LQG) scenario.

To make the problem of a collapsing dust cloud more manageable, we further assume a homogeneous evolution of the dust cloud in the marginally bound case. We then make use of the techniques from LQG to quantize the interior of the dust cloud. The loop quantization of the spacetimes with spherical symmetry dates back to quantization of the interior of the Schwarzschild black hole.[1–6] Later it was extended to include both interior and exterior of the black hole[7–13] as well as the dynamical collapsing spacetime which is filled with a massless scalar or the dust.[14–18] All these studies focussed on the resolution of the central singularity by incorporating quantum geometry effects via holonomies and/or inverse triad modification but not on comparaing the physics in different quantizations. Further, none of the forementioned models have been directly derived from LQG. Therefore, it is reasonable to ask whether the physical predictions in those models are robust when further modifications from LQG are included in the dynamics. To answer this question, we have studied two distinct quantization prescriptions for the interior of a collapsing dust cloud and compared their physical implications.[19] The first quantization employs holonomy and triads while in the second quantization, in addition to the holonomy corrections, we have also considered the contributions from the gauge-covariant fluxes[20] which have beenr ecently explored in the cosmological setting.[21–23] Our results have shown that although the resolution of the central singularity is robust against different quantization strategies, the resulting dynamics also exhibit distinctive features for each quantization ansatz. In particular, the first quantization results in black hole-white hole twins while the second quantization strategy generally leads to black hole-white hole asymmetry.

2. The classical dust shell model

The classical dust shell model describes the dynamics of the outermost shell of a collapsing dust cloud. It is based on the LTB model which is obtained from a spherically symmetric solution of Einstein's equations in GR with non-rotational dust as a matter source. The metric of LTB spacetime is given by

$$ds^2 = -d\tau^2 + \frac{(R')^2}{1+2f}dx^2 + R^2 d\Omega^2, \tag{1}$$

where R is the areal radius, $f(x)$ is the total energy of a unit mass at x and a prime denotes derivative with respect to the radial coordinate x. Depending on the sign of f, there are three distinct cases: the marginally bound case with $f = 0$, the bound case with $f < 0$ and the unbound case with $f > 0$. In the following, we focus on the marginally bound case. The corresponding Hamiltonian constraint for the LTB dust model is explicitly given in terms of both canonical ADM variables and the Ashtekar variables for a general inhomogeneous dust cloud in.[24] The classical dust shell model can then be obtained from the LTB model by a homogeneous reduction where the energy density of the dust cloud is assumed to depend only on time.[25] Thanks to the spherical symmetry and the homogeneity of the dust cloud interior, one can integrate the classical Hamiltonian constraint of the LTB model along the radial direction as well as the angular part and arrive at the Hamiltonian constraint for the outermost dust shell which can be expressed in terms of a canonical pair consisting of the radial components of the extrinsic curvature and the densitized triad.[19] It turns out that for the homogeneous dust collapse, the interior spacetime is isometric to the spatially-flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime. As a result, the classical Hamiltonian constraint of the outermost dust shell can be related with the one for the spatially-flat FLRW spacetime.[19]

For a complete description of thiS collapsing model, in addition to the dust interior, one still needs to specify the exterior spacetime which is glued to the dust interior at the boundary. We choose the exterior spacetime to be the generalized

Vaidya spacetime[26] so that it allows a non-vanishing effective dust pressure after quantum geometrical effects are taken into account.[14] The matching conditions are worked out under the requirements that the first and the second fundamental forms should be continuous across the boundary. In this way, it turns out that in the marginally bound case the Vaidya mass which is the mass of the dust cloud as seen by an outside observer differs from the mass function of the dust cloud by a factor of $1/2G$ with G denoting the Newton's constant.

3. Loop quantization of the dust shell model with holonomy and triad variables

The loop quantization of the dust shell model is based on the classical Hamiltonian constraint of the outermost dust shell which is expressed in terms of the Ashtekar variables.[19] Due to the quantization ambiguities, there are different quantization prescriptions which can result in distinct effective dynamics. Since the interior of a homogeneous dust cloud is isometric to a spatially-flat FLRW spacetime, one can apply techniques developed in loop quantum cosmology (LQC)[27] in a straightforward way. After quantizing the outermost dust shell by employing holonomies and the triads with the $\bar{\mu}$ scheme,[28] we find that the classical central singularity is resolved and replaced with a bounce which takes place at a fixed maximum energy density in the Planck regime. Meanwhile, the collapse of the dust cloud is succeeded by a re-expansion after the bounce point. The formation of the trapped surfaces during the contraction and the expansion of the dust cloud depends solely on the initial dust mass. There also exists a threshold dust mass M^* below which no black hole or white hole can form during the evolution of the dust cloud. On the other hand, when the initial dust mass is larger than M^*, a dynamical black hole can form during the collapse of the dust cloud and correspondingly a white hole can form during the re-expansion of the dust cloud. The evolution of the black hole and the white hole is always symmetric with respect to the bounce point and in particular the mass of the black hole is the same as the mass of the white hole. Moreover, right at the bounce, the trapped surfaces vanishes and the Vaidya mass becomes zero which implies that an asymptotic flat Minkowski spacetime emerges at the bounce point due to the quantum repulsive force. This repulsive force is also reflected by a negative effective pressure near the bounce point when the energy density is in the Planck regime.

4. Loop quantization of the dust shell model with holonomy and gauge covariant flux variables

Apart from the holonomy and triad quantization, we also study the physical consequences of using holonomy and the gauge covariant fluxes[20] which is motivated from the need to go beyond symmetry reduced triads in to obtain an effective Hamiltonian with loop quantum modifications from LQG using suitable coherent states. Its physical implications have been explored for the spatially-flat FLRW model recently.[21–23] This quantization strategy leads to distinct dynamical evolution of the outermost dust shell as compared with the one discussed in the last section.

In particular, although the classical central singularity is resolved and replaced with a quantum bounce which also takes place in the Planck regime, the formation/ evolution of the black hole and the white hole is no longer symmetric with respect to the bounce point. Due to quantum gravity effects there exist two characteristic dust masses M_1 and M_2 ($M_1 < M_2$). When the initial dust mass is less than M_1, no trapped surfaces can form during the entire evolution of the dust cloud. On the other hand, when the initial dust mass lies between M_1 and M_2, only a black hole can form during the collapse of the dust cloud. Finally, when the initial dust mass is larger than M_2, both black hole and the white hole can form. In the last case, we find the evolution of the black hole and the white hole is not symmetric with respect to the bounce point and in particular the effective mass of the white hole is only $2/\pi$ of the one for the black hole and the black hole always outlives the white hole in the proper time. This asymmetry is due to the difference in the classical limits of the pre- and post-bounce regimes of the effective dynamics. As a result, black hole-white hole twins do not exist in this quantization strategy.

5. Summary

The gravitational collapse of a homogeneous dust cloud provides a platform to test different loop quantizations and investigate their resulting physical implications. We find even in this simple setting, quantization ambiguities can lead to very distinct phenomenological effects. In particular, we have studied two quantization strategies in this context, the one employing holonomies and triads and the other using holonomies and gauge covariant fluxes. Although in both cases, the central singularity is resolved and replaced with a quantum bounce, there are qualitative differences between two quantization prescriptions. In the former, when the dust mass is larger than the threshold mass, there are black hole-white hole twins. While in the latter, a black hole and white hole twin system is not possible and the the mass of the white hole is only $2/\pi$ of the mass of the black hole because of quantum gravity effects. Further, there can be situations in which only black hole forms during the collapse of the dust cloud.

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