# Improved Least-Squares Migration through Double Sweeping Solver

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Right Running Head: Migration with Double-Sweeping Solver

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Original paper date of submission: September 23, 2021

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## ABSTRACT

Based on a recently developed approximate wave equation solver, we propose a methodology to reduce the computational cost of seismic migration in the frequency domain. The proposed approach divides the domain of interest into smaller subdomains, and the wavefield is computed using a sequential process to determine the downward propagating and upward propagating wavefields – hence named *double sweeping solver*. A sequential process becomes possible using a special approximation of the interface conditions between subdomains. The proposed method is incorporated into the least-squares migration framework as an approximate solver. The associated computational effort is comparable to one-way wave equation approaches, yet, as illustrated by numerical examples, the accuracy and convergence behavior are comparable to that of the full-wave equation.

**Keywords**: depth migration, least-squares, frequency-domain, wave equation

# 1

## INTRODUCTION

Migration is the process of converting seismic reflection data into an image of the subsurface, given a background velocity model. Migration methods can be classified into three major groups: Kirchhoff migrations (French, 1975; Schneider, 1978), one-way-wave-equation (OWWE) migrations (Claerbout, 1970; Berkhout, 1979; Ristow and Rühl, 1997), and full-wave-equation migrations such as reverse-time migration (RTM) (Baysal et al., 1983). Kirchhoff methods are efficient migration algorithms that result in accurate images for simple velocity models. The effectiveness of Kirchhoff methods is significantly reduced with velocity models with large contrasts but are still useful in performing target-oriented imaging. As indicated by the name, OWWE only allows wave propagation in a specific direction, either downward or upward. OWWE-based migration methods can handle complex velocity models efficiently, but they do not preserve amplitudes of the wavefield (Zhang et al., 2005) and cannot handle multiples (Mulder and Plessix, 2004b). Although there are many ways to improve the accuracy of OWWE migration (see, e.g., Joncour et al., 2005; Zhang et al., 2005; Kiyashchenko et al.,2005 and 2007; and You, Wu, and Liu 2018), the most accurate migration algorithms are based on the full-wave equation that accurately simulates wave propagation in complex media. However, the computational cost of full-wave simulation is high, especially for large-scale problems. Alternatively, a reflectivity model can be obtained based on linearized least-squares optimization to minimize the error between modeled scattered wavefields and observed data (Lailly, 1983; Tarantola, 1984; Schuster, 1993; Nemeth et al., 1999; Chavent and Plessix, 1999; Mulder and Plessix, 2004b). Such least-squares migration methods enable high-resolution imaging for complex structures, even in the presence of noise or limited data (Nemeth et al., 1999). This method, however, requires multiple forward and backward simulations at each iteration, leading to a significant computational expense. Therefore, least-square migration may benefit from an approximate wave solver with the accuracy close to the full-wave equation but with reduced computational cost.

Least-squares migration (LSM) is performed either in the frequency domain (e.g., see Plessix and Mulder 2004c) or in the time domain, i.e., least-squares reversed time migration (LSRTM, e.g., Dai, Fowler, and Schuster, 2012, Zeng, Dong, and Wang, 2014, and Chen and Sacchi, 2017). If a proper subset of frequencies is used, a frequency-domain migration can be computationally more efficient than in a time-domain approach (Mulder and Plessix, 2004a). Frequency-domain computation has multiple additional advantages: numerical dispersion is more easily controlled in the frequency domain (e.g., Guddati and Yue 2004). Wave attenuation due to material damping is more accurately modeled with the help of complex-valued, frequency-dependent wave velocity fields; and the discretization can be frequency-dependent, resulting in computational efficiency. Despite these advantages, high-frequency wave simulations are still computationally demanding for large-scale seismic imaging. OWWEs provide an efficient framework to perform forward modeling in the frequency domain, but the standard OWWE methods do not preserve the amplitudes of the wavefield; thus, they are not effective for the least-square migration of field data. It would be ideal to develop a method that has the efficiency of OWWE and the accurate amplitude predictability of the full-wave equation so that it can be used as a solver in a least-squares migration framework.

There are multiple approaches to improve the amplitude accuracy of OWWE-based wave simulations; Kiyashchenko et al. (2005) proposed an iterative procedure to solve the full-wave equation by applying multiple OWWE operators. This method introduces a new right-hand side term to OWWE representing heterogeneities in the velocity model, and thus leading to improved wave amplitudes. Also, Stanton and Sacchi (2017) proposed a least-squares migration method where OWWE is used to improve the computation of the forward and adjoint operators. In their method, the OWWE wavefield decomposition is based on the eigenvalues and eigenvectors of the Christoffel equation associated with elastic media. The solution is extended to heterogeneous cases using the phase-shift plus interpolation and split-step correction methods (Gazdag and Sguazzero, 1984; Stoffa et al., 1990).

This paper proposes an alternative least-squares migration approach based on an approximate solver in the frequency domain, which results in an accurate image close to that from full-wave equation migration while being as computationally efficient as the OWWE. The proposed approach is based on splitting the computational domain into horizontal subdomains and then solving for the wavefield inside those subdomains using a two-step procedure. The first step involves computing the downward propagating wavefield by solving for the wavefield inside the subdomains sequentially from top to bottom (downward sweeping); the key here is the use of special interface conditions that preserve the amplitude during transmission at the subdomain interfaces. In the second step, the reflected wavefield due to the material discontinuities is backpropagated by solving for the wavefield inside the subdomains from bottom to top, again using special interface conditions. It is worth noting that the upward sweeping may make it possible to model the diving and prismatic waves by adding the upward going wavefield – which is not possible using the OWWE (including the diving and prism waves may improve accuracy of subsalt migration and the full waveform inversion).

