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Index Modulation with Channel Training: Spectral Efficiency and Optimal Antenna Alphabets

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***Abstract*—Index modulation is a MIMO technology where some transmit antennas are idle during each transmission interval, but the receiver requires knowledge of all link gains at all times. Even though index modulation is particularly sensitive to the cost of estimating the channel state information (CSI), the impact of CSI cost and imperfections on the capacity of index modulation has not been adequately characterized until now. As a result, the marginal cost/benefit of each additional antenna, and the optimal antenna alphabet have remained unclear. This study computes the spectral efficiency of index modulation subject to training and determines optimal antenna alphabets. Our approach involves a comprehensive examination of the influence of pilot power and degrees of freedom on the achievable rate of index modulation. The results include 2.5dB improvement over the best previously known bound for 4 × 2 spatial modulation at 6b/s/Hz. Additionally, we determine the conditions under which single-antenna transmission is superior to spatial modulation, and vice versa. Moreover, this training-based spectral efficiency analysis is extended to the multiuser uplink, identifying the number of users that can be accommodated while maximizing the uplink sum-rate under spatial modulation.**

***Index Terms*—Index modulation, spatial modulation, pilots, channel training, channel estimation**

I. Introduction

Spatial modulation selects one out of M available transmit antennas per channel use, and transmits a modulation symbol from the selected antenna [1], [2], [3]. The selection index and modulation symbol both carry information [4], [5]. The motivation for spatial modulation is often reducing hardware complexity, but in the process the achievable rate may be reduced. Generalized spatial modulation allows $L > 1$ transmit antennas to be selected at each time [6], [7], providing more options in the tradeoff between complexity and performance. The idea of spatial modulation has been extended to the selection of sub-carriers and time slots; this collection of techniques that subsumes spatial modulation is known as *index modulation.*

A large part of the literature on spatial modulation assumes free CSI at the receiver [4], [5], [1], including spectral efficiency calculations with free and perfect CSI [8], [9], [10], [11]. Practical systems, however, obtain CSI via pilots and estimation, introducing two important features: first, acquiring CSI incurs a *cost* in power and transmission time (degrees of freedom). The cost of CSI can be a predominant design issue in some spatial modulation scenarios. Second, the CSI obtained by pilots and estimation is imperfect and contains some noise. In several studies of the *bit-error performance* of spatial modulation [12], [13], [14], [15], [16], [17], imperfect CSI has featured prominently but the *cost* of pilots has been disregarded.

In the analysis of *spectral efficiency,* the effect of these two features (the cost of CSI and its imperfection) cannot be meaningfully separated from one another, requiring a more careful analysis. Rajashekar *et al.* [9] derived a lower bound on the training-based capacity of spatial modulation which used a loose approximation for the rate of the index (see Section II), and disregarded the influence of the rate of the index in data/pilot power optimization. Both these issues are resolved in the present work. Furthermore, the results of [9] are neither formally extended to generalized spatial modulation, nor is any straightforward extension of [9] self-evident. He *et al.* [18] analyzed spectral efficiency in the multiuser uplink. As acknowledged in [18, proof of Lemma 2], the analysis lacks a tractable term for the rate of the index, and substitutes heuristics for it. Further, the approximation for the modulation symbol rate introduces weakness in the results, as explained in Sec. IV of the present paper. Zhang *et al.* [19] extend the analysis of [18] to generalized spatial modulation, but unfortunately it also inherits the limitations of [18].

To summarize, the existing works on spectral efficiency of index modulation subject to pilots and training [9], [18], [19] have not provided sufficiently accurate bounds on modulation and coding rates, which are necessary for the efficient operation of practical systems. Furthermore, due to the difficulties encountered in their analysis, important design and operational questions have remained unanswered. Among them: it has been unclear when spatial modulation is superior to SIMO and vice versa, and how many antennas must optimally participate in index modulation (antenna alphabets). The manner of dependence of these issues on the channel dynamics and SNR is of significant interest, but has remained open thus far.

A. Contributions of this Work

This work presents an accurate characterization of the spectral efficiency of spatial and generalized spatial modulation via the derivation of a tight lower bound that accounts for training overhead and training error. In contrast to [9], the present work develops and utilizes an exact expression for the rate carried by the index. For example, at 6 bits/s/Hz, our lower bound is 2.5 dB tighter than the best available bound [9] for 4×2 spatial modulation.

In addition to the practical importance of accurate modulation and coding rates, our results also address broader

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operational questions that have so far remained unanswered. For instance: when should we use spatial modulation, and when should we simply use fixed single-antenna signaling? In the former, the RF chain will switch among antennas, allowing a component rate to be transmitted via the antenna index, but it also requires pilots for each antenna. In the latter, the RF chain is always connected to the same antenna, reducing the training requirement, but also giving up on the rate that could be emitted via the antenna index. It has been speculated that SIMO is preferable to spatial modulation in highly dynamic channels and at low SNR; our analysis for the first time corroborates this intuition with rigor, and produces the boundary of SNR vs. coherence length that characterizes which approach should be chosen.

Our results also determine the optimal antenna alphabet for spatial modulation, i.e., the number of antennas that must participate in spatial modulation to yield maximal spectral efficiency, subject to training and pilots.[1] It is not always optimal to utilize all the available antennas; sometimes it is better to leave some of them completely unused and save the pilots that would be required for them. We show that the optimal antenna alphabet size grows with SNR and channel coherence time, and show the exact manner of this dependence.

The analysis of spectral efficiency[2] is also extended to the multiuser uplink, with users employing index modulation to communicate with a multi-antenna base station. Unlike [18], [19], we offer an exact analysis for the index and modulation symbols. We illustrate that our results produce a better and more reliable estimate of the required system resources, compared with the lower bound of [18]. For example, let us consider 5 mobiles, each employing two antennas for spatial modulation, operating at 1.6 bits/s/Hz at 0dB. The result of [18] estimates that 16 antennas are needed at the base station, while our sharpened results show that 12 antennas can be sufficient, if properly optimized as shown herein.[3] Thus, our results have a tangible impact on system design. As in the point-to-point setting, our results also illustrate the regimes where SIMO is preferable to spatial modulation, or alternatively, a k-transmit antenna MIMO is preferable to k-RF-chain generalized spatial modulation. Our results also indicate the optimal number of users that achieve the maximum sum-rate, all other operating parameters being fixed. An early version of some results of this paper appeared in [20].

The rest of the paper is organized as follows. The training-based spectral efficiency of point-to-point spatial modulation is analyzed in Sec. II, and that of generalized spatial modulation is analyzed in Sec. III. The spectral efficiency results are extended to the multiuser index modulation in Sec. IV. Finally, the paper is concluded in Sec. V.

[1]This can be considered a generalization of the question of SIMO vs. spatial modulation.

[2]Throughout the paper, whenever not mentioned explicitly, the existence of pilots, training, and estimation errors is implicit in the analysis of spectral efficiency.

[3]A similar comparison applies at other settings, too.

II. Spatial Modulation

Consider a multi-antenna system with M transmit and N receive antennas. The $M \times N$ spatial modulation activates one transmit antenna per channel use and transmits a symbol. The system model is characterized by

$$\mathbf{y} = \sqrt{\rho}\left(\sum_{i=1}^{M} \mathbf{g}_i v_i\right) z + \mathbf{w}, \tag{1}$$

where ρ denotes the signal-to-noise ratio, $\mathbf{g}_i$ is the channel gain vector from transmit antenna i to N receive antennas, v_i is a binary variable that is zero if antenna i is inactive and one if antenna i is activated, and z denotes the modulation symbol transmitted from the active antenna. The noise $\mathbf{w}$ and the channel gain $\mathbf{g}_i$ are $N \times 1$ vectors whose components are i.i.d. and obey $\mathcal{CN}(0, 1)$. The system model in (1) can be written compactly as follows

$$\mathbf{y} = \sqrt{\rho}\mathbf{G}\mathbf{v}z + \mathbf{w}, \tag{2}$$

where $\mathbf{G} = [\mathbf{g}_1 \; \mathbf{g}_2 \; \cdots \; \mathbf{g}_M]$ and $\mathbf{v} = [v_1 \; v_2 \; \cdots \; v_M]^T$ such that $\mathbf{v}$ belong to the canonical basis $\{\mathbf{e}_i\}$. We assume a block-fading channel that remains constant over one coherence interval and changes independently in the subsequent coherence interval. Transmission is carried out in frames of length equal to one coherence interval, with each frame divided into a training phase followed by a data transmission phase. In the training phase, pilot signals are transmitted, and the channel is estimated at the receiver. In the data transmission phase, spatial modulation vectors are transmitted, which are decoded at the receiver based on the received signal and the knowledge of the estimated channel.

A. Spectral Efficiency with Estimated CSIR

Let $\widehat{\mathbf{G}}$ denote the MMSE estimate of $\mathbf{G}$ and $\tilde{\mathbf{G}}$ the estimation error, which are uncorrelated, zero-mean complex Gaussian matrices. The variance of the entries of $\tilde{\mathbf{G}}$ is denoted σ_e^2, thus entries of $\widehat{\mathbf{G}}$ have variance $1-\sigma_e^2$. The system model is equivalent to

$$\begin{aligned}\mathbf{y} &= \sqrt{\rho}\widehat{\mathbf{G}}\mathbf{v}z + \sqrt{\rho}\tilde{\mathbf{G}}\mathbf{v}z + \mathbf{w} \\ &= \sqrt{\rho}\widehat{\mathbf{G}}\mathbf{v}z + \tilde{\mathbf{w}},\end{aligned} \tag{3}$$

where $\tilde{\mathbf{w}} \triangleq \sqrt{\rho}\tilde{\mathbf{G}}\mathbf{v}z + \mathbf{w}$ is the effective noise at the receiver. Let $\widehat{\mathbf{g}}_i$ denote the column i of $\widehat{\mathbf{G}}$.

