A note on co-dimension 2 defects in $N = 4, d = 7$ gauged supergravity

Michael Gutperle *, Nicholas Klein

Mani L. Bhaumik Institute for Theoretical Physics, Department of Physics and Astronomy, University of California, Los Angeles, CA 90095, USA

Abstract

In this note we present a solution of $N = 4, d = 7$ gauged supergravity which is holographically dual to a co-dimension two defect living in a six dimensional SCFT. The solution is obtained by double analytic continuation of a two charge supersymmetric black hole solution. The condition that no conical deficits are present in the bulk and on the boundary is satisfied by a one parameter family of solutions for which some holographic observables are computed.

1. Introduction

The construction and study of extended conformal defects is an important subject in the investigation of superconformal field theories (SCFT). Defects are characterized by the broken and preserved symmetries. In a d-dimensional SCFT, a p-dimensional conformal defect preserves a $SO(p, 2) \times SO(d - p)$ subgroup of the $SO(d, 2)$ conformal group. The first factor is the conformal symmetry acting on the world volume of the defect and the second factor is the rotational symmetry in the transverse directions, which acts like a global symmetry on the degrees of freedom localized on the defect.

If the SCFT has a holographic dual it is interesting to look for the holographic description of such defects, which fall into two categories: First, a brane is placed in the bulk spacetime which

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* Corresponding author.
 E-mail address: gutperle@physics.ucla.edu (M. Gutperle).

ends on the boundary at the p dimensional defect [1,2]. In a probe approximation the gravitational back reaction of such the brane is neglected, but the embedding is determined by solving the world volume equations of motion or the BPS-condition following from world volume kappa symmetry [3]. Second, a fully back reacted solution of the supergravity can be constructed using an ansatz of AdS and sphere factors warped over a base space (which can be a line or a Riemann surface with boundary). Solutions can either be constructed in lower dimensional gauged supergravities [4,5] and in favorable circumstances be uplifted ten or eleven dimensions, or alternatively solutions can be constructed in ten or eleven dimensions where the solution is constructed by a warped product of AdS (and sphere factors) over a base manifold. For example in [6–9] the base manifold is a non compact Riemann surface with boundary and solutions preserve half the supersymmetry, in [10,11] the base manifolds are higher dimensional Kaehler-Einstein spaces and the amount of preserved supersymmetries is reduced. A careful analysis of the uplifted solutions would be needed to investigate a possible relation to the solutions [10,11].

The former solutions are easier to obtain but the later are more general and in many cases give a top down understanding of the defects as backreacted solutions of intersecting brane systems, which allow us to identify the gauge theories, often of quiver type, which flow to the SCFTs.

In this note we consider the holographic description of $p = 4$ dimensional defects in $d = 6$ dimensional SCFTs. We construct solutions in a truncation of maximal $SO(5)$ gauged supergravity in seven dimensions with $U(1) \times U(1)$ gauge symmetry. These solutions are related by a double analytic continuation to extremal supersymmetric black hole solutions. They are also closely related to compactifications of the seven dimensional theory on spindles - two dimensional compact surfaces with conical deficits which have been studied extensively in the past two years (see e.g. [12–20]). Both constructions start with a ansatz $AdS_5 \times S^1$ warped over a real coordinate. For the spindle solution the real coordinate takes values on a compact interval and the circle closes off at either end of the interval where there are conical singularities. The spindle solutions correspond to novel compactifications dual to four dimensional SCFTs where supersymmetry is preserved not by a topological twist but due to a novel mechanism.

In our case the real coordinate takes values on a half-line. The S^1 closes off at the end of the half line and the $AdS_5 \times S^1$ decompactifies at infinity to an asymptotic AdS_7 space. The solution therefore describes a holographic dual conformal a four dimensional defect living inside a six dimensional SCFT. One of the main results of the present paper is that for the two charge extremal solution we find a one parameter family of completely regular solutions where there is no conical deficit at the location where the circle closes off.

The structure of this note is as follows. In section 2 we describe the seven dimensional gauged supergravity and the relevant solutions which are obtained from double analytic continuation of black hole solutions. In section 3 we perform a regularity analysis based on the absence of conical singularities in the bulk and boundary and obtain a one parameter family of regular solutions, as well as solutions with conical singularities in the bulk related to spindles which have been actively investigated recently. In section 4 we perform some holographic calculations using the regular solutions, in particular we calculate the on-shell action of the solution, as well as the expectation value of the stress tensor and conserved R-symmetry currents. In section 5 we briefly discuss the uplift of the solution to eleven dimensions which is used to identify the R-symmetry currents of the six dimensional SCFT to which the seven dimensional gauge fields are dual. We close with a discussion of our results and leave some details of calculations to an appendix.