The idea of splitting the wavefield into upward and downward propagators is similar to OWWEs, but the proposed method better preserves amplitudes and results in an accurate yet efficient migration algorithm for complex velocity models. This method is already implemented as a preconditioner to solve the Helmholtz equation (Eslaminia and Guddati 2016) and improve full-waveform inversion's computational efficiency (Eslaminia *et al.* 2022).

This study shows that the proposed approach also enables efficient 2D least-squares migration with a convergence behavior similar to that of a full-wave equation method.

This paper is organized as follows. In the next section, the least-squares migration formulation is briefly reviewed. We then present the proposed approximate solver. A 2D numerical experiment is used to illustrate the accuracy of the proposed method as a forward solver in the following section. Then, the effectiveness of the proposed method is demonstrated for the least-squares migration using several synthetic examples. The paper is concluded with some closing remarks.

## LEAST-SQUARES MIGRATION

A least-squares migration formalism can be derived by minimizing the least-squares error (misfit) between recorded data $\boldsymbol{d}$ and a synthetic wavefield $\boldsymbol{u}$, extracted at the receiver locations for all sources and frequencies (Lailly, 1983; Tarantola, 1984). An inverted image can be determined by solving the following normal equation (e.g., see Nemeth et al., 1999 and Chavent and Plessix 1999):

$$\mathbf{m_s} = (\Re\{\mathbf{L}^\dagger\mathbf{L}\})^{-1}\Re\{\mathbf{L}^\dagger(\mathbf{d}-\mathbf{P}\mathbf{u_0})\}, \tag{1}$$

In the context of LSM, $\Re\{\mathbf{L}^\dagger\mathbf{L}\}$ is the Hessian matrix, whereas the term $\Re\{\mathbf{L}^\dagger(\mathbf{d}-\mathbf{P}\mathbf{u_0})\}$ corresponds to the gradient vector, where † denotes the conjugate transpose, $\mathbf{P}$ is a projection operator to extract simulated wavefield at the location of receivers and $\mathbf{u}_0$ is the wavefield corresponding to the background velocity model $\mathbf{m_0}$, i.e., $\mathbf{S}_0\mathbf{u}_0 = \mathbf{f}$ with $\mathbf{S}_0 = \mathbf{S}(\mathbf{m}_0)$ where $\mathbf{S}$ is the full-wave operator. Using the Born approximation, $\mathbf{L}$ is the linearized forward modeling operator:

$$\mathbf{L} = -\mathbf{P}\mathbf{S}_0^{-1} \frac{\partial \mathbf{S}}{\partial \mathbf{m}}\bigg|_{\mathbf{m}_0} \mathbf{u}_0. \tag{2}$$

The gradient term $\Re\{\mathbf{L}^{\dagger}(\mathbf{d} - \mathbf{P}\mathbf{u}_0)\}$ becomes the cross-correlation between the forward-propagating wavefield $\mathbf{u}_0$ and the back-propagated wavefield of the residual, which is equivalent to the imaging principle in the conventional prestack depth migration methods (see, e.g., Claerbout, 1985; Mulder and Plessix, 2004b). The inverse Hessian $(\Re\{\mathbf{L}^{\dagger}\mathbf{L}\})^{-1}$ in equation 1 is an illumination operator that increases the accuracy of the subsurface image with depth. However, computing and inverting the full Hessian matrix is practically impossible for large-scale problems. Therefore, an iterative scheme must be used to perform least-squares minimization. We can write the residual at the $k^{\text{th}}$ iteration as

$$\mathbf{r}^k = \mathbf{P}\mathbf{u} - \mathbf{d} = \mathbf{P}\mathbf{u}_0 + \mathbf{L}\mathbf{m}_s^k - \mathbf{d}, \tag{3}$$

and the iterative solution becomes

$$\mathbf{m}_s^{k+1} = \mathbf{m}_s^k - \alpha(\mathbf{M}^k)^{-1}\mathbf{g}^k, \tag{4}$$

where $\mathbf{M}^k$ is an approximation of the Hessian at the current iteration, $\alpha$ is the step size, and $\mathbf{g}^k$ is the current gradient vector given by:

$$\mathbf{g}^k = \Re\{\mathbf{L}^{\dagger}\mathbf{r}^k\} = \Re\{\mathbf{L}^{\dagger}(\mathbf{P}\mathbf{u}_0 + \mathbf{L}\mathbf{m}_s^k - \mathbf{d})\}. \tag{5}$$

While several methods exist for determining the approximate Hessian (e.g., Plessix and Mulder 2004), we utilize the BFGS method. The step size is also determined using a cubic line search algorithm. An alternative approach is to use the preconditioned Conjugate Gradient (CG) method but is not considered in this paper.

