

Participatory Equity in One Undergraduate Linear Algebra Class

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Inquiry and active learning instructional methods have largely been regarded as equitable and beneficial for students. However, researchers have highlighted math classrooms as racialized and gendered spaces that can negatively impact marginalized students' experiences in such spaces. In this study, I examine the development of one argument, and whose ideas are solicited and leveraged, in an inquiry-oriented linear algebra course with an eye toward participatory equity. I found that gender related most to the inequity of participation in argumentation and that only men participated in generalizing activity. This study adds to the growing literature addressing equity in inquiry and active learning math settings.

Keywords: Inquiry-oriented instruction, Participatory equity, Argumentation, Linear Algebra

Researchers have found active learning to be beneficial for students (Esmonde, 2009; Freeman et al., 2014; Laursen et al., 2014; Rasmussen et al., 2020; Yackel & Cobb, 1996) and link active and cooperative learning environments to higher student achievement and success in STEM courses compared to traditional, lecture-based teaching methods (Freeman et al., 2014; Theobald et al., 2020; Zakaria & Daud, 2010). These student-centered approaches generally involve argumentation; however, argumentation can take many forms depending on who contributes and connects between ideas in the arguments. While many of these approaches have been linked to positive outcomes, researchers have also found whole-class and small-group discussions in mathematics classes to be gendered and racialized spaces, which can create negative mathematical experiences for women and students of color (Boaler, 2008; Ernest et al., 2019; Hand et al., 2012; Jackson & Cobb, 2010; Leyva et al., 2020; Reinholz et al., 2022; Wilson et al., 2019). Some students might have their mathematical methods consistently explored more often or more thoroughly, while others might feel as though their ideas are overlooked. The emphasis on certain ways of participating in mathematical activity involve cultural assumptions and expectations about what is normal and acceptable (Boaler, 2002; Nasir & Cobb, 2007).

Studies examining students' success, achievement, or growth in active learning mathematics classrooms often omit the gendered and racialized experiences that transpire in these classrooms. Much of the research detailing the latter tend to background the mathematics. This study fills a gap in the research by analyzing argumentation in an inquiry-oriented linear algebra course while attending to racialized and gendered interactions. My goal is to illuminate ways in which certain interactions in student-centered classrooms, such as who gets called on or whose ideas get taken up, can lead to racialized and gendered experiences in mathematical practices, such as argumentation. One research question guided this analysis: *In the context of an inquiry-oriented linear algebra class, how are arguments being developed and whose ideas are solicited and leveraged during this development?*

Theoretical Background: Equitable Participation and Argumentation

Equitable Participation

Leyva et al. (2020) theorize the ways in which mathematics classrooms are racialized and gendered by locating classroom events at the intersection of societal discourses about ability in STEM and explicit and implicit institutional logics related to access and authority (e.g., use of

certain mathematics courses to prevent students from entering certain majors). With this lens, they highlight the ways in which students experience a range of classroom events that provide different opportunities and support to students in math class that vary systematically by race and gender. Reinholz and Shah (2018) refer to equity related to who is allowed to participate in classroom practices, such as discussions, as participatory equity. Ernest et al., 2019 describe participatory equity as concerning “the fair distribution of both participation and opportunities to participate in core aspects of the learning process” (p. 155). Reinholz and Shah (2018) explain that participating in mathematical discussions is part of the learning process, which can present a variety of settings for inequities. They explain that while it might not be intentional, women and students of color tend to be systematically given easier tasks and fewer opportunities to participate in doing mathematics.

Argumentation

I use Toulmin’s (1958/2003) model of argumentation not only to examine the mathematics being discussed by students and the instructor but also to organize the instructor’s prompts for contributions and responses to those contributions. Toulmin originally used this model to examine “argumentation in the traditional sense of one person convincing an audience of the validity of a claim” (Conner et al., 2014, p. 404). Mathematics education researchers (e.g., Andrews-Larson et al., 2019; Conner et al., 2014; Rasmussen & Stephan, 2014) have shifted the way this model is used, to examine the development of ideas through argumentation. An argument is a series of statements involving a combination of claims, data, warrants, rebuttals, and backings as defined below (Conner et al., 2014; Toulmin, 2003).