2. 7-dim gauged supergravity

We consider a truncation of maximal $N = 4$, $SO(5)$ gauged supergravity in seven dimensions [21] with $U(1) \times U(1)$ gauge symmetry and two scalars [22–24]. There exists a consistent up-lift of the seven dimensional solutions to eleven dimensional supergravity [22]. The solutions we consider are double analytic solutions of charged non-rotating black hole solutions [23,24], where the S^5 factor is replaced by a AdS_5 factor and the time coordinate is replaced by a space-like compact circle coordinate. The black hole solution depends on a non-extremality parameter and two charges. The extremal solution preserves either half or a quarter of the thirty-two super-symmetries of the gauged supergravity theory for one or two nonzero charges respectively [24]. It was shown in [13] that the analytically continued extremal solutions also preserve the same amount of supersymmetry.

We follow the conventions of [13] to facilitate a comparison with their analysis. The action for the bosonic fields of $U(1) \times U(1)$ gauged supergravity in seven dimensions is given by

$$S = -\frac{1}{16\pi G_N} \int d^7 x \sqrt{-g} \left(R - g_c^2 V(\phi) - \frac{1}{2} \sum_{i=1}^{2} \partial_\mu \phi_i \partial^\mu \phi_i - \frac{1}{4} e^{\sqrt{2}\phi_1 + \sqrt{\frac{2}{5}}\phi_2} F_1^2 \right.$$
$$\left. - \frac{1}{4} e^{-\sqrt{2}\phi_1 + \sqrt{\frac{2}{5}}\phi_2} F_2^2 \right) \tag{2.1}$$

where $F_i = dA_i$, $i = 1, 2$ and the potential for the scalar fields is given by

$$V(\phi) = 2g_c^2 e^{-\sqrt{\frac{2}{5}}\phi_2} \left(-8 + e^{\sqrt{10}\phi_2} - 8e^{\sqrt{\frac{5}{2}}\phi_2} \cosh\frac{\phi_1}{\sqrt{2}} \right) \tag{2.2}$$

The solution given in [13] can be expressed in term of the following functions

$$\begin{aligned}
h_i(y) &= y^2 + q_i, \quad i = 1, 2 \\
P(y) &= h_1(y)h_2(y) \\
Q(y) &= -y^3 - \mu y + g_c^2 h_1(y)h_2(y)
\end{aligned} \tag{2.3}$$

and is given by

$$ds^2 = \left(yP(y) \right)^{\frac{1}{5}} \left\{ ds_{AdS_5}^2 + \frac{y}{4Q(y)} dy^2 + \frac{Q(y)}{P(y)} dz^2 \right\}$$

$$A_i = \left(\frac{\sqrt{1 - \frac{\mu}{q_i}} q_i}{h_i(y)} + \alpha_i \right) dz, \quad i = 1, 2$$

$$e^{\phi_1} = \left(\frac{h_1(y)}{h_2(y)} \right)^{\frac{1}{\sqrt{2}}}, \quad e^{\phi_2} = \frac{(h_1(y)h_2(y))^{\frac{1}{\sqrt{10}}}}{y^{2\sqrt{\frac{2}{5}}}} \tag{2.4}$$

It is easy to verify that the equations of motion following from the variation of the action (2.1) are satisfied for such a solution. Here q_1, q_2 are related to the charges and μ is a non-extremality parameter which we set to $\mu = 0$. This choice corresponds to a supersymmetric solution as in [13]. We will also set $g_c = 1$ for simplicity. For these choices the solution with $q_1 = q_2 = 0$ corresponds to a unit radius AdS_7, using $AdS_5 \times S^1$ slicing coordinates.

3. Regularity analysis

In this section we present the conditions that regularity imposes on the solution. The analysis follows the general strategy employed in other cases of holographic description of defects [25–27]. It is also closely related to the construction of holographic calculations of Renyi-entropies [28,29], compactifications on spindles [12–15] and related constructions [30,31].

In [13] the solution presented in section 2 was used to construct a AdS_5 compactification of seven dimensional supergravity on a two dimensional compact space, a so-called spindle. A spindle is topologically a two sphere with two conical deficits at the north and south poles respectively. A spindle exists if the function $Q(y)$, defined in (2.3) has two real zeros and in between the zeros both $Q(y)$ and $P(y)$ are positive. The regularity, supersymmetry and the quantization of the deficit angle coming from a consistent interpretation of the uplift to eleven dimensions impose conditions on the parameters of the solution which were worked out in [13].