## METHODOLOGY

At each iteration of LSM algorithm (defined by equations 3, 4 and 5), we need one forward solve for computation of the misfit in equation 3, i.e., application of the forward operator $\mathbf{L}$, and another backward solve for the computation of the backpropagated misfit, i.e., application of the adjoint operator $\mathbf{L}^{\dagger}$. In addition, one forward solve is required per each iteration of the cubic line search, i.e., application of the forward operator $\mathbf{L}$. Although all wave simulations involve a constant operator $\mathbf{S}_0$ associated with the background velocity model, it might not be practical to factorize, store, and reuse the LU factorization for large-scale problems. An alternative is to solve the wave simulations iteratively using a preconditioned generalized minimal residual (GMRES) method (Saad and Schultz, 1986; Erlangga *et al.,* 2004; Engquist and Ying, 2011, Demanet *et al.* 2012; Eslaminia and Guddati, 2016). However, the computational cost associated with iterative solvers is still significant since each input source must be simulated independently. The basic idea of the proposed framework is to introduce an approximate solver with a computational cost comparable to conventional OWWEs to quickly perform all wavefield simulations without significant loss in accuracy of the wavefield. The details are discussed in the remainder of this section.

### Modeling approach

To simplify the discussion, we consider the case where the forward problem is modeled using the time-harmonic acoustic (scalar) wave equation (Helmholtz equation),

$$-\nabla^2 u - \frac{\omega^2}{c(x,z)^2} u = f \quad \text{in } (x,z) \in \Omega, \tag{6}$$

where $u$ is the scalar wavefield, $\omega$ is the temporal frequency, and $c(x,z)$ is the wave velocity within the domain of interest Ω. Absorbing boundary conditions (ABC) are utilized at the boundaries as follows:

$$-\frac{\partial u}{\partial \mathbf{n}} = \lambda u \quad \text{on} \quad \partial\Omega, \tag{7}$$

where $\boldsymbol{n}$ is a unit vector perpendicular to the boundary and $\lambda$ is the Dirichlet-to-Neumann (DtN) map, which is essentially the half-space stiffness.

Before moving on to the proposed approach, we discuss on the choice of the DtN map $\lambda$, as it plays an important role in interface conditions in the proposed method. For example, if we consider the boundary parallel to the $x$-axis, the $\lambda$ is formally written as,

$$\lambda = -i\sqrt{\frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2}}, \tag{8}$$

which is nonlocal, computationally expensive and often approximated. The approximations are typically classified into two categories: the first class is based on rational approximations (Lindman, 1975; Engquist and Majda, 1979; Higdon, 1986), and the second is related to perfectly matched layers (Berenger, 1994; Chew and Weedon, 1994). These seemingly disparate classes are shown to be linked (Asvadurov et al. 2003), and the resulting approximation, called Perfectly Matched Discrete Layers (PMDL, Guddati et al., 2008), inherits the respective advantages of the two classes. While more details of PMDL can be found in Guddati and Lim (2006), the basic idea is to replace the half-space with a small number of complex-length finite element layers with midpoint integration. It is shown that three to five PMDL layers are often sufficient to capture wide-angle propagations (Guddati, 2006; Guddati and Lim, 2006; Guddati and Heidari, 2007), and the idea is applicable to more complicated elastic wave equations (Guddati, 2006; Savadatti and Guddati, 2012a 2012b). Therefore, we adapt PMDL not only as the absorbing boundary condition but also as interface conditions, as discussed in the next section. We emphasize, however, that the proposed methodology is not limited to PMDL, can be used in conjunction with other DtN approximations.

**Approximate wave equation solver**

The basic idea is to split the wavefield into a downward propagating component $u_d$ and an upward-propagating component $u_u$ and approximate each by ignoring multiple reflections. Thus, $u_d$ is the downward-propagating wavefield obtained by the proper transmission of amplitudes across material interfaces, while $u_u$ is the upward propagating wavefield resulting from primary reflections at the same interfaces. With such an approximation, $u_d$ can be obtained by a sequential solution from the top of the domain to the bottom. Then $u_u$ can be calculated from the bottom of the domain to the top. A critical aspect of this procedure is to compute the transmission and reflection amplitudes accurately. In what follows, we present the mathematical framework for solving for both $u_d$ and $u_u$.

The domain of interest $\Omega$ is divided into horizontal subdomains $\Omega^j$ (for $j$ = 1 to $n$, starting from the top to the bottom) to facilitate a sequential solution. The horizontal boundaries of $\Omega^j$ are denoted by $\Gamma_t^j$ at the top and $\Gamma_b^j$ at the bottom of a subdomain $j$. We use the superscript $j$ to represent entities associated with $\Omega^j$, and subscripts $t$ and $b$ to represent the variables at the top and the bottom, respectively. The following boundary conditions are applicable for the $j^{\text{th}}$ subdomain:

$$\begin{aligned} u_t^j &= u_b^{j-1}, \qquad \left.\frac{\partial u^j}{\partial z}\right|_{\Gamma_t^j} = \left.\frac{\partial u^{j-1}}{\partial z}\right|_{\Gamma_b^{j-1}} \\ u_b^j &= u_t^{j+1}, \qquad \left.\frac{\partial u^j}{\partial z}\right|_{\Gamma_b^j} = \left.\frac{\partial u^{j+1}}{\partial z}\right|_{\Gamma_t^{j+1}} \end{aligned} \quad . \tag{9}$$

The conditions in equation 9 represent the continuity of the wavefield and traction across the interfaces and cause the solution to be coupled across all the subdomains. The basic idea of the proposed method is to approximate the interface conditions in equation 9 to relax the coupling between interface conditions and facilitate a sequential solution of the wavefield. This idea is not new, and in fact, is the basis for traditional OWWE (Guddati and Heidari, 2007). The key difference is that the proposed method preserves amplitudes associated with transmission and primary reflections at material interfaces.