Proposition 1. *The capacity of spatial modulation with estimated CSIR satisfies the following lower bound:*

$$\begin{aligned}C \geq &\mathbb{E}_{\widehat{\mathbf{g}}_1}\left[\log_2\left(1 + \frac{\|\widehat{\mathbf{g}}_1\|^2\rho}{1+\rho\sigma_e^2}\right)\right] + \log_2 M - \\ &\mathbb{E}_{\mathbf{y},\widehat{\mathbf{G}}}\left[\sum_{i=1}^{M} \frac{f_i(\mathbf{y},\widehat{\mathbf{G}})}{\sum_{j=1}^{M} f_j(\mathbf{y},\widehat{\mathbf{G}})} \log_2 \frac{\sum_{j=1}^{M} f_j(\mathbf{y},\widehat{\mathbf{G}})}{f_i(\mathbf{y},\widehat{\mathbf{G}})}\right],\end{aligned} \tag{4}$$

where $z \sim \mathcal{CN}(0,1)$ and

$$f_i(\mathbf{y},\widehat{\mathbf{G}}) \triangleq \mathbb{E}_z\left[\frac{1}{(\pi(1+|z|^2\rho\sigma_e^2))^N} \exp\left(\frac{-\|\mathbf{y}-\sqrt{\rho}\widehat{\mathbf{g}}_i z\|^2}{1+|z|^2\rho\sigma_e^2}\right)\right]. \tag{5}$$

Proof. We aim to characterize a channel whose input is excited with a pilot sequence, antenna index, and modulation symbol, and whose output is the channel observations due to the pilot and data. Since channel estimates are a deterministic function of channel observations during pilot transmissions, by data processing inequality:

$$\begin{aligned} C &\geq \max_{p(\mathbf{v},z):\mathbb{E}(|z|^2)\leq 1} I(\mathbf{v},z;\mathbf{y},\widehat{\mathbf{G}}) \\ &\overset{(a)}{=} \max_{p(\mathbf{v},z):\mathbb{E}(|z|^2)\leq 1} [I(\mathbf{v},z;\mathbf{y}|\widehat{\mathbf{G}}) + I(\mathbf{v},z;\widehat{\mathbf{G}})] \\ &\overset{(b)}{\geq} I(\mathbf{v},z;\mathbf{y}|\widehat{\mathbf{G}}) + I(\mathbf{v},z;\widehat{\mathbf{G}}) \\ &\overset{(c)}{=} I(\mathbf{v},z;\mathbf{y}|\widehat{\mathbf{G}}), \end{aligned} \tag{6}$$

where (a) follows from chain rule, (b) replaces the optimal input distribution with an arbitrary distribution satisfying $\mathbb{E}(|z|^2) \leq 1$, and (c) follows from the independence of transmitted data (z and $\mathbf{v}$) from channel gains, and therefore of its estimate. We now calculate $I(\mathbf{v},z;\mathbf{y}|\widehat{\mathbf{G}})$ when $\mathbf{v}$ is uniformly distributed, and $z \sim \mathcal{CN}(0,1)$ independent of $\mathbf{v}$.

$$I(\mathbf{v},z;\mathbf{y}|\widehat{\mathbf{G}}) = I(z;\mathbf{y}|\mathbf{v},\widehat{\mathbf{G}}) + I(\mathbf{v};\mathbf{y}|\widehat{\mathbf{G}}). \tag{7}$$

The first term is a SIMO mutual information subject to estimated CSIR:

$$\begin{aligned} I(z;\mathbf{y}|\mathbf{v},\widehat{\mathbf{G}}) &\overset{(a)}{=} I(z;\mathbf{y}|\mathbf{v}=\mathbf{e}_1,\widehat{\mathbf{G}}) \\ &\overset{(b)}{=} I(z;\mathbf{y}|\mathbf{v}=\mathbf{e}_1,\widehat{\mathbf{g}}_1) \\ &\overset{(c)}{\geq} \mathbb{E}_{\widehat{\mathbf{g}}_1}\left[\log_2\left(1+\frac{\|\widehat{\mathbf{g}}_1\|^2\rho}{1+\rho\sigma_e^2}\right)\right], \end{aligned} \tag{8}$$

where (a) follows from the statistical symmetry between transmit antennas, (b) from the independence of channels from different transmit antennas, and (c) follows from the worst-case noise property [21] while utilizing the expectation form for conditional mutual information[4]. We now evaluate $I(\mathbf{v};\mathbf{y}|\widehat{\mathbf{G}})$ as follows:

$$\begin{aligned} I(\mathbf{v};\mathbf{y}|\widehat{\mathbf{G}}) &= h(\mathbf{v}|\widehat{\mathbf{G}}) - h(\mathbf{v}|\widehat{\mathbf{G}},\mathbf{y}) \\ &= \log_2 M - h(\mathbf{v}|\widehat{\mathbf{G}},\mathbf{y}), \end{aligned} \tag{9}$$

where we used independence of the uniformly distributed antenna index from channel gains. We now tend to the second term in (9)

$$h(\mathbf{v}|\widehat{\mathbf{G}},\mathbf{y}) = \mathbb{E}_{\mathbf{y},\widehat{\mathbf{G}}}\big[\sum_{i=1}^{M} p(\mathbf{v}=\mathbf{e}_i|\widehat{\mathbf{G}},\mathbf{y})\log_2\frac{1}{p(\mathbf{v}=\mathbf{e}_i|\widehat{\mathbf{G}},\mathbf{y})}\big], \tag{10}$$

where

$$\begin{aligned} p(\mathbf{v}=\mathbf{e}_i|\widehat{\mathbf{G}},\mathbf{y}) &= \frac{p(\mathbf{y}|\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}})p(\mathbf{v}=\mathbf{e}_i|\widehat{\mathbf{G}})}{\sum_{j=1}^{M} p(\mathbf{y}|\mathbf{v}=\mathbf{e}_j,\widehat{\mathbf{G}})p(\mathbf{v}=\mathbf{e}_j|\widehat{\mathbf{G}})} \\ &= \frac{p(\mathbf{y}|\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}})}{\sum_{j=1}^{M} p(\mathbf{y}|\mathbf{v}=\mathbf{e}_j,\widehat{\mathbf{G}})}, \end{aligned}$$

using once again the uniform distribution $p(\mathbf{v}=\mathbf{e}_i|\widehat{\mathbf{G}}) = \frac{1}{M}$.

$$\begin{aligned} p(\mathbf{y}|\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}}) &= \int_z p(\mathbf{y},z|\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}})dz \\ &= \int_z p(\mathbf{y}|z,\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}})p(z|\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}})dz \\ &= \mathbb{E}_z\big[p(\mathbf{y}|z,\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}})\big], \end{aligned}$$

where the last equality follows since $p(z|\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}}) = p(z)$ due to the independence of z from $\mathbf{v}$ and $\widehat{\mathbf{G}}$. Expressing $p(\mathbf{y}|\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}})$ as an expectation of $p(\mathbf{y}|z,\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}})$ enables further calculations, because from the model (3), $p(\mathbf{y}|z,\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}}) \sim \mathcal{CN}(\sqrt{\rho}\widehat{\mathbf{g}}_i z,(1+|z|^2\rho\sigma_e^2)\mathbf{I})$, therefore

$$\begin{aligned} &p(\mathbf{y}|\mathbf{v}=\mathbf{e}_i,\widehat{\mathbf{G}}) = \\ &\quad \mathbb{E}_z\left[\frac{1}{(\pi(1+|z|^2\rho\sigma_e^2))^N}\exp\left(\frac{-\|\mathbf{y}-\sqrt{\rho}\widehat{\mathbf{g}}_i z\|^2}{1+|z|^2\rho\sigma_e^2}\right)\right] \\ &\quad \triangleq f_i(\mathbf{y},\widehat{\mathbf{G}}). \end{aligned}$$

Therefore,

$$p(\mathbf{v}=\mathbf{e}_i|\widehat{\mathbf{G}},\mathbf{y}) = \frac{f_i(\mathbf{y},\widehat{\mathbf{G}})}{\sum_{j=1}^{M} f_j(\mathbf{y},\widehat{\mathbf{G}})}. \tag{11}$$

Using (11) in (10), combining the resulting expression with (9), and substituting (9) and (8) in (7) proves the proposition. □

B. Accounting Training Cost

Let T be the coherence interval of the channel in number of channel uses. Let T_τ and T_d denote the training interval and data transmission interval, respectively. Also, let ρ_τ and ρ_d denote the training and data SNR, respectively. Then, by conservation of time and energy

$$\begin{aligned} T &= T_\tau + T_d, \\ \rho T &= \rho_\tau T_\tau + \rho_d T_d. \end{aligned} \tag{12}$$

We now express the spectral efficiency of spatial modulation after accounting for the training time and energy in Proposition 1. With an abuse of notation, we redefine f_i from Eq. (5) by replacing ρ with ρ_d:

$$\begin{aligned} &f_i(\mathbf{y},\widehat{\mathbf{G}}) = \\ &\quad \mathbb{E}_z\left[\frac{1}{(\pi(1+|z|^2\rho_d\sigma_e^2))^N}\exp\left(\frac{-\|\mathbf{y}-\sqrt{\rho_d}\widehat{\mathbf{g}}_i z\|^2}{1+|z|^2\rho_d\sigma_e^2}\right)\right]. \end{aligned} \tag{13}$$

Then,

$$\begin{aligned} C_\tau \geq \frac{T-T_\tau}{T}&\left[\mathbb{E}_{\widehat{\mathbf{g}}_1}\left[\log_2\left(1+\frac{\|\widehat{\mathbf{g}}_1\|^2\rho_d}{1+\rho_d\sigma_e^2}\right)\right] + \log_2 M - \right. \\ &\left.\mathbb{E}_{\mathbf{y},\widehat{\mathbf{G}}}\left[\sum_{i=1}^{M}\frac{f_i(\mathbf{y},\widehat{\mathbf{G}})}{\sum_{j=1}^{M} f_j(\mathbf{y},\widehat{\mathbf{G}})}\log_2\frac{\sum_{j=1}^{M} f_j(\mathbf{y},\widehat{\mathbf{G}})}{f_i(\mathbf{y},\widehat{\mathbf{G}})}\right]\right], \end{aligned} \tag{14}$$

where [22]

$$\sigma_e^2 = \frac{1}{MN}\mathrm{Tr}\left\{\left(\frac{1}{N}\mathbf{I}_M + \frac{\rho_\tau}{N}\mathbf{X}_\tau\mathbf{X}_\tau^H\right)^{-1}\right\}, \tag{15}$$

[4]The worst-case noise property applies here since, given the index, the channel between z and $\mathbf{y}$ is similar to [21], i.e., a Rayleigh fading SIMO channel with imperfect channel state information at the receiver.

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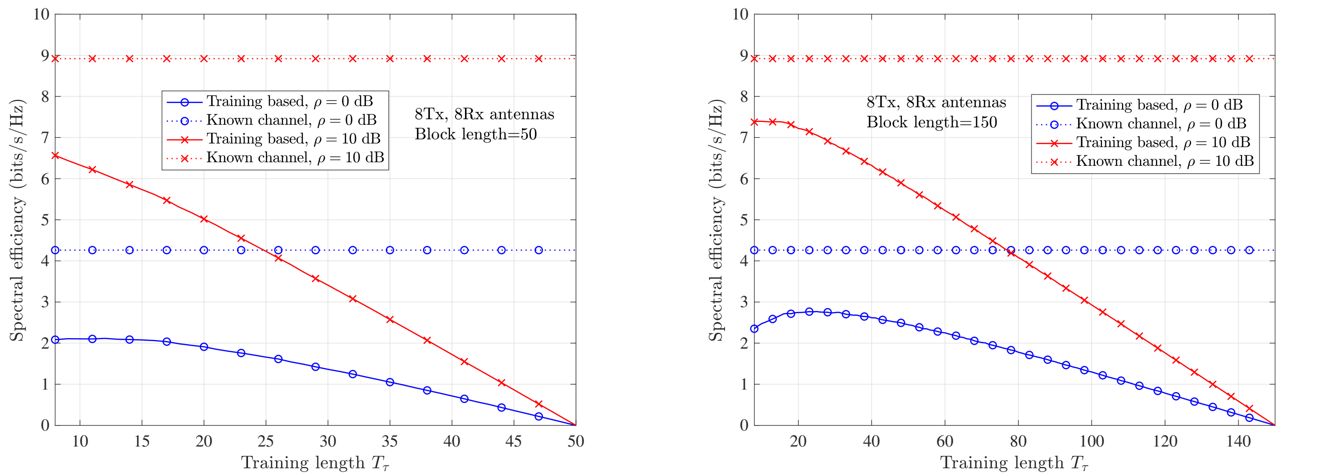

Fig. 1: Spectral efficiency of 8×8 spatial modulation as a function of training length T_τ when training and data powers are equal ($\rho_\tau = \rho_d$).

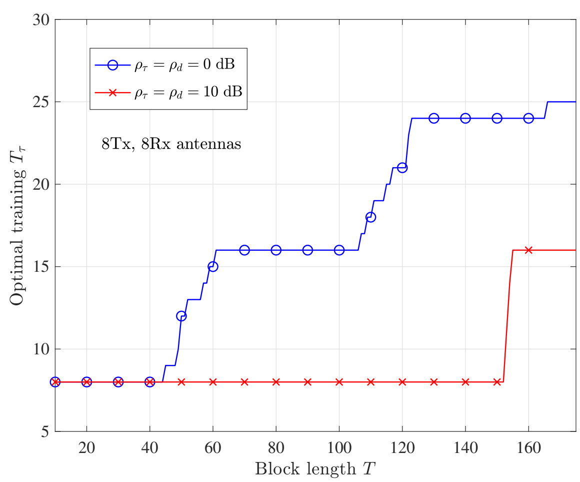

Fig. 2: Optimal training length as a function of block length with equal training and data power.

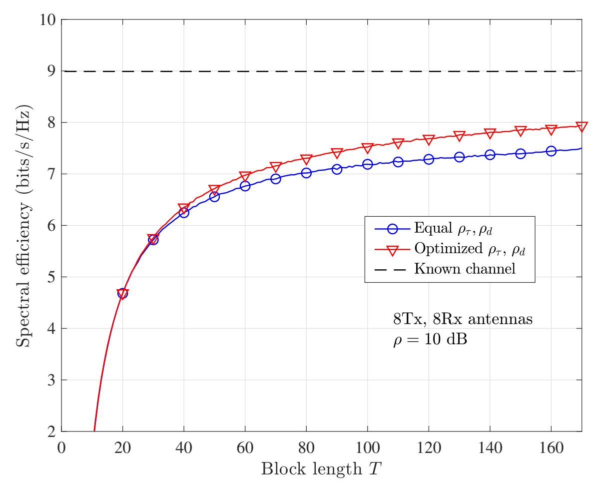

Fig. 3: Spectral efficiency of spatial modulation as a function of block length.

where $\mathbf{X}_\tau$ is the $M \times T_\tau$ pilot sequence[5].

Figure 1 shows the spectral efficiency of 8×8 spatial modulation (Eq. (14)) as a function of training duration T_τ for coherence intervals (also referred to as block lengths) $T = 50$ and 150, when the data and training powers are equal. The spectral efficiency with perfectly known CSIR is also shown in the figure. At low SNR (0 dB in the figure), the optimal training requires transmitting more pilots than transmit antennas, and the optimal training duration increases with block length T. For example, the optimal training at 0 dB and $T = 150$ requires $T_\tau = 24$ pilots, which is three times the number of transmit antennas. Whereas, at high SNR (10 dB in the figure), optimally, as many training symbols are employed as transmit antennas even at $T = 150$. To further illustrate these observations, Fig. 2 shows the optimal training length as a function of block length at 0 dB and 10 dB SNR. At 0 dB the optimal training length increases rapidly with block length. Whereas, at 10 dB, the optimal training length is equal to the number of transmit antennas ($T_\tau = M = 8$) up to block length 150, beyond which it increases to $T_\tau = 2M = 16$.

Figure 3 shows the spectral efficiency of 8×8 spatial modulation as a function of block length under two cases: i) equal ρ_τ, ρ_d (optimized over training time) and ii) optimized ρ_τ, ρ_d. The spectral efficiency with known CSIR is also shown for reference. Allowing the training and data powers to vary improves the spectral efficiency compared with equal training and data powers. Also, the gain with optimized ρ_τ, ρ_d compared with equal ρ_τ, ρ_d increases with block length.

In Fig. 4, we compare the spectral efficiency bound in Eq. (14) with Rajashekar *et al.* [9]. Both the results are optimized with respect to training and data powers. The bound in Eq. (14) is tighter than Rajashekar *et al.* bound across all SNR values.

[5]Orthogonal pilots are considered for all the numerical evaluations in this paper.

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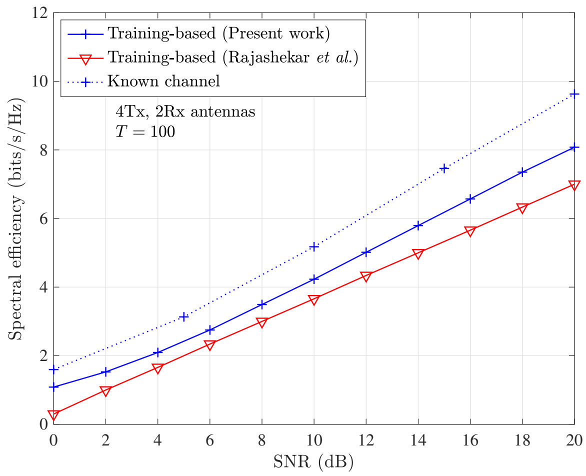

Fig. 4: Comparing Eq. (14) with Rajashekar *et al.* bound [9].

For example, at 6 bits/s/Hz, our bound is tighter by 2.5 dB.

Figure 5 shows the spectral efficiency of spatial modulation (with optimal ρ_τ, ρ_d) as a function of SNR, for block lengths $T = 50$ and 200. The figure also shows the spectral efficiency of spatial modulation with CSIR, SIMO with training [23], and SIMO with CSIR. Up to a certain critical SNR, which depends on the block length, the training-based spectral efficiency of spatial modulation is less than SIMO (e.g., 4 dB for $T = 50$). Figure 6 shows this critical SNR as a function of block length, demonstrating that spatial modulation has smaller spectral efficiency than SIMO in highly dynamic channels. This is a natural consequence of the higher training overhead of spatial modulation (M pilots) compared with SIMO (one pilot). Spatial modulation has the same training overhead as MIMO, but lacks its spatial multiplexing gain. Fig. 6 shows that spatial modulation achieves higher spectral efficiency compared with SIMO in slowly varying channels.

As indicated by the above results, spatial modulation is inferior to SIMO at low-SNR and in highly dynamic channels. The spectral efficiency of spatial modulation can be further improved by choosing the optimal antenna alphabet. For a given coherence interval and operating SNR, the optimal antenna alphabet achieves the best tradeoff between the training overhead and the achievable rate. When $M^* = 1$, the optimal transmission scheme reduces to SIMO. Figure 7 shows the optimal antenna alphabet for spatial modulation as a function of SNR and channel dynamics. While SIMO is optimal in low-SNR and low-block length regime, the optimal antenna alphabet size increases with SNR and block length. Therefore, if M antennas are available at the transmitter, the transmitter-receiver pair can agree on employing $M^*(1 \leq M^* \leq M)$ antennas for spatial modulation based on SNR and channel dynamics.

III. Generalized Spatial Modulation

Consider an $M \times N$ multi-antenna system. Generalized spatial modulation uses L $(1 \leq L \leq M)$ transmit RF chains and activates L transmit antennas out of the M available transmit antennas in a channel use. From the activated antennas, L symbols $z_1, \ldots, z_L$ are transmitted. The system model is given by

$$\mathbf{y} = \sqrt{\rho}\mathbf{G}\mathbf{V}\mathbf{z} + \mathbf{w}, \tag{16}$$

where $\mathbf{z} = [z_1, z_2, \ldots, z_L]^T$ is the $L \times 1$ vector of symbols transmitted from L active antennas, $\mathbf{V}$ is the $M \times L$ antenna activation matrix which when multiplied to $\mathbf{G}$ extracts the L columns of $\mathbf{G}$ corresponding to the L active antennas. If the antenna i of the transmitter $(1 \leq i \leq M)$ is the active antenna j $(1 \leq j \leq L)$, then $\mathbf{V}_{ij} = 1$, and rest of the entries of $\mathbf{V}$ are zeros. The number of possible $\mathbf{V}$ matrices is $\binom{M}{L}$, denoted by $\{\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_{\binom{M}{L}}\}$. The system model with estimated channel can be written as

$$\mathbf{y} = \sqrt{\rho}\widehat{\mathbf{G}}\mathbf{V}\mathbf{z} + \sqrt{\rho}\tilde{\mathbf{G}}\mathbf{V}\mathbf{z} + \mathbf{w}. \tag{17}$$

Let $\widehat{\mathbf{G}}_i \triangleq \widehat{\mathbf{G}}\mathbf{V}_i$ denote the $N \times L$ channel matrix corresponding to the antenna activation matrix $\mathbf{V}_i$.

Proposition 2. *The capacity of generalized spatial modulation with estimated CSIR satisfies the following lower bound:*

$$C \geq \mathbb{E}_{\widehat{\mathbf{G}}_1}\left[\log_2 \det\left(\mathbf{I}_L + \frac{\widehat{\mathbf{G}}_1^H\widehat{\mathbf{G}}_1\rho}{L(1+\rho\sigma_e^2)}\right)\right] + \log_2\binom{M}{L} - \mathbb{E}_{\mathbf{y},\widehat{\mathbf{G}}}\left[\sum_{i=1}^{\binom{M}{L}} \frac{u_i(\mathbf{y},\widehat{\mathbf{G}})}{\sum_{j=1}^{\binom{M}{L}} u_j(\mathbf{y},\widehat{\mathbf{G}})} \log_2 \frac{\sum_{j=1}^{\binom{M}{L}} u_j(\mathbf{y},\widehat{\mathbf{G}})}{u_i(\mathbf{y},\widehat{\mathbf{G}})}\right], \tag{18}$$

where $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \frac{1}{L}\mathbf{I}_L)$ *and*

$$u_i(\mathbf{y},\widehat{\mathbf{G}}) \triangleq \tag{19}$$

$$\mathbb{E}_{\mathbf{z}}\left[\frac{1}{(\pi(1+\|\mathbf{z}\|^2\rho\sigma_e^2))^N} \exp\left(\frac{-\|\mathbf{y}-\sqrt{\rho}\widehat{\mathbf{G}}_i\mathbf{z}\|^2}{1+\|\mathbf{z}\|^2\rho\sigma_e^2}\right)\right]. \tag{20}$$

Proof. The proof is similar to that of Proposition 1 and is omitted for brevity. □

We now express the spectral efficiency of generalized spatial modulation after accounting for the training time and energy in Proposition 2. With an abuse of notation, we redefine u_i from Eq. (20) by replacing ρ with ρ_d:

$$u_i(\mathbf{y},\widehat{\mathbf{G}}) = \mathbb{E}_{\mathbf{z}}\left[\frac{1}{(\pi(1+\|\mathbf{z}\|^2\rho_d\sigma_e^2))^N} \exp\left(\frac{-\|\mathbf{y}-\sqrt{\rho_d}\widehat{\mathbf{G}}_i\mathbf{z}\|^2}{1+\|\mathbf{z}\|^2\rho_d\sigma_e^2}\right)\right].$$

Then,

$$C_\tau \geq \frac{T-T_\tau}{T}\left[\mathbb{E}_{\widehat{\mathbf{G}}_1}\left[\log_2 \det\left(\mathbf{I}_L + \frac{\widehat{\mathbf{G}}_1^H\widehat{\mathbf{G}}_1\rho_d}{L(1+\rho_d\sigma_e^2)}\right)\right] + \log_2\binom{M}{L} - \mathbb{E}_{\mathbf{y},\widehat{\mathbf{G}}}\left[\sum_{i=1}^{\binom{M}{L}} \frac{u_i(\mathbf{y},\widehat{\mathbf{G}})}{\sum_{j=1}^{\binom{M}{L}} u_j(\mathbf{y},\widehat{\mathbf{G}})} \log_2 \frac{\sum_{j=1}^{\binom{M}{L}} u_j(\mathbf{y},\widehat{\mathbf{G}})}{u_i(\mathbf{y},\widehat{\mathbf{G}})}\right]\right], \tag{21}$$

where the estimation error σ_e^2 is as given in Eq. (15).

Figure 8 shows the spectral efficiency of 8×8 generalized spatial modulation with four active antennas, as a function of SNR for $T = 50$ and 200. The figure also shows the

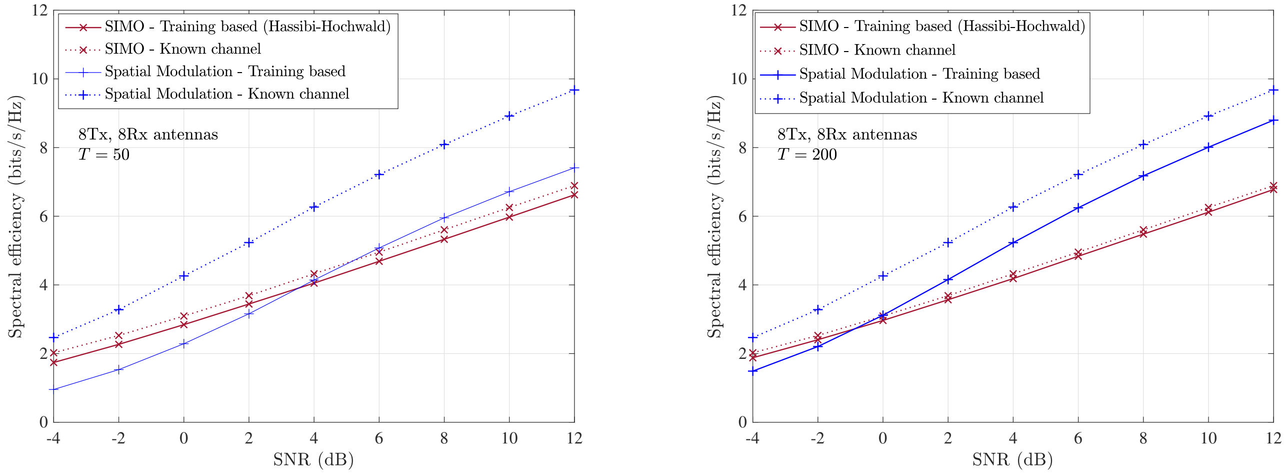

Fig. 5: Spectral efficiency of 8×8 spatial modulation as a function of SNR.

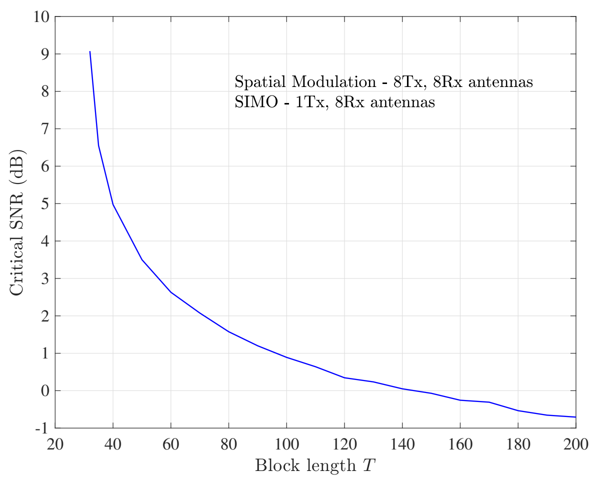

Fig. 6: Critical SNR for SIMO vs. spatial modulation. A highe. rate is achieved by spatial modulation above the critical SNR, and by SIMO blow the critical SNR.

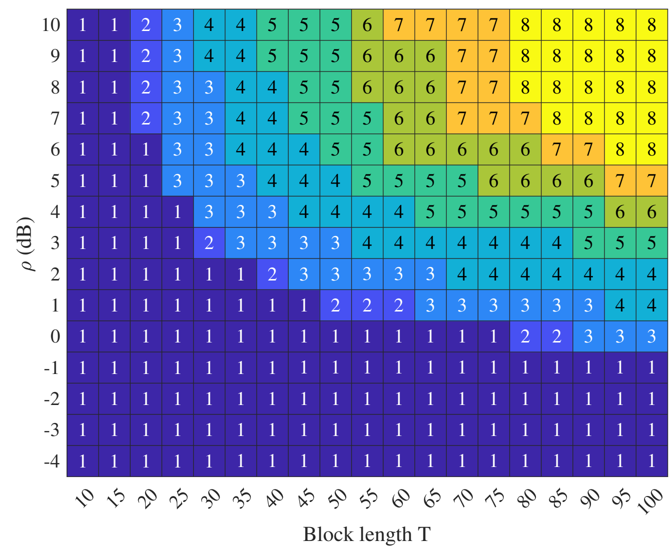

Fig. 7: Optimal antenna alphabet with one RF chain.

spectral efficiency of generalized spatial modulation with a known channel, training-based spectral efficiency of 4×8 MIMO (matching the number of RF chains with generalized spatial modulation), and spectral efficiency of 4×8 MIMO with known channel. The training-based spectral efficiency of generalized spatial modulation is less than 4×8 MIMO at low-SNR and smaller block length. This happens because generalized spatial modulation has the same training requirement as MIMO, but offers smaller spatial multiplexing

Generalized spatial modulation benefits from optimizing the antenna alphabet. Figure 9 shows the optimum antenna alphabet when two RF chains are available at the transmitter. At low-SNR and high channel dynamics, it is favorable to employ 2×8 MIMO instead of employing generalized spatial modulation. The optimal antenna alphabet size M^* increases with SNR and block length.

IV. Index Modulation for Multiuser Uplink

A. Multiuser Spatial Modulation

Let K denote the number of users, each equipped with M transmit antennas. The users employ spatial modulation to send data to a base station equipped with N receive antennas. Let $\mathbf{x}_k = \boldsymbol{u}_k z_k$ denote the transmit spatial modulation signal of user k, where $\boldsymbol{u}_k \in \{\mathbf{e}_i,\ i = 1, \ldots, M\}$ is the index vector and z_k is the modulation signal. Let $\mathbf{G}_k \in \mathbb{C}^{N \times M}$ denote the channel matrix from user k to the base station. The channel gains follow $\mathcal{CN}(0, 1)$. The $N \times 1$ received signal at the base station is given by

$$\begin{aligned}\mathbf{y} &= \sqrt{\rho}(\mathbf{G}_1\mathbf{x}_1 + \mathbf{G}_2\mathbf{x}_2 + \cdots + \mathbf{G}_K\mathbf{x}_K) + \mathbf{w} \\ &= \sqrt{\rho}\mathbf{G}\mathbf{x} + \mathbf{w}, \end{aligned} \tag{22}$$

where $\mathbf{G} = [\mathbf{G}_1\ \mathbf{G}_2\ \ldots\ \mathbf{G}_K]$ and $\mathbf{x} = [\mathbf{x}_1^T\ \mathbf{x}_2^T \ldots\ \mathbf{x}_K^T]^T$. Let $\widehat{\mathbf{G}}_k$ denote the MMSE estimate of $\mathbf{G}_k$ and $\tilde{\mathbf{G}}_k$ denote the estimation error. Also, let $\widehat{\mathbf{G}} \triangleq [\widehat{\mathbf{G}}_1\ \widehat{\mathbf{G}}_2\ \ldots\ \widehat{\mathbf{G}}_K]$ and

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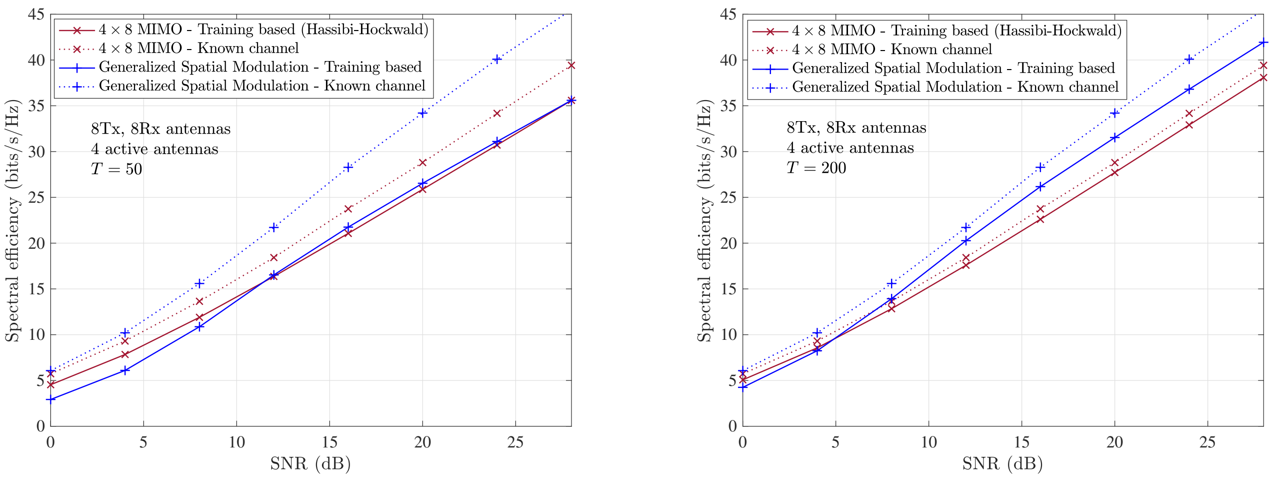

Fig. 8: Spectral efficiency of 8×8 generalized spatial modulation as a function of SNR.

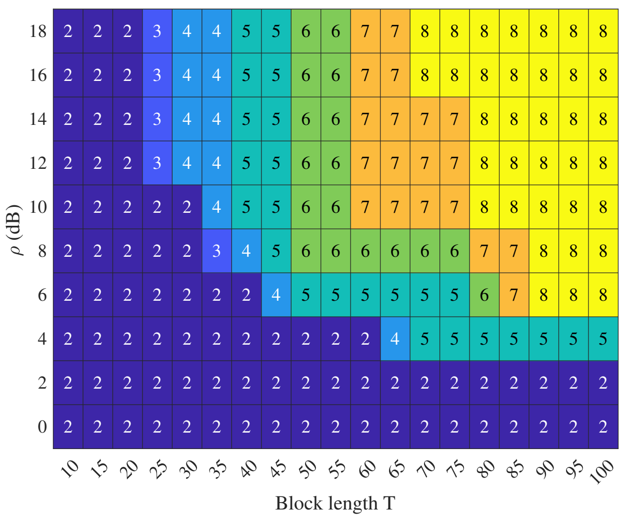

Fig. 9: Optimal antenna alphabet with two RF chains.

$\tilde{\mathbf{G}} \triangleq [\tilde{\mathbf{G}}_1 \ \tilde{\mathbf{G}}_2 \ \ldots \ \tilde{\mathbf{G}}_K]$. The received signal is equivalently written as

$$\mathbf{y} = \sqrt{\rho}\widehat{\mathbf{G}}\mathbf{x} + \sqrt{\rho}\tilde{\mathbf{G}}\mathbf{x} + \mathbf{w}. \tag{23}$$

At the base station, zero-forcing (ZF) combining is performed on $\mathbf{y}$ using $\widehat{\mathbf{G}}$ to cancel multiuser interference. The received signal after ZF combining is given by

$$\begin{aligned}\tilde{\mathbf{y}} &= \sqrt{\rho}\widehat{\mathbf{G}}^{\dagger}\widehat{\mathbf{G}}\mathbf{x} + \sqrt{\rho}\widehat{\mathbf{G}}^{\dagger}\tilde{\mathbf{G}}\mathbf{x} + \widehat{\mathbf{G}}^{\dagger}\mathbf{w} \\ &= \sqrt{\rho}\mathbf{x} + \tilde{\mathbf{w}},\end{aligned} \tag{24}$$

where $\tilde{\mathbf{w}} = \sqrt{\rho}\widehat{\mathbf{G}}^{\dagger}\tilde{\mathbf{G}}\mathbf{x} + \widehat{\mathbf{G}}^{\dagger}\mathbf{w}$ is the $MK \times 1$ effective noise after combining. From (24), the received signal corresponding to user k is given by

$$\begin{aligned}\tilde{\mathbf{y}}_k &= \sqrt{\rho}\mathbf{x}_k + \tilde{\mathbf{w}}_k \\ &= \sqrt{\rho}\mathbf{u}_k z_k + \tilde{\mathbf{w}}_k,\end{aligned} \tag{25}$$

where $\tilde{\mathbf{w}}_k$ is the $M \times 1$ effective noise vector that affects the signal of user k.

Proposition 3. *In the uplink of a multiuser spatial modulation system with estimated channel at the base station, the following rate can be achieved for User k:*

$$\begin{aligned}R_k =& \mathbb{E}\left[\log_2\left(1 + \frac{\rho}{(1+\rho K\sigma_e^2)\left[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}\right]_{(kM,kM)}}\right)\right] + \\ & \log_2 M - \\ & \mathbb{E}\left[\sum_{i=1}^{M}\frac{f_{i,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})}{\sum_{j=1}^{M} f_{j,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})}\log_2\frac{\sum_{j=1}^{M} f_{j,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})}{f_{i,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})}\right],\end{aligned} \tag{26}$$

where $\left[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}\right]_{(kM,kM)}$ is the diagonal element kM of $(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}$, $f_{i,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})$ is given by

$$f_{i,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}}) = \frac{\exp\left(-(\tilde{\mathbf{y}}_k)^H\left(\rho\mathbf{e}_i\mathbf{e}_i^H + \mathbf{R}_{\tilde{w}_k}\right)^{-1}\tilde{\mathbf{y}}_k\right)}{|\rho\mathbf{e}_i\mathbf{e}_i^H + \mathbf{R}_{\tilde{w}_k}|},$$

where $\mathbf{R}_{\tilde{w}_k} = (1+\rho K\sigma_e^2)\left[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}\right]_k$ with $\left[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}\right]_k$ being the diagonal block k when $(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}$ is expressed as a block matrix composed of $M \times M$ sized blocks.

Proof. See Appendix A. □

The estimation error σ_e^2 for the multiuser system is given by [22]

$$\sigma_e^2 = \frac{1}{KMN}\mathrm{Tr}\left\{\left(\frac{1}{N}\mathbf{I}_{KM} + \frac{\rho}{N}\mathbf{X}_\tau\mathbf{X}_\tau^H\right)\right\}. \tag{27}$$

Under orthogonal pilots,[6] the minimal training duration is $T_\tau = MK$, leading to an achievable sum-rate:

$$R_{sum} = \left(1 - \frac{T_\tau}{T}\right)\sum_{k=1}^{K} R_k. \tag{28}$$

[6]The developments in this section assume orthogonal pilots for different users in the uplink, as is common in many contemporary systems. The results are most salient in the wide set of scenarios where the uplink channel has per-user training requirements that are both necessary and reasonable. This holds true, for example, when the uplink and downlink operate on different frequencies, or more generally when training based on channel reciprocity is either infeasible or unreliable.

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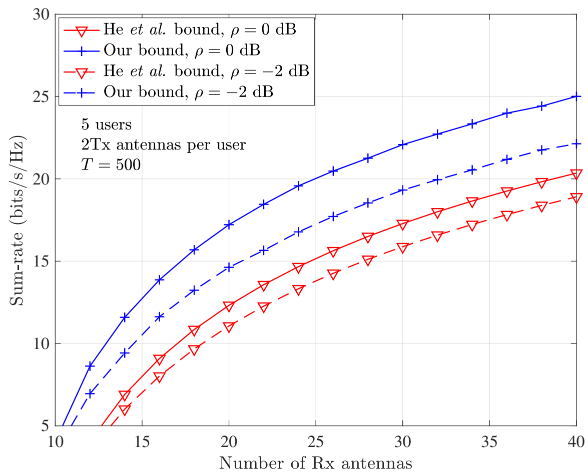

Fig. 10: Comparing Eq. (28) with He *et al.* bound [18].

Figure 10 shows the sum-rate of uplink spatial modulation (Eq. (28)) as a function of the number of base station antennas for five user system when users are equipped with two transmit antennas. The figure also shows the sum-rate derived by He *et al.* [18]. The sum rate in Eq. (28) is tighter than that of [18] and provides a better estimate of the operational requirements. For example, [18] suggests that 16 base station antennas are necessary for achieving a rate of 1.6 bits/s/Hz per user at 0 dB, whereas our bound shows that 12 antennas are sufficient. The imprecision in [18] is due to Jensen's loss in evaluating the rate of the modulation symbol, energy detection, and hard thresholding in evaluating the rate carried by the index.

Figure 11 shows the training-based sum-rate of uplink spatial modulation as a function of SNR when eight users, each equipped with four transmit antennas, communicate with a base station with 64 receive antennas, at block lengths $T = 200$ and 500. The training-based sum-rate of SIMO, spatial modulation with known CSIR, and SIMO with known CSIR are also shown in the figure. For $T = 200$, the sum-rate of spatial modulation is less than SIMO at all SNR, whereas at $T = 500$, spatial modulation achieves a higher rate. This is because spatial modulation has a training overhead of 32 pilots compared to 8 pilots in SIMO. Therefore, at $T = 200$, the rate loss in spatial modulation due to training overhead dominates over the additional rate gain due to index bits, which results in a lesser overall rate than SIMO. Whereas, at $T = 500$, the effect of training overhead is reduced due to the availability of more data slots, which enables spatial modulation to achieve a higher rate than SIMO through the additional index information.

Unlike perfect CSIR, the rate achieved by the training-based system increases with the number of users up to a certain value, beyond which it decreases (with perfect CSIR, the rate growth becomes sublinear after a certain number of users). The number of users at which the peak rate is achieved is a function of both the block length and the number of receive antennas. Therefore, characterizing the optimal number of users is essential in determining the number of receive antennas required for supporting users with a certain guaranteed rate. It also provides knowledge about how the channel dynamics limit the number of users that can be supported.

Figure 12 shows the sum-rate of multiuser spatial modulation as a function of the number of users K for 64 and 128 base station receive antennas, when users are equipped with four antennas, SNR is 10 dB, and the block length is 500 time-slots. The sum-rate increases up to a certain number of users K^*, beyond which it decreases. This is because the training overhead is directly proportional to K, and therefore, beyond K^* the rate loss due to training overhead dominates over the rate gain due to more users. Also, K^* is higher for a base station with more receive antennas. For example, $K^* = 13$ when the base station has 64 antennas and $K^* = 26$ when the base station has 128 receive antennas.

To gain further insight, Fig. 13 shows K^* as a function of base station antennas for block lengths $T = 200$ and 500, at 10 dB SNR. The K^* increases at a higher rate for $T = 500$ compared to $T = 200$ since the fraction of slots available for data decreases faster in a channel with a smaller T. Figure 14 shows K^* as a function of SNR for 32, 64, and 128 base station antennas when $T = 500$ slots. At a given SNR, K^* is higher for a base station with more antennas and K^* increases with SNR.

B. Multiuser Generalized Spatial Modulation

Consider an uplink multiuser system with users equipped with M transmit antennas and $L(1 < L < M)$ RF chains. The users employ generalized spatial modulation to send data to a base station equipped with N receive antennas. Let $\mathbf{x}_k = \boldsymbol{U}_k \mathbf{z}_k$ denote the transmit generalized spatial modulation signal for user k with $\boldsymbol{U}_k \in \{\mathbf{V}_i,\ i = 1, \dots, \binom{M}{L}\}$ being the antenna activation matrix as discussed in Sec. III and $\mathbf{z}_k$ being the transmitted signal vector from the L active antennas. Let $\mathbf{G}_k \in \mathbb{C}^{N \times M}$ denote the channel matrix from user k to the base station, and let $\mathbf{G}_{k,i} \triangleq \mathbf{G}_k \mathbf{V}_i \in \mathbb{C}^{N \times L}$ denote the channel of user k when antenna activation matrix $\mathbf{V}_i$ is selected. The channel gains follow $\mathcal{CN}(0, 1)$. The $N \times 1$ received signal at the base station is given by

$$\begin{aligned} \mathbf{y} &= \sqrt{\rho}(\mathbf{G}_1\mathbf{x}_1 + \mathbf{G}_2\mathbf{x}_2 + \cdots + \mathbf{G}_K\mathbf{x}_K) + \mathbf{w} \\ &= \sqrt{\rho}(\mathbf{G}_1\boldsymbol{U}_1\mathbf{z}_1 + \mathbf{G}_2\boldsymbol{U}_2\mathbf{z}_2 + \cdots + \mathbf{G}_K\boldsymbol{U}_K\mathbf{z}_K) + \mathbf{w} \\ &= \sqrt{\rho}\mathbf{G}\mathbf{x} + \mathbf{w}, \end{aligned} \tag{29}$$

where $\mathbf{G} = [\mathbf{G}_1\ \mathbf{G}_2\ \dots\ \mathbf{G}_K]$ and $\mathbf{x} = [\mathbf{x}_1^T\ \mathbf{x}_2^T \dots\ \mathbf{x}_K^T]^T$. The received signal with estimated channel $\widehat{\mathbf{G}}$ is given by

$$\begin{aligned} \mathbf{y} &= \sqrt{\rho}\widehat{\mathbf{G}}\mathbf{x} + \sqrt{\rho}\tilde{\mathbf{G}}\mathbf{x} + \mathbf{w} \\ &= \sqrt{\rho}\widehat{\mathbf{G}}\mathbf{x} + \tilde{\mathbf{w}}. \end{aligned} \tag{30}$$

The signal of user k after ZF processing at the receiver is given by

$$\begin{aligned} \tilde{\mathbf{y}}_k &= \sqrt{\rho}\mathbf{x}_k + \tilde{\mathbf{w}}_k \\ &= \sqrt{\rho}\boldsymbol{U}_k\mathbf{z}_k + \tilde{\mathbf{w}}_k. \end{aligned} \tag{31}$$

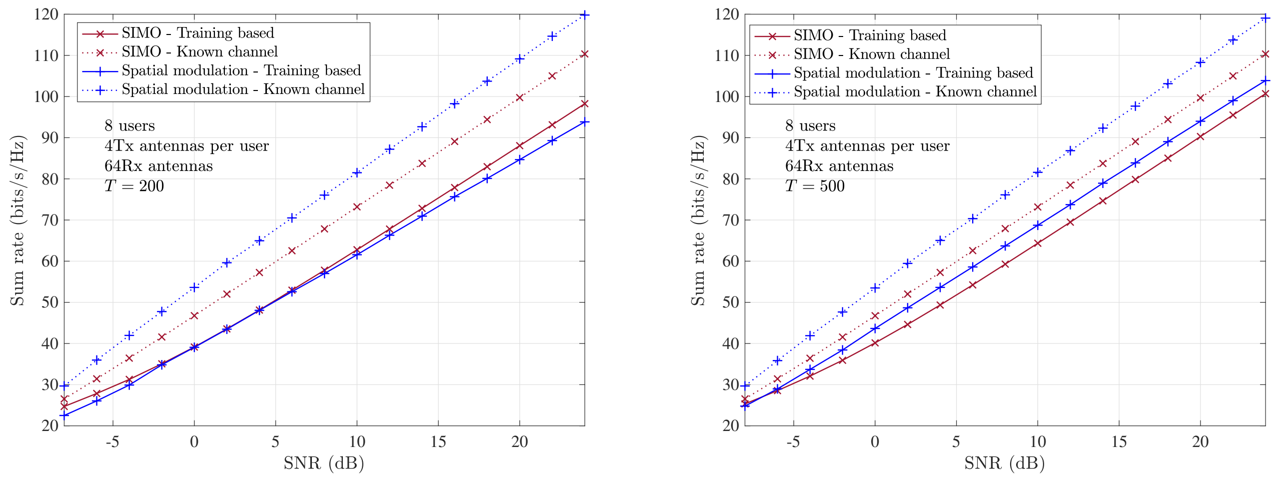

Fig. 11: Sum-rate of multiuser spatial modulation uplink as a function of SNR.

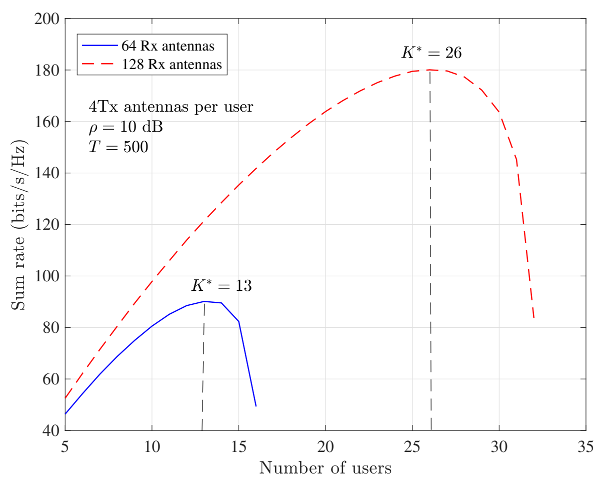

Fig. 12: Sum-rate of multiuser spatial modulation uplink as a function of number of users.

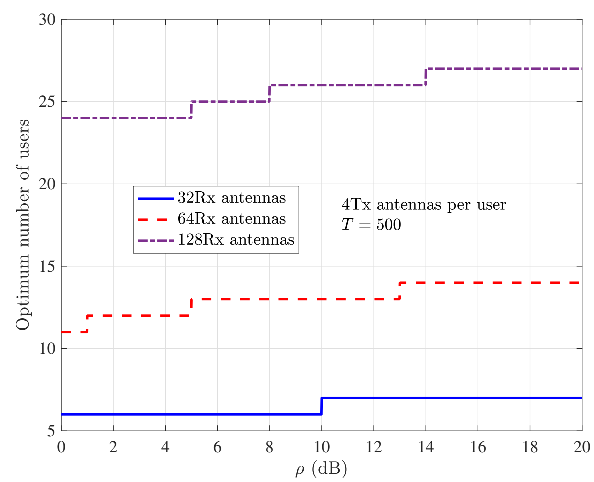

Fig. 14: Optimum number of users versus SNR for multiuser spatial modulation.

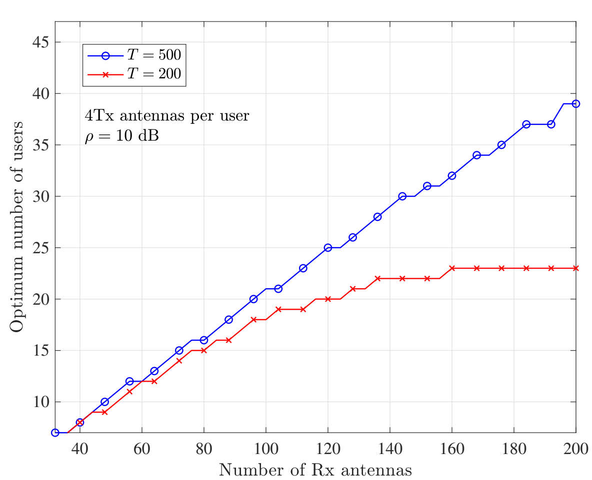

Fig. 13: Optimum number of users versus the number of receive antennas for multiuser spatial modulation.

Proposition 4. *In the uplink of a multiuser generalized spatial modulation system with estimated channel at the base station, the following rate can be achieved for User k:*

$$\begin{aligned} R_k =& L\mathbb{E}\left[\log_2\left(1+\frac{\rho}{L(1+\rho K\sigma_e^2)\left[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}\right]_{(kM,kM)}}\right)\right] \\ &+\log_2\binom{M}{L}- \\ &\mathbb{E}\left[\sum_{i=1}^{\binom{M}{L}}\frac{u_{i,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})}{\sum_{j=1}^{\binom{M}{L}}u_{j,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})}\log_2\frac{\sum_{j=1}^{\binom{M}{L}}u_{j,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})}{u_{i,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})}\right], \end{aligned} \tag{32}$$

where $\left[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}\right]_{(kM,kM)}$ *is the diagonal element* kM *of* $(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}$, $u_{i,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})$ *is given by*

$$u_{i,k}(\tilde{\mathbf{y}}_k,\widehat{\mathbf{G}})=\frac{\exp\left(-(\tilde{\mathbf{y}}_k)^H\left(\frac{\rho}{L}\mathbf{V}_i\mathbf{V}_i^H+\mathbf{R}_{\tilde{w}_k}\right)^{-1}\tilde{\mathbf{y}}_k\right)}{\left|\frac{\rho}{L}\mathbf{V}_i\mathbf{V}_i^H+\mathbf{R}_{\tilde{w}_k}\right|},$$

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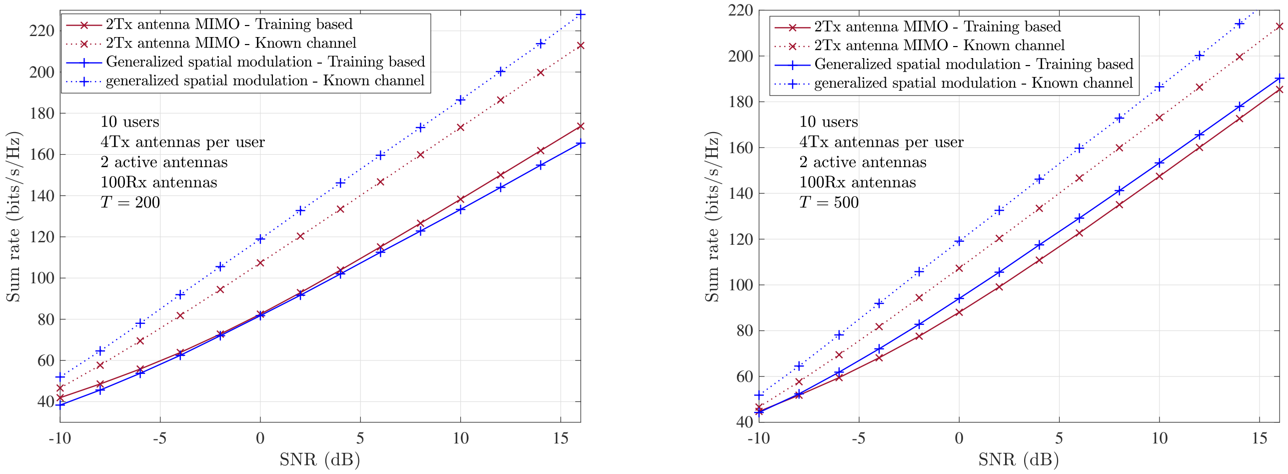

Fig. 15: Sum-rate of multiuser generalized spatial modulation as a function of SNR.

where $\mathbf{R}_{\tilde{w}_k} = (1 + \rho K \sigma_e^2) \left[(\widehat{\mathbf{G}}^H \widehat{\mathbf{G}})^\dagger\right]_k$ *with* $\left[(\widehat{\mathbf{G}}^H \widehat{\mathbf{G}})^\dagger\right]_k$ *being the diagonal block* k *when* $(\widehat{\mathbf{G}}^H \widehat{\mathbf{G}})^\dagger$ *is expressed as a block matrix composed of* $M \times M$ *sized blocks.*

Proof. The proof is similar to Proposition 3 and is omitted for brevity. □

The estimation error σ_e^2 in the above proposition is the same as given in Eq. (27). The sum-rate after accounting for training overhead is given by

$$R_{sum} = \left(1 - \frac{T_\tau}{T}\right) \sum_{k=1}^{K} R_k. \tag{33}$$

Figure 15 shows the training-based sum-rate of multiuser generalized spatial modulation as a function of SNR for $T = 200$ and 500, when ten users, each equipped with four transmit antennas and two RF chains, communicate with a base station having 100 receive antennas. The figure also shows the sum-rate when users employ MIMO with two transmit antennas (matching RF chains with generalized spatial modulation). For smaller coherence interval ($T = 200$), the achievable rate of generalized spatial modulation is less than two-transmit antenna MIMO at all SNR, whereas for longer coherence interval ($T = 500$ in the figure) generalized spatial modulation achieves a higher rate.

Figure 16 shows K^* as a function of base station antennas. The K^* grows at a higher rate for $T = 500$ compared to $T = 200$. Figure 17 shows K^* as a function of SNR for 32, 64, and 128 base station antennas. At any given SNR, K^* is higher when the base station has more antennas. For a given number of receive antennas, K^* increases with SNR.

V. Conclusions

We studied the spectral efficiency of spatial and generalized spatial modulation with channel training accounting for the training overhead and error. Our results showed that in highly dynamic channels and at low-SNR, spatial modulation has

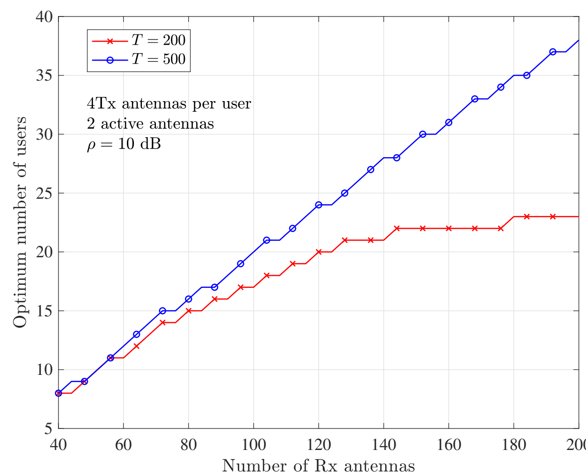

Fig. 16: Optimum number of users versus the number of receive antennas for multiuser generalized spatial modulation.

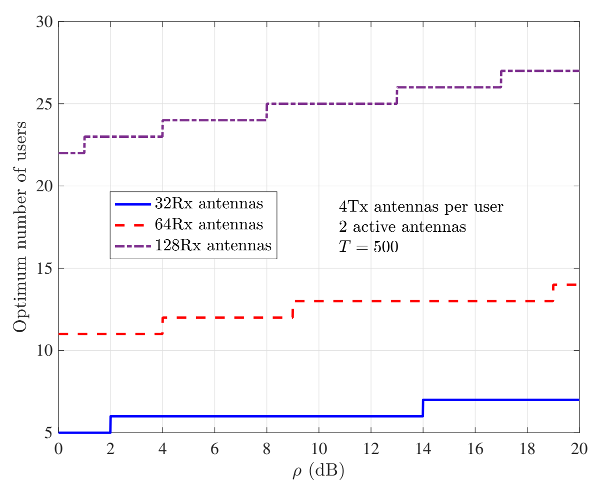

Fig. 17: Optimum number of users versus SNR for multiuser generalized spatial modulation.

lesser spectral efficiency than SIMO and L-RF-chain generalized spatial modulation has lesser spectral efficiency than L-MIMO. Our results also provided the optimal antenna alphabet for spatial and generalized spatial modulation subject to pilots and training. Finally, our results were extended to the multiuser uplink where users employ index modulation to transmit data to a base station. Our results characterized the optimal number of users that maximize the sum-rate, when all other operating conditions are fixed.