In our case the two dimensional space will be non-compact and we will look at the region from the largest positive zero of $Q(y)$ to infinity, which is a region where Q is positive. In the following we will investigate the regularity conditions imposed on the solution. For convenience we write out the functions which determined the regularity (recall we have set $\mu = 0$).

$$Q(y) = -y^3 + (y^2 + q_1)(y^2 + q_2) = y^4 - y^3 + (q_1 + q_2)y^2 + q_1 q_2$$
$$P(y) = (y^2 + q_1)(y^2 + q_2) \tag{3.1}$$

As $y \to \infty$ we approach an asymptotic AdS_7 region, with a six dimensional boundary. In this limit the metric takes the form

$$\lim_{y \to \infty} ds^2 = y \, ds^2_{AdS_5} + y \, dz^2 + \frac{1}{4y^2} dy^2 + \cdots$$
$$= \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} \left(ds^2_{AdS_5} + dz^2 \right) + \cdots \tag{3.2}$$

where we defined the Fefferman-Graham coordinate ζ as $y = 1/\rho$ and the dots denote subleading terms in y and ρ, which are determined in appendix A. The metric is asymptotic to AdS_7. Since the z direction parameterizes a circle, the holographic boundary of the asymptotic AdS space is of the form $AdS_5 \times S^1$. The six dimensional metric on the boundary is given by

$$ds^2_6 = \frac{dr^2 - dt^2 - \sum_{i=1}^3 dx_i^2}{r^2} + dz^2$$
$$= \frac{1}{r^2} \left(d\zeta^2 - dt^2 - \sum_{i=1}^3 dx_I^2 + r^2 dz^2 \right) \tag{3.3}$$

which is conformal to $R^{1,5}$ if the coordinate z has periodicity 2π. For a different periodicity of z the boundary has a conical singularity at $r = 0$. In the standard formulation of AdS/CFT the boundary theory does not have dynamical gravity and hence a co-dimension two defect does not induce a conical deficit, as a cosmic string would in a gravitational theory. Consequently the condition of the absence of a conical deficit on the boundary fixes the periodicity of the S^1 coordinate z to be 2π.

We now seek conditions on q_1, q_2 such that there is at least one positive zero and that it is not a double zero. Once we have such a y_+, we can guarantee that in the range $[y_+, \infty)$ both metric functions $Q(y) > 0$ and $P(y) > y^3 > 0$ are positive and the metric is regular. An important quantity for the nature of the zeros of Q is the discriminant
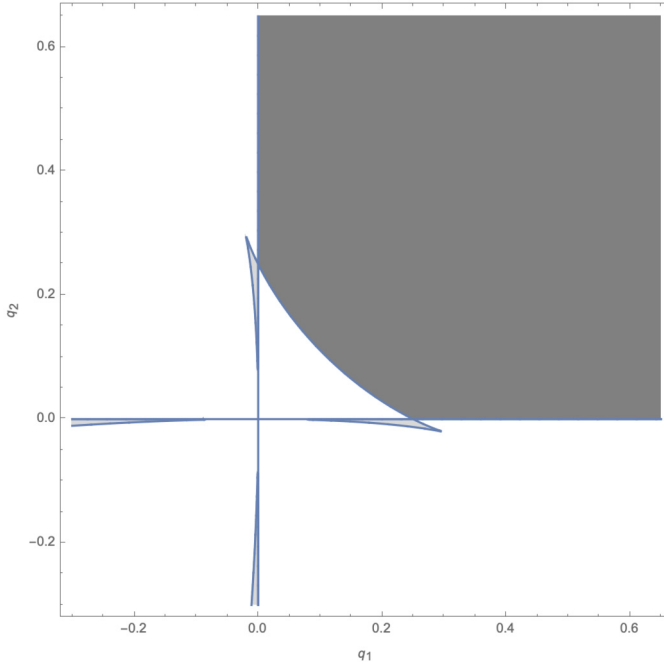
Fig. 1. Sign of the discriminant (3.4) of the polynomial $Q(y)$ in the (q_1, q_2) plane. The dark gray region is excluded.

$$D = q_1 q_2 \Big(16(q_1^4 + q_2^4) - 4(q_1^3 + q_2^3) - 64(q_1^3 q_2 + q_1 q_2^3) + 96 q_1^2 q_2^2 + 132(q_1^2 q_2 + q_1 q_2^2)$$

$$- 27 q_1 q_2 \Big) \tag{3.4}$$

Note that the vanishing of the discriminant implies the presence of a real double zero and for $D > 0$ we have either four or no real zeros whereas for $D < 0$ we have two real and two complex conjugate roots. We show a plot of the sign of the discriminant as a function of q_1, q_2 in Fig. 1, where locus of vanishing discriminant is represented by the blue curve and regions of positive discriminant are shaded gray.