The proposed approximate solution is performed in two steps. First, the downward-propagating (downward-continued) wavefield $u_d$ is computed considering only downward transmissions at the interfaces. Then the wavefields are corrected by adding the upward-propagating (upward-continued) wavefield $u_u$, resulting from primary reflections at all the interfaces. The details are discussed below.

**Downward propagating wavefield**

The downward propagating wavefield $u_d$ is determined by solving the subproblems from top to bottom, starting with the first subdomain. As illustrated in Figure 1, the basic idea is to compute the wavefield in the $j^{\text{th}}$ subdomain based on the full-wave equation while assuming the only incoming wavefield is from the $j$-1$^{\text{st}}$ subdomain ($u_{d,b}^{j-1}$ in Figure 1b). This approximate solution is achieved by applying the following two conditions:

1. The incoming wavefield from the $j$-1$^{\text{st}}$ subdomain must be propagated into the current subdomain without any reflections at the top interface.
2. The change in material properties (e.g., wave velocity) must be considered to get accurate transmission effects of the wavefield at the bottom interface (and reflection, which will be used for upward propagation in the next section).

The first condition is enforced by adding a half-space at the top interface, with materials properties matching with the top boundary $\Gamma_t^j$. The second condition is enforced by using the full-wave equation within the subdomain and adding another half-space at the bottom interface but with material properties consistent with the top boundary of the next subdomain $\Gamma_t^{j+1}$ (to obtain the appropriate reflection and transmission at the interface). This half-space approximation eliminates the dependency on the wavefield from the next subdomains. It also provides an accurate interface condition to model the reflections due to any material change (e.g., a sharp change in the material or discretization of the velocity model) at the vicinity of the bottom interface. Note that if the velocity changes and the half-space is matched with the bottom boundary of the current subdomain $\Gamma_b^j$, there will be no reflection, and the transmission will be inaccurate; thus, we would not get the desired downward propagating wavefield.

Mathematically, attaching the bottom half-space translates to

$$\frac{\partial u_d^j}{\partial z} + \lambda_t^{j+1} u_d^j = 0 \text{ on } \Gamma_b^j, \tag{10}$$

where $\lambda_t^{j+1}$ is the stiffness of half-space matching with the top of the next subdomain. Equation 10 simplifies the bottom interface conditions in equation 9 by removing dependency on the wave solution from the next subdomain. It is also worth noting that the equation 10 is in fact the standard OWWE for the downward propagating wavefield.

The boundary condition at the top is slightly more involved. In addition to attaching a half-space, we have an incident wave coming from the previous subdomain, which is the downward-propagating wavefield at the bottom boundary of the previous subdomain $\Gamma_b^{j-1}$. The top half-space can be modeled using half-space stiffness as in equation 7, while the incident wave is modeled using standard scattering formalism (e.g., Colton and Kress, 2013). Thus, the top interface condition can be written as

$$-\left.\frac{\partial u_d}{\partial z}\right|_{\Gamma_t^j} + \lambda_t^j u_{d,t}^j = -\left.\frac{\partial u_d^{j-1}}{\partial z}\right|_{\Gamma_b^{j-1}} + \lambda_t^j u_{d,b}^{j-1}, \tag{11}$$

where the scattered wave $u_s = u_{d,t}^j - u_{d,b}^{j-1}$ satisfies one-way propagation at the interface, i.e., $\left.\frac{\partial u_s}{\partial z}\right|_{\Gamma_t^j} = \lambda_t^j u_s$. Using equation 10 from the previous subdomain, we have,

$$\left.\frac{\partial u_d^{j-1}}{\partial z}\right|_{\Gamma_b^{j-1}} = -\lambda_t^j u_{d,b}^{j-1}, \tag{12}$$

and equation 11 simplifies to,

$$-\frac{\partial u_d^j}{\partial z} + \lambda_t^j u_d^j = 2\lambda_t^j u_{d,b}^{j-1} \text{ on } \Gamma_t^j. \tag{13}$$

In summary, the downward propagating wavefield is solved using the local boundary value problem:

$$\begin{aligned} &-\nabla^2 u_d^j - \frac{\omega^2}{c(x,z)^2} u_d^j = f(x,z) \quad \text{in } \Omega^j, \\ &-\frac{\partial u_d^j}{\partial z} + \lambda_t^j u_d^j = 2\lambda_t^j u_{d,b}^{j-1} \text{ on } \Gamma_t^j, \\ &\frac{\partial u_d^j}{\partial z} + \lambda_t^{j+1} u_d^j = 0 \text{ on } \Gamma_b^j, \end{aligned} \tag{14}$$

where $j = 1...n$ and $u_{d,b}^{j-1}$ is the downgoing wavefield at $\Gamma_b^{j-1}$, computed in the $j$-1th subdomain and zero for $j = 1$.

