Interaction of Pilot Reuse and Channel State Feedback under Coherence Disparity

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***Abstract*—When the individual links in a downlink channel have different coherence intervals, previous studies have shown that pilot reuse can achieve not only rate gains, but also gains in degrees of freedom (DoF). Channel state feedback is another source of gains, but combining beamforming with pilot reuse presents new and interesting design questions. The performance of such a scheme has been an open problem under coherence disparity, the regime in which pilot reuse is most promising. We propose a new non-orthogonal transmission scheme for pilots and data that harmoniously combines with either perfect or imperfect channel state feedback. The proposed scheme employs both product superposition and zero-forcing beamforming within the same framework, and improves the resulting achievable rates. The developments include careful pilot placement and an efficient pilot reuse strategy under channel state feedback in a multi-user downlink channel. Numerical results are presented to corroborate our findings.**

I. Introduction

Pilot reuse is an important tool in the continued pursuit of better bandwidth usage, especially with highly varying channels. Recent work has shown that pilot reuse is stunningly effective in multi-user networks where links have different coherence intervals [1]. However, the highly effective pilot reuse techniques in this regime (e.g., product superposition) have been on first impression incompatible with effective usage of channel state feedback. The present work explores this important open question.

We propose a novel non-orthogonal transmission scheme combining product superposition and zero-forcing beamforming that allows simultaneous transmission of pilots and data to different users. The main contribution of this work is to address the apparent incompatibility of product superposition with beamforming, to allow them to be used together. Product superposition operates by presenting to one user an effective channel that is a product of one link gain with another user's data [1]. This leads to a major challenge for beamforming since the user with the virtual channel is unable to measure the true (physical) link gain.

In this paper, we consider L users experiencing different coherence intervals. For convenience, we assume that any two coherence intervals have integer ratio. However, this is only for convenience and there exist techniques for extending integer to non-integer ratios, which are omitted in this manuscript for brevity. Furthermore, we assume channel state feedback is available from mobiles to the base station, either perfectly or imperfectly, but without delay. We explain the proposed transmission strategy and derive the achievable rate expressions. We also calculate both channel estimation error and total noise power in our proposed scheme under different feedback links and present numerical results to illustrate the gains achieved through the proposed method.

This work was supported in part by the grant 1718551 from the National Science Foundation.

A brief outline of relevant literature is as follows: Pilot reuse has been studied in massive multiple-input multiple-output (MIMO) networks [2]–[4] and device-to-device (D2D) systems [5], [6] to reduce pilot overhead and achieve rate gains. This issue has also been highlighted in [7]–[12] for broadcast channel by introducing product superposition as an efficient pilot reuse strategy that improves not only rate gains, but also gains in degrees of freedom (DoF). In the absence of channel state information (CSI), [7]–[9] investigated the use of product superposition for a two-user broadcast channel with different coherence times. The DoF region for a multi-user broadcast channel with coherence disparity was investigated via product superposition in [10], [11] assuming no CSI feedback to the transmitter. In [12], product superposition is combined with CSI at the transmitter, but only two channel coherence lengths are assumed to be present, and one of them is a static channel.

Notation: Matrices and vectors are denoted by bold capital letters and bold small letters, respectively. $[.]^T$, $[.]^H$, $[.]^*$, tr$(.)$ and $\mathbb{E}(.)$ denote transpose, Hermitian, conjugate, trace and statistical expectation, respectively. $\mathbb{C}^{p\times q}$ denotes the set of $p \times q$ complex matrices. diag$(\mathbf{a})$ denotes a diagonal matrix whose entries are the elements of the vector $\mathbf{a}$.

II. System Model

We consider a multiple-input single-output (MISO) broadcast channel whose transmitter with M transmit antennas serves L single-antenna receivers which have unequal coherence times (see Fig. 1). Throughout the paper, the receiving terminals are denoted 'receiver' or 'user.' The channel coefficient vector from the transmitter to User ℓ is denoted $\mathbf{h}_\ell \in \mathbb{C}^{M\times 1}$ having independent identically distributed (i.i.d.) entries with the distribution $\mathcal{CN}(0, 1)$. In this paper, we restrict our attention to $M = L$. The system operates under