We can use Descartes' rule of signs to show that in the region with either one or both q_1 and q_2 negative, we have two real roots in the (white) region where $D < 0$ and four real roots in the (gray) region where $D > 0$. In the region where both q_1, q_2 are positive we have two real zeros in the white region where $D < 0$ and no real zeros in the (dark gray) region, where $D > 0$. This implies that the dark gray region of charges is excluded since $Q(y)$ is never zero here and we will produce a naked singularity when y goes to zero and the Ricci scalar diverges.

Note that if $y = y_0$ is a double zero the metric will approach the following form near $y = y_0 + \rho$

$$ds^2 \sim (y_0 P(y_0))^{\frac{1}{5}} \Big(ds_{AdS_5}^2 + \frac{y_0}{\gamma \rho^2} d\rho^2 + \frac{\gamma \rho^2}{P(y_0)} dz^2 \Big) \tag{3.5}$$

where $\gamma = \frac{1}{2} Q''(y) |_{y=y_0}$. This produces a singularity at $\rho = 0$. (We will see that we will never have to worry about this case for q_1, q_2 which satisfy the other regularity conditions.)

Now we assume that we are in the allowed region of the q_1, q_2 plane and consider the $y \to y_+$ limit where y_+ is the largest positive zero of the function $Q(y)$. Letting $y = y_+ + \rho$, we have that

$$
\begin{aligned}
Q(y) &\approx Q'(y_+)\rho \\
P(y) &\approx P(y_+) = (Q(y_+) + y_+^3) = y_+^3
\end{aligned}
\tag{3.6}
$$

Plugging these into the metric (2.4) and defining the new radial coordinate $r = \rho^{\frac{1}{2}}$, we obtain

$$
\left(yP(y)\right)^{\frac{1}{5}}\left(\frac{y}{4Q(y)}dy^2 + \frac{Q(y)}{P(y)}dz^2\right) \sim \frac{y_+^{\frac{9}{5}}}{Q'(y_+)}\left(dr^2 + \left(\frac{Q'(y_+)}{y_+^2}\right)r^2 dz^2\right)
\tag{3.7}
$$

As discussed above the absence of a conical deficit on the boundary fixes the periodicity of z to be 2π.

$$
\frac{Q'(y_+)}{y_+^2} = \frac{1}{n}
\tag{3.8}
$$

gives us the metric on a which is regular everywhere except at the location $y = y_+$ where the circle shrinks to zero size, where there is a conical deficit angle $2\pi(1 - \frac{1}{n})$.

Using the explicit form of Q, we obtain the following constraint on the charges:

$$
y_+\left(4y_+^2 - (3 + \frac{1}{n})y_+ + 2(q_1 + q_2)\right) = 0
\tag{3.9}
$$

Note that the value of the largest root y_+ also depends on the charges q_1, q_2 and the resulting expression does not have a compact explicit expression. It is however clear that the condition will constrain the charges q_1, q_2 to lie on a one dimensional curve, which depends on the value of the conical deficit. In Fig. 2 we illustrate the curves of allowed charges for the case $n = 1$ which corresponds to a completely nonsingular spacetime, and $n = 2, 3$ which corresponds to spaces with conical deficits π and $\frac{2}{3}\pi$ respectively.

We note that there is no completely regular solution (i.e. a solution without conical deficit at the boundary or in the bulk) with one of the q_1 and q_2 charges set to zero. Hence all completely regular solutions preserve eight of the thirty two supersymmetries of the AdS_7 vacuum of the gauged supergravity Consequently, the dual four dimensional defect preserves $N = 1, d = 4$ superconformal symmetry. A solution preserving $N = 2, d = 4$ supeconformal symmetry would necessarily have a conical deficit in the seven dimensional spacetime.

We can choose that constant α_i in the expression for the gauge fields $A_i, i = 1, 2$ such that the gauge field vanishes at $y = y_+$, where the circle shrinks to zero size. In contrast to the compact spindle case, this does not impose any additional conditions since there is only one value of y instead of two, where the circle shrinks.