**Upward propagating wavefield**

The body force and material discontinuities cause an upward propagating wavefield that must be added to the downward propagating wavefield. The upward propagation step discussed in this section essentially captures the effect of primary reflections and external excitation from the subdomain below. As illustrated in Figure 2, the reflection is simply the difference between the transmitted wave (downward propagating wavefield at the top of $j$+1st subdomain,$u_{d,t}^{j+1}$) and the incident wave (downward propagating wavefield at the bottom of the $j$th subdomain,$u_{d,b}^{j}$). This reflected wavefield should be added to the upward propagating wavefield $u_{u,t}^{j+1}$ from the bottom subdomain to obtain the total upward incident wavefield for the $j$th subdomain $u_{inc}^{j}$ as follows:

$$u_{inc}^{j} = u_{d,t}^{j+1} - u_{d,b}^{j} + u_{u,t}^{j+1}. \tag{15}$$

Note that body forces are not considered here, since they are already included in generating the downward sweeping step, which is included in $u_{d,t}^{j+1}$, thus indirectly affecting the upward propagating wavefield.

Once the incident wavefield is determined according to equation 15, the solution procedure for the upward propagating wavefield is similar to the previous section but in the reverse direction presented in Figure 1c. Specifically, at the top of the subdomain, we attach a half-space that matches material properties with the bottom of the next (upper) subdomain. At the bottom of the subdomain, the scattering formalism is applied by attaching a matching half-space with the total incident wavefield given in equation 15.

Following equation 14, the upward propagating subproblem for subdomain $j$ is written as:

$$\begin{aligned}
&-\nabla^2 u_u^j - \frac{\omega^2}{c(x,z)^2} u_u^j = 0 \quad \text{in } \Omega^j \\
&-\frac{\partial u_u^j}{\partial z} + \lambda_b^{j-1} u_u^j = 0 \;\; \text{on } \Gamma_t^j \\
&\frac{\partial u_u^j}{\partial z} + \lambda_b^j u_u^j = 2\lambda_b^j \left(u_{d,t}^{j+1} - u_{d,b}^j + u_{u,t}^{j+1}\right) \text{ on } \Gamma_b^j
\end{aligned} \tag{16}$$

We reemphasize that the external body force is zero for upward sweeping since its effects are already considered as part of the upward incident wavefield in the scattering boundary condition.

Note that the sequence of downward and upward sweeps can be reversed, in which case body force should be considered in the upward sweep. In addition, there are situations where only single sweep is considered, where the body force effect must be accordingly included.

## Implementation

Algorithm 1 illustrates the implementation of the double-sweeping solver. The implementation consists of three main steps: initialization, downward sweeping, and upward sweeping. The initialization step involves dividing the domain into smaller subdomains, assembling the wave equation matrix (the dynamic stiffness matrix) associated with the sub-domain interior and PMDL padding, and utilizing LU factorization to factorize the stiffness matrix. This step is performed once per frequency.

**Algorithm 1: Double-Sweeping Solver**

**Initialization (perform once for a given frequency)**

1. Divide the domain into $N_s$ horizontal subdomains
2. Add PMDL layers to represent the top and bottom interface boundary conditions.
3. Factorize the downward and the upward matrices using an efficient LU factorization algorithm.

**Downward sweeping (repeat for sources)**

**for** $j = 1 \; to \; N_s$

1. Evaluate the right-hand side in equation 14.
2. Solve equation 14 for $u_d^j$

**Upward sweeping (repeat for sources)**

**for** $j = N_s \; to \; 1$

1. Evaluate the right-hand side in equation 16.
2. Solve equation 16 for $u_u^j$.

Approximate Wavefield: $u_{approx}^j = u_d^j + u_u^j$

Although the proposed (approximate) double-sweeping solver can be incorporated in any least-squares migration algorithm to speed up the imaging process, in this study, we take advantage of the LSM in the frequency domain and utilize a cascaded least-squares algorithm. As shown in Algorithm 2, the target frequency range is divided into a series of frequency groups, and the least-squares migration is progressively performed after updating the background image using the result from the lower frequency range. Also, to improve the accuracy of the forward solver (especially for the lower frequency range), we utilize an adaptive decomposition approach for dividing the computational domain, where a lower number of the subdomains is used for the low frequencies.

**Algorithm 2: Least-Squares Migration based on the Double Sweeping Solver**

**Preprocessing**

1. Divide the target frequency range to $n_{group}$ frequency groups.
2. Setup number of sub-domains for each frequency group.

**Least-squares migration**

**Initialization**

- Given the background velocity model $\mathbf{m_0}$, and corresponding background wavefield $\mathbf{u_0}$, compute the initial gradient $\mathbf{g}^0$ defined in equation 5 given the scattering model resulted from *the preceding group of frequency* $\mathbf{m}_\mathrm{s}^0$, (for the first group, $\mathbf{m}_\mathrm{s}^0$ can be initialized as $\mathbf{0}$.

**Minimization (BFGS iterations)**

- Minimize the misfit iteratively over all frequencies in the group and all shots, i.e., update model $\mathbf{m}_s^k$ until convergence. Each iteration (*k*) involves the following steps:

1. **Gradient $\mathbf{g}^k$ computation**
   - Compute the gradient in equation 5 by approximating the action of operators **L** and $\mathbf{L}^\dagger$ using the double-sweeping solver in *Algorithm 1*.
2. **BFGS step calculation**
   - Update the inverse of the Hessian operator using the algorithm described in Nocedal and Wright (2006).
   - Compute $\mathbf{m}_s^k$.
3. **If $\mathbf{m}_s^k$ does not decrease the misfit significantly, backtrack on that using cubic line search.**
   Minimize the misfit using cubic line search, each iteration (*j*) in the cubic line search involves the following:
   - Compute the misfit in equation 5 by evaluating $\mathbf{L}\mathbf{m}_\mathrm{s}^j$ using the double-sweeping solver in *Algorithm 1*.
4. **Update the current scattering model $\mathbf{m}_s^{k+1}$.**

**Repeat until convergence.**

**Computational cost**

An effective way to solve the governing equation 6 (Helmholtz equation) in the full 2D domain is using multifrontal direct LU factorization methods (Duff and Reid, 1983; Liu, 1992). For a given $n \times n$ 2D mesh (total number of unknowns $N = n^2$), multifrontal methods require $O\left(N^{3/2}\right) = O(n^3)$ flops and $O(N\, log\, N) = O(n^2\, log\, n)$ storage. In the case of OWWEs, the solution to the wave equation is reduced to series of 1D or quasi 1D problems (i.e., for the $n \times n$ mesh *n* systems with the size of *n*). These systems can be efficiently solved using banded factorization methods with the total computational complexity of $O(n^2) = O(N)$. For the proposed method, each subdomain can also be factorized using banded LU methods assuming the thickness of subdomains is small compare to *n*. For a given subdomain with a thickness of *m* and *p* PMDL layers, the bandwidth is $2(m + 2p)$ and the computational cost of factorization is $O(4n(m + 2p)^2)$ with required storage of $O\big(2n(m + 2p)\big)$. Multiplying the cost of a single subdomain by number of subdomains (i.e., $n/m$ subdomains for the downward sweep and $n/m$ subdomains for the upward sweep), the total cost of the factorization becomes $(8n^2(m + 2p)^2/m)$. If the same subdomains are used for both downward and upward sweeps, the factorization cost becomes $O(4n^2(m + 2p)^2/m)$. Assuming *n* to be large, the computational complexity of the proposed method (for all subdomains) becomes $O(n^2) = O(N)$, which is clearly in the same order of OWWEs, albeit with different constants of multiplication that depend on the individual subdomain thickness.

*Remarks on the frequency and the computational cost:* Our numerical experiments show that the double-sweeping solver produces a more accurate wavefield for lower frequencies if we utilize thicker subdomains. The proposed method is more expensive for lower frequencies than for higher frequencies if the mesh resolution is kept constant. However, the grid resolution gradually increases with frequency in practical applications. Therefore, if we use the same number grid points per subdomains for all frequencies, the computational cost of the double-sweeping solver increases with frequency, just like the full-wave equation.

Now, although the proposed method has not been implemented for 3D problems and recognizing that the final computational cost would depend on implementation details, we can estimate the computational cost for a $n \times n \times n$ mesh with a total number of unknowns equal to $N = n^3$. Starting with the full-wave equation, the multifrontal methods for 3D problems cost $O(N^2) = O(n^6)$ flops and $O\left(N^{4/3}\right) = O(n^4)$ storage, which is impractical for large-scale wave propagation simulations. For OWWE, the simulation is reduced to a series (essentially) of 2D systems ($n$ systems with $n \times n$ mesh for each system). Each 2D system may be solved using multifrontal methods that require $O(n^3)$ flops and $O(n^2 \, log \, n^2)$ storage. Thus, the total cost of 3D OWWE becomes $O(n^4) = O\left(N^{4/3}\right)$ flops and $O(n^3 \, log \, n^2) = O(N \, log \, N)$.

For the proposed approach, the factorization of a given subdomain can be done efficiently using multifrontal methods. Assuming that the subdomains are thin, the computational cost associated with each subdomain becomes similar to 2D multifrontal methods. Therefore, the computation cost of the proposed method becomes $O(n^4) = O\left(N^{4/3}\right)$ flops and $O(n^3 \, log \, n^2) = O(N \, log \, N)$ – which is similar to OWWE, though with different constants.

*The parameterization of the subdomain thickness*: To ensure that the overhead associated with the half-space approximations for each subdomain does not dominate the computational cost, and to maximize accuracy by capturing intra-subdomain multiple reflections, it is better to choose thicker subdomains. In fact, for practical (3D) applications, it may be best to choose the largest subdomain thickness for which direct multifrontal solution can be efficiently performed.

## NUMERICAL ILLUSTRATION OF THE PROPOSED SOLVER

We demonstrate the accuracy of the proposed approximation using a 2D wave propagation simulation with the velocity model shown in Figure 3a. The excitation force is a Ricker pulse located at $x = 0.5$ km and $z = 0$ km with a central frequency of 20 Hz. For the full-wave equation modeling approach, the domain of interest is discretized using a 200 × 200 finite element mesh, while we use 20 horizontal subdomains with a thickness of ten finite element layers. The modified integration rule proposed in Guddati and Yue (2004) is used to reduce numerical dispersion errors. The stiffness of the half-space is approximated using five PMDL layers. We perform a time-harmonic simulation for the frequency ranging from 0.4 to 100 Hz with the increment of 0.4 Hz, and then time histories are computed using an inverse Fourier transform.

Figure 3 and 4 compare the snapshots and wiggle plots of the wavefields from the full-wave equation and the proposed methods. This comparison illustrates that the proposed double sweeping method is in good agreement with the full-wave solution. It is worth noting that there is a small phase-shift between the approximation solution and the full-wave wavefield. As clearly seen in Figure 4, the time-shift gradually increases in the direction of the sweep. Therefore, we hypothesize that the time-shifts are attributed to the sweeping process. An approximate way to think about sweeping (with thin slabs) is that it is like forward differencing, while single coupled (full-wave) solutions are somewhat similar to central differencing; naturally the former would have higher dispersion. Consequently, the error can be reduced by increasing the thickness of the subdomains (reducing number of sweeping steps) but it will lead to higher computational cost. Alternatively, the grid steps can be reduced, which too is associated with increased computational cost.

In addition, the result from the single downward-sweep algorithm is also shown in Figure 3c and 3e and the wiggle plots in Figure 4; as expected, the method captures the downward propagation effect while preserving the amplitude, while the upward sweep adds the primary reflections to the solution (the slight time-shift is likely due to numerical dispersion). This difference can be seen at depth 0.2 km in Figures 4a and 4c, and between depth 0.3 and 0.5 km in Figures 4b and 4d. While in some cases it may be beneficial to capture all the multiples, we have observed through numerical experiments that capturing the primary reflections is sufficient for migration purposes, as illustrated in the next section. Using thick subdomains allows reflections within each subdomain, which do not propagate to the upper subdomains, leading to a discontinuous wavefield as seen in Figure 3c. While this is not of concern in our application to migration, to confirm this phenomenon, we repeat the numerical example using one-element-thick subdomains to reduce this effect; as seen in Figure 3e, in comparison to Figure 3c, the resulting wavefield appears more continuous, containing only downward propagating modes.

## MIGRATION EXAMPLES

The approximate solver is implemented using an Intel C++ Compiler with Ox optimization. The lightweight EIGEN library (Guennebaud, 2010) enhanced by the Intel Math Kernel Library (Intel® MKL, 2018) is used to perform linear algebra operations. For the reference solutions, all forward simulations at a given BFGS iteration of the least-squares migration process utilizes the GMRES solver, where the proposed double-sweeping solver is used as a preconditioner (Eslaminia and Guddati 2016). The relative tolerance for the GMRES convergence is set to 0.001. For the proposed migration framework, we utilize the double-sweeping solver as the main forward modeling step to speed up the gradient computation and the cubic line-search step, as presented in Algorithm 2. The LU factorization of the subdomain matrices (Initialization Step in Algorithm 1) is performed using the multi-core PARDISO solver (Intel® PARDISO). The computation associated with multiple frequencies is distributed over multiple processors using the Intel Message Passing Interface (MPI) library. All numerical simulations have been carried on North Carolina State University "Henry 2" cluster. For these numerical experiments, we have considered the following three scenarios:

1. Full-wave LSM: It represents the reference solution where the background wavefield $\mathbf{u}_0$ , the application of both the forward operator $\mathbf{L}$, and the adjoint operator $\mathbf{L}^\dagger$ are all computed using the full-wave equation.

2. Proposed LSM: The LSM based on the proposed double-sweep method, where the background wavefield $\mathbf{u}_0$, the application of both the forward operator $\mathbf{L}$, and the adjoint operator $\mathbf{L}^\dagger$are all computed using the proposed double-sweep solver.

3. One-way LSM: The LSM based on a one-way solver, where (a) the application of the forward operator $\mathbf{L}$ is computed using a single upward sweep with $\mathbf{m}_s^k$ acting as a body force to bring primary reflections to the receivers, (b) the application of the adjoint operator $\mathbf{L}^\dagger$ is computed using a single downward sweep, while (c) the background wavefield $\mathbf{u}_0$ is computed using the proposed double-sweep solver to prevent any interference of the background velocity on the residual calculation.

**Migration of a 30° reflector**

The first migration example involves a three-layer structure with a 30°-angle reflector, as shown in Figure 5a. The domain is 4 km × 2 km, and the data acquisition is carried out using 61 shots and 124 receivers distributed uniformly at the top of the domain boundary. The domain is extended on both sides by 0.5 km to have a wider aperture. A 0.4 km buffer layer is added on the top boundary to remove first arrivals. The velocity model is defined on a 250 × 120 grid resulting in 30,000 parameters. The background velocity model is generated by smoothing the true velocity model with a Gaussian filter with a window size of 10 pixels (see Figure 5b). The true reflectivity shown in Figure 5c is computed as the difference between the true and background velocity model. The domain is discretized using 500 × 240 bilinear dispersion-reduced finite elements (Guddati and Yue, 2004). The sources are Ricker pulses with a central frequency of 15 Hz, and the synthetic data are obtained for frequencies ranging from 1 to 20 Hz with an increment of 0.25 Hz. The migration is performed progressively for the same frequency range in four groups: 1-5 Hz, 6-10 Hz, 11-15 Hz, and 16-20 Hz. We limit the maximum number of the BFGS iterations per frequency group to 50. To improve the inversion accuracy (especially for the lower frequency range), we utilize an adaptive decomposition approach for dividing the computational domain. Specifically, we use three subdomains for 1-5 Hz group, five subdomains for 6-10 Hz group, and ten subdomains for 11-15 Hz and 16-20 Hz groups. The adaptive decomposition approach might also benefit a multiscale inversion scheme by reducing the computational cost at higher frequency ranges using a larger number of subdomains (otherwise, the subdomain mesh size increases with frequency, making the forward solves increasingly expensive). The DtN map is approximated using five PMDL layers with a constant length of $2c/\omega$, where $c$ is the velocity of the adjacent finite element in the interior (physical subdomain).

The migrated images for all three scenarios and their convergence behaviors at different frequency groups are shown in Figures 6 and 7, respectively. These results illustrate that the proposed double-sweeping result is as accurate as the reference images without altering the convergence behavior for the entire frequency range. In the case of one-way LSM, although the single sweep approximation of $\mathbf{L}$ and $\mathbf{L}^{\dagger}$ has minimal effects on the convergence behavior in the lower frequency range, it alters the convergence behavior in higher frequency ranges as illustrated in Figure 7. We hypothesize that the reason for this is that the exclusion of the upcoming wavefield removes the contribution of the multiples from the residual – especially in the presence of sharp velocity variation in this numerical experiment.

This numerical example shows that including the upward propagating wavefield improves convergence behavior for velocity models with large velocity contrasts (for which the full-wave equation is usually needed) with relatively small increase in computational cost.

At the end, the CPU time for the proposed double-sweeping method is about 14 hours of CPU time per processor, while the full-wave based LSM requires about 42 hours, representing a 67% reduction in the computational cost.

**Migration of an inclusion**

The second example involves the migration of inclusion with large velocity contrast, embedded in a two-layered system as shown in Figure 8a. The migration velocity model is generated by smoothing the true velocity model around the inclusion with a Gaussian filter with a window size of 10 pixels shown in Figure 8b. The true reflectivity is shown in Figure 8c and the migration setup is the same as in the previous experiment. The migrated images in Figures 8d, 8e, 8f respectively present the migrated images for the full-wave LSM, the proposed method, and LSM with one-way. Similar to the previous example, the double-sweep LSM is as accurate as the reference image from full-wave LSM. On the other hand, the one-way LSM results in higher level of artifacts (see Figure 8f). This example illustrates that, whenever multiples are significant, the proposed LSM would be a better alternative to the one-way LSM while being computationally more efficient than the Full-wave LSM.

**Migration of the Marmousi model**

The third example involves the migration of the Marmousi model shown in Figure 9a. The domain is 5.0 km × 2.5 km, and the data acquisition is carried out using 61 shots and 124 receivers distributed uniformly at the top of the domain boundary. To have a wider aperture, the domain is extended on both sides by 0.625 km. Like the previous example, a 0.5 km buffer layer is added to the top boundary to remove the first arrivals. The final domain dimensions are 6.25 km × 3.0 km. The velocity model is defined on a 250 × 150 grid (37,500 parameters). The background velocity model is generated by smoothing the true velocity model with a Gaussian filter with a window size of 15 pixels shown in Figure 9b. The true reflectivity is the difference between the true and the background velocity models shown in Figure 9c. The domain is discretized using 500 × 300 bilinear dispersion-reduced finite elements. The synthetic data are obtained for the frequency range of 1 to 20 Hz with an increment of 0.25 Hz. The sources are Ricker pules with the central frequency of 15 Hz. Similar to the previous example, the migration is performed in four frequency groups: 1-5 Hz, 6-10 Hz, 11-15 Hz, and 16-20 Hz. We limit the maximum number of BFGS iterations per frequency group to 50. To improve the inversion accuracy (especially for the lower frequency range), we utilize an adaptive decomposition approach with three subdomains for 1-5 Hz group, five subdomains for 6-10 Hz group, and ten subdomains for 11-15 Hz and 16-20 Hz groups.

The migrated images and the convergence behaviors at different frequency groups are shown in Figures 10 and 11, respectively. Full-wave LSM takes about 81 hours CPU time per processor, the proposed method requires only about 15 hours, leading to 80% reduction in computational cost. As illustrated, both double-sweep and one-way LSM result in accurate solutions, close to the reference image, without altering the convergence behavior. We hypothesize that excluding the upcoming wavefield does not significantly impact the residual, and the effect of the multiples is negligible in the specific case of the Marmousi model. Accordingly, the proposed one-way LSM could be utilized in similar cases. However, if the upcoming wavefields are required for accurate imaging, e.g. the scenario in Figure 8, the proposed double-sweep LSM is an efficient alternative to the full-wave LSM.

## CONCLUSION

We develop a new least-squares migration algorithm in the frequency domain based on an approximate sequential solver for the wave equation. The basic idea of the approximate solver is to split the domain into smaller horizontal subdomains and solve separately for downgoing and upcoming wavefields. Such a separation and sequential solutions are made possible by approximating the interface conditions between the subdomains by neglecting the multiples while preserving the amplitudes of primary reflections and transmissions. The downward-propagating wavefield is first computed by solving subdomains from top to bottom, followed by the solution of primary reflections and other upward propagating waves from bottom to top. The resulting (approximate) double-sweeping solver is utilized as the primary solver into the least-squares migration framework. As illustrated by 2D numerical examples, the resulting least-squares migration algorithm is shown to be as accurate as the full-wave equation-based migration. The proposed method also converges with almost the same behavior as the full-wave equation approach while only requiring a fraction of computational effort.

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## FIGURES' CAPTION

Figure 1. Schematic representation of the proposed method: a) subdomains with color-coