

Percolation transitions in urban mobility networks in America's 50 largest cities

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ARTICLE INFO

Keywords:

Mobility network
Percolation analysis
Disruption
Critical transition
Early warning signal
Urban connectivity

ABSTRACT

Urban mobility can be significantly disrupted by various extreme events. The disruptions threaten urban spatial connectivity and affect people's ability to access various essential services. Accurate characterization and timely alert of the critical transitions of urban mobility networks can help mitigate the above risks. However, there lacks an approach to characterize the critical transition state of urban mobility networks and warn their transitions during extreme events. The universality of the characteristics of disrupted mobility networks across different cities is another fundamental question that remains underexplored. By mining big geodata, we construct the mobility networks of the 50 most populous Metropolitan Statistical Areas (MSAs) in the U.S., and study their disruption patterns by conducting network percolation analysis. We find that all mobility networks experience abrupt transitions when reaching a universal critical threshold, at which the giant components of neighborhoods suddenly collapse and dissolve into small clusters. We also develop an indicator, by analyzing the neighborhood cluster distributions, that approximates how far a mobility network is to the critical threshold and provides an early warning of its critical transition. Our findings provide insights into mobility and neighborhood connectivity in cities, which can provide guidance for transportation management, epidemic control, and emergency evacuation.

1. Introduction

Cities are highly complex systems composed of numerous components whose interactions are responsible for a broad spectrum of dynamics and evolution in cities (Batty, 2016). Urban mobility, which is rooted in these complexities and dynamics, provides a novel approach to understanding how a city is organized and how different components of a city interact with each other (Batty, 2013; Zhong et al., 2014). Taking disaster scenarios as an example, urban mobility plays a vital role in the resilience of cities to natural or manmade disasters by enabling populations to access from their residential areas the various critical facilities distributed in the city, such as grocery stores, gas stations, hospitals, and pharmacies (Logan & Guikema, 2020). Understanding human mobility is critical to identify high-risk communities, tailor public policies, guide emergency responses and ultimately, construct a more sustainable and resilient city.

The increasing accessibility to digital traces of human whereabouts in cities, made available by recent advancements of mobile and

ubiquitous computing technologies, has offered numerous new opportunities for exploring patterns and applications of human mobility behaviors. It is therefore not surprising that the study of human mobility has attracted considerable interest in recent years, with particular attention being paid to the empirical analysis of urban mobility patterns (Hong et al., 2021; Wang & Taylor, 2014), fine-grained individual mobility modeling (Pappalardo et al., 2015; Schlöpfer et al., 2021), and traffic or crowd flow prediction (Saber et al., 2020; Simini et al., 2021).

Despite the growing volume of research on urban mobility, the characteristics of disrupted urban mobility, which can result from various disruptive events such as natural disasters and pandemics, is still largely underexplored. When these events happen, they usually trigger massive abnormal activities of urban dwellers within the affected areas, and cause significant disruptions to urban mobility (Saja et al., 2020; Wang & Taylor, 2016; Yabe & Ukkusuri, 2020; Zhao et al., 2022). In extreme cases, the disruptive events may even lead to the critical transitions of urban mobility networks and full loss of urban spatial connectivity, paralyzing human flows in cities (Bailey et al., 2018; Li et al.,

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<https://doi.org/10.1016/j.scs.2023.104435>

Received 30 October 2022; Received in revised form 2 January 2023; Accepted 28 January 2023

Available online 31 January 2023

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2021b). Motivated by the significance of understanding the disrupted urban mobility and managing the associated risks, a number of studies have been conducted in recent years, primarily within the COVID-19 pandemic context, to investigate the disruption patterns of urban mobility networks. These studies have reported reduction of mobility flows, significant correlation between mobility flows and the disruptive events, and occurrence of critical status transitions of the mobility networks. Analysis of this existing, yet limited literature reveals that there are several important questions that have still remained to be explored. First of all, there lacks an approach to characterize the critical transition state of urban mobility networks and alert their transitions during extreme events. Particularly, complex urban systems, such as the financial systems and ecology systems, often emit early warning signals before their critical transitions (Dai et al., 2012; Kéfi et al., 2014; Scheffer et al., 2009, 2012; Squartini et al., 2013), and yet this phenomenon has not been empirically explored for large-scale mobility networks. Second, the universality of characteristics of disrupted mobility networks across different cities, especially beyond the specific COVID-19 pandemic context and related to more general forms of disruptions, is another fundamental question that remains underexplored. Answers to the above questions will deepen our understanding of the characteristics of disrupted urban mobility, and have important practical implications for evacuation, emergency response, and long-term recovery operations in cities (Egerer et al., 2020; Olazabal et al., 2018).

Motivated by the above knowledge gaps, this study aims to examine urban mobility patterns under perturbations using large-scale mobility datasets from American cities. The specific objectives of this study are threefold: (1) to capture critical transitions in urban mobility under perturbations, (2) to compare disrupted mobility patterns across cities, and (3) to design a universal early warning indicator that can signal the approaching transition of urban mobility. To achieve these research objectives, we adopt an approach originated from the percolation theory to investigate complex urban mobility. Empirical data from the 50 most populous Metropolitan Statistical Areas (MSAs) in the United States, including large-scale mobility data collected from mobile phones in 2019 and the 2019 Census block group data, are used for our analysis. We construct urban mobility networks of these MSAs, and perform large-scale computational experiments on these networks. The findings are expected to advance our understanding about how disrupted urban mobility evolves, and guide the sustainable urban planning and emergency management practices to achieve improved resilience against potential extreme events.