4. Holographic calculations

The solutions describe holographic co-dimension two defects in the six dimensional SCFT. In this section we calculate some holographic observables and discuss the implications for the defects imposed by regularity constraints. As discussed in section 3 the solution approaches AdS_7 asymptotically where the six dimensional boundary is $AdS_5 \times S^1$. While the boundary is conformal to $R^{1,5}$, it is simpler to work with the $AdS_5 \times S^1$ form of the boundary which is natural given the metric (2.4). All holographic calculations can be mapped to a flat boundary using the conformal mapping described in section 3.
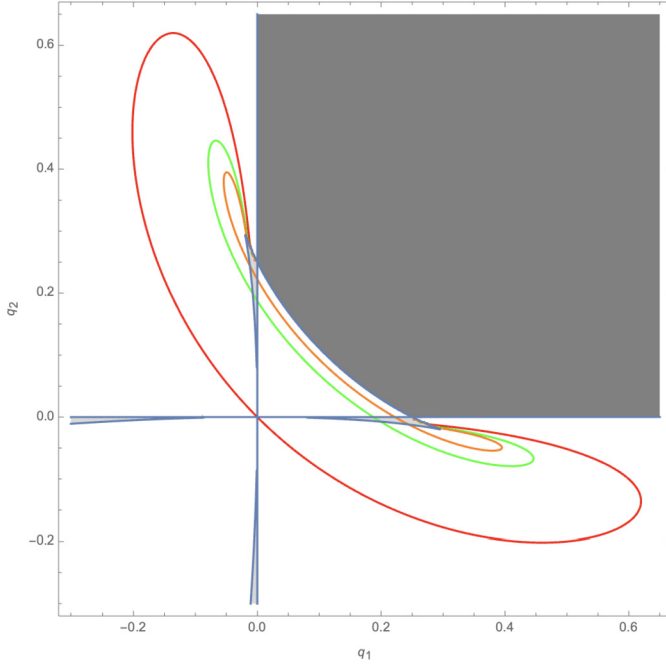
Fig. 2. Allowed charges for different values of conical deficits: $n = 1$ (red) is the completely regular solution and conical singularities with $n = 2$ (green) and $n = 3$ (orange).

4.1. On shell action

To evaluate the on shell action we have to add a Gibbons-Hawking term to the action (2.1) which is needed for a good variational principle. Using the trace of the Einstein equation the on-shell action can be expressed as

$$S_{\text{on-shell}} = -\frac{1}{16\pi G_N} \int_M \sqrt{-g} \left(\frac{2}{5} V - \frac{1}{10} e^{\sqrt{2}\phi_1 + \sqrt{\frac{2}{5}}\phi_2} F_1^2 - \frac{1}{10} e^{-\sqrt{2}\phi_1 + \sqrt{\frac{2}{5}}\phi_2} F_2^2 \right)$$

$$+ \frac{1}{8\pi G_N} \int_{\partial M} \sqrt{-h}\,\Theta \tag{4.1}$$

The Gibbons-Hawking term is obtained from the trace of the second fundamental form

$$\Theta_{\mu\nu} = -\frac{1}{2} \left(\nabla_\mu n_\nu + \nabla_\nu n_\mu \right) \tag{4.2}$$

Here h_{ab} is the induced metric and n_μ is the outward pointing normal vector at the cut-off surface. For the solution discussed in the paper we choose the cutoff surface at large $y = y_c$. Furthermore since the spacetime closes off at the larges zero y_+ of $Q(y)$, the integral of the coordinate y in the action (4.1) is on $y \in [y_+, y_c]$. The on-shell action becomes

$$S_{\text{on-shell}} = \frac{\text{Vol}_{AdS_5}}{16\pi G_N} \left(-10 y_c^3 + 10 y_c^2 - 6(q_1 + q_2) y_c - \frac{4}{5}(q_1 + q_2) \right)$$

$$- \frac{2q_1 q_2}{5y_+} - \frac{6(q_1 + q_2)y_+}{5} - 2y_+^3 + \frac{4}{5} \frac{q_1^2}{q_1 + y_+^2} + \frac{4}{5} \frac{q_2^2}{q_2 + y_+^2} \Big) + o(y_c^{-1}) \tag{4.3}$$

Here Vol_{AdS_5} is the regularized volume of AdS_5. The regularized on shell action is divergent in the limit $y_c \to \infty$ which removes the cutoff. In order to get a finite renormalized action we have to add covariant counter terms at the cutoff surface [32–35]

$$S_{\mathrm{ct}} = \frac{1}{8\pi G_N} \int\limits_{y=y_c} \sqrt{-h} \Big(W(\phi_1, \phi_2) + \frac{1}{8} R[h] + \frac{1}{64} (R[h]_{ab} R[h]^{ab} - \frac{3}{10} R[h]^2) \Big)$$

$$= \frac{\mathrm{Vol}_{AdS_5}}{16\pi G_N} \Big(10y_c^3 - 10y_c^2 + 6(q_1 + q_2)y_c + \frac{5}{8} \Big) + o(y_c^{-1}) \tag{4.4}$$

Here $R[h]_{ab}$, $R[h]$ are the Ricci tensor and scalar respectively calculated from the induced metric at the cutoff surface. $W(\phi)$ is the superpotential

$$W(\phi_1, \phi_2) = e^{2\sqrt{\frac{2}{5}}\phi_2} + 2e^{-\frac{1}{\sqrt{2}}\phi_1 + \frac{1}{\sqrt{10}}\phi_2} + 2e^{-\frac{1}{\sqrt{2}}\phi_1 - \frac{1}{\sqrt{10}}\phi_2} \tag{4.5}$$

Which is related to the scalar potential defined in (2.2) by

$$V = 2 \sum_{i=1,2} \Big(\frac{\partial W}{\partial \phi_i} \Big)^2 - \frac{6}{5} W^2 \tag{4.6}$$

The renormalized action is the given by

$$S_{ren} = \lim_{y_c \to \infty} \Big(S_{\mathrm{on-shell}} + S_{\mathrm{ct}} \Big)$$

$$= \frac{\mathrm{Vol}_{AdS_5}}{16\pi G_N} \Big(\frac{5}{8} - \frac{4}{5}(q_1 + q_2) - \frac{2}{5}\frac{q_1 q_2}{y_+} - \frac{6}{5}(q_1 + q_2)y_+ - 2y_+^3 + \frac{4}{5}\frac{q_1^2}{q_1 + y_+^2}$$

$$+ \frac{4}{5}\frac{q_2^2}{q_2 + y_+^2} \Big) \tag{4.7}$$

and when we include the relationship between the q_i's and y_+ implied by $Q(y_+) = 0$, we obtain a remarkably simple result:

$$S_{ren} = \frac{\mathrm{Vol}_{AdS_5}}{16\pi G_N} \Big(\frac{5}{8} - 2y_+^2 \Big) \tag{4.8}$$

As discussed above, our solutions describe holographic co-dimension 2 defects. In particular, when $q_1, q_2 = 0$ ($y_+ = 1$), we just obtain the AdS_7 vacuum which must be subtracted in order to identify the quantity above with the expectation value of the defect.

$$S_{ren} - S_{ren}|_{q_1, q_2 = 0} = \frac{\mathrm{Vol}_{AdS_5}}{8\pi G_N} \Big(1 - y_+^2 \Big) \tag{4.9}$$

Note that the volume of AdS_5 has to be regularized and will contain a scheme independent logarithmic divergent term. We interpret the coefficient (4.9) as a central charge [37] associated with the four dimensional defect.

4.2. Stress tensor and currents

The expectation value of the renormalized holographic stress tensor was derived in [33,35,36] and can be obtained from the renormalized action

$$\langle T_{ab}\rangle_{\text{ren}} = \frac{2}{\sqrt{\det(g_{(0)})}} \frac{\partial S_{ren}}{\partial g_{(0)}^{ab}} \tag{4.10}$$

Where $g_{(0)}$ is the asymptotic boundary metric in Fefferman-Graham coordinates.

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ab}(x,\rho) dx^a dx^b \tag{4.11}$$

with

$$g_{ab}(x,\rho) = g_{(0),ab} + \rho g_{(2),ab} + \rho^2 g_{(2),ab} + \rho^3 g_{(3),ab} + h_{(3),ab}\rho^3 \log\rho + \cdots \tag{4.12}$$

Here the asymptotic boundary is at $\rho = 0$. We defer the details of the calculation to the appendix A but note one of the features of the expansion (4.12) is the absence of the logarithmic term, i.e. we find $h_{(3),ab}$ vanishes. The final result for the expectation value of the stress tensor is

$$\langle T_{ab}\rangle_{\text{ren}} dx^a dx^b = h_D ds^2_{AdS_5} - 5h_D ds^2_{S^1}, \qquad h_D = \left(\frac{1}{18} - \frac{2}{15}(q_1 + q_2)\right) \tag{4.13}$$

which is traceless, indicating a vanishing six dimensional trace anomaly, which is in accordance with the absence of a logarithmic term in (4.12). The coefficient h_D can be called the defect's conformal dimension in analogy with other defects such as surface defects in four dimensions [38–40].[1]

The gauge fields are dual to conserved currents and from the asymptotic behavior of A_i given in (2.4), we can read off the source and expectation value using the standard AdS/CFT dictionary.

$$\lim_{\rho \to 0} A_i = \left(q_i \rho^4 + \cdots\right) dz, \quad i = 1,2 \tag{4.14}$$

which implies that there is no source for the conserved currents and the expectation value of the currents is given by

$$\langle J_i \rangle = q_i dz \tag{4.15}$$

Since the currents are dual to the $U(1) \times U(1)$ R-symmetry, we have a non-vanishing holonomy around the S^1. Recall that the regularity conditions derived in section 3 constrain the charges and hence the holonomies to a one parameter family.

