

A new signal injection-based method for estimation of position in interior permanent magnet synchronous motors

ISSN 1755-4535

Received on 13th December 2019

Revised 15th February 2020

Accepted on 16th March 2020

E-First on 21st April 2020

doi: 10.1049/iet-pel.2019.1558

www.ietdl.org

Bowen Yi^{1,2}, Slobodan N. Vukosavić³, Romeo Ortega^{2,4}, Aleksandar M. Stanković⁵, Weidong Zhang¹ ✉¹Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China²Laboratoire des Signaux et Systèmes, CNRS - CentraleSupélec, Gif-sur-Yvette 91192, France³Electrical Engineering Department, University of Belgrade, Belgrade 11000, Serbia⁴Department of Control Systems and Informatics, ITMO University, St. Petersburg, Russia⁵Department of Electrical Engineering and Computer Science, Tufts University, Medford, MA 02155, USA

✉ E-mail: wdzhang@sjtu.edu.cn

Abstract: Several heuristic procedures to estimate the rotor position of interior permanent magnet synchronous motors (IPMSM) via signal injection have been reported in the application literature, and are widely used in practice. These methods, based on the use of linear time-invariant (LTI) high-pass/low-pass filtering, are instrumental for the development of sensorless controllers. To the best of authors' knowledge, no theoretical analysis of these methods has been carried out. The objectives of this note are (i) to invoke some recent work on the application of averaging techniques for injection-based observer design to develop a theoretical framework to analyse the LTI filtering used in sensorless methods and (ii) to propose a new method that on the one hand ensures improved accuracy and on the other hand can be related to the current filtering technique. An additional advantage of the new method is that it relies on the use of linear operators, implementable with simple computations. The effectiveness of the proposed scheme is assessed by experiments on an IPMSM platform driven by a 521 V DC bus with a 5-kHz pulse-width modulation.

Nomenclature

Symbols

$\alpha - \beta$	stationary axis reference frame quantities
$d - q$	synchronous axis reference frame quantities
n_p	number of pole pairs
R_s	stator resistance, Ω
ω	angular velocity, rad/s
Φ	magnetic flux, Wb
J	drive inertia, kg m^2
T_L	load torque, N m
f	friction constant
θ	rotor flux angle, rad
L_d, L_q	d - and q -axis inductances, H
v, i	stator voltage and current [V, A]
ω_h	angular frequency of injection signal, rad/s
ε	period of injection signal ($\varepsilon = 2\pi/\omega_h$), s
V_h	amplitude of injection signal, V
$\ \cdot\ $	Euclidean norm
s	Laplace transform symbol
p	differential operator $p = d/dt$
$y(t) = \mathcal{H}[u(t)]$	bounded-input bounded-output operator $\mathcal{H}$ acting on the input signal $u(t)$ to generate the output $y(t)$
y_v	virtual output
$i_{\alpha\beta}$	$[i_\alpha, i_\beta]^T$
$v_{\alpha\beta}$	$[v_\alpha, v_\beta]^T$
$\mathbf{I}, \mathcal{I}$	identity matrix on $\mathbb{R}^{2 \times 2}$ and $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Superscripts

r	actual reference frame
$\hat{r}$	estimated reference frame
r^*	reference value
r^h	high-frequency component

r^L	low-frequency component
$v_{\alpha\beta}^C$	low-frequency control input

1 Introduction

Permanent magnet synchronous motors (PMSMs) are widely used in industrial applications because of their superior power density and high efficiency. One of the more—practically and theoretically—challenging open problems for PMSMs is the design of controllers without rotational sensors, the so-called sensorless control. Two different types of sensorless control methodologies are currently being used in practice. The first one is a model-based method, which is known in applications as the back-emf or flux-linkage estimation. In this method, the fundamental components of the electrical signals are used to design a back-emf observer or a flux observer [1–9]. The second one is a saliency-tracking-based method, in which information is extracted from the high-frequency components of stator currents via high-frequency signal injections [10–13].

It is well-known that because of the loss of observability at standstill [8], observer-based methods cannot be used in the low-speed region [11]. On the other hand, the performance of the saliency tracking-based method, which utilises the anisotropy due to the rotor saliency and/or magnetic saturation, is not degraded at low speeds. In this study, we address the problem of position estimation for PMSMs at low speeds or standstill, using a signal injection-based method.

There are two kinds of PMSMs, surface PMSMs (SPMSMs) or interior PMSMs (IPMSMs), the difference being the location of the permanent magnets, either on the surface of the rotor, for the former, or buried in the cavities of the rotor core, for the latter. There are several technical reasons why IPMSMs are more convenient in applications than SPMSMs, see [14, Table 6.2]. On the other hand, the magnetic characteristics of IPMSM, and consequently the dynamic model is, far more complicated than one of the SPMSMs—see the discussion in [14, Subsection 6.2.2] and [7, Section 6]. As a matter of fact, because of the inability to deal

with this complex dynamics, the overwhelming majority of papers published by the control community in sensorless control operating in medium- or high-speed regions are concerned with SPMSMs, see e.g. [2–5, 8, 9]—with [15, 16] a notable exception. In contrast, the saliency-tracking methods at low speeds have received more attention from the applied journals in the context of IPMSMs.

Owing to the rotor saliency, the signal injection method is more efficiently applied in IPMSMs than in SPMSMs. For this reason, we consider in this study IPMSMs. In the last two decades, signal injection-based approaches have been successfully applied, in a heuristic manner, in various applications, some of them reported in application journals [10–12, 14, 17, 18]. The classical approach is, first, to inject high-frequency probing signals into the motor terminal; then, extract the high-frequency components of the stator currents to get position estimates. Besides the question of the choice of the injection signal and its mode of application, the key problem is the signal processing of the measured stator currents to extract the desired information of the mechanical coordinates. This task is usually achieved via the combination of linear time-invariant (LTI) high pass-filters (HPFs) and low-pass filters (LPFs) [14]—an approach that is justified by a series of technique-oriented practical considerations, hard to fit into a rigorous theoretical framework. To the best of our knowledge, no theoretical analysis of these heuristic methods has been reported in the literature.

Our contributions in this study are threefold.

- Provide a theoretical framework for the analysis of conventional LTI filtering methods used in injection-based sensorless control of IPMSMs.
- Propose a new method for the extraction of the information on the mechanical coordinates that, using the aforementioned framework, is shown to be superior to the existing LTI filtering technique. An important aspect of this point is that the increase in computational complexity with respect to the current HPF/LPF practice should be negligible.
- Prove that the new proposed method admits an HPF/LPF interpretation. This is an important issue since it shows the connection—and downwards-compatibility—of the new method with standard industry practice.

Towards this end, we rely on the recent work [13, 19–21], where invoking averaging techniques, a rigorous theoretical analysis of injection-based methods for observer design has been carried out. The importance of disposing of rigorous analytic results can hardly be overestimated, since it allows, on the one hand, to carry out a quantitative performance assessment while, on the other hand, it provides guidelines to make more systematic and simplify the parameter tuning procedure. We underscore that the IPMSM model adopted in the study is widely accepted by the drives community since it precisely describes the behaviour of the machine in the absence of magnetic saturation, a phenomenon that is conspicuous by its absence when the load in the motor is within the normal operating range.

The remainder of paper is organised as follows. In Section 2, we recall the mathematical model of IPMSMs and formulate the problem of estimation of position using signal injection. Section 3 discusses the classical frequency-based accuracy analysis of the position estimators used for the conventional methods, and highlights their theoretical limitations. In Section 4, the new method is proposed, and then some comparisons and similarities with the conventional methods are given in Section 5. Simulation and experimental results are given in Section 6. The paper is wrapped up with some concluding remarks in Section 7.