2. Literature review

2.1. Understanding urban mobility for more sustainable and resilient cities

Knowledge about human mobility patterns in cities is valuable for a wide range of advanced applications in sustainable and resilient cities (Wang et al., 2022b). For instance, an array of studies have used individuals' trajectories to identify urban structures from a functional perspective (Liu et al., 2015; Zhang et al., 2019c; Zhang et al., 2021), reveal social disparities (Hu et al., 2022; Wang et al., 2018), and model the spread of pandemic (Chang et al., 2021), providing a better understanding of urban dynamics. Particularly, understanding human mobility patterns can help improve the resilience of cities by providing a quantifiable and continuous reflection and assessment of the responses of various urban systems during disasters (Ilbeigi, 2019; Roy et al., 2019), such as urban transportation infrastructures (Nogal & Honfi, 2019; Zhang et al., 2019a), urban social systems (Li et al., 2022; Yabe et al., 2021), and so on.

Recently, an increasing number of studies have investigated urban mobility networks, which are typically constructed using spatial blocks such as neighborhoods as nodes and time-varying mobility flows between blocks as links. By leveraging the complex network theory and

techniques, these studies have brought new insights to the relationships between the urban structure, spatial distribution of urban facilities, and citizen's mobility demand (Gong et al., 2017; Liu et al., 2021; Wang et al., 2022a). In addition, boosted by the COVID-19 pandemic, a number of studies have been conducted to track overall mobility reduction (Schlosser et al., 2020), reveal the relationship between the mobility network structure and the distribution of pandemic (Freitas et al., 2020; Jia et al., 2020), and model and predict the pandemic spread based on mobility network attributes (Chang et al., 2021; Zeng et al., 2020). In sum, the findings of the above mobility network-based studies reflect the distribution and division of collective activities of citizens. Such information has then been proven useful in developing better solutions to boosting commercial activities, selecting sites for urban facilities, fostering social interactions, improving urban disaster resilience, and so on.

In summary, prior studies have demonstrated broad applications and significant values of incorporating human mobility knowledge to support the development of sustainable and resilient cities. Meanwhile, the literature review also suggests that disruptions to human mobility behaviors, which can be caused by disruptive events such as natural disasters, has drawn limited attention. Although there is abundant empirical evidence that indicates that individuals' mobility patterns may significantly differ between normal states and perturbed states (Wang & Taylor, 2014; Zhang & Li, 2022), more research is needed to understand the disrupted mobility behaviors, so as to better support sustainable urban planning and improve the resilience of cities during disruptive events.

2.2. Percolation analysis of urban systems using big geodata

Percolation theories are studied extensively in network science, revealing important network characteristics of various types of complex networks. Inspired by the growing availability of big geodata, such as GPS traces and geotagged social media posts, percolation analysis is playing an increasing role in urban studies, contributing to various urban problems, including identifying latent hierarchical urban structures, detecting phase transitions in complex urban systems, and robustness assessment of urban networks, which are summarized as follows. First of all, urban systems usually present hierarchical structures at many different scales, which can be revealed by percolation analysis. Urban structures, in the form of community structures, sub-centers and boundaries of urban areas, can be uncovered by percolation analysis on road networks (Arcaute et al., 2016) and mobility networks (Cao et al., 2020; Sarkar et al., 2019). Second, percolation analysis is a useful approach for studying disruptions and transitions of different urban networks. For example, the percolation processes were investigated on mobility networks (Deng et al., 2021) and traffic systems (Li et al., 2015a; Olmos et al., 2018) to capture the discontinuous phase transition from a globally connected network into isolated local flows. Third, the percolation threshold can be used as a reliability indicator for the operational limits of networks, which can help to understand and manage the network failure behavior (Li et al., 2015b). For instance, percolation threshold was integrated into reliability assessment of road network (Dong et al., 2020; Zhou et al., 2019) and public transit network (Hamedmoghadam et al. 2021) from a topological perspective.

As the big geodata about human mobility is becoming increasingly accessible, a few recent studies have attempted to apply percolation analysis to mobility networks to demonstrate their hierarchical topology and reveal their changes during the COVID-19 pandemic (Deng et al., 2021; He et al., 2022). However, in the realm of human mobility networks research, quantitative approaches for detecting and warning the critical transitions are still lacking. This has hindered both in-depth interpretation of observed empirical evidence of disrupted mobility patterns, and the early warning of such disruptions. In addition, limited efforts have been made in the existing literature to compare the outcomes of percolation analysis on mobility networks across different

urban contexts. As a result, limited is known about the heterogeneity between different cities with respect to the responses of their mobility networks to external disruptions. Therefore, more research is needed to investigate the percolation process of mobility networks across different cities, so as to better understand the resilience of mobility networks and guide the protection of urban connectivity during disruptive events.

3. Mobility data and mobility networks

3.1. Data sets and preprocessing

This study employs an anonymized and aggregated mobility dataset from [SafeGraph \(2020\)](#) that measures travel flows within the United States at the census block group (CBG) level in 2019. Mobility data from [SafeGraph \(2020\)](#) are accessed and used to extract mobility flows (more details are shown in Supplementary Material). Prior studies using SafeGraph data have found that the data are generally representative of the U.S. population ([Chang et al., 2021](#); [Klise et al., 2021](#); [Perra, 2021](#); [Squire, 2019](#)).

In this study, we focus on mobility flows in the largest 50 MSAs in the United States, based on their total population rank in 2020. The list of the 50 most populous U.S. MSAs (Table S1) is based on the 2020 Census data from the U.S. Census Bureau. The boundaries of the MSAs and CBGs are from the shapefiles of the “Combined Statistical Areas” and “Block Groups” data in the 2019 Census data ([US Census Bureau, 2019](#)).

To construct the mobility networks that represent the year of 2019, while avoiding excessive computational loads, daily mobility flow data for the above 50 MSAs from Monday through Thursday in the second week of March, June, September and December of 2019 (a total of 16 weekdays) are extracted from the SafeGraph data. For each MSA, the mobility flow data are preprocessed before analysis as follows: (1) mobility flows from the large airports by total passenger boardings in the United States are excluded. It is because mobility from airports to destinations are not daily flows concerned in this study; (2) the geographic scope of each mobility network is limited to the corresponding MSA, and flows out of the geographic boundaries of the MSA are excluded.

3.2. Mobility network construction

To conduct percolation analysis, we first determine the strength of each spatial interaction within each MSA. Here we use the normalized weighted flow derived from daily mobility data to indicate the strength of connection between CBGs. For a given connection between a CBG pair, its strength is represented by the daily travel volume. Then we normalize the volume of each connection by dividing it by the count of devices whose home locations are in the origin CBG. Mathematically, the undirected normalized weighted flow of link between CBG(*i*) and CBG(*j*), denoted as r_{ij} , can be calculated as:

$$r_{ij} = \frac{v_{ij}}{n_i} + \frac{v_{ji}}{n_j} \quad (1)$$

where v_{ij} is the daily volume from CBG(*i*) to CBG(*j*), and n_i is the number of devices whose homes were in CBG(*i*) during the study period.

Then, we build daily mobility-based networks of 16 workdays in the year of 2019 for 50 MSAs in the United States. For each network, every CBG is taken as a node, and an undirected weighted link between each CBG pair is created using the strength of r_{ij} as its weight. In the percolation analysis, for a given weight threshold, q , all links e_{ij} in the network can be classified into two categories: if the normalized weight of a link is larger than q , the link is considered connected, and when the normalized weights of the links are lower than q , the links are considered disconnected and removed.

$$e_{ij} = \begin{cases} 1, & r_{ij} \geq q \\ 0, & r_{ij} < q. \end{cases} \quad (2)$$

In this way, a mobility network can be constructed from links with normalized weights higher than q . The size of the giant component (GC) and the size of the second-largest component (SGC) in each mobility network are calculated after link removal. The GC and SGC are normalized by MSAs' network size as follows to allow comparison across MSAs:

$$GC = \frac{N_{GC}}{N} \quad (3)$$

$$SGC = \frac{N_{SGC}}{N} \quad (4)$$

where N_{GC} is the number of nodes in the giant component, N_{SGC} is the number of nodes in the second giant component, and N is the total number of nodes in the MSA network.

In the percolation analysis, the less-weighted links are removed first and the network becomes more fragmented as the value of q increases, and it eventually becomes completely fragmented (the representativeness of this percolation method in real-world cases is further explained in Supplementary Material). The critical threshold q_c is defined as the value of q when the size of SGC reaches maximal, according to percolation theory ([Li et al., 2021a](#)). Given that direct flow could exist between any pair of nodes in the mobility networks, the above percolation process is a long-range percolation ([Grimmett, 1999](#)), which has been widely adopted to simulate a number of significant real-world scenarios, such as human mobility and social interactions ([He et al., 2022](#); [Li et al., 2011](#)).

4. Analysis and results

4.1. Percolation process in MSAs

We simulate percolation on the daily mobility networks of the 50 MSAs for 16 workdays. The percolation processes in nine representative MSAs, selected based on their diversities in geographical attributes, areas and population sizes, are shown in [Fig. 1\(a-i\)](#). The size of GC (blue line) and the size of SGC (red line) in the percolation process are plotted. Two maps in each subfigure show the transition of GC at the critical threshold q_c : immediately before q approaches q_c , the size of GC is large; at q_c , GC significantly shrinks and the size of SGC reaches its maximal. The GC before the transition, and GC and SGC after the transition are shown in each map. Of special interest is the critical threshold q_c , at which the GC suddenly collapses and dissolves into clusters with smaller sizes.

[Fig. 1\(a\)](#) of New York MSA is used as an example to demonstrate the percolation process. At $q \ll q_c$, GC includes almost all the nodes, each representing a CBG, in the original mobility network. Many of these nodes in GC are connected by links with relatively low normalized weights. As q increases, these links are removed gradually. Thus, the size of GC shrinks and small clusters begin to emerge. When the value of q reaches q_c , the network experiences sudden fragmentation with the size of GC dropping from 30.9% of all the nodes to 15.9%, shedding half of its size. Simultaneously, the size of SGC peaks. When $q > q_c$, in contrast, GC is disintegrated, and there are only small clusters of connected nodes. These small clusters contain links with relatively high normalized weights, and yet they cannot maintain the global connectivity of the mobility network.

The same pattern is observed in other cities in [Fig. 1](#) (more details are shown in Supplementary Material Fig. S5–13). Urban mobility networks become fragile under increasing perturbation (e.g., reduction in mobility between neighborhoods), and disintegrate abruptly at critical points q_c 's. Specifically, as q increases, the size of GC decreases and that of SGC increases. At q_c , SGC reaches its maximum size and the size of GC experiences an abrupt drop simultaneously. Reaching q_c signifies the phase transition for network connectivity in the MSAs. According to percolation theory ([Hamedmoghadam et al., 2021](#); [Li et al., 2015a](#); [Zeng](#)

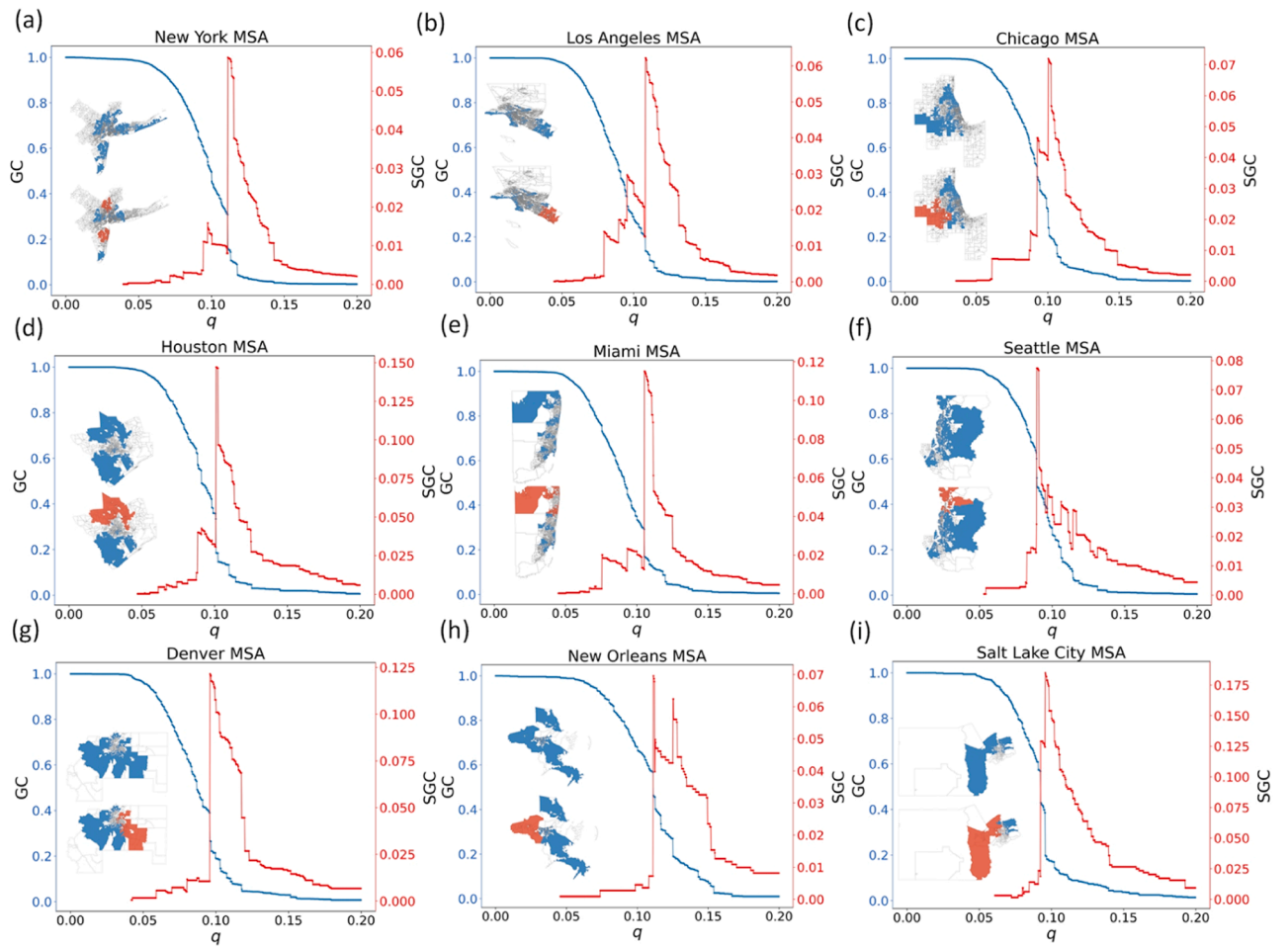


Fig. 1. Percolation patterns of nine representative MSAs in the U.S. The left y-axis reflects the size of GC (blue line) and right y-axis reflects the size of SGC (red line). The spatial distributions of GC (blue) before the critical transition (the upper map), and GC (blue) and SGC (red) at the critical transition (the lower map) are also shown in the figure.

et al., 2019), the size of GC has been widely used as an indicator of global connectivity of networks, and the critical threshold q_c is an informative measure of the resilience characteristics of network connectivity.

4.2. Universal patterns in the percolation process

Despite the diversity of the nine MSAs in geographical attributes, population density, mobility network topologies, and so on, we observe similar percolation processes in terms of the size changes in GCs and SGCs in Fig. 1. To further test the universality of such patterns, we analyze the percolation processes in the 50 largest MSAs in the U.S. These percolation processes are illustrated in Fig. 2(a) and (b).

We observe that the mobility networks follow a similar percolation process across the 50 MSAs. The GCs of these networks stay stable at the beginning of link removal. They start to shrink as q continues to increase before eventually reaching zero (Fig. 2(a)).

Surprisingly, as shown in Fig. 2(a) and (b), q_c 's from different MSAs are distributed within a narrow range. In fact, our analysis shows that q_c 's extracted from daily mobility networks of the 50 MSAs follow a normal distribution (Fig. 3) where the mean value is 0.096 with a mere standard deviation of 0.012 (KS test, p -value = 0.905). The surprising finding reveals that different MSAs share a universal critical threshold at which their mobility networks would lose global connectivity due to the collapses of the largest components.

The existence of a universal critical threshold across all the MSAs might be a manifestation of the “percolation threshold saturation” phenomenon that prior research discovered in the long-range

percolation of networks with high average node degree (Zhukov et al., 2018), although the plausibility of this hypothesis under the specific percolation approach used in this study still requires further exploration. Universal characteristics found in our study may be the key to understanding complex mobility networks' response to perturbations (Barzel & Barabási, 2013), and could inform more accurate and universal modeling of urban connectivity in future research.

4.3. Distribution of cluster sizes (s) at the critical point

Next, we examine how GCs break down at the critical points. We analyze the cluster sizes (s) of the daily mobility networks at q_c 's. The distributions of s of the 50 MSAs in the 16 workdays are shown in a log-log plot in Fig. 4, with each color representing a different MSA. The red diamonds show the aggregate distribution of s from all the 50 MSAs at their respective q_c 's. The results show that the probability distributions of s in the 50 MSAs follow power law distributions: $P(s) \sim s^{-\beta}$, where s is the cluster size, and the exponent parameter $\beta = 1.96 \pm 0.40$. The aggregate distribution also follows a power law distribution with $\beta = 2.03$. The above consistency demonstrates a surprising homogeneity across the 50 MSAs. Following the steps in Clauset et al., (2009) and Zhang et al., (2019b), we conduct the Kolmogorov-Smirnov test, and results indicate that the aggregate distribution and individual distributions of the cluster size at critical points all follow the power law.

Similar scale-invariant behaviors near critical transition points have been observed in various complex systems, such as land degradations (Tirabassi et al., 2014), traffic networks (Zeng et al., 2019; Zhang et al.,

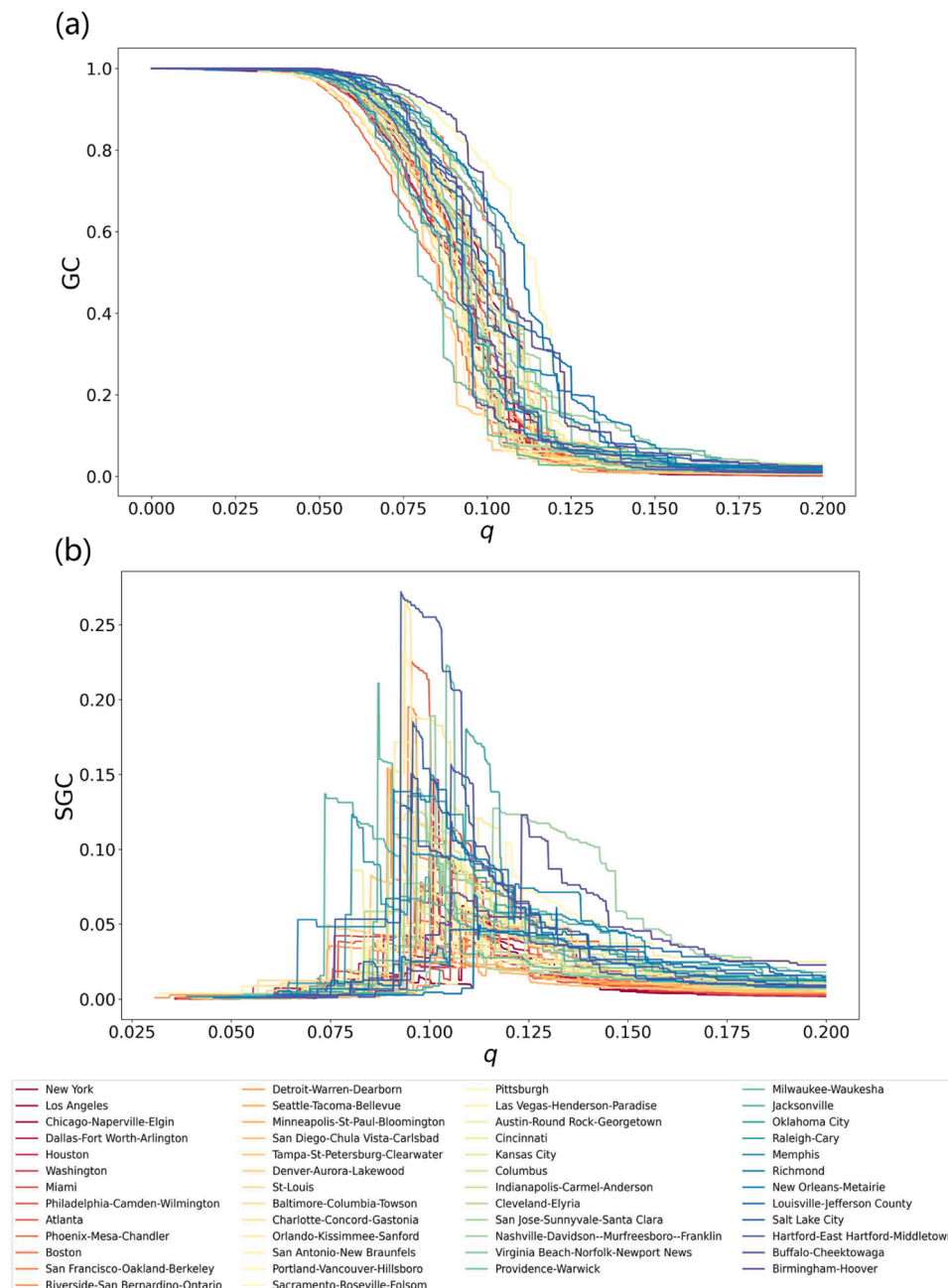


Fig. 2. Percolation curves of GC and SGC in 50 MSAs. (a) The size of GC in 50 MSAs during percolation on Sep. 9, 2019 as an example; (b) The size of SGC in 50 MSAs during percolation on Sep. 9, 2019 as an example.

2019b), land-cover patterns (Zurlini et al., 2014), public opinions (Ramos et al., 2015), and so on. Our study provides one of the first pieces of empirical evidence that the critical transitions also exist in large-scale urban mobility networks across diverse geographical contexts.

Such a scale-invariant structure may have arisen from the local positive feedback, i.e., the connection strength of a node to a cluster tends to increase with the size of the cluster (Scanlon et al., 2007). The main drivers of this positive feedback probably include the resources (Meekan et al., 2017), opportunities (Cummings et al., 2015) and social relationships (Axhausen, 2005) in cities. These resources are ampler in larger clusters and thus drive people to travel to and within them. However, the growth of large clusters is constrained globally by the total amount of resources and opportunities and the total trips that people can make. The competing forces between the positive feedback and growth constraints have likely co-produced the power law distribution of the

cluster size.

After observing the power law distribution of s for mobility networks of all MSAs at the percolation criticality, the aggregate cluster distributions before, near, at and after the percolation criticality are further calculated. The results are shown in Fig. 5. The distributions are drawn at different q values: $q_c - 0.05$ (a), $q_c - 0.02$ (b), q_c (c), and $q_c + 0.05$ (d). When $q \ll q_c$, for each MSA, there is a giant component and a number of small clusters. The aggregate distribution of the 50 MSAs is characterized by the co-existence of a small number of large clusters and many more small clusters (Fig. 5(a)). When q approaches q_c , for each MSA, the size of GC decreases together with an increase in the number of small clusters. Consequently, the aggregate distribution of s gradually approaches the power law distribution (Fig. 5(b)). When q is near or at q_c , the aggregate distribution of s exhibits power law distribution (Fig. 5). After q_c , giant component quickly breaks down. The aggregate

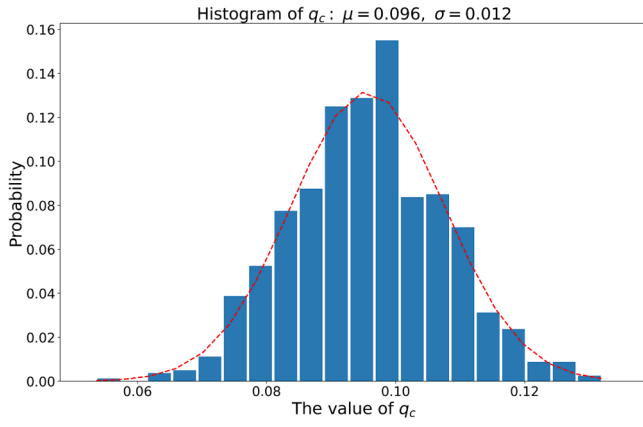


Fig. 3. Distributions of q_c in 50 MSAs of 16 workdays.

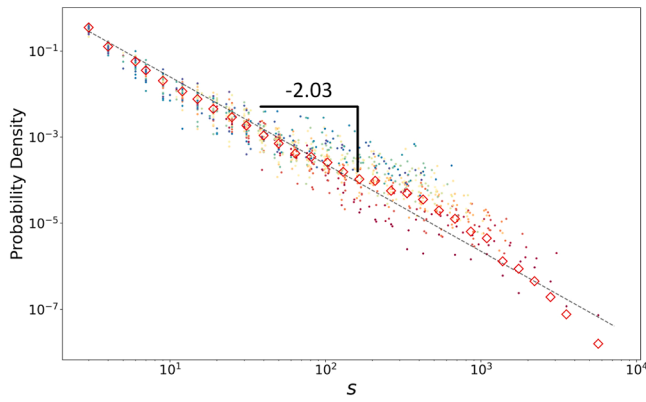


Fig. 4. Distributions of s at q_c in 50 MSAs of 16 workdays. The colors of legends for MSAs are the same as in Fig. 2.

distribution of s continues to follow power law but with a much shorter tail (Fig. 5(d)). The same trends described above are also observed for each of the 50 MSAs individually (details are shown in Supplementary Material Fig. S14–22).

4.4. Early warning signal of the critical transition

Prior research has pointed out that, as gradual change in an external forcing factor (in this case, the percolation process) drives a system closer to a critical transition, the distribution of the states of the units in the system (e.g., the clusters in the mobility network in our study) may change in characteristic ways, exhibiting scale-invariant distribution (Foti et al., 2013). Such change can be viewed as an early warning because the system may shift permanently to an alternative state if the external forcing factor persists (Scheffer et al., 2009). To explore such a phenomenon in mobility networks, we calculate the Kolmogorov-Smirnov distance (Massey, 1951):

$$D_{ks} = \max |F_0(x) - S_N(x)| \quad (5)$$

between $S_N(x)$ which is the observed cumulative step-functions of s for individual MSAs, and $F_0(x)$ which is the cumulated distribution functions of corresponding power law distributions. In this way, D_{ks} quantifies the distance between the empirical distribution function of samples and the cumulative distribution function of the fitted power law distribution. Fig. 6(a) shows the K-S distance for power-law distribution of cluster size during percolation for 50 MSAs. The median values extracted from the 50 MSAs are plotted (red curves) in Fig. 6(a) to show the trend. The values of q are normalized by subtracting the q_c values of the corresponding MSAs and dates. As such, the zero value of the normalized q in Fig. 6 represents the point of critical phase transition.

The D_{ks} of the cluster size distributions for individual MSAs (Fig. 6 (a)) shows a significant decreasing trend before the critical phase transition. The decreasing value of D_{ks} reflects increasing similarity between the cluster size distribution and the fitted power law distribution. Furthermore, we calculate the derivative of D_{ks} with respect to q , denoted as m , for each MSA:

$$m = \frac{\Delta D_{ks}}{\Delta q} \quad (6)$$

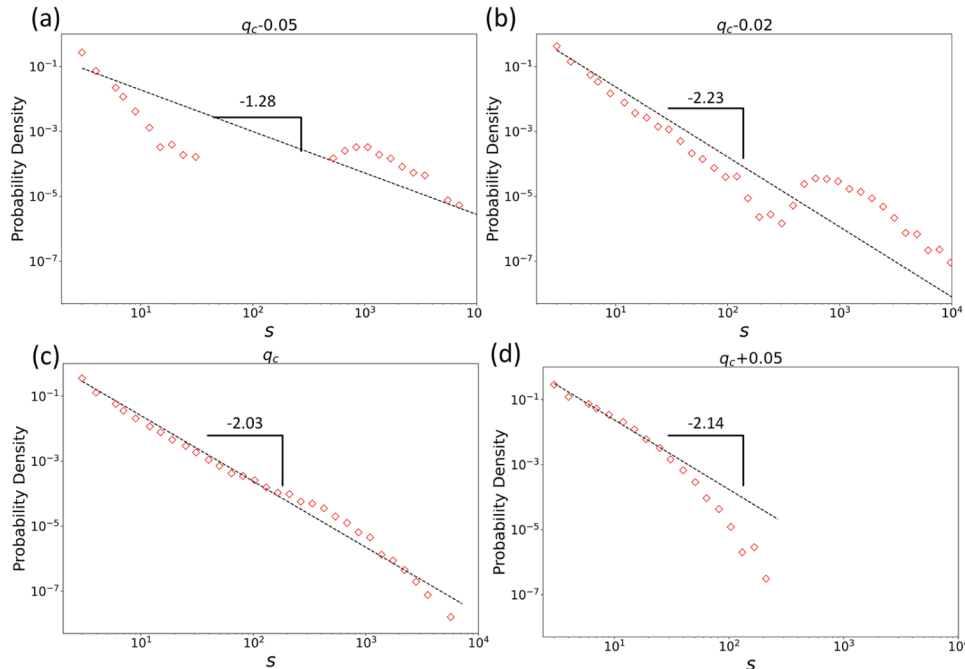


Fig. 5. Aggregate distributions of cluster sizes during percolation for all 50 MSAs.

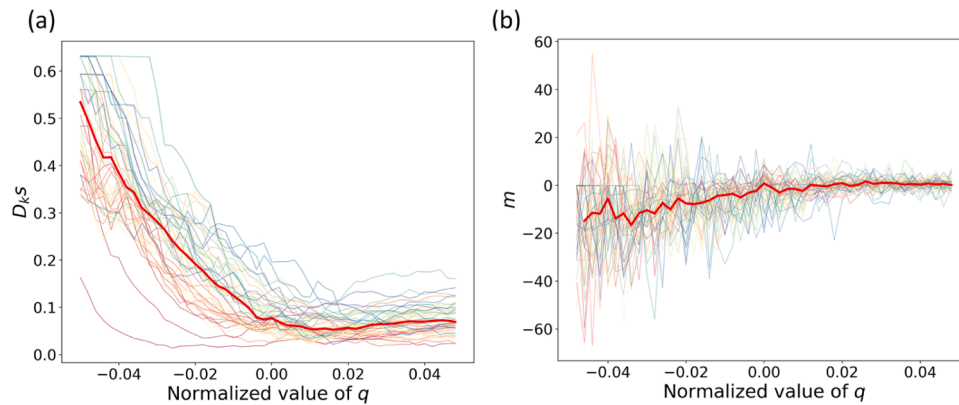


Fig. 6. K-S distance and the value of indicator m during percolation for 50 MSAs.

Fig. 6(b) shows the values of indicator m for all MSAs during percolation. The values of q are normalized the same way as in Fig. 6(a). The median values of m are extracted (red curve). As a general trend shown in Fig. 6(b), the value of m is far from zero at the beginning of the percolation process, but quickly converges towards zero as the value of q approaches the percolation criticality. The value of m approaching to zero indicates a potential early warning signal of the critical transition; m approximately indicates the distance between the current state of a mobility network and its critical point of state transition. It reflects a particular spatial configuration of complex systems arising before critical transition (Kéfi et al., 2014).