Appendix A
Proof of Proposition 3

The rate of user k with estimated channel $\widehat{\mathbf{G}}$ at the base station is given by

$$\begin{aligned} R_k &= I(\boldsymbol{u}_k, z_k; \tilde{\mathbf{y}}_k, \widehat{\mathbf{G}}) \\ &\overset{(a)}{=} I(\boldsymbol{u}_k, z_k; \tilde{\mathbf{y}}_k|\widehat{\mathbf{G}}) + I(\boldsymbol{u}_k, z_k; \widehat{\mathbf{G}}) \\ &\overset{(b)}{=} I(\boldsymbol{u}_k, z_k; \tilde{\mathbf{y}}_k|\widehat{\mathbf{G}}), \end{aligned} \tag{34}$$

where (a) follows from chain rule and (b) follows from the independence of transmitted data from channel gains, and therefore of its estimate. By chain rule

$$I(\boldsymbol{u}_k, z_k; \tilde{\mathbf{y}}_k|\widehat{\mathbf{G}}) = I(z_k; \tilde{\mathbf{y}}_k|\boldsymbol{u}_k, \widehat{\mathbf{G}}) + I(\boldsymbol{u}_k; \tilde{\mathbf{y}}_k|\widehat{\mathbf{G}}). \tag{35}$$

We evaluate the chain rule terms when $\boldsymbol{u}_k$ is distributed uniform and $z_k \sim \mathcal{CN}(0,1)$ independent of $\boldsymbol{u}_k$. The effective noise $\tilde{\mathbf{w}}$ is approximated by Gaussian distribution [24] whose statistics follows from Eq. (24):

$$\mathbb{E}(\tilde{\mathbf{w}}|\widehat{\mathbf{G}}) = \sqrt{\rho}\mathbb{E}(\mathbf{x}) + \sqrt{\rho}\widehat{\mathbf{G}}^{\dagger}\mathbb{E}(\tilde{\mathbf{G}})\mathbb{E}(\mathbf{x}) + \widehat{\mathbf{G}}^{\dagger}\mathbb{E}(\mathbf{w}) = 0.$$

and

$$\begin{aligned} \mathbb{E}(\tilde{\mathbf{w}}\tilde{\mathbf{w}}^H|\widehat{\mathbf{G}}) =& \rho\widehat{\mathbf{G}}^{\dagger}\mathbb{E}(\tilde{\mathbf{G}}\mathbf{x}\mathbf{x}^H\tilde{\mathbf{G}}^H)(\widehat{\mathbf{G}}^{\dagger})^H + \\ & \widehat{\mathbf{G}}^{\dagger}\mathbb{E}(\mathbf{w}\mathbf{w}^H)(\widehat{\mathbf{G}}^{\dagger})^H, \end{aligned} \tag{36}$$

with

$$\begin{aligned} \mathbb{E}(\tilde{\mathbf{G}}\mathbf{x}\mathbf{x}^H\tilde{\mathbf{G}}^H) &= \mathbb{E}\Big(\sum_{\ell=1}^{K}\sum_{m=1}^{K} \tilde{\mathbf{G}}_\ell \boldsymbol{u}_\ell z_\ell z_m^* \boldsymbol{u}_m^H \tilde{\mathbf{G}}_m^H\Big) \\ &\overset{(a)}{=} \sum_{\ell=1}^{K} \mathbb{E}(\tilde{\mathbf{G}}_\ell \boldsymbol{u}_\ell \boldsymbol{u}_\ell \tilde{\mathbf{G}}_\ell^H) \\ &\overset{(b)}{=} \sum_{\ell=1}^{K} \mathbb{E}_{\boldsymbol{u}_l}\left[\mathbb{E}_{\tilde{\mathbf{G}}}(\tilde{\mathbf{G}}\boldsymbol{u}_\ell \boldsymbol{u}_\ell^H \tilde{\mathbf{G}}_\ell^H|\boldsymbol{u}_\ell)\right] \\ &= \frac{1}{M}\sum_{\ell=1}^{K}\sum_{i=1}^{M} \mathbb{E}(\tilde{\mathbf{g}}_{\ell,i}\tilde{\mathbf{g}}_{\ell,i}^H) \\ &= \frac{1}{M} MK\sigma_e^2\mathbf{I} = K\sigma_e^2\mathbf{I}, \end{aligned} \tag{37}$$

where (a) follows since $\mathbb{E}(z_\ell z_m^*) = 1$ for $\ell = m$ and 0 otherwise, (b) follows from the property of conditional expectations, and the rest of the equalities follow from direct evaluation. Using (37) in (36), we obtain

$$\mathbb{E}(\tilde{\mathbf{w}}\tilde{\mathbf{w}}^H|\widehat{\mathbf{G}}) = (1+\rho K\sigma_e^2)(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}}^H)^{\dagger}. \tag{38}$$

From (38), the noise covariance corresponding to user k is given by

$$\begin{aligned} \mathbf{R}_{\tilde{w}_k} &\triangleq \mathbb{E}(\tilde{\mathbf{w}}_k\tilde{\mathbf{w}}_k^H|\widehat{\mathbf{G}}) \\ &= (1+\rho K\sigma_e^2)\big[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}}^H)^{\dagger}\big]_k, \end{aligned} \tag{39}$$

where $\big[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}}^H)^{\dagger}\big]_k$ is the diagonal block k when $(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}}^H)^{\dagger}$ is written as a block matrix composed of $M \times M$ sized blocks. Now, using (39) and by symmetry, the first term in (35) is given by

$$I(z_k; \tilde{\mathbf{y}}_k|\boldsymbol{u}_k, \widehat{\mathbf{G}}) = \mathbb{E}\left[\log_2\!\left(1 + \frac{\rho}{(1+\rho K\sigma_e^2)\left[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}\right]_{(kM,kM)}}\right)\right], \tag{40}$$

where $\left[(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}\right]_{(kM,kM)}$ denotes the diagonal element kM of $(\widehat{\mathbf{G}}^H\widehat{\mathbf{G}})^{\dagger}$. The second term in (35) can be evaluated as follows:

$$\begin{aligned} I(\boldsymbol{u}_k; \tilde{\mathbf{y}}_k|\widehat{\mathbf{G}}) &= h(\boldsymbol{u}_k|\widehat{\mathbf{G}}) - h(\boldsymbol{u}_k|\widehat{\mathbf{G}}, \tilde{\mathbf{y}}_k) \\ &= \log_2 M - h(\boldsymbol{u}_k|\widehat{\mathbf{G}}, \tilde{\mathbf{y}}_k). \end{aligned} \tag{41}$$

We have

$$h(\boldsymbol{u}_k|\widehat{\mathbf{G}}, \tilde{\mathbf{y}}_k) = \mathbb{E}\left[\sum_{i=1}^{M} p(\boldsymbol{u}_k = \mathbf{e}_i|\widehat{\mathbf{G}}, \tilde{\mathbf{y}}_k)\log_2 \frac{1}{p(\boldsymbol{u}_k = \mathbf{e}_i|\widehat{\mathbf{G}}, \tilde{\mathbf{y}}_k)}\right], \tag{42}$$

where

$$\begin{aligned} p(\boldsymbol{u}_k = \mathbf{e}_i|\widehat{\mathbf{G}}, \tilde{\mathbf{y}}_k) &= \frac{p(\tilde{\mathbf{y}}_k|\boldsymbol{u}_k = \mathbf{e}_i, \widehat{\mathbf{G}})p(\boldsymbol{u}_k = \mathbf{e}_i|\widehat{\mathbf{G}})}{\sum_{j=1}^{M} p(\tilde{\mathbf{y}}_k|\boldsymbol{u}_k = \mathbf{e}_i, \widehat{\mathbf{G}})p(\boldsymbol{u}_k = \mathbf{e}_j|\widehat{\mathbf{G}})} \\ &= \frac{p(\tilde{\mathbf{y}}_k|\boldsymbol{u}_k = \mathbf{e}_i, \widehat{\mathbf{G}})}{\sum_{j=1}^{M} p(\tilde{\mathbf{y}}_k|\boldsymbol{u}_k = \mathbf{e}_i, \widehat{\mathbf{G}})}, \end{aligned} \tag{43}$$

where the second equality follows since $p(\boldsymbol{u}_k = \mathbf{e}_i|\widehat{\mathbf{G}}) = \frac{1}{M}$ for $i = 1, \ldots, M$. From (25) and (39), we have $\mathbb{E}(\tilde{\mathbf{y}}|\boldsymbol{u}_k = \mathbf{e}_i, \widehat{\mathbf{G}}) = 0$ and

$$\mathbb{E}(\tilde{\mathbf{y}}_k\tilde{\mathbf{y}}^{k^H}|\boldsymbol{u}_k = \mathbf{e}_i, \widehat{\mathbf{G}}) = \rho\mathbf{e}_i\mathbf{e}_i^H + \mathbf{R}_{\tilde{w}_k}.$$

Therefore,

$$p(\tilde{\mathbf{y}}_k|\boldsymbol{u}_k = \mathbf{e}_i, \widehat{\mathbf{G}}) = \frac{\exp\big(-(\tilde{\mathbf{y}}_k)^H(\rho\mathbf{e}_i\mathbf{e}_i^H + \mathbf{R}_{\tilde{w}_k})^{-1}\tilde{\mathbf{y}}_k\big)}{\pi^M|\rho\mathbf{e}_i\mathbf{e}_i^H + \mathbf{R}_{\tilde{w}_k}|}.$$

Substituting the above equation in (43), and combining the equations (43), (42), and (41) gives

$$I(\boldsymbol{u}_k;\tilde{\mathbf{y}}_k|\widehat{\mathbf{G}}) = \log_2 M - \mathbb{E}\left[\sum_{i=1}^{M}\frac{f_{i,k}(\tilde{\mathbf{y}}_k, \widehat{\mathbf{G}})}{\sum_{j=1}^{M} f_{j,k}(\tilde{\mathbf{y}}_k, \widehat{\mathbf{G}})}\log_2\frac{\sum_{j=1}^{M} f_{j,k}(\tilde{\mathbf{y}}_k, \widehat{\mathbf{G}})}{f_{i,k}(\tilde{\mathbf{y}}_k, \widehat{\mathbf{G}})}\right], \tag{44}$$

where

$$f_{i,k}(\tilde{\mathbf{y}}_k, \widehat{\mathbf{G}}) = \frac{\exp\big(-(\tilde{\mathbf{y}}_k)^H(\rho\mathbf{e}_i\mathbf{e}_i^H + \mathbf{R}_{\tilde{w}_k})^{-1}\tilde{\mathbf{y}}_k\big)}{|\rho\mathbf{e}_i\mathbf{e}_i^H + \mathbf{R}_{\tilde{w}_k}|}.$$

Using (40) and (44) in (35) proves the proposition.

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