Another holographic observable which can be calculated is the entanglement entropy in the presence of a defect (see e.g. [42–45]). General arguments relate this quantity to the ones already calculated in this section [46,47].

5. Uplift to 11 dimensions

The seven dimensional solutions presented in section 2 can be uplifted to solutions of eleven dimensional supergravity [13,22]

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[1] See [41] for an in depth discussion of anomalies for co-dimension two conformal defects.

$$ds_{11}^2 = \Omega^{\frac{1}{3}} ds_7^2 + \Omega^{-\frac{2}{3}}\left(e^{-\sqrt{\frac{8}{5}}\phi_2} d\mu_0^2 + e^{\frac{\phi_1}{\sqrt{2}}+\frac{\phi_2}{\sqrt{10}}}(d\mu_1^2 + \mu_1^2(d\phi_1 + A_1)^2) \right.$$

$$\left. + e^{-\frac{\phi_1}{\sqrt{2}}+\frac{\phi_2}{\sqrt{10}}}(d\mu_2^2 + \mu_2^2(d\phi_2 + A_2)^2) \right) \tag{5.1}$$

Where Ω is defined as

$$\Omega = e^{\sqrt{\frac{8}{5}}\phi_2}\mu_0^2 + e^{-\frac{\phi_1}{\sqrt{2}}-\frac{\phi_2}{\sqrt{10}}}\mu_1^2 + e^{\frac{\phi_1}{\sqrt{2}}-\frac{\phi_2}{\sqrt{10}}}\mu_2^2 \tag{5.2}$$

The coordinates ϕ_i, $i = 1, 2$ are angular coordinates with periodicity 2π and the coordinates μ_i satisfy the constraint $\sum_{i=0}^{2}\mu_i^2 = 1$. The four form antisymmetric tensor flux is given by

$$*_{11}F_4 = \left(2\sum_{a=0}^{2}(X_a^2\mu_a^2 - \Omega X_a) + \Omega X_a \right)\text{vol}_7 + \frac{1}{2}\sum_{a=0}^{2}\frac{1}{X_a}(*_7 dX_a) \wedge d(\mu_a^2)$$

$$+ \frac{1}{2}\sum_{a=1}^{2}\frac{1}{X_a^2}d(\mu_a^2) \wedge (d\phi_a + A_a) \wedge *_7 F_a \tag{5.3}$$

Here $*_{11}$ is the Hodge dual with respect to the eleven dimensional metric (5.1) whereas $*_7$ and vol$_7$ are the Hodge dual and volume with of to the seven dimensional metric (2.4) respectively. Note that the AdS_7 vacuum solution $q_1 = q_2 = 0$ gives the $AdS_7 \times S^4$ solution of eleven dimensional supergravity, dual to the vacuum of the six dimensional SCFT. Since the gauge fields A_i, $i = 1, 2$ twist the two angular coordinates ϕ_i in the metric (5.1) we can identify the gauge fields as dual to $U(1) \times U(1)$ R-symmetry currents inside the $SO(5)$ R-symmetry of the $N = (0, 2)$ six dimensional SCFT.

6. Discussion

In this note we constructed holographic solutions of $N = 4$, $d = 7$ gauged supergravity which describe four dimensional defects living inside a six-dimensional SCFT. The solutions are closely related to AdS_5 compactifications on spindles of the same theory [13]. The main difference lies in the fact that the two dimensional space transverse to the AdS_5 factor is compact in the spindle case, whereas in our case the space is non-compact and the solution has an asymptotic AdS_7 boundary. Consequently instead of describing a holographic dual for a four dimensional CFT, our solutions are holographic to a four dimensional super conformal defects living inside a six dimensional SCFT. For the spindle [13] the two dimensional space is a sphere with two conical singularities at the north and south pole, which is a compact space. The main result of the present paper is that for the two charge extremal solutions it is possible to find completely regular solutions without any conical deficits in the bulk or on the asymptotic boundary. These solutions form a one parameter family in the space of extremal solutions and a quarter of the supersymmetries of the AdS_7 vacuum. Another class of solutions are the "half-spindle" solutions of [30,31] where the two dimensional space has the topology of a punctured disk with one conical singularity in the center and smeared M5-brane sources. It is possible to also have solutions with conical singularity in the bulk with have a non-compact space with the topology of the punctured plane. In particular if we set one of the charges to zero there will always have a conical singularity, however the amount of supersymmetry is now doubled and the uplifted solutions fall into the (local) M-theory solutions with AdS_5 factors, such as [48–50]. It would be interesting to see whether its possible how to incorporate solutions with a noncompact direction leading to an asymptotic AdS_7 boundary and hence describing a four dimensional defect embedded in a

six dimensional SCFT. The uplift and embedding of these theories will also be helpful to argue for the integer quantization of the conical deficits, which is not possible from a purely seven dimensional point of view.

The asymptotic boundary of the spacetime is $AdS_5 \times S^1$ which is conformal to $R^{1,5}$ under this map the circle parameterizes the angular direction of the transverse R^2. Since our solution has a non-vanishing expectation value of the $U(1) \times U(1)$ R-symmetry currents we can interpret the defect as a homolomy defect for the R-symmetry currents. Examples of such defects have been constructed for free field theories [51–55]. For surface defects in four dimensional $N = 4$ SYM such defects can be related to probe brane and fully back reacted LLM geometries [56–58] and some observables were matched in [40]. It would be interesting to see whether such a relation exists for four dimensional defects in the six dimensional SCFT, in particular whether there is a field theory analogue of the regularity condition relating the two charges or holonomies that we found. We leave these interesting questions for future work.

CRediT authorship contribution statement

Both authors contributed equally to this work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. Calculation of holographic stress tensor

In this section we calculate the expectation value of the holographic stress tensor following [35]. The metric (2.4) has the following large y expansion

$$
\begin{aligned}
ds^2 = \Big(&\frac{1}{y^2} + \frac{1}{y^3} + \frac{5 - 4(q_1 + q_2)}{5y^4} + \cdots \Big)\frac{dy^2}{4} \\
&+ \Big(y + \frac{q_1 + q_2}{5y} + \frac{-2q_1^2 - 2q_2^2 + q_1 q_2}{25y^3} + \cdots \Big)ds^2_{AdS_5} \\
&+ \Big(y + \frac{q_1 + q_2}{5y} + \frac{4q_1 + 4q_2}{5y^2} + \frac{-2q_1^2 - 2q_2^2 + q_1 q_2}{25y^3} + \cdots \Big)dz^2
\end{aligned}
\tag{A.1}
$$

where the dots denote terms which go faster to zero in the limit $y \to \infty$. The following coordinate transformation brings the metric into Fefferman-Graham form

$$y = \frac{1}{\rho} + \frac{1}{2} + \frac{5 - 16(q_1 + q_2)}{80}\rho - \frac{q_1 + q_2}{30}\rho^2 + o(\rho^3) \tag{A.2}$$

Which takes the following form

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}g_{ab}(x, \rho)dx^a dx^b$$

$$g_{ab}(x, \rho) = g_{(0),ab} + \rho g_{(2),ab} + \rho^2 g_{(4),ab} + \rho^3 g_{(6),ab} + h_{(6),ab}\rho^3 \log\rho + \cdots \tag{A.3}$$

The g_{ab} the takes the following form in Fefferman-Graham coordinates

$$g_{ab}(x, \rho)dx^a dx^b = \left(1 + \frac{1}{2}\rho + \frac{1}{16}\rho^2 - \frac{2q_1 + 2q_2}{15}\rho^3 + \cdots\right)ds^2_{AdS_5}$$

$$+ \left(1 - \frac{1}{2}\rho + \frac{1}{16}\rho^2 + \frac{2(q_1 + q_2)}{3}\rho^3 + \cdots\right)dz^2 \tag{A.4}$$

From which we can read off the $g_{(i),ab}$, $i = 0, 2, 4, 5$. Note that there is no term logarithmic in ρ and hence $h_{(6),ab} = 0$ for the solution considered in this paper. The expectation value of the holographic stress tensor is then given by

$$\langle T_{ab}\rangle = g_{(6),ab} - A_{(6),ab} + \frac{1}{24}S_{ab} \tag{A.5}$$

Where A_6 and S are expressed in terms of $g_{(0)}, g_{(2)}, g_{(4)}$ and their derivatives. Explicit expressions can be found in [35] and evaluating them for our background gives

$$\langle T_{ab}\rangle_{\text{ren}}dx^a dx^b = \left(\frac{1}{18} - \frac{2}{15}(q_1 + q_2)\right)ds^2_{AdS_5} + \left(-\frac{5}{18} + \frac{2}{3}(q_1 + q_2)\right)dz^2 \tag{A.6}$$

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