Caveat. An abridged version of this paper has been presented in [22].

2 Model and problem formulation

The voltage equations of the IPMSM in the stationary frame are given by [14]

$$v_{\alpha\beta} = [R_s I + L(\theta)p - 2n_p \omega L_1 Q(\theta) \mathcal{F}] i_{\alpha\beta} + n_p \omega \Phi \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}, \quad (1)$$

where we define the mappings

$$L(\theta) := L_0 I + L_1 Q(\theta) \\ Q(\theta) := \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix},$$

with the averaged inductance L_0 and the inductance difference value L_1 as

$$L_0 := \frac{1}{2}(L_d + L_q), \quad L_1 := \frac{1}{2}(L_d - L_q).$$

The stationary model (1), together with the mechanical dynamics, can be expressed in the standard state-space form as follows:

$$\begin{aligned} L(\theta) \frac{d}{dt} i_{\alpha\beta} &= F(i_{\alpha\beta}, \theta, \omega) + v_{\alpha\beta} \\ \frac{d}{dt} \theta &= n_p \omega \\ J \frac{d}{dt} \omega &= n_p \Phi (i_\beta \cos \theta - i_\alpha \sin \theta) - f \omega - T_L, \end{aligned} \quad (2)$$

where we define the mapping

$$F(\cdot) := (2n_p \omega L_1 Q(\theta) \mathcal{F} - R_s I) i_{\alpha\beta} + n_p \omega \Phi \begin{bmatrix} \sin \theta \\ -\cos \theta \end{bmatrix}.$$

Problem formulation (position estimation via signal injection): Assume there is a stabilising controller operator Σ_C measuring only $i_{\alpha\beta}$, and define its output as

$$v_{\alpha\beta}^C(t) := \Sigma_C[i_{\alpha\beta}(t)].$$

Inject a high-frequency signal to one axis of the control voltage, say, the α -axis, i.e.

$$v_{\alpha\beta} = v_{\alpha\beta}^C + \begin{bmatrix} V_h \sin \omega_h t \\ 0 \end{bmatrix}, \quad (3)$$

where $\omega_h := 2\pi/\varepsilon$, with $\varepsilon > 0$ small, and $V_h > 0$. The problem is to define an operator

$$\Sigma_E: i_{\alpha\beta} \mapsto \hat{\theta}$$

such that

$$\limsup_{t \rightarrow \infty} \left| \hat{\theta}(t) - \theta(t) \right| \leq \mathcal{O}(\varepsilon), \quad (4)$$

where $\mathcal{O}$ is the uniform big O symbol. [That is, $f(z, \varepsilon) = \mathcal{O}(\varepsilon)$ if and only if $|f(z, \varepsilon)| \leq C\varepsilon$, for a constant C independent of z and ε . Clearly, $\mathcal{O}(1)$ means the boundedness of a signal.]

It is well-known that high-frequency probing signals have almost no effect on the motor mechanical coordinates. However, due to the rotor saliency, it induces different high-frequency responses in the α - and β -axes currents. This fact provides the possibility to recover the angle from the high-frequency components of stator currents.

3 Frequency decomposition and quantitative analysis of conventional methods

In this section, we give the analysis of frequency decomposition of the stator currents $i_{\alpha\beta}$, which is instrumental for the design and analysis of position estimators.

3.1 Conventional frequency analysis

First, we recall the conventional frequency decomposition in the technique-oriented literature, which relies on the *ad-hoc*

application of the superposition law [11, 14], i.e. suppose the electrical states consist of high-frequency and low-frequency components as

$$(\cdot)_{\alpha\beta} = (\cdot)_{\alpha\beta}^h + (\cdot)_{\alpha\beta}^l.$$

If $\omega \simeq 0$, the current responses can be separated as

$$v_{\alpha\beta}^l + v_{\alpha\beta}^h = (R_s I + L(\theta)p)(i_{\alpha\beta}^l + i_{\alpha\beta}^h). \quad (5)$$

For the approximated high-frequency model

$$v_{\alpha\beta}^h \simeq L(\theta)p i_{\alpha\beta}^h,$$

neglecting the stator resistance, the angle θ can be regarded as a constant, thus the high-frequency response contains the information of θ , namely, for the input (3) we have

$$\begin{aligned} i_{\alpha}^h &= \frac{V_h(L_0 - L_1 \cos 2\theta)}{L_d L_q p} [\sin \omega_h t] \\ i_{\beta}^h &= \frac{-(V_h L_1 \sin 2\theta)}{L_d L_q p} [\sin \omega_h t]. \end{aligned}$$

Substituting $p = (d/dt)$, we approximately get the high-frequency components of the stator current as

$$i_{\alpha\beta}^h = \frac{1}{\omega_h L_d L_q} \begin{bmatrix} -L_1 \cos 2\theta + L_0 \\ -L_1 \sin 2\theta \end{bmatrix} (-V_h \cos \omega_h t). \quad (6)$$

The derivation of the above high-frequency model is based on two assumptions, namely, the superposition law and the slow angular velocity $\omega \simeq 0$, regarding which, the following remarks are in order.

R1 The dynamics (2) is highly non-linear. It is well-known that non-linear systems ‘mix’ the frequencies, making the superposition law not applicable. Although using the classical decomposition (5) to estimate position may work in practice, it fails to reliably provide, neither a framework for a quantitative performance assessment nor guidelines to tune parameters.

R2 The assumption $\omega \simeq 0$ implies that the decomposition above is applicable only at standstill or very low speeds.

3.2 Frequency analysis via averaging

Averaging analysis provides a rigorous and elegant decomposition of the measured currents as follows. We refer the reader to [23, 24] for the basic theory on averaging analysis. Applying averaging analysis, it is shown in [13] that with $\omega_h > 0$ large enough

$$i_{\alpha\beta} = \bar{i}_{\alpha\beta} + \varepsilon y_{\alpha\beta} S + \mathcal{O}(\varepsilon^2), \quad (7)$$

where we defined the signal

$$S(t) := -\frac{V_h}{2\pi} \cos(\omega_h t), \quad (8)$$

the (so-called) virtual output

$$y_v := \frac{1}{L_d L_q} \begin{bmatrix} -L_1 \cos 2\theta + L_0 \\ -L_1 \sin 2\theta \end{bmatrix}, \quad (9)$$

and $\bar{i}_{\alpha\beta}$ is the current of the closed-loop system with $v_{\alpha\beta} = v_{\alpha\beta}^c$, i.e. without signal injection. From (9) it is clear that the angular position θ can be recovered from the virtual output y_v , i.e.

$$\theta = \frac{1}{2} \arctan \left\{ \frac{y_{v2}}{y_{v1} - \frac{L_0}{L_d L_q}} \right\}.$$

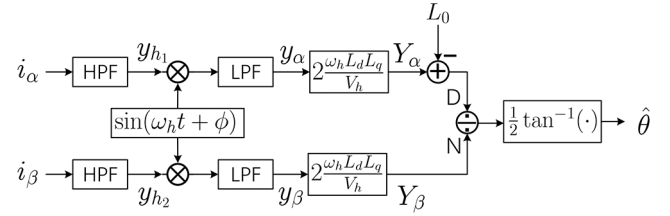


Fig. 1 Block diagram of the conventional signal injection method [14]

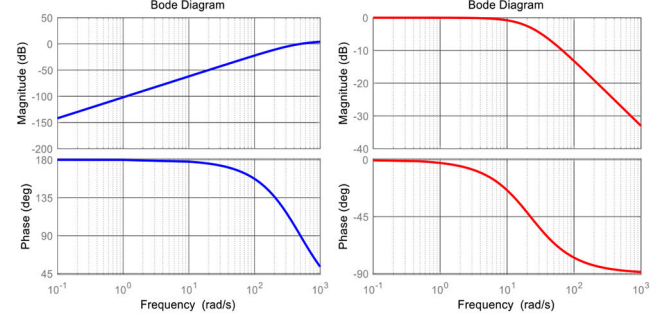


Fig. 2 Bode diagram of the HPF/LPF (10) ($\omega_h = 500$, $\omega_* = 1$)

Hence the position estimation problem is translated into the estimation of y_v . Towards this end, we notice that from a frequency viewpoint, $i_{\alpha\beta}$ contains fundamental frequency component $\bar{i}_{\alpha\beta}$ and high-frequency component $\varepsilon y_v S$. It should be noticed that the high-frequency term $\varepsilon y_v S$ coincides with the one in (6), but the averaging analysis characterises all the components in $i_{\alpha\beta}$ quantitatively.

It is natural, then, that to ‘reconstruct’ y_v —out of measurements of $i_{\alpha\beta}$ —we need to separate these components via some sort of HPF and LPF operations. This is the rationale underlying most of the existing position estimators reported in the literature, see [14] for a recent review.

R3 The tiny term $\mathcal{O}(\varepsilon^2)$ in (7) is caused by second-order periodic averaging analysis, which is concerned with solving a perturbation problem in a properly selected time scale.

R4 Rigorously, the function $\arctan$, adopted in the study for convenience, should be replaced by the two-argument arctangent function $\text{atan2}(\cdot, \cdot)$, the fact widely known in the drives community.

3.3 Quantitative results of conventional methods

In [14], the position estimation method, for low rotation speeds, shown in Fig. 1 is proposed. To evaluate the performance of the classical method depicted in Fig. 1, without loss of generality, select the LTI filters as the transfer function [Given a smooth signal $y(t)$ and its Laplace transform $Y(s)$, the following operators in the time domain can be written as $\text{HPF}[y(t)] = \frac{2p^2}{(\lambda_h + p)^2} [y(t)]$ and $\text{LPF}[y(t)] = \frac{\lambda_\ell}{\lambda_\ell + p} [y(t)]$. For brevity, we make a slight abuse of notation using both $\text{HPF}[y(t)]$ and $\text{HPF}(s)Y(s)$ below.]

$$\begin{aligned} \text{HPF}(s) &= \frac{2s^2}{(\lambda_h + s)^2} \\ \text{LPF}(s) &= \frac{\lambda_\ell}{\lambda_\ell + s}, \end{aligned} \quad (10)$$

with parameters

$$\lambda_h = \omega_h, \quad \lambda_\ell = \max \{ \sqrt{\omega_h \omega_*}, 1 \}. \quad (11)$$

The Bode diagrams of two filters are given in Fig. 2 with $\omega_h = 500$ and $\omega_* = 1$.

Applying averaging analysis at reduced speeds and setting $\phi = 0$, we have the following.

Proposition 1: For the IPMSM model (2), suppose the control $v_{\alpha\beta}^C$ guarantees all the states bounded, with the speed

$$|(\bar{\omega}, \dot{\bar{\omega}}, \dot{v}_{\alpha\beta}^C)| \leq \ell_\omega$$

for some constant ℓ_ω independent of ε . If the filters are selected as (10) and (11), then the signal processing procedure depicted in Fig. 1, namely

$$\begin{aligned} y_h &= \text{HPF}[i_{\alpha\beta}] \\ Y &= 2 \frac{\omega_h}{V_h} L_q L_q \text{LPF}[y_h \sin(\omega_h t + \phi)] \\ \hat{\theta} &= \frac{1}{2} \arctan \left\{ \frac{Y_\beta}{Y_\alpha - L_0} \right\} \end{aligned} \quad (12)$$

with $\phi = 0$ and

$$Y := \text{col}(Y_\alpha, Y_\beta),$$

guarantees

$$\limsup_{t \rightarrow \infty} |\hat{\theta}(t) - \theta(t)| = n\pi + \mathcal{O}(\varepsilon^{\frac{1}{2}})$$

for $n \in \mathbb{Z}$, when $\omega_h \geq \omega_h^*$ for some $\omega_h^* > 0$, with $\varepsilon = 2\pi/\omega_h$.

Before presenting the proof, let us say a few words regarding the proposition intuitively. The widely popular LTI filtering technique for signal injection, illustrated in Fig. 1, is compactly expressed in (12). To analyse it quantitatively, we make a mild assumption of bounded angular velocity and its time derivative. Proposition 1 figures out the steady-state accuracy $\mathcal{O}(\varepsilon^{1/2})$ of the conventional LTI filtering method with the suggested parameters.

Proof: Applying the operator HPF to (7), we have [We omit the exponentially decaying term ε_i of filtered signals in the following analysis.]

$$\text{HPF}[i_{\alpha\beta}] = \text{HPF}[\bar{i}_{\alpha\beta}] + \frac{2\pi}{\omega_h} \text{HPF}[D(\theta)S] + \mathcal{O}(\varepsilon^2) + \varepsilon_i, \quad (13)$$

with the definition

$$D(\theta) := L^{-1}(\theta) \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

For the first term of (13), we have

$$\begin{aligned} \text{HPF}[\bar{i}_{\alpha\beta}] &= \frac{2}{(\omega_h + p)^2} [r_1(t)] \\ r_1(t) &:= \frac{\partial \mathcal{F}}{\partial i_{\alpha\beta}} \cdot (\mathcal{F} + L^{-1} v_{\alpha\beta}^C) + \left(\frac{\partial \mathcal{F}}{\partial \bar{\omega}} + L^{-1} \right) \mathcal{O}(\ell_\omega) \\ &\quad + n_p \bar{\omega} \left(\frac{\partial \mathcal{F}}{\partial \theta} + \frac{\partial L^{-1}}{\partial \theta} v_{\alpha\beta}^C \right) \end{aligned}$$

where we have used the assumption

$$|(\bar{\omega}, \dot{\bar{\omega}}, \dot{v}_{\alpha\beta}^C)| \leq \ell_\omega$$

in the last term, with

$$\mathcal{F}(i_{\alpha\beta}, \theta, \omega) := L^{-1}(\theta) F(i_{\alpha\beta}, \theta, \omega).$$

There always exists a constant $\omega_h^* \in \mathbb{R}_+$ such that for $\omega_h > \omega_h^*$

$$\text{HPF}[\bar{i}_{\alpha\beta}] = \frac{2}{\omega_h^2} \cdot \frac{\omega_h^2}{(\omega_h + p)^2} [\mathcal{O}(1)].$$

Some basic linear system analysis shows

$$\left| \frac{\omega_h^2}{(\omega_h + p)^2} [\mathcal{O}(1)] \right| = \mathcal{O}(1),$$

thus yielding

$$\text{HPF}[\bar{i}_{\alpha\beta}] = \mathcal{O}(\varepsilon^2).$$

For the second term in the right-hand side of (13), we have

$$\begin{aligned} \frac{2\pi}{\omega_h} \text{HPF}[D(\theta)S] &= -\frac{2V_h}{\omega_h} \frac{1}{(\omega_h + p)^2} [r_2(t)] \\ r_2(t) &:= a_1 \cos(\omega_h t) + a_2 \omega_h \sin(\omega_h t) \\ &\quad - a_3 \omega_h \sin(\omega_h t) - \omega_h^2 D(\theta) \cos(\omega_h t). \end{aligned}$$

with

$$\begin{aligned} a_1 &:= \frac{d}{dt} (n_p \omega D'(\theta)), \\ a_2 &:= n_p \omega D'(\theta), \\ a_3 &:= \omega_h D'(\theta) n_p \omega, \end{aligned}$$

the derivatives of which are all bounded. If the parameter ω_h is large enough, we have

$$\frac{2\pi}{\omega_h} \text{HPF}[D(\theta)S] = \frac{1}{\omega_h} V_h D(\theta) \sin(\omega_h t) + \mathcal{O}(\varepsilon^2).$$

Therefore, the currents filtered by the HPFs become

$$\begin{aligned} y_h &:= \text{HPF}[i_{\alpha\beta}] \\ &= \frac{1}{\omega_h} V_h D(\theta) \sin(\omega_h t) + \mathcal{O}(\varepsilon^2). \end{aligned} \quad (14)$$

Multiplying $\sin(\omega_h t + \phi)$ on both sides with $\phi = 0$, we get

$$\sin(\omega_h t) y_h = \frac{V_h}{2\omega_h} D(\theta) - \frac{V_h}{2\omega_h} D(\theta) \cos(2\omega_h t) + \mathcal{O}(\varepsilon^2), \quad (15)$$

where we have used the trigonometric identity

$$\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta).$$

Applying the LPF to (15), for the first term we have

$$\text{LPF} \left[\frac{V_h}{2\omega_h} D(\theta) \right] = \frac{V_h}{2\omega_h} D(\theta) + \mathcal{O}(\varepsilon^{3/2}).$$

For the second term, we have

$$\text{LPF} \left[\frac{V_h}{2\omega_h} D(\theta) \cos(2\omega_h t) \right] = \mathcal{O}(\varepsilon^{3/2}),$$

with straightforward calculations and the swapping lemma.

Therefore, the filtered signal satisfies

$$\begin{bmatrix} y_\alpha \\ y_\beta \end{bmatrix} := \text{LPF}[\sin(\omega_h t) y_h] = \frac{V_h}{2\omega_h} D(\theta) + \mathcal{O}(\varepsilon^{3/2}).$$

Notice the explicit form of $D(\theta)$, thus we having

$$\begin{aligned} \begin{bmatrix} Y_\alpha \\ Y_\beta \end{bmatrix} &:= \begin{bmatrix} \frac{2\omega_h L_d L_q}{V_h} y_\alpha \\ \frac{2\omega_h L_d L_q}{V_h} y_\beta \end{bmatrix} \\ &= \begin{bmatrix} L_0 - L_1 \cos 2\theta \\ -L_1 \sin 2\theta \end{bmatrix} + \mathcal{O}(\varepsilon^{1/2}). \end{aligned}$$

It is obvious that when time t goes to infinity, the identity

$$\tan 2\theta = \frac{Y_\beta}{Y_\alpha - L_0} + \mathcal{O}(\varepsilon^{1/2})$$

holds. Thus,

$$\theta = \frac{1}{2} \arctan \left\{ \frac{Y_\beta}{Y_\alpha - L_0} \right\} + n\pi + \mathcal{O}(\varepsilon^{1/2}) + \varepsilon_t,$$

with $n \in \mathbb{Z}$. It completes the proof. $\square$

The saliency-tracking-based method has an angular ambiguity of π . It is possible to utilise the saturation effect in the d -axis of machine, as well as y_v to conduct the magnetic polarity identification. The problem is out of scope of the study, and we refer the readers to [12, 18] for more details. It should be underscored that the above quantitative analysis does not rely the constant speed assumption, making it also applicable to transient stages in low speed region.

When the IPMSM is working at standstill, the estimation accuracy at the steady stage becomes also $\mathcal{O}(\varepsilon)$. A corollary at standstill is given as follows.

Corollary 1: For proposition (1) with $\omega \equiv 0$, we have

$$\limsup_{t \rightarrow \infty} \left| \hat{\theta}(t) - \theta(t) \right| = n\pi + \mathcal{O}(\varepsilon)$$

with $n \in \mathbb{Z}$.

Proof: It follows clearly with $\lambda_\ell = 1$. $\square$

The above corollary underscores that the position estimation at standstill admits the same order of accuracy with the one of the proposed virtual output estimator. This is because the angular position θ degenerates as a constant for such a case.

4 Proposed estimation method

4.1 New design

In this section, we propose a new estimator following the methodology in [25, 26]. Before presenting the new design, we define three bounded-input bounded-output-stable, linear operators,

- first, the delay operator $\mathcal{D}_d$, with parameter $d > 0$

$$\mathcal{D}_d[u(t)] = u(t - d); \quad (16)$$

- second, the weighted zero-order-hold operator $\mathcal{Z}_w$, parameterised by $w > 0$, and defined as

$$\begin{aligned} \dot{\chi}(t) &= u(t) \\ \mathcal{Z}_w[u(t)] &= \frac{1}{w} [\chi(t) - \chi(t - w)]; \end{aligned} \quad (17)$$

- third, the linear time-varying (LTV) operator $\mathcal{G}_{\text{grad}}$ defined as

$$\begin{aligned} \dot{x}(t) &= -\gamma S^2(t)x(t) + \gamma S(t)u(t) \\ \mathcal{G}_{\text{grad}}[u(t)] &= \frac{1}{\varepsilon} x(t), \end{aligned} \quad (18)$$

where $\gamma > 0$ is a tuning gain.

These operators are instrumental for the following design.

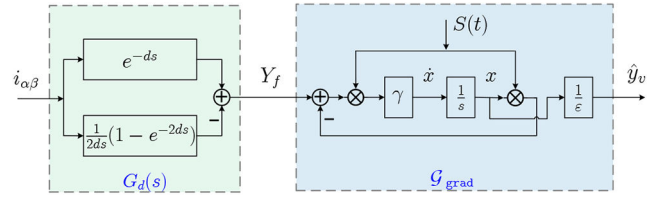


Fig. 3 Block diagram of the proposed estimation method

To construct the estimator, apply the first two operators to the currents as follows:

$$Y_f(t) := (\mathcal{D}_d - \mathcal{Z}_{2d})[i_{\alpha\beta}(t)]. \quad (19)$$

We make the observation that using the Laplace transform, the action of (19) may be represented in the frequency domain as

$$Y_f(s) = G_d(s)i_{\alpha\beta}(s),$$

where we defined the transfer function

$$G_d(s) := e^{-ds} + \frac{1}{2ds}(e^{-2ds} - 1). \quad (20)$$

The description of the estimator is completed applying the third operator $\mathcal{G}_{\text{grad}}$ to Y_f to generate the estimate of y_v , denoted as $\hat{y}_v$, i.e.

$$\hat{y}_v(t) = \mathcal{G}_{\text{grad}}[Y_f(t)]. \quad (21)$$

See Fig. 3. The measured stator current $i_{\alpha\beta}$ is first filtered by the transfer function $G_d(s)$ defined in (20), then going through a gradient descent operator $\mathcal{G}_{\text{grad}}$, and we will get the estimate of the virtual output y_v . As discussed in Section 3.2, it is equivalent to obtaining the angular position θ , which is our target. This is the signal processing procedure of our new design, the properties of which will be introduced below.

Using the analysis reported in [25], with $d = \varepsilon$ it is shown that the estimator (19), (21) verifies

$$\lim_{t \rightarrow \infty} \left| \hat{y}_v(t) - y_v(t) \right| \leq \mathcal{O}(\varepsilon).$$

A rigorous statement is given as follows.

Proposition 2: For the dynamical model of IPMSMs (2), suppose the control $v_{\alpha\beta}^C$ guarantees all states bounded and the speed

$$|\dot{y}_v| \leq \ell_v \quad (22)$$

for some constant ℓ_v , there exist constants $\omega_h^*, \gamma^* > 0$ such that for $\omega_h > \omega_h^*$ and $\gamma > (\gamma^*/\varepsilon)$, the estimate satisfies

$$\limsup_{t \rightarrow \infty} \left| \mathcal{G}_{\text{grad}}^\gamma \circ G_d(s)[i_{\alpha\beta}(t)] - y_v(t) \right| = \mathcal{O}(\varepsilon)$$

where y_v is defined in (9) with $d = \varepsilon$.

Proof: We give a brief outline here. With the Taylor expansion, we can obtain the time-varying regressor

$$Y_f(t) = S(t)\theta_v(t - d) + \mathcal{O}(\varepsilon^2), \quad (23)$$

with

$$Y_f(t) := G_d(s)[i_{\alpha\beta}(t)], \quad \theta_v(t) = \varepsilon y_v(t).$$

Define the error signal

$$\tilde{\theta}_v := \hat{\theta}_v - \varepsilon \gamma_v.$$

Invoking (23) and Assumption (22), if $\gamma > (\gamma^*/\varepsilon)$ we get

$$\dot{\tilde{\theta}}_v := -\gamma S^2(t) \tilde{\theta}_v + \mathcal{O}(\varepsilon). \quad (24)$$

Clearly, the signal $S(t)$ is of persistent excitation, that is,

$$\int_t^{t+\frac{1}{\varepsilon}} S^2(\tau) d\tau \geq S_0,$$

for all $t \geq 0$ and some $S_0 > 0$. Invoking Krasovskii's theorem and carrying-out some basic perturbation analysis, it completes the proof. $\square$

Thus, defining the angle estimate as

$$\hat{\theta} = \frac{1}{2} \arctan \left\{ \frac{\hat{y}_{v2}}{\hat{y}_{v1} - (L_0/L_d L_q)} \right\}, \quad (25)$$

the required asymptotic accuracy (4) is achieved.

R5 The above analysis shows the accuracy enhancement of the new design, compared with the conventional LTI filtering technique. It should be pointed out that all the operators used in the new design are linear, making the algorithm highly efficient. Indeed, the proposed algorithm can be regarded as an LTV operator.

4.2 Parameter tuning

For the implementation of the proposed estimator, there are three tunable parameters, namely, γ , ω_h , and V_h . We are in position to give some discussions on their selections.

D1 A larger gain γ yields a faster convergence speed of the estimation error. The performance assessment, including the transient and steady-state stages, of the virtual output estimates is instrumental for its tuning.

D2 For the frequency parameter ω_h , there is a trade-off between the estimation accuracy and the sensitivity to unavoidable high-frequency measurement noises [27]. On the one hand, a higher frequency increases the accuracy. On the other hand, the measurement is

$$i_{\alpha\beta} = \tilde{i}_{\alpha\beta} + \varepsilon \gamma_v S + \mathcal{O}(\varepsilon^2) + \nu, \quad (26)$$

where ν is the measurement noise. Thus, a higher frequency will decrease the signal-to-noise ratio.

D3 The amplitude V_h shares some similar effects on the estimation performance with ω_h , due to (26), since, a larger amplitude V_h increases the signal-to-noise ratio. However, if the probing amplitude V_h is sufficiently large, the oscillation in mechanical coordinates is not neglectable.

4.3 Model with magnetic saturation

To clarify the underlying mechanism of conventional LTI filtering methods we consider in this study the basic IPMSM model (1). The proposed analysis may be adapted, almost *verbatim*, to more

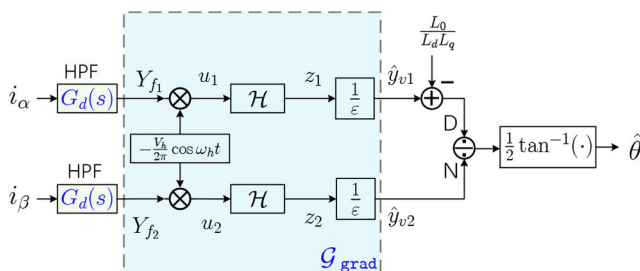


Fig. 4 Equivalent block diagram of the proposed estimation method

general models, e.g. the model with magnetic saturation, which we proceed to discuss below.

Considering the flux linkage $\lambda \in \mathbb{R}^2$ and Faraday's law, we have

$$\dot{\lambda} = v_{\alpha\beta} - R i_{\alpha\beta}, \quad (27)$$

which holds for the IPMSM models. The stator current satisfies the constitutive relation

$$i_{\alpha\beta} = \nabla_{\lambda} H_E(\lambda, \theta), \quad (28)$$

where $H_E(\lambda, \theta)$ is the magnetic energy stored in the inductors. The two equations above hold true independently of the consideration of magnetic saturation, whose effect is captured in the energy function. The model of the unsaturated IPMSM model (1) is completed replacing

$$H_E(\lambda, \theta) = \frac{1}{2} [\lambda - c(\theta)\Phi]^T L^{-1}(\theta) [\lambda - c(\theta)\Phi] \quad (29)$$

with $c(\theta) := \text{col}(\cos \theta, \sin \theta)$, with (27) and (28). As pointed out in [13] under high-load condition, magnetic saturation should be taken into account in the model. In such a case, the magnetic energy is more complicated than the quadratic form (29), see [13, 28–32] for more details.

The analysis of virtual outputs extraction in Sections 3–4 is exactly the same for (29) and non-quadratic magnetic energy $H_E(\lambda, \theta)$. The difference appears on the way how to recover the angular position θ from y_v . For the latter, it can be formulated as the following optimisation problem:

$$\hat{\theta} = \min_{\hat{\theta} \in [0, 2\pi)} \left\| \hat{y}_v - \nabla_{\lambda} H_E(\lambda, \theta) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\|,$$

whose analytical solution, if it exists, depends on the particular modelling of magnetic saturation. See [13] for the discussions about saturated models.

5 Frequency interpretation of new estimator

We have shown that the new estimator (4) achieves performance enhancement with a higher accuracy. In this section, we show the structural and functional similarities between two methods.

The new estimator proposed in this study exactly coincides with the block diagram in Fig. 1 assigning $\phi = (3\pi/2)$ and the filters as follows:

$$\begin{aligned} \text{HPF} &= \mathcal{D}_d - \mathcal{Z}_{2d} \\ \text{LPF} &= \frac{1}{2} \left(\frac{V_h}{2\pi} \right)^2 \mathcal{H}, \end{aligned} \quad (30)$$

where $\mathcal{H}$ is the single-input single-output LTV filter

$$\begin{aligned} \dot{z}(t) &= -\gamma S^2(t) z(t) + \gamma u(t) \\ \mathcal{H}[u(t)] &= z(t). \end{aligned} \quad (31)$$

See Fig. 4.

To illustrate the high-pass and low-pass filtering properties of (30) and (31), we show in Fig. 5 the Bode diagram of the transfer function $G_d(s)$, defined in (20). From the figure it is clear that $G_d(s)$ verifies a high-pass property.

The frequency response of the operator $\mathcal{H}$, by fixing $\gamma = 1$ and $V_h = 1$, is the same as the linear time periodic (LTP) system below

$$\dot{y} = -(\cos \omega_h t)^2 y + u. \quad (32)$$

Introducing the change of state coordinate

$$y(t) = \tilde{P}^{-1}(t) r(t)$$

with

$$\bar{P}(t) := \exp\left(-\frac{1}{4}\omega_h \sin 2\omega_h t\right), \quad (33)$$

we get the LTV system

$$\dot{r}(t) = -\frac{1}{2}r(t) + \bar{P}(t)u(t). \quad (34)$$

Therefore, the LTP system (32) can be represented as in Fig. 6. From (33) it is clear that the matrix $\bar{P}(t)$ is almost identity for sufficiently large ω_h , while the transfer function $(1/s + 0.5)$ admits the low-pass property for a large ω_h . Hence, the LTP system (32), as well as the operator $\mathcal{H}$, is an LPF. In Fig. 1, the signal $\sin(\omega_h t + \phi)$, entering before the LPF, has different parameters ϕ for the classical design and the proposed one, which are 0 and $(3\pi/2)$, respectively. As shown in Figs. 2 and 5, this is caused by the different phase lags $(+\pi/2$ and $-\pi/2)$ of the second-order LTI filter (10) and the delayed LTI filter $G_d(s)$ at the frequency ω_h .

6 Simulations and experiments

6.1 Simulations

The proposed estimator is first tested by means of simulations in Matlab/Simulink. We use the parameters of Table 1, the current-feedback controller Σ_C given below, together with the proposed estimator.

- (i) Position estimator in Fig. 3 with (25).
- (ii) Rotation between $\alpha\beta$ -coordinates and misaligned dq -coordinates, namely

$$i_{dq} = e^{-j\hat{\theta}} i_{\alpha\beta}, \quad v_{\alpha\beta} = e^{j\hat{\theta}} v_{dq}.$$

- (iii) Speed regulation proportional–integral loops

$$i_{dq}^* = \left(K_p + K_i \frac{1}{p}\right)(\omega^* - \hat{\omega}),$$

where ω^* is the reference speed, and $\hat{\omega}$ is an estimate of the rotor speed obtained via the following phase-locked loop (PLL)-type estimator

$$\begin{aligned} \dot{\eta}_1 &= K_p(\hat{\theta} - \eta_1) + K_i \eta_2 \\ \dot{\eta}_2 &= \hat{\theta} - \eta_1 \\ \dot{\hat{\omega}}_p &= K_p(\hat{\theta} - \eta_1) + K_i \eta_2 \\ \hat{\omega} &= \frac{1}{n_p} \hat{\omega}_p. \end{aligned} \quad (35)$$

- (iv) Current regulation loops

$$\begin{aligned} v_d &= \left(K_p + K_i \frac{1}{p}\right)(i_d^* - i_d^{\ell}) - L n_p \hat{\omega} i_q \\ v_q &= \left(K_p + K_i \frac{1}{p}\right)(i_q^* - i_q^{\ell}) + L n_p \hat{\omega} i_d + n_p \hat{\omega} \Phi, \end{aligned}$$

where i_{dq}^{ℓ} are filtered signals of i_{dq} by some LPFs.

We operate the motor at the slow speed of 30 rad/min with $T_L = 0.5$ Nm and the parameters $\varepsilon = 10^{-3}$, $\gamma = 10^4$, $V_h = 1$, $\omega^* = 0.5$ and those in Table 2. Fig. 7 shows the simulation results. In Fig. 7a, we also give the position estimate obtained from the conventional LTI filters, denoted $\hat{\theta}_{LTI}$. Considering the root-mean-square deviation (RMSD)

$$\text{RMSD} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |\hat{\theta}(s) - \theta(s)|^2 ds}$$

with $\theta, \hat{\theta} \in \mathcal{S}^1$, we calculate the RMSDs for two methods in the interval [3, 17] s. They are 0.0872 and 0.1411 for the proposed design and the conventional LTI filtering method, respectively. We conclude that the new design outperforms the conventional LTI filtering method with a higher accuracy. It is also observed that the sensorless control law regulates the angular velocity at the desired value. In Figs. 7e and f, we also test the position estimator, as well as closed-loop performance, with different injection frequencies, 500 and 600 Hz, verifying the claim in D2.

6.2 Experiments

Losses and compensations. Before introducing the experimental results, let us say something about the loss of phase shift, which is unavoidable, as well as its compensations.

The excitation signal is injected into the modulation signal for the stationary α -axis. The excitation in the α -axis is, indeed, also affected by the inverter imperfections, for instance, the lockout time. Further on, the current i_α responding to the excitation is phase-shifted by 90° with respect to the voltage only in an ideal case, whereas the winding does not have any resistance. In practice, it should be a non-zero winding resistance, with the phase shift lower than 90° . Additional impact is due to the excitation-frequency eddy-currents in the magnetic circuit. Acting as a short-circuited secondary winding in a transformer-like electromagnetic setup, the iron losses introduce an additional change of the phase shift. Thus, these factors, but not limited to them, cause the phase of high-frequency component in $i_{\alpha\beta}$, see the term $S(\cdot)$ in (7), to be different from the one in the test signal.

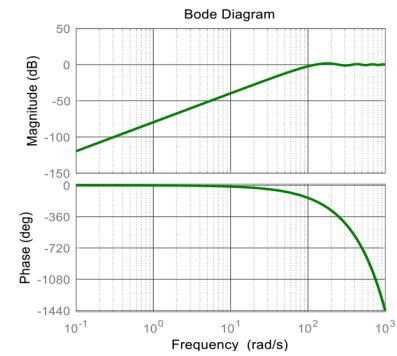


Fig. 5 Bode diagram of the transfer function $G_d(s)$ ($\omega_h = 500$, $n = 2$)

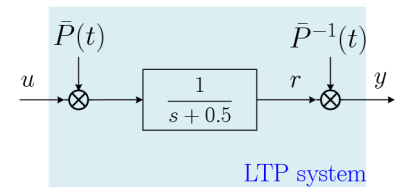


Fig. 6 Equivalent block diagram of the LTP system (32)

Table 1 Parameters of the IPMSM: simulation (first column) and experiments (second column)

Number of pole pairs (n_p)	6	3
permanent magnet flux linkage constant (Φ), Wb	0.11	0.39
d -axis inductance (L_d), mH	5.74	3.38
q -axis inductance (L_q), mH	8.68	5.07
stator resistance (R_s), Ω	0.43	0.47
drive inertia (J), kg m ²	0.01	≥ 0.01

Table 2 Parameters of the controller and the PLL estimator

$[K_p, K_i]$ in the speed loop	[1, 5]
$[K_p, K_i]$ in the current loop	[5, 5]
$[K_p, K_i]$ in the PLL estimator	[5, 0.01]

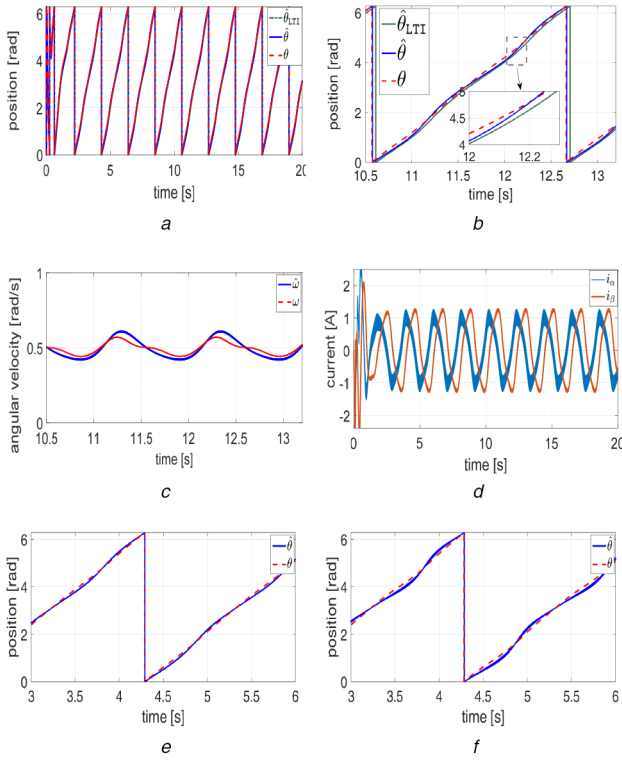


Fig. 7 Simulation results

(a) The angle θ and its estimates (test frequency 1000 Hz), (b) The angle θ and its estimates (test frequency 1000 Hz), (c) Angular velocity and its estimate, (d) Stator currents $i_{\alpha\beta}$, (e) The angle θ and its estimates (test frequency 600 Hz), (f) The angle θ and its estimates (test frequency 500 Hz)

Fig. 8 illustrates the effect of the phase shift loss on the virtual output estimate $\hat{y}_{v1}$ via simulations, namely, adding ‘artificial’ phase shifts in the high-frequency components of (7) to study the changes of $\hat{y}_{v1}$. In terms of (9), the virtual output y_{v1} admits the form $y_{v1} = a \cos 2\theta + b$ with some constants a and b . In Fig. 8 we observe the phase shift causes the drifts of amplitude a and bias b . A natural compensation method is using the signal $S(t) = -V_h \cos(\omega_h t + \phi_p)$ rather than (8) in the gradient descent operator $\mathcal{G}_{grad}$, with ϕ_p tuned in $[0, 2\pi)$.

Here we introduce an alternative approach to compensate all the losses, which not only contain the phase shifts but also the inductance values, at the ports of virtual output estimates, i.e. using the compensated virtual outputs

$$\hat{y}_{v1}^p := \ell_1 \hat{y}_{v1} + \ell_2, \quad \hat{y}_{v2}^p := \ell_3 \hat{y}_{v2},$$

and corresponding angle estimate

$$\hat{\theta} = \frac{1}{2} \arctan \left\{ \frac{\hat{y}_{v2}^p}{\hat{y}_{v1}^p - \frac{L_o}{L_d L_q}} \right\}.$$

The parameter adjustment principle of ℓ_i ($i = 1, 2, 3$) is to make the infimums and supremums of the signals $\hat{y}_{v1}^p$ and $\hat{y}_{v2}^p$ coincide with the ones calculated from (9). For more details, we first run the motor at a constant speed, with the obtained estimates $\hat{y}_v$ approximating some periodic signals, the means and amplitudes of which may differ from the calculations from (9). We then tune ℓ_i in order to let $\hat{y}_v$ have the same means and amplitudes with the ones in (9). It is clear that ℓ_1 and ℓ_3 are proportional to the amplitudes, and ℓ_2 affects the mean value. Such a procedure can be done offline, and then we apply the well-tuned parameters for online estimation.

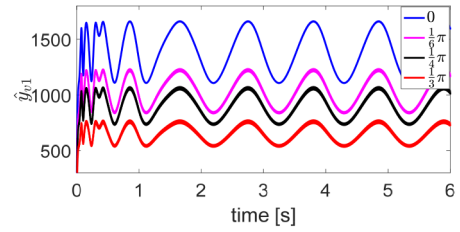


Fig. 8 Loss effect of shift drifts

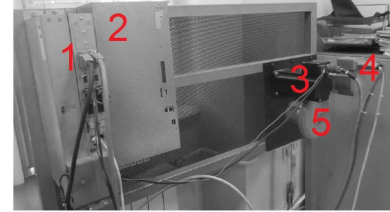


Fig. 9 Experimental testing setup

System configuration. The scheme developed in this study was tested on an IPMSM platform, shown in Fig. 9. The test IPMSM is a FAST IPMSM, whose parameters are given in Table 1. It has a 72 V line-to-line peak at 1000 revolutions per minute (RPM). The voltage of DC bus is 521 V, with the frequency of pulse-width modulation (PWM) 5 kHz.

The experimental setup comprises two synchronous motors, one of which runs in the speed control mode, used to maintain the speed at the desired level. The motors are coupled by means of a toothed belt, which also connects an inertial wheel. In Fig. 9, the experimental setup is also equipped with two mechanically coupled, inverter supplied brushless DC motors: (i) main power supply unit comprising the line rectifier and two three-phase PWM inverters with control circuits, (ii) DC-bus support with dynamic braking, (iii) speed controlled motor, (iv) torque controlled motor, (v) inertia coupled with both motors. The motor under the test is obtained by taking an industry-standard FAST motor and introducing changes into the rotor magnetic circuit so as to obtain the difference (2:3) between the d -axis and q -axis inductance. This motor runs in the torque-control mode. The speed and position are obtained through the high-speed digital serial link from standard industrial high-resolution sensors mounted on the shaft. The sampling time is $T_s = 300$ ns, and the acquisition time is set to cover at least two electrical periods. The three-phase currents, voltages and the rotor position were measured from the drive measurement system—a ‘Sincoder’ shaft sensor.

In experiments, we only test the proposed estimator, thus the estimated signals are not used in the closed-loop system, whose bandwidth of the speed loop is larger than 100 Hz. The test signal is injected into the modulating signal in the α -axis. The test motor is driven by another motor which kept the speed at 60 RPM.

Experimental results. In the first experiment, the amplitude of the test signal is 2 V, with the frequency 400 Hz with zero reference currents (i_d^d and i_q^d), in this way we can increase the resolution of the measurement system. The parameters of the estimator are selected as $\gamma_\alpha = 1.25 \times 10^4$ and $\gamma_\beta = 2.5 \times 10^4$. Let us first consider a normal operation mode with a 400 Hz test signal and zero load, and run the motor at a constant speed. For such a case, Fig. 10a shows the performance of the proposed position estimator, which generally works well. The test signal was only injected to the α -axis, which is illustrated in Fig. 10b after Clarke transform. Under the same conditions, we compare the proposed angular estimator with the conventional LTI filter in Fig. 11a, in which we observe the accuracy enhancement of the new design. This is probably because the phase lag in the virtual output estimates of the proposed design is smaller than that of the LTI filter, see Fig. 11b. Of course, the performance of the conventional methods may be further improved via some particular technique-oriented tricks in the applied literature, which, however, are out of scope of this study.

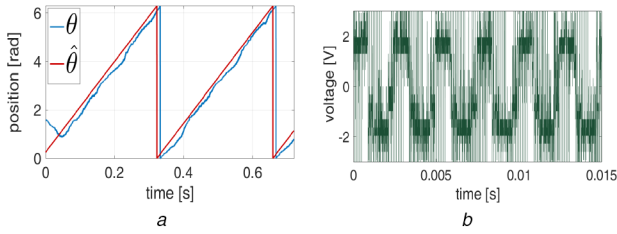


Fig. 10 Experimental results (zero load, 400 Hz injection signal, constant speed)

(a) Angle θ and its estimates, (b) Stator voltage u_α

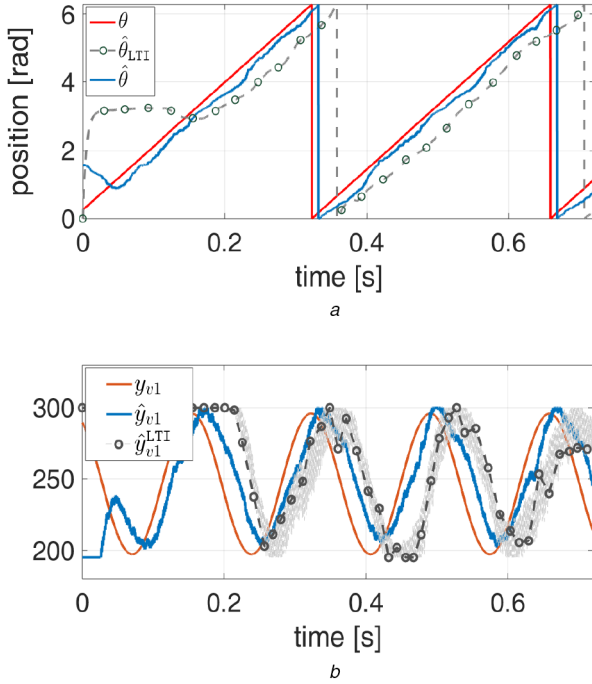


Fig. 11 Comparison between the proposed estimator and the conventional LTI filter via experiments

(a) Angle θ and its estimates, (b) Virtual output y_{v1} and its estimates $\hat{y}_{v1}$

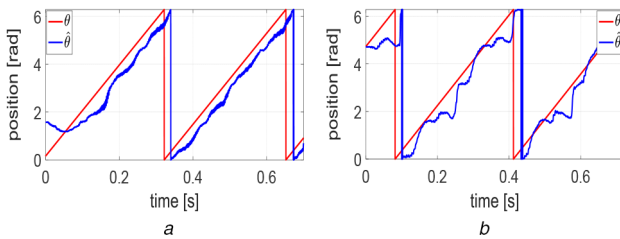


Fig. 12 Effects of the injection frequency

(a) Test frequency 200 Hz, (b) Test frequency 100 Hz

The motor platform utilised 5 kHz PWM, thus we should select the frequency of the test signal less than one-tenth of this value, i.e. 500 Hz, in order to be able to ‘generate’ the desired sinusoidal signals. We first test an 800 Hz probing signal, but unfortunately, it does not work as expected. We present some experimental results in Fig. 12 to illustrate estimation performance with the injection frequencies 200 and 100 Hz, respectively. We conclude that the performances degenerate with the frequency varying from 400 to 100 Hz. A possible interpretation, as discussed in D2, is that such a case has a low signal-to-noise ratio with a large ω_h . A similar result was observed for magnetic levitation systems in [25]. Therefore, the injection frequency is suggested to be selected in [200, 400] Hz for the tested motor in our experimental testing setup. For any other equipment, the probing frequency can be selected following the same procedure above, considering both the frequency of PWM signals and the signal-to-noise ratio.

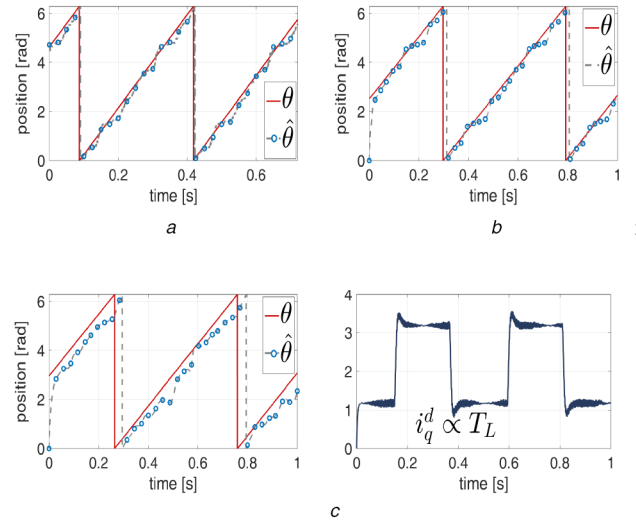


Fig. 13 Estimation performance with loads

(a) $\omega = 60$ RPM with constant load ($i_q^d \approx 1$ A), (b) $\omega = 40$ RPM with constant load ($i_q^d \approx 1$ A), (c) Time-varying load at $\omega = 40$ RPM

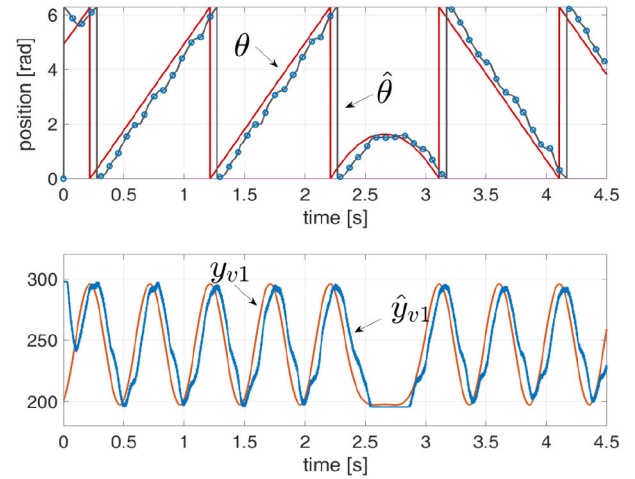


Fig. 14 Speed reversal from +20 RPM to -20 RPM ($i_q^d \approx 1.8$ A)

To evaluate the proposed HPF, we consider the cases with non-zero reference current i_q^d , which is proportional to the load T_L , with angular velocities at 60 and 40 RPM, respectively. The corresponding results with constant loads are illustrated in Figs. 13a and b, in which we get relatively satisfactory performances. We then test the performance with time-varying loads at 40 RPM in Fig. 13c, where a slight distortion may be observed probably due to a relatively heavy load. [To further improve estimation performance, magnetic saturations should be taken into account, but it is out of scope of this paper.] Fig. 14 shows the results when the motor under a constant load is reversing from +20 to -20 RPM in around one second. It is sufficient to verify the effectiveness of the proposed method in several demanding conditions.

7 Conclusion

This study addresses the problem of position estimation of IPMSMs at low speeds and standstill. Although the saliency-tracking-based methods are effective and widely-studied, the theoretical analysis of the conventional methods, taking into account the non-linear dynamics of IPMSMs, was conspicuous by its absence. This study attempts to fill in this gap analysing the stator current $i_{\alpha\beta}$ via the averaging method, with guaranteed error with respect to the injection frequency ω_h . Also, with the key identity (7), we develop a new position estimator, which ensures an improved accuracy. Moreover, we establish the connection between the new method and the conventional one, showing that they can

be unified in the HPF/LPF framework from the perspective of signal processing.

The following extensions and issues are of interest to be further explored.

- For the sake of clarity, we only study the basic case of signal-injection methods for the IPMSM model (1). The proposed method can also be extended to other motor models, for instance, saturated interior (or surface mounted) PMSMs.
- It is of interest to couple the proposed method with some model-based (non-invasive) techniques, for instance the gradient descent observer in [5, 7], in order to be able to operate the sensorless controller over a wide speed range. Such an approach has been pursued in [15, 16].
- The proposed method is relatively sensitive to power converters dead times. It is of practical interest to develop a self-commissioning methods by means of adaptive observers, which is promising to obtain a more practically useful scheme.

8 Acknowledgments

The authors would like to thank François Malrait at Schneider Electric for some technical clarifications. This study is supported by the National Natural Science Foundation of China (61473183, U1509211, and 61627810), the National Key R&D Program of China (SQ2017YFGH001005), the China Scholarship Council, the Government of the Russian Federation (074U01), and the Ministry of Education and Science of Russian Federation (14.Z50.31.0031, goszadanie no. 8.8885.2017/8.9).

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