5. Discussions

Prior studies have suggested that a small change could cause a regime shift in a complex system. In this study, we reveal that such critical transitions also exist in large-scale urban mobility networks. Knowing the existence of this critical transition phenomena can inform public policies and engineering strategies to prevent urban connectivity from experiencing abrupt state changes and drastic connectivity losses. Moreover, we discover two universalities in percolation transitions among different urban contexts: (1) the critical thresholds q_c 's at phase transitions in the 50 MSAs follow a normal distribution, indicating that these MSAs share a similar critical threshold around $q_c = 0.096$; (2) scale-invariant behaviors near critical transition points have been observed in mobility networks. The universalities in urban connectivity patterns observed in this study are of remarkable theoretical and practical significance, suggesting a certain level of generalizability of findings obtained by studying the most tractable perturbation cases to other mobility networks. These insights could be applied in urban resilience-related policy making to support human mobility under disruptive events. Specifically, the percolation analysis of mobility networks in our study can not only measure the resilience characteristics of mobility networks, but also shed light on emergency management in real-world scenarios, such as epidemics, winter storms and heavy rains, in which weak links are vulnerability and likely to be disconnected first. Our percolation process has revealed how human mobility behaviors are gradually affected and finally approaching to transitions under external disruptions. During the process, critical links, vulnerable nodes, and high connectivity clusters can be identified from the percolation simulation (Hamedmoghadam et al., 2021; Li et al., 2015a). Such information could be used to inform the design of disaster response schemes and control methods via protection or enhancement of a minimal set of links, as well as to guide the prioritization of link reconstruction during post-disaster recovery. In addition, this study also reveals the spatial structure of node cluster distributions in perturbed mobility networks, which can play a major role in informing efficient resource mobilization schemes and prioritizing vulnerable neighborhoods and communities

during disaster events (Sun & Zhang, 2020; Zhang et al., 2019b).

Moreover, our analysis takes one of the first steps to devise a quantifiable early warning signal for mobility networks based on the proposed indicator, whose distance to the zero value can indicate how far a mobility network constructed from mobility flows is to its point of critical state transition. This novel indicator can be used as a warning signal on the closeness of a mobility network to its transition, and therefore has significant implications for protecting the connectivity of urban mobility networks through improved risk assessment and scenario planning. For instance, it could inform policymakers of the neighborhoods at potential risk of losing connectivity, who could then take targeted measures, such as improving public transportation, to prevent populations at risk from being segregated and ensure their access to essential services during extreme events. In addition, the early warning signal devised in this study, evaluated on data collected from 50 MSAs in the U.S., demonstrates promising generalizability across different geographical contexts. Our early warning signal method may also inspire new opportunities for future intelligent applications, such as intelligent emergency decision making support system, for sustainable and resilient urban planning and management (Foltýnová et al., 2020).

Our study has several limitations that are noteworthy. We investigated the connectivity of mobility networks under simulated perturbations of percolation and provided scientific insights for possible real-world scenarios. However, the network disruptions caused by natural hazards or extreme events could be more complicated than the theoretical percolation process. The percolation process in this study uses a global threshold and does not reflect localized perturbations, such as disaster-induced local power outages or traffic jams. In future work, we will localize the settings of threshold q depending on the perturbation intensity in each neighborhood and evaluate our findings against real-world events. In addition, we used open-source datasets, in which anonymized location data were collected and aggregated from numerous mobile devices. Certain groups, such as the elderly and children, are likely to be less represented in the dataset. Although the data has been used in multiple studies to understand human movements at various spatiotemporal scales in the U.S. (Chang et al., 2021; Kang et al., 2020; Li et al., 2021b), more efforts are needed to evaluate the reliability of our findings at different geographic regions by comparing with other data sources.

6. Conclusions

Mobility networks, which play a fundamental role in maintaining urban connectivity, may disintegrate during disruptive events. To investigate this phenomenon, we studied the percolation transitions in mobility networks in the top 50 MSAs in the U.S. Our research revealed that mobility networks universally experienced abrupt transitions under simulated perturbations. When undergoing these transitions, the

distributions of cluster sizes in the mobility networks fundamentally changed. Thus, the shift in distribution could be devised as an early warning signal that alerts when the mobility networks would be approaching their critical thresholds. Moreover, we found that the critical thresholds across different cities were almost identical after mobility flows were normalized by population. Building upon the extensive literature on human mobility in cities, this study takes one of the first steps in revealing the percolation transitions of mobility networks under the influence of perturbations. The findings can help to better evaluate the stress on urban mobility imposed by extreme events, predict the size of the damage to urban mobility or even its collapse, and support more informed risk mitigation and resilience enhancement strategies for the urban mobility and urban planning.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

Acknowledgments

This material is based upon work supported by the National Natural Science Foundation of China (NSFC) under Grant No. 71974105, and the National Science Foundation (NSF) under Grants No. 2027744, 2125326, and 2228533. The authors are grateful for the support of NSFC and NSF. Any opinions, findings, conclusions or recommendations expressed in the paper are those of the authors and do not necessarily reflect the views of the funding agencies.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.scs.2023.104435.

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