Subscription Models for Differential Access to Real-time Information

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1. Introduction

Uncertainties in transportation supply strongly influence traffic operations and planning, and designing policies that account for travelers' responses to these uncertainties is critical for the success of transportation systems. Real-time information through vehicle- and infrastructure-based sensors and services built on those sensors such as Wejo's Real-Time Traffic Intelligence [12] offer the potential to quantify the uncertainty and influence travelers' response to uncertainties. Long-term impacts of information systems are studied using various game-theoretic models: user equilibrium with recourse [11], Markovian congestion games [2, 15], policy-based dynamic traffic assignment [5], and Bayesian congestion games [13]. The underlying tenet for these models replaces the traveler's route choice with a policy or strategy that adaptively determines the downstream arc at a node given the revealed information. Lately, efficient hyperbush algorithms have also been proposed [14] that can solve for strategy-based equilibrium flow efficiently on city-wide networks.

In this research, we take the perspective of a private agency that has a plethora of data on traffic systems and answer the question–**how to assess the system level impact for different individual subscription rates for available data?** Evaluating the success of a subscription rate requires a model that explains the choices of subscribed and unsubscribed travelers. Extending on our previous work [4], we contribute to the strategic equilibrium literature by presenting a novel multiclass assignment model under the presence of real-time information. Unlike prior assumptions that select a fleet of travelers to comply with the prescribed routes for improving system's efficiency [3, 9], we take a near-term perspective where a traveler subscribes to more information only if they can improve their utility (such as lower travel time, reduce decision making burden, or reliable travel experience avoiding extreme events). Accounting for travelers' responses, the data agency can then determine the level of subscription rates that are beneficial both to the system (lower system-wide delay) and the agency (high revenue).

A subscription model predicts the system level impact for a chosen subscription amount, a framework for which is shown in Figure 1. For a chosen subscription amount the subscription-choice model determines the percent of travelers in the population who subscribe to the additional travel information, which then determines the multiclass equilibrium solution and the expected total cost savings obtained from opting for subscriptions. The system-wide impact for the chosen subscription amount is determined using the fixed point solution to this framework.

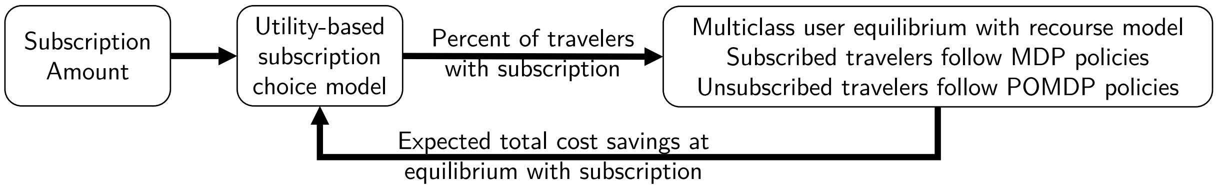

Figure 1 Framework for the subscription model to real-time Information

2. Problem Formulation

We focus on the static UER settings introduced in [11]. Let $G = (N, A)$ represent a directed network with N as set of nodes and A as set of links. Let $\mathcal{I}_i^+$ denote the set of all outgoing arcs and $Z \subseteq N$ denote the set of nodes where trips begin or end. For each $(u, v) \in Z^2$, let d_{uv} denote the number of travelers from origin u to destination v. Each link in the network exists in different *link-states*. Let S_{ij} be set of all link-states for link $(i, j) \in A$ with a certain discrete probability mass function. We assume that only the downstream links of a node have correlated travel times while other links have independent travel times (A#1), referred to as the node-independent arc travel time assumption in the literature [8]. Also define $S = \bigcup_{(i,j)\in A} S_{ij}$.

Let x_s denote the total vehicular flow using link-state s. Let $c_s(x_s)$ denote the generalized cost in link-state s as a function of total flow in the link-state x_s. Similar to the assumptions on link costs for standard traffic assignment, we assume that functions $c_s(\cdot)$ are separable by link-state flows, and are positive, strictly increasing, continuous, and differentiable functions of flow for all $s \in S$ (A#2).

Routing model for travelers with information subscription: Upon arrival at a node $i \in N$, subscribed travelers draw a realization of downstream arcs from the joint probability distribution of downstream arc travel times. Each realization is referred as a *node state*. Let $\Theta_i = \bigtimes_{j\in\mathcal{I}_i^+} S_{ij}$ denote the set of all possible node states at node i and the probability of occurrence of node state $\theta \in \Theta_i$ is denoted by q_i^θ. At each node-state $(i, \theta) \in \Phi$, a traveler chooses a downstream node $j \in \mathcal{I}_i^+$. This decision making for a traveler is captured by a *policy* $\pi : \Phi \to N$ that maps each node-state to a downstream node or a terminal node if the node corresponding to the node-state is a destination. We focus our attention on non-waiting and terminating policies. Let $\hat{\Pi}_{\Uparrow}^v$ denote the non-waiting policies terminating at destination $v \in Z$ and $\hat{\Pi}_{\Uparrow} = \bigcup_{v\in Z} \hat{\Pi}_{\Uparrow}^v$ be the set of all policies for subscribed travelers. Optimal policy given link-state costs is a solution to a Markov decision process (MDP) [1].

Routing model travelers without information subscription: Upon arrival at a node $i \in N$, unsubscribed travelers receive partial information about uncertainties through publicly available sources or past experiences. For example, on express lane systems, travelers without subscription might only see the toll but will not see the experienced travel times. We call this an *observation* of a node state. Let z denote the observation made at node i using the publicly displayed information and let $\mathcal{Z}_i$ denote the set of possible observations at node $i \in N$. Let $o(z|\theta)$ denote the probability of making observation $z \in \mathcal{Z}_i$ conditional to the true node state being $\theta \in \Theta_i$ for all $i \in N$. By the definition of the set of all possible observations,

$\sum_{z\in\mathcal{Z}_i} o(z|\theta) = 1$ for all $\theta \in \Theta_i$. Policies for unsubscribed travelers are modeled as a partially observable Markov-decision process (POMDP). Extending the work in POMDP literature [7, 10], we prove the following proposition.

Proposition 1. *Under the node-independent arc travel time assumption, observation at a node is a sufficient statistic for finding the optimal policy given partial information and uncertainties.*

Let $\mathcal{O}$ denote the set of all observations. Unsubscribed travelers follow a deterministic policy $\pi : \mathcal{O} \to N$ that maps the current observation of a node state to a downstream node. We use $\hat{\Pi}_{\Downarrow}$ and $\hat{\Pi}^v_{\Downarrow}$ to denote the set of all POMDP policies and policies terminating at destination $v \in Z$ for unsubscribed travelers.

Example Consider the network shown in Figure 2 where at node 1, four states are possible corresponding to reduced capacities on link (1,3). For two of low capacity states, the toll in the corresponding state on link (1,2) is high (\$3) and for the remaining two states, the toll is low (\$1).

Table 1 Node states at node 1 in the Example Network

Node State	Link (1,2) with constant capacity = 600	Link (1,3) with no toll
θ_1	Toll = \$3	Capacity = 300
θ_2	Toll = \$3	Capacity = 600
θ_3	Toll = \$1	Capacity = 900
θ_4	Toll = \$1	Capacity = 1200

For subscribed travelers, upon arrival at a node, the node state is revealed as real-time information through the subscription platform. For unsubscribed travelers, only the observation about tolls is available, resulting in policies with higher costs. The network transformation approach can be used to find optimal policies using dynamic programming methods [6].

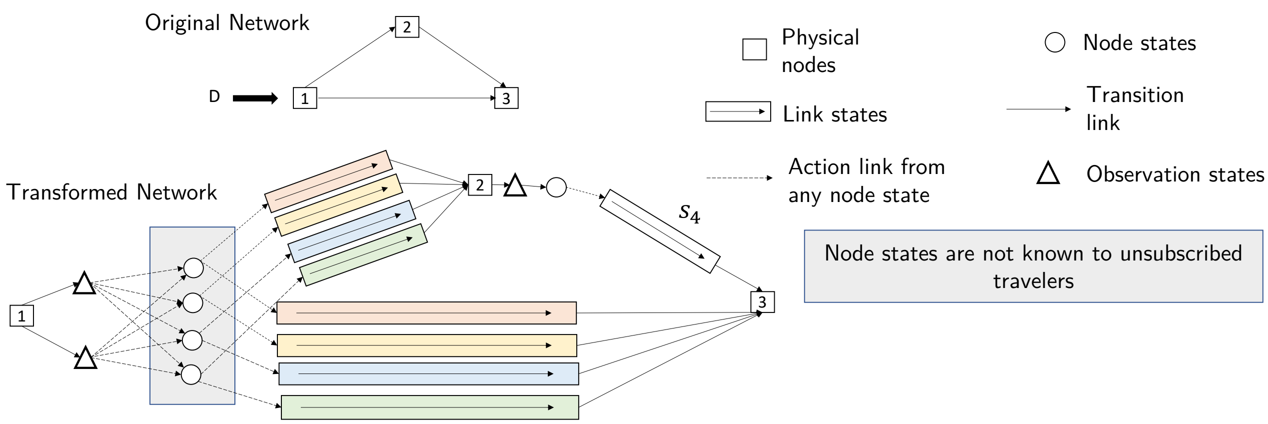

Figure 2 Network Transformation for Optimal Routing of Subscribed and Unsubscribed Travelers

Multiclass equilibrium model: We consider that both travel groups behave strategically and adapt chosen policies in response to other traveler's responses. For this modified congestion game, we are interested in knowing the equilibrium point (referred to as Multiclass-UER or MUER). For computing this equilibrium, we assume that the node-state and observation probability distributions are known in advance to travelers (A#3). Define y_u^π be the flow originating from node $u \in Z$ following policy π and $\mathbf{y}$ as a vector of all y_u^π for all $u \in Z$, $\pi \in \hat{\Pi}_\Uparrow^v \bigcup \hat{\Pi}_\Downarrow^v$, and $v \in Z$. At MUER, all used policies between an origin and a destination have equal and minimal expected costs, for each subscribed and unsubscribed traveler, that is:

$$y_u^\pi > 0 \Rightarrow C_u^\pi(\mathbf{y}) = \min_{\bar{\pi}\in\hat{\Pi}_\Uparrow^v} C_u^{\bar{\pi}}(\mathbf{y}) \quad \forall v \in Z, \pi \in \hat{\Pi}_\Uparrow^v \quad \text{and} \quad y_u^\pi > 0 \Rightarrow C_u^\pi(\mathbf{y}) = \min_{\bar{\pi}\in\hat{\Pi}_\Downarrow^v} C_u^{\bar{\pi}}(\mathbf{y}) \quad \forall v \in Z, \pi \in \hat{\Pi}_\Downarrow^v \tag{1}$$

It can be shown that MUER policy flows satisfying this principle are solution to the following convex program:

$$\min_{\mathbf{y}} Z(\mathbf{y}) = \sum_{s\in S} \int_0^{x_s} c_s(w)dw \tag{2}$$

$$\text{s.t. Demand conservation across policy flows} \tag{3}$$

$$\text{Policy flow and total link-state flow relationship (details skipped)} \tag{4}$$

$$\text{Non-negativity across policy flows} \tag{5}$$

Proposition 2. *Under assumptions A#1–A#3, the optimal solutions to the convex program* (2)–(5) *correspond exactly to the policy flows satisfying the MUER definition in Equation* (1)*.*

Proposition 3. *At equilibrium, the total link-state flows are unique while the flows for unsubscribed and subscribed travelers are not unique.*

Subscription choice model: Upon solving the MUER, we obtain the unique expected travel cost for subscribed ($C_\Uparrow^{*,uv}$) and unsubscribed ($C_\Downarrow^{*,uv}$) travelers for OD pair $(u,v) \in Z^2$. Let $C_{\text{savings}}(p)$ denote the demand-weighted average cost across all OD pairs at equilibrium: $C_{\text{savings}}(p) = \left(\sum_{(u,v)\in Z^2}\left(C_\Downarrow^{*,uv} - C_\Uparrow^{*,uv}\right)d_{uv}\right)/\sum d_{uv}$, where p denotes the proportion of subscribed travelers. We then use a binary Logit model to determine the proportion as a function of cost savings and subscription rate β:

$$p = \frac{1}{1+\exp(\beta - C_{\text{savings}}(p))} \quad p \in [0,1] \tag{6}$$

Proposition 4. *For a given subscription rate, there exists a unique fixed point to Equation* (6)*.*

3. Solution Algorithms and Conclusions

Solving the fixed point for the subscription model in Figure 1 requires an iterative structure as shown below:

Algorithm 1 Iterative algorithm for solving the subscription model for a given subscription rate β

Step 1: Start with a guess $p \in (0,1)$

Step 2: Solve MUER using hyperbush algorithm in [14]

Step 3: Compute $C_{\text{savings}}(p)$ and updated value of proportion using Equation (6). If the percent change is below a threshold, stop. Else, go back to Step 2.

Computational results are skipped for brevity. Broadly, in this abstract, we presented the model and propositions for determining the effect of subscription rates. The proposed subscription model can be used for designing optimal subscription rates in various settings where real-time information can be a valuable tool such as express lanes, parking systems, roadside delivery, and routing of vulnerable road users.

Acknowledgments

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References

[1] Boyles SD, Rambha T, 2016 *A note on detecting unbounded instances of the online shortest path problem. Networks* 67(4):270–276.

[2] Calderone DJ, 2017 *Models of competition for intelligent transportation infrastructure: Parking, ridesharing, and external factors in routing decisions* (University of California, Berkeley).

[3] Chen Z, Lin X, Yin Y, Li M, 2020 *Path controlling of automated vehicles for system optimum on transportation networks with heterogeneous traffic stream. Transportation Research Part C: Emerging Technologies* 110:312–329.

[4] Folsom L, Park H, Pandey V, 2022 *Dynamic routing of heterogeneous users after traffic disruptions under a mixed information framework. Frontiers in Future Transportation* 3.

[5] Gao S, 2012 *Modeling strategic route choice and real-time information impacts in stochastic and time-dependent networks. IEEE Transactions on Intelligent Transportation Systems* 13(3):1298–1311.

[6] Miller-Hooks E, 2001 *Adaptive least-expected time paths in stochastic, time-varying transportation and data networks. Networks: An International Journal* 37(1):35–52.

[7] Pandey V, 2021 *Adaptive routing algorithms on express lanes with partial online information. Presented at 100th Transportation Research Board Annual Meeting*, na, na (Washington D.C.: TRB).

[8] Provan JS, 2003 *A polynomial-time algorithm to find shortest paths with recourse. Networks: An International Journal* 41(2):115–125.

[9] Sharon G, Albert M, Rambha T, Boyles S, Stone P, 2018 *Traffic optimization for a mixture of self-interested and compliant agents. Proceedings of the AAAI Conference on Artificial Intelligence*, volume 32.

[10] Striebel C, 1965 *Sufficient statistics in the optimum control of stochastic systems. Journal of Mathematical Analysis and Applications* 12(3):576–592.

[11] Unnikrishnan A, Waller ST, 2009 *User equilibrium with recourse. Networks and Spatial Economics* 9(4):575–593.

[12] Wejo, 2022 *Real-time traffic data for smart mobility intelligence.* URL `https://www.wejo.com/products/real-time-traffic-intelligence`.

[13] Wu M, Amin S, Ozdaglar AE, 2021 *Value of information in bayesian routing games. Operations Research* 69(1):148–163.

[14] Xu Z, Xie J, Liu X, Nie Y, 2022 *Hyperbush algorithm for strategy-based equilibrium traffic assignment problems. Transportation Science* .

[15] Zimmermann M, Frejinger E, Marcotte P, 2021 *A strategic markovian traffic equilibrium model for capacitated networks. Transportation Science* 55(3):574–591.