- *Claim*: statement to be validated
- *Data*: evidence validating the claim
- *Warrant*: statement connecting data to the claim
- *Rebuttal*: statements asserting when the warrant would not be valid
- *Backing*: “support that give warrants authority” (usually unstated) (Andrews-Larson et al., 2019, p. 4)

Data or warrants can become a *Data/Claims* or *Warrant/Claims*, respectively. For instance, if a claim that once needed validation becomes data to validate another claim, it is a *Data/Claim*.

Methods

Study Context, Participants, and Data Sources

This study focuses on a single instructional unit (systems of linear equations) in one undergraduate linear algebra class at a public institution in the Southeastern United States. According to institutional data, this university is categorized as a primarily Black institution. The class was taught using a hybrid format, where some students were in-person and some online via Teams. Students could decide each day how they wanted to join the class (in-person or online). The class used OneNote, a platform where the instructor and students could see the tasks, insert text or images, write notes, and see what each group did every class (making sharing group work easier in the hybrid format). The instructor for this course, Dr. Pi (all names are pseudonyms), used inquiry-oriented instruction to teach most, if not all, of the topics. All gender and race data were self-reported by students via survey. The participants were students enrolled in the course who consented to participate, which included four women, eight men, and no students self-identifying in any other gender categories; six students self-identified as Black, two as two or more races, one as Asian, one as Hispanic or Latinx, one as White, and one preferred not to

respond. The data sources were video and audio recordings of whole class discussions (across 12 days), records of the chat in Teams, and field notes.

Data Sources and Analysis

Equitable participation analysis. To examine the equitable participation in the arguments described below, I coded students' contributions using Reinholz and Shah's (2018) EQUIP observation codes. The unit of analysis was a *sequence* of talk. A sequence begins when a student initially speaks and ends when another student speaks, thus starting a new sequence of talk. Because of the reliance on another student speaking, sequences vary in length depending on how long the instructor and student are speaking. If two students are going back and forth in conversation, then each time a student speaks starts a new line of code. For this study, I used Author's (2019) modified EQUIP codes. I generally focused on Dr. Pi's Solicitation Method and Evaluation. I counted the number of sequences involved in argumentation and the number of sequences involving contributions from women and from men. I used these numbers to calculate equity ratios (Reinholz & Shah, 2018), which compare the percentage of sequences involving women or men to their percentage in the population. An equity ratio of one means the ratio of a group's contributions in discussions (in this case, argumentation) was proportional to their representation in the class. An equity ratio less than one means a group is underrepresented in argumentation, where an equity ratio greater than one means an overrepresentation.

Argumentation analysis. I identified 12 different arguments taking place throughout the systems unit, which was eight class days long. For a discussion to be identified as an argument, students had to make vocal mathematical contributions. The arguments were large in grain size, where I allow for subarguments in the development of arguments (Conner et al., 2014). I identified the arguments by topic using my knowledge of the tasks and what happened in class. I identified all mathematical whole class discussions in the video data and selected which ones I wanted to further analyze based on observed trends, which will be described later in this section. After transcribing, I did the argumentation analysis first, which involved identifying the different arguments. I then examined the overall flow of ideas, later adding the evidence provided by students and Dr. Pi. I then labelled the different contributions as components in the arguments. In the 12 arguments I identified, I noticed some important trends that were relevant to the research question as they directly related to equitable or inequitable participation in argument development. The trend highlighted in this paper is that no women contributed to any arguments after the second half of the fourth day. This happened to be when the class started moving toward general activity (Gravemeijer, 1999), in which they generalized knowledge they had developed about solution sets so far in the unit. Instead, women only participated in situational activity in which students worked with systems in the context of a meal plan.

Findings

EQUIP Findings

Equity ratios were calculated based on a population of 8 people (due to consent), 2 women and 6 men. There were 98 sequences of talk throughout the unit, 5 (5%) from women and 93 (95%) from men. The 5 sequences involving women were all made within the first 4 (of 8) days of the systems unit, when the task sequence intended for mostly situational activity. Women had 5% of the total sequences but comprised 25% of the total population, as compared to men having 95% of the sequences while comprising 75% of the population (shown in Table 1). This

produces an equity ratio of .2 for women and 1.267 for men. Thus, men were participating in argumentation far more than their representation in class, where women were involved in argumentation much less than their representation. Overall, men spoke more in general and, when women did speak, it was earlier in the unit, pointing to more contributions from women during situational activity than generalizing activity. When I examined the solicitation method by gender, I found that women were called on and not called on two times each (50% of the time for each code) (shown in Table 2). Table 2 shows men spoke most of the time without being called on (66.7%), but also spoke a fair amount being called on (29.4%). Sometimes men were called on was because they had previously spoken without being called on and Dr. Pi later asked them to elaborate or made a connection back to their idea. Women were also only solicited to give contributions answering a “what” type question, where men were solicited to give answer what, how, and/or why questions.

Table 1. Sequences and equity ratios by gender.

Gender	% Of Sequences	% Of Population	Equity Ratio
Women (2)	5%	25%	.2
Men (6)	95%	75%	1.267

Table 2. Solicitation method by gender.

Gender	Called On	Not Called on	Called on Volunteer	Called on Group
Women (2)	2 (50%)	2 (50%)	-----	-----
Men (6)	30 (29.4%)	68 (66.6%)	2 (2%)	2 (2%)

I found that the groups by race that participated in arguments almost proportionally to their representation in the class were the Asian student (equity ratio of 1.2 shown in Table 3) and the Black students (.82). The four Black students consisted of two women and two men. One man, Noel, had about 83% of the sequences within this group. He was both called on and not called on about equally as often. It could be argued that the White student had an equity ratio relatively close to 1 (.72). Alberto, who identified as multiracial, contributed considerably more than his representation in the class. He contributed to arguments more than three times his representation in the class. He was often not called on and had some of the longest sequences of talk. Lastly, the Hispanic/Latinx man, Jorge, was involved in argumentation much less than his representation in the class. He was generally called on and often had short sequences of talk.

Table 3. Sequences and equity ratios by race/ethnicity.

Race/Ethnicity	% Of Sequences	% Of Population	Equity Ratio
Black (4)	41%	50%	.82
Asian (1)	15%	12.5%	1.2
Hispanic/Latinx (1)	3%	12.5%	.24

White (1)	9%	12.5%	.72
2 or More Races (1)	32%	12.5%	2.56

Argumentation Findings: Contributions from only men during generalizing activity

The selected argument (shown in Figure 2) was chosen because it highlights and is representative of the type of generalization that occurred in the class, where students were generalizing knowledge about solution sets of systems by drawing on what they learned previously in the meal plans task. It lasted about 16 minutes and took place on the sixth day of the unit. Students were discussing a task in which they match four different systems to images of what their graphs would look like before solving the system or using technology to graph. In particular, students were discussing which graph out of six choices corresponded with System 1,

$$\begin{cases} x + y + z = 210 \\ 5x + 7y + 10z = 1500 \\ x + y + z = 500 \end{cases}$$
 Dr. Pi started the discussion by looking at and reading out each group's choices for which graph matches:

“So y'all say that system one, the third equation will be parallel to equation one, but intersects equation two [Data/Claim 11]. So, we believe it's graph E [Claim 12] ... They said for D, E, F (shown in Figure 1), that they have no common points of intersection [Data 10] ... But due to the way systems three and four equations were made, they had connection either to D, E, or F.”

All the groups agreed that System 1 has no solution (Data/Claim 1); thus, System 1 corresponds with D, E, or F (Data/Claim 2). Dr. Pi then explained what the same group said about two of the equations, “we know that $x + y + z = 210$ and the money equation, $[5x + 7y + 10z = 1500]$ do intersect,” (Rebuttal 3, Data/Claim 11) with which the class agreed. The group also said that “the scalars of $x + y + z = 210$ and $x + y + z = 500$ are the same and would probably lie on the same plane or be parallel to each other” (Data/Claim 3). This was revoiced by Dr. Pi and other students, notably Noel who pointed out which planes in the graph were parallel.

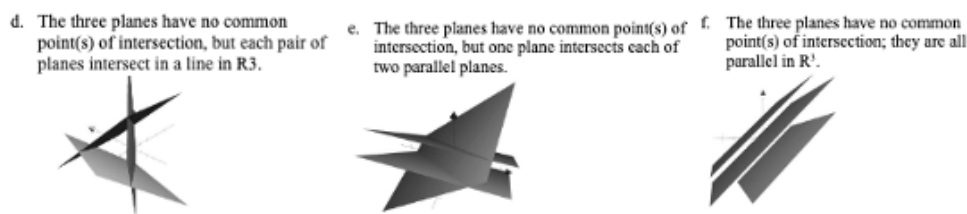


Figure 1. Options D, E, and F as possibilities for the graph of System 1.

This argument was driven by students' ideas. Dr. Pi voiced some of the argument by reading out each group's decision and explanation and drew in students to explain their reasoning or reason with other students' ideas. Dr. Pi did not make many connections during this argument, but rather engaged students in sensemaking about their own and others' decisions and making connections between those. In the end, students articulated what it means for a system to have no solution: there are no intersections between any of the three planes, or there is no common intersection between all three planes. Dr. Pi emphasized the latter as correct, and Aaron concluded E represents System 1.

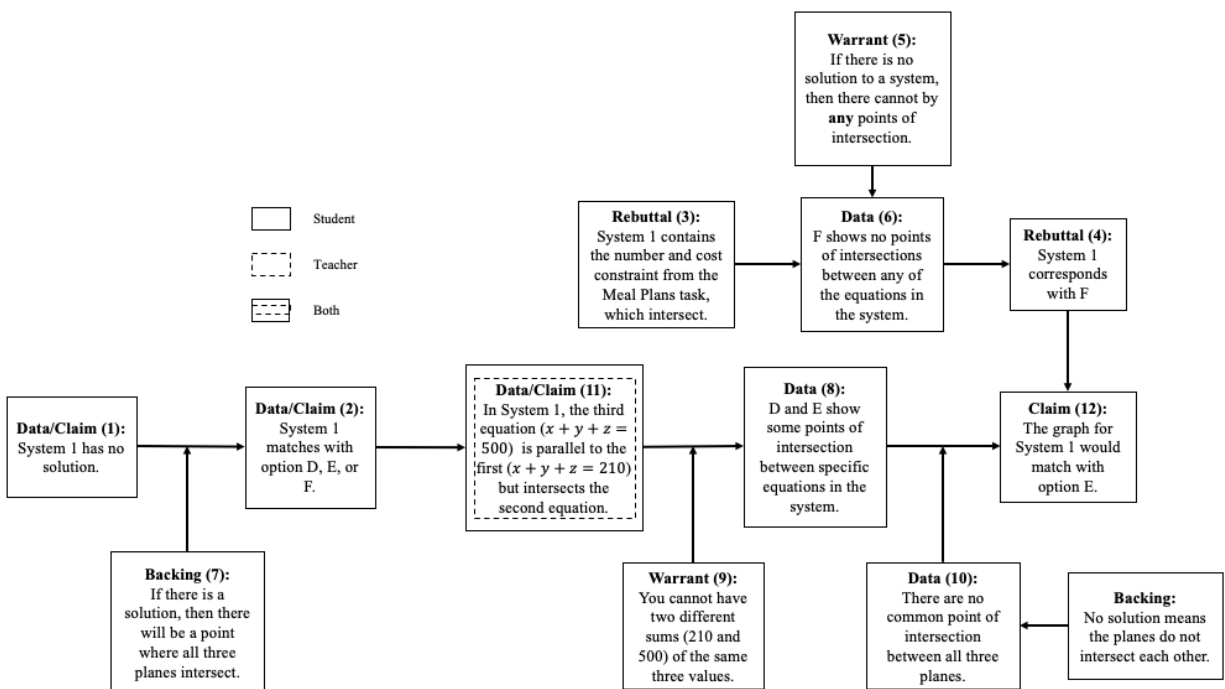


Figure 2. Map of the argument: Contributions from only men during generalizing activity.

Argument with EQUIP. Recall that this argument was selected because it is representative of the generalizing argumentation in which students engaged later in the unit. Also recall that no women verbally contributed to this type of argumentation during this unit, which I noticed during the argumentation analysis but confirmed through the EQUIP analysis. The EQUIP analysis showed that many of the students who did participate in developing this argument spoke without being called on, which could contribute to women not participating because men tend to dominate open discussions (Reinholz et al., 2021; Author, 2019). There was one instance in which Dr. Pi called on a group to see if they wanted to change their answer, which still elicited a response from a man. These findings of more men participating when Not Called On or Called On Group are consistent with findings from Author (2019).

Dr. Pi started the discussion by reading out different groups' ideas, so several argument contributions were provided by students but voiced by Dr. Pi. He started with a group's idea that System 1 has no solution because two of the equations are parallel planes, so System 1 must match with D, E, or F. To further think about this, the class looked at a graph of the system that a group developed to find which two planes were parallel. When discussing the colors of the planes, Alberto said, "*Leave the color identification to the girls.*" Dr. Pi made an uncomfortable noise in an effort to push back a bit on this comment. Alberto added that he said this "*because [he is] colorblind.*" This presented a moment when the women in the class could have felt uncomfortable and framed as less capable than the men, or like they do not belong. Alberto was suggesting that men need to make more mathematical contributions while the women can be left to decide colors. There were no women attending in-person in class that day and students joining virtually always had their cameras off, so it was hard to tell women's reactions. This comment created a slight disruption in the class discussion before Dr. Pi continued.

At one point after reading the groups' decisions, Dr. Pi asked if the group from Breakout Room 1 wanted to change their answer. Aaron spoke on behalf of the group and explained that he still thinks System 1 matches F because there should be no intersections when a system has no

solution. Alberto interjected, without being called on, that a solution to a system would be somewhere *all three planes* intersect, not just two of them. In the end, Dr. Pi leveraged Alberto's contributions leading Aaron to agree that System 1 matches with E. In this argument, Alberto, Noel, and Aaron had the largest number of talk sequences. Alberto had 33% of the 15 sequences of talk, Aaron had 26.67% of the sequences, and Noel had 20% of the sequences, so they comprised 79.67% of the sequences of talk were contributed by three students in the class. This tended to be the trend throughout the generalizing arguments, where only a few students, always men, contributed most of the arguments. Alberto, who made a comment that suggested women's role in a mathematics classroom is to identify colors on a graph, was ultimately positioned as making the mathematically correct contribution that was publicly accepted by the authority figure, Dr. Pi. Not only were women not part of the development of the argument, but they were also learning mathematics in a context where a fellow student (a man) who made a sexist comment regularly played a central role in the mathematics that was developed.

Discussion

In this paper, I found that the data showed that the most distinct participation inequities in this class related to gender more than race. Women contributed between zero and three times per class, while provided between 4 to 30 sequences. Women's overall representation was substantially less than what would be considered equal, let alone equitable. Women participated in argumentation during situational activity and not formalizing activity. This could be related gender stereotypes in which women are framed as procedural and algorithmic thinkers as opposed to creative and abstract (Leyva, 2017). This could also be related to a larger stereotypical narrative in society (and STEM classrooms) around what women can and cannot do. Furthermore, this could contribute to similar outcomes between women enrolled in inquiry courses and women enrolled in non-inquiry courses (Johnson et al., 2020), as abstraction and generalization are core components of inquiry-oriented instruction. The current study adds to literature highlighting the nuance and complexities of active learning and discussions in mathematics classrooms.

Additionally, Alberto tended to dominate discussions and, because he usually spoke without being called on, there was limited space for other students to contribute. Instructors can sometimes implicitly, and unintentionally, allow students to drive the mathematical discussion, even when the other students provide more mathematically sound arguments (Black, 2004). Alberto also publicly commented during whole class discussion, saying Dr. Pi and Noel should "*leave color identification to the girls.*" This was a multi-faceted situation in that Alberto felt comfortable enough in the class to make this comment but there are also implications for how this could have affected women in the class. It was normal for Dr. Pi and students to make jokes during class, but these jokes generally did not reference anyone's gender or race. This comment implied that men's role is to do the mathematics and women can take on a more superficial role (e.g., identify colors). Despite making this comment, Alberto spoke more than most students in the class and had his mathematical reasoning heavily publicly featured in almost every argument. These findings together point to the need for further examinations into the nature of interactions in inquiry and active learning classrooms, even when there appears to be strong argumentation taking place. Exploring these interactions can reveal inequities taking place when using a type of instruction thought to be the most equitable.

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