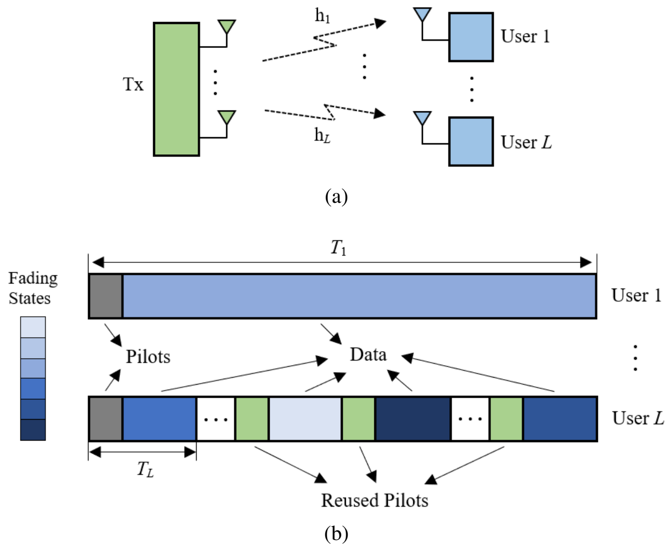

Fig. 1: (a) MISO broadcast channel, and (b) ordering the link coherence times, and pilot placement in the proposed scheme

block-fading, where $\mathbf{h}_\ell$ remains unchanged for T_ℓ time slots, and change independently across blocks. We order the link coherence times, in descending order (see Fig. 1):

$$T_L < \cdots < T_1$$

For simplicity, we assume that $\frac{T_\ell}{T_{\ell+1}}$ is an integer greater than 1. This assumption may be relaxed using techniques developed in [10], but are omitted in this paper for brevity and to concentrate on the most important parts of the development. Common pilots and reused pilots are respectively indicated by gray and green in Fig. 1. The received signal is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} = \left[\mathbf{h}_1, \cdots, \mathbf{h}_L\right]^T$ is the channel matrix collecting all channel vectors, $\mathbf{n} \in \mathbb{C}^L$ is additive Gaussian noise with i.i.d. entries $\mathcal{CN}(0, N_0)$, and $\mathbf{x} \in \mathbb{C}^M$ is the transmit signal. The transmit signal is subject to the average power constraint $\mathbb{E}[\|\mathbf{x}\|^2] \leqslant P$ at each time slot.

The transmit signal is $\mathbf{x} = \mathbf{WQs}$, such that $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_L] \in \mathbb{C}^{M\times L}$ is a zero-forcing precoding matrix $\mathbf{W} = \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}$ [13], $\mathbf{Q} = \mathrm{diag}\left(\sqrt{\rho_1}, \cdots, \sqrt{\rho_L}\right)$ is the allocated powers to all users, and $\mathbf{s} = [s_1, \cdots, s_L]^T \in \mathbb{C}^L$ contains L independent normalized symbols intended for L users. Additive noises at the receivers are denoted $\mathbf{n}_\ell$ and are i.i.d. Gaussian $\mathcal{CN}(0, N_0)$.

III. Non-Orthogonal Transmission Scheme and Achievable Rates

The integer coherence time ratios allow us to concentrate on a single time period of length T_1, with all operations repeating every T_1 time slots. Assuming the smallest coherence time is T_L, we need $\frac{T_1}{T_L}$ pilot transmissions within T_1, because User L experiences $\frac{T_1}{T_L}$ different channel realizations in the duration. User 1, on the other hand, needs only one pilot during this time, since its channel realization remains unchanged in the duration. Other users fall somewhere in between. After each of the pilot intervals, some users will refresh their channel estimates, all new channel estimates are fed back to the transmitter,[1] and the transmitter updates its beamforming vectors.

Any time a user exploits/reuses a pilot interval for data transmission, it will be unable to estimate the channel during that time interval. Therefore, we must keep track of the pilot intervals needed for each user, and how many data transmission opportunities are available for that user. At each of the candidate pilot intervals, an arbitrary User ℓ may have a channel that, depending on T_ℓ, either has transitioned to a new value since the last time it estimated the channel, or it has not. If the link for User ℓ is experiencing a new realization since the last refresh, the user needs to estimate the new channel and therefore needs the pilot, and cannot reuse the pilot interval for data. But if its channel has remained unchanged since the last refresh, User ℓ does not need another pilot at this time. The same effect holds for all users.

Given that overall understanding, during the first T_L time slots (please consult the figure), all users need to estimate their channel and need a pilot, therefore the transmit signal is

$$\mathbf{X} = \left[\mathbf{P}, \, \mathbf{WQS}\right], \tag{2}$$

where $\mathbf{P} \in \mathbb{C}^{M\times M}$ is the pilot matrix. $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_L]^T \in \mathbb{C}^{L\times(T_L - M)}$ is the codeword carrying data for L users. The corresponding received signal for each user during the first T_L time slots is

$$\mathbf{y}_\ell = \left[\mathbf{h}_\ell^T\mathbf{P}, \, \mathbf{h}_\ell^T \sum_{i=1}^{L} \mathbf{w}_i\sqrt{\rho_i}\,\mathbf{s}_i\right] + \mathbf{n}_\ell. \tag{3}$$

All users estimate their channels during the M time slots in the pilot $\mathbf{P}$, and return it to the transmitter through perfect or imperfect feedback link, instantaneously. Then, data transmission will occur for $T_L - M$ time slots, via the precoder that the transmitter chooses according to the channel state feedback.

In the remaining $T_1 - T_L$ time slots, transmission occurs over $\frac{T_1}{T_L} - 1$ blocks each with length T_L. In each of these blocks the transmitted signal has the following form:

$$\mathbf{X} = \left[\mathbf{U}, \, \mathbf{WQS}\right], \tag{4}$$

with the corresponding received values

$$\mathbf{y}_\ell = \left[\mathbf{h}_\ell^T\mathbf{U}, \, \mathbf{h}_\ell^T \sum_{i=1}^{L} \mathbf{w}_i\sqrt{\rho_i}\,\mathbf{s}_i\right] + \mathbf{n}_\ell. \tag{5}$$

Clearly the main difference compared with Eq. (2) is in $\mathbf{U}$, a signal component that is designed to carry data for some users, but still be able to serve for channel estimation for other users, in the following manner. We build $\mathbf{U}$ to carry data via zero-

[1] We shall see this can be an estimate of a physical link gain, or an estimate of an *equivalent* channel that is the product of one link gain and another users' transmitted data.

forcing beamforming, as follows:

$$\mathbf{U} = \mathbf{W}'\mathbf{Q}'\mathbf{S}' \tag{6}$$

During the times that the transmitter is emitting $\mathbf{U}$, User ℓ receives a noisy version of $\mathbf{h}_\ell^T\mathbf{U}$. If User ℓ already knows $\mathbf{h}_\ell$ and does not need channel estimation at this time, it can attempt to decode $\mathbf{U}$.[2] If User ℓ does *not* know $\mathbf{h}_\ell$ at this time, it will attempt to estimate $\widetilde{\mathbf{h}}_\ell = \mathbf{h}_\ell^T\mathbf{U}$ and feed it back to the transmitter. The transmitter has full knowledge of transmitted value $\mathbf{U}$, therefore can estimate the true channel $\mathbf{h}_\ell$ and use it for beamforming.[3]

We now outline an accounting of time slots needed for rate calculations. The first M time slots in both Eqs. (2), (4) are referred to as *pilot phase* while the remaining $T_L - M$ time slots are called the *data phase*. User ℓ, within one of its coherence intervals T_ℓ, has $T_\ell - (\frac{T_\ell}{T_L})M$ time slots in data phase, but also can transmit data during $(\frac{T_\ell}{T_L} - 1)M$ time slots under pilot phase. We allow different powers in data and pilot phases. The rate for User ℓ can be achieved as

$$R_\ell = \frac{M}{T_\ell}\sum_{i=1}^{\frac{T_\ell}{T_L}-1}\log(1+\frac{\rho'_{\ell i}}{N'_{\ell i}}) + (\frac{T_L - M}{T_\ell})\sum_{j=1}^{\frac{T_\ell}{T_L}}\log(1+\frac{\rho_{\ell j}}{N_{\ell j}}), \tag{7}$$

where $N_{\ell j}$ denotes the total noise power at User ℓ during the data phase of block j, $N'_{\ell i}$ denotes the total noise power at User ℓ during pilot i. Both $N_{\ell j}$ and $N'_{\ell i}$ need to be calculated in each block based on the receiver noise and channel estimation error which will be calculated later in this section for different feedback links. $\rho'_{\ell i}$ and $\rho_{\ell j}$ denote the allocated powers to User ℓ over pilot i and the data phase of block j, respectively. The first term in (7) is the achieved rate over $(\frac{T_\ell}{T_L} - 1)$ pilot phases, each with length M. The allocated powers to the users are not the same in all pilot phases. The second term in (7) is the achieved rate over the data phases, which can be obtained by averaging over $\frac{T_\ell}{T_L}$ data phases, each with length $T_L - M$. User L with the shortest coherence time T_L does not receive data over the pilot phase, since its channel needs to be estimated every T_L time slots. Thus, the rate for User L can be achieved as

$$R_L = (1 - \frac{M}{T_L})\log(1+\frac{\rho_L}{N}), \tag{8}$$

where N denotes the total noise power at the receiver and needs to be calculated according to the receiver noise and channel estimation error.

In the following, we discuss perfect and imperfect feedback channels. We show how they operate in the proposed transmission scheme and how they affect the achievable rates in (7) and (8) by carefully estimating the true channel at the transmitter and calculating the total noise power at the receivers.

A. Perfect Feedback

In this part, we consider a perfect feedback link, even though the channel estimation at each receiver is not perfect (i.e., there exists a channel estimation error modeled as Gaussian noise). Each user feeds back the estimate of its true or equivalent channel to the transmitter through an errorless feedback link immediately after downlink training. In the first block with length T_L, User ℓ receives (3) and estimates $\mathbf{h}_\ell$ during the common pilot phase with length M. Assuming $\mathbf{P} = \mathbf{I}_M$, User ℓ estimates its channel from the observation

$$\mathbf{r}_\ell = \sqrt{P}\,\mathbf{h}_\ell + \overline{\mathbf{n}}_\ell, \tag{9}$$

where $\overline{\mathbf{n}}_\ell \in \mathbb{C}^M$ is additive Gaussian noise with i.i.d. entries $\mathcal{CN}(0, N_0)$. The minimum mean square error (MMSE) estimate $\overline{\mathbf{h}}_\ell$ of the true channel $\mathbf{h}_\ell$ can be obtained as [14]

$$\begin{aligned}\overline{\mathbf{h}}_\ell &= \mathbb{E}\big[\mathbf{h}_\ell\mathbf{r}_\ell^H\big]\,\mathbb{E}\big[\mathbf{r}_\ell\mathbf{r}_\ell^H\big]^{-1}\mathbf{r}_\ell \\ &= \frac{\sqrt{P}}{N_0 + P}\mathbf{r}_\ell,\end{aligned} \tag{10}$$

with the estimation error $\boldsymbol{\omega}_\ell = \mathbf{h}_\ell - \overline{\mathbf{h}}_\ell$ that is Gaussian and uncorrelated with $\overline{\mathbf{h}}_\ell$. The covariance of $\boldsymbol{\omega}_\ell$ is $\sigma_\ell^2\mathbf{I}_M$ with

$$\sigma_\ell^2 = \frac{N_0}{N_0 + P}. \tag{11}$$

The estimate channel matrix $\overline{\mathbf{H}} = \big[\overline{\mathbf{h}}_1, \cdots, \overline{\mathbf{h}}_L\big]^T$ is perfectly available at the transmitter to design the precoding matrix $\mathbf{W}$. The true channel matrix $\mathbf{H}$ can be written in terms of the estimate channel and estimation error as

$$\mathbf{H} = \overline{\mathbf{H}} + \boldsymbol{\Omega}, \tag{12}$$

where $\boldsymbol{\Omega} = \big[\boldsymbol{\omega}_1, \cdots, \boldsymbol{\omega}_L\big]^T$ is the channel estimation error matrix that is uncorrelated with $\overline{\mathbf{H}}$. Replacing (12) in (1), the received signal by L users at each time slot of the data phase can be written as

$$\begin{aligned}\mathbf{y} &= \mathbf{HWQs} + \mathbf{n} \\ &= \big[\overline{\mathbf{H}} + \boldsymbol{\Omega}\big]\mathbf{WQs} + \mathbf{n} \\ &= \mathbf{Qs} + \boldsymbol{\Omega}\mathbf{WQs} + \mathbf{n} \\ &= \mathbf{Qs} + \hat{\mathbf{n}},\end{aligned} \tag{13}$$

where $\hat{\mathbf{n}} = \boldsymbol{\Omega}\mathbf{WQs} + \mathbf{n}$ denotes the total noise at the receivers which combines the additive noise $\mathbf{n}$ and residual channel estimation error. The covariance of the total noise at the receivers can be calculated as

$$\begin{aligned}\mathbb{E}[\hat{\mathbf{n}}\,\hat{\mathbf{n}}^H] &= \mathbb{E}\Big[\big(\boldsymbol{\Omega}\mathbf{WQs} + \mathbf{n}\big)\big(\boldsymbol{\Omega}\mathbf{WQs} + \mathbf{n}\big)^H\Big] \\ &= \mathbb{E}\Big[\mathrm{tr}\big(\mathbf{WQss}^H\mathbf{Q}^H\mathbf{W}^H\big)\Big]\sigma_\ell^2\mathbf{I}_L + N_0\mathbf{I}_L \\ &= \bigg(\frac{2PN_0 + N_0^2}{P + N_0}\bigg)\mathbf{I}_L,\end{aligned} \tag{14}$$

where we used the fact $\mathbb{E}[\boldsymbol{\Gamma}\mathbf{A}\boldsymbol{\Gamma}^H] = \mathrm{tr}(\mathbf{A})\mathbf{I}_N$ for any $\mathbf{A} \in \mathbb{C}^{N\times N}$ [15], [16] when the entries of $\boldsymbol{\Gamma} \in \mathbb{C}^{N\times N}$ are

[2] For the purposes of achieving capacity, there is an outer code that allows the collection of symbols and length of the codeword to be sufficiently large. However, for simplicity of expression, we simply say "decode $\mathbf{U}$" when certain symbols belong to a codeword that can be decoded. Well-known assumptions under CSIT, such as separable coding of states, are implicit.

[3] We note that $\mathbf{U}$ is a full-rank square matrix, because the zero-forcing matrix $\mathbf{W}'$ is full-rank with probability one, the Gaussian codeword $\mathbf{S}'$ is full-rank with probability one, and the diagonal power matrix $\mathbf{Q}'$ is also full-rank.

i.i.d. complex Gaussian with zero mean and unit variance. We also used $\mathbb{E}\big[\mathrm{tr}\big(\mathbf{W}\mathbf{Q}\mathbf{s}\mathbf{s}^H\mathbf{Q}^H\mathbf{W}^H\big)\big] = P$ due to the total power constraint at the transmitter, and $\mathbb{E}\big[\mathbf{n}\mathbf{n}^H\big] = N_0\mathbf{I}_L$.

In the remaining blocks, each with length T_L, User ℓ receives (5) and if/when its channel experiences a transition, it estimates the equivalent channel $\widetilde{\mathbf{h}}_\ell = \mathbf{h}_\ell^T\mathbf{U}$ from the observation

$$\widetilde{\mathbf{r}}_\ell = \widetilde{\mathbf{h}}_\ell + \overline{\mathbf{n}}_\ell. \tag{15}$$

The MMSE estimate of $\widetilde{\mathbf{h}}_\ell$ is denoted $\breve{\mathbf{h}}_\ell$:

$$\begin{aligned}\breve{\mathbf{h}}_\ell &= \mathbb{E}\Big[\widetilde{\mathbf{h}}_\ell\widetilde{\mathbf{r}}_\ell^H\Big]\mathbb{E}\Big[\widetilde{\mathbf{r}}_\ell\widetilde{\mathbf{r}}_\ell^H\Big]^{-1}\widetilde{\mathbf{r}}_\ell \\ &= \frac{P}{N_0+P}\big(\widetilde{\mathbf{h}}_\ell + \overline{\mathbf{n}}_\ell\big).\end{aligned} \tag{16}$$

The channel estimate $\breve{\mathbf{h}}_\ell$ is communicated with the transmitter via a perfect channel. The transmitter computes the estimate $\widehat{\mathbf{h}}_\ell$ of the true channel $\mathbf{h}_\ell$ multiplying $\breve{\mathbf{h}}_\ell$ by $\mathbf{U}^{-1}$. As explained earlier, $\mathbf{U}$ is full rank and therefore invertible. Thus, $\widehat{\mathbf{h}}_\ell$ can be obtained as

$$\widehat{\mathbf{h}}_\ell = \frac{P}{N_0+P}\big(\mathbf{h}_\ell^T + \overline{\mathbf{n}}_\ell\mathbf{U}^{-1}\big), \tag{17}$$

with the estimation error

$$\widehat{\boldsymbol{\omega}}_\ell = \mathbf{h}_\ell - \widehat{\mathbf{h}}_\ell^T = \frac{N_0}{N_0+P}\mathbf{h}_\ell - \frac{P}{N_0+P}(\mathbf{U}^{-1})^T\overline{\mathbf{n}}_\ell^T. \tag{18}$$

The covariance of the error is then calculated as

$$\begin{aligned}\mathbb{E}\Big[\widehat{\boldsymbol{\omega}}_\ell\widehat{\boldsymbol{\omega}}_\ell^H\Big] =& \bigg(\frac{N_0^2}{(N_0+P)^2} \\ &+ \frac{P^2N_0}{(N_0+P)^2}\mathbb{E}\Big[\mathrm{tr}\big(\mathbf{U}^*\mathbf{U}^T\big)^{-1}\Big]\bigg)\mathbf{I}_M,\end{aligned} \tag{19}$$

which can be used in (12)-(14) to calculate the total noise power at the receivers and then calculate the achievable rates using (7) and (8). The calculated covariance in (19) shows that the pilot reuse increases the error on available CSI at the transmitter.

B. Imperfect Feedback

In this part, we assume that the feedback channel is imperfect which increases the noise level in received signals. We consider analog feedback scheme [17] where each user transmits on the feedback channel a scaled version of its downlink training observation. Therefore, over the first block with length T_L, User ℓ transmits the scaled version of its observation in (9). The received signal at the transmitter is given by

$$\begin{aligned}\mathbf{z}_\ell &= \frac{\sqrt{P}}{\sqrt{N_0+P}}\mathbf{r}_\ell + \mathbf{m} \\ &= \frac{P}{\sqrt{N_0+P}}\mathbf{h}_\ell + \frac{\sqrt{P}}{\sqrt{N_0+P}}\overline{\mathbf{n}}_\ell + \mathbf{m} \\ &= \frac{P}{\sqrt{N_0+P}}\mathbf{h}_\ell + \widetilde{\mathbf{m}},\end{aligned} \tag{20}$$

where $\mathbf{m}$ denotes additive Gaussian noise at the transmitter with i.i.d. entries $\mathcal{CN}(0, N_0)$ and is independent of $\overline{\mathbf{n}}_\ell$. $\widetilde{\mathbf{m}}$ denotes the total Gaussian noise at the transmitter with covariance

$$\begin{aligned}\mathbb{E}\Big[\widetilde{\mathbf{m}}\,\widetilde{\mathbf{m}}^H\Big] &= \Big(\frac{P}{N_0+P}\Big)\mathbb{E}\Big[\overline{\mathbf{n}}_\ell\overline{\mathbf{n}}_\ell^H\Big] + \mathbb{E}\Big[\mathbf{m}\mathbf{m}^H\Big] \\ &= N_0\Big(\frac{2P+N_0}{N_0+P}\Big)\mathbf{I}_M.\end{aligned} \tag{21}$$

The transmitter observes $\mathbf{z}_\ell$ and computes the MMSE estimate $\overline{\mathbf{h}}_\ell$ of the true channel $\mathbf{h}_\ell$ as

$$\overline{\mathbf{h}}_\ell = \frac{P}{\sqrt{N_0+P}(N_0+P)}\mathbf{z}_\ell, \tag{22}$$

with the estimation error

$$\overline{\boldsymbol{\omega}}_\ell = \mathbf{h}_\ell - \overline{\mathbf{h}}_\ell = \frac{N_0^2+2PN_0}{(N_0+P)^2}\mathbf{h}_\ell - \frac{P}{(N_0+P)\sqrt{N_0+P}}\widetilde{\mathbf{m}}. \tag{23}$$

The covariance of the error can be calculated as

$$\mathbb{E}\Big[\overline{\boldsymbol{\omega}}_\ell\overline{\boldsymbol{\omega}}_\ell^H\Big] = \frac{(N_0+2PN_0)^2 + P^2N_0(2P+N_0)}{(N_0+P)^4}\mathbf{I}_M. \tag{24}$$

Similar to (12), the true channel $\mathbf{H}$ can be written in terms of the estimate channel $\overline{\mathbf{H}} = \big[\overline{\mathbf{h}}_1, \cdots, \overline{\mathbf{h}}_L\big]^T$ and estimation error $\overline{\boldsymbol{\Omega}} = \big[\overline{\boldsymbol{\omega}}_1, \cdots, \overline{\boldsymbol{\omega}}_L\big]^T$ as $\mathbf{H} = \overline{\mathbf{H}} + \overline{\boldsymbol{\Omega}}$. The entries of $\overline{\mathbf{H}}$ and $\overline{\boldsymbol{\Omega}}$ are given in (22) and (23), respectively. The transmitter designs the precoding matrix $\mathbf{W}$ to transmit data. The received signal by L users at each time slot of the data phase can be written as

$$\begin{aligned}\mathbf{y} &= \mathbf{H}\mathbf{W}\mathbf{Q}\mathbf{s} + \mathbf{n} \\ &= \big[\overline{\mathbf{H}} + \overline{\boldsymbol{\Omega}}\big]\mathbf{W}\mathbf{Q}\mathbf{s} + \mathbf{n} \\ &= \mathbf{Q}\mathbf{s} + \overline{\boldsymbol{\Omega}}\mathbf{W}\mathbf{Q}\mathbf{s} + \mathbf{n} \\ &= \mathbf{Q}\mathbf{s} + \breve{\mathbf{n}},\end{aligned} \tag{25}$$

where $\breve{\mathbf{n}} = \overline{\boldsymbol{\Omega}}\mathbf{W}\mathbf{Q}\mathbf{s} + \mathbf{n}$ is the total noise at the receivers and its covariance can be calculated through the same manner in (14) as

$$\begin{aligned}\mathbb{E}\Big[\breve{\mathbf{n}}\,\breve{\mathbf{n}}^H\Big] =& \bigg(\frac{P(N_0+2PN_0)^2}{(N_0+P)^4} \\ &+ \frac{P^3N_0(2P+N_0)}{(N_0+P)^4} + N_0\bigg)\mathbf{I}_L.\end{aligned} \tag{26}$$

In the remaining blocks, each with length T_L, User ℓ whose channel has experienced a transition, receives (15) during the first M time slots and returns its scaled version to the transmitter. The received signal at the transmitter is given by

$$\begin{aligned}\widetilde{\mathbf{z}}_\ell &= \frac{\sqrt{P}}{\sqrt{N_0+P}}(\widetilde{\mathbf{h}}_\ell + \overline{\mathbf{n}}_\ell) + \mathbf{m} \\ &= \frac{\sqrt{P}}{\sqrt{N_0+P}}\widetilde{\mathbf{h}}_\ell + \frac{\sqrt{P}}{\sqrt{N_0+P}}\overline{\mathbf{n}}_\ell + \mathbf{m} \\ &= \frac{\sqrt{P}}{\sqrt{N_0+P}}\widetilde{\mathbf{h}}_\ell + \widetilde{\mathbf{m}},\end{aligned} \tag{27}$$

where $\widetilde{\mathbf{m}}$ is the total Gaussian noise at the transmitter with the covariance calculated in (21). The transmitter first computes the MMSE estimate of the equivalent channel $\widetilde{\mathbf{h}}_\ell$ from the

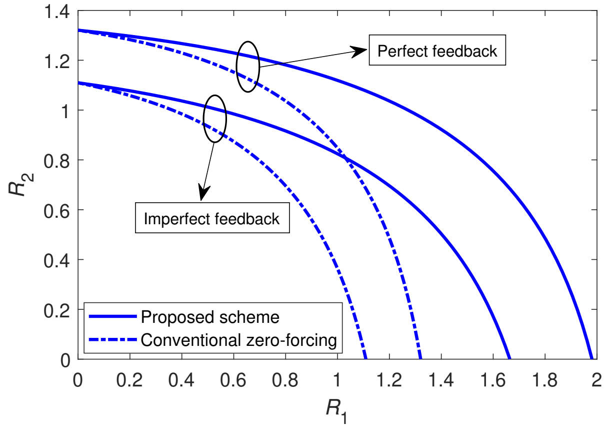

Fig. 2: Achievable rate regions via proposed transmission scheme and conventional zero-forcing considering perfect and imperfect feedback channels

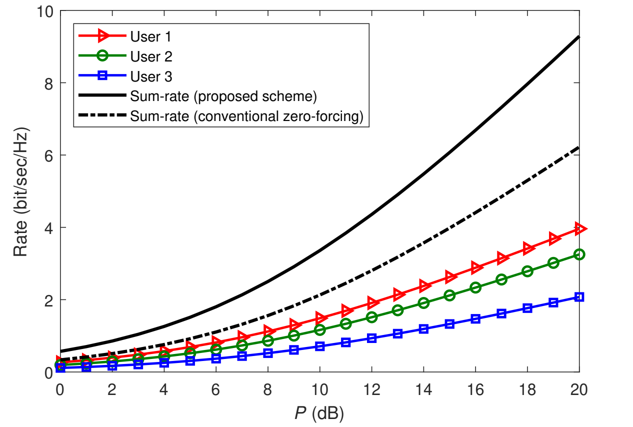

Fig. 3: Achievable sum-rates via proposed transmission scheme and conventional zero-forcing in a three-user MISO broadcast channel

observation $\widetilde{\mathbf{z}}_\ell$ as

$$\breve{\mathbf{h}}_\ell = \frac{P\sqrt{P(P+N_0)}}{P^2+N_0(2P+N_0)}\Big(\frac{\sqrt{P}}{\sqrt{N_0+P}}\widetilde{\mathbf{h}}_\ell+\widetilde{\mathbf{m}}\Big). \tag{28}$$

Then, it obtains the estimate $\widehat{\mathbf{h}}_\ell$ of the true channel $\mathbf{h}_\ell$ multiplying $\breve{\mathbf{h}}_l$ by $\mathbf{U}^{-1}$ as

$$\widehat{\mathbf{h}}_\ell = \frac{P\sqrt{P(P+N_0)}}{P^2+N_0(2P+N_0)}\Big(\frac{\sqrt{P}}{\sqrt{N_0+P}}\mathbf{h}_\ell^T+\widetilde{\mathbf{m}}\mathbf{U}^{-1}\Big), \tag{29}$$

with the estimation error

$$\widehat{\boldsymbol{\omega}}_\ell = \mathbf{h}_\ell - \widehat{\mathbf{h}}_\ell^T = \frac{N_0(2P+N_0)}{P^2+N_0(2P+N_0)}\mathbf{h}_\ell - \frac{P\sqrt{P(P+N_0)}}{P^2+N_0(2P+N_0)}(\mathbf{U}^{-1})^T\widetilde{\mathbf{m}}^T. \tag{30}$$

The covariance of the error is then calculated as (31) which can be used to calculate the total noise power at the receivers and obtain the rate results.

IV. Numerical Results

In this section, we present numerical results to demonstrate the achievable rate gains through the proposed transmission scheme. For the sake of comparison, we also include the results for conventional zero-forcing without product superposition in which only pure pilots are sent to all users (i.e., pilots and data are not overlapping). Unless stated otherwise, we assume $M = L$, $N_0 = 1$ W, $P = 10$ dB, and uniform power allocation across users.

In Fig. 2, we present the achievable rate regions through the proposed transmission scheme and conventional zero-forcing under both perfect and imperfect feedback links. Here, we consider a two-user MISO broadcast channel where User 1 has coherence time $T_1 = 8$ and User 2 has coherence time $T_2 = 4$. It is observed that under equal system parameters, the proposed transmission scheme provides a significant gain over the conventional zero-forcing in achievable rate region. It is also observed that the achievable rate region is degraded through imperfect feedback link. This is due to the fact that the imperfect feedback channel increases the error on available CSI at the transmitter.

In Fig. 3, we present the individual rates and sum-rate achieved through the proposed transmission scheme for a three-user MISO broadcast channel assuming imperfect feedback link. We also present the achieved sum-rate through conventional zero-forcing. We assume the coherence times $T_1 = 24$, $T_2 = 12$ and $T_3 = 6$ for User 1, User 2 and User 3, respectively. It is observed that User 1 achieves higher rate than other users due to the fact that it has the longest coherence time and contains more reused pilots to receive data. It is also observed that the proposed scheme provides a significant gain over the conventional zero-forcing in achievable sum-rate. For example, at the target sum-rate of 6 bit/sec/Hz, a gain of 4.6 dB is achieved through the proposed scheme.

V. Conclusion

In this paper, we have proposed a new non-orthogonal transmission scheme for a multi-user MISO broadcast channel with coherence disparity under channel state feedback. We have carefully designed the transmission scheme based on pilot reuse and beamforming strategies and derived the resulting achievable rates. The proposed scheme is an efficient solution for the downlink pilot reuse problem under channel state feedback that allows both product superposition and beamforming to work within the same framework. Our results indicated that the proposed transmission scheme outperforms the conventional zero-forcing in achievable rates.

$$\mathbb{E}\Big[\widehat{\boldsymbol{\omega}}_\ell\widehat{\boldsymbol{\omega}}_\ell^H\Big] = \Bigg(\Big(\frac{N_0(2P+N_0)}{P^2+N_0(2P+N_0)}\Big)^2 + \Big(\frac{P\sqrt{P(P+N_0)}}{P^2+N_0(2P+N_0)}\Big)^2\Big(\frac{2PN_0+N_0^2}{N_0+P}\Big)\mathbb{E}\Big[\mathrm{tr}\big(\mathbf{U}^*\mathbf{U}^T\big)^{-1}\Big]\Bigg)\mathbf{I}_M \tag{31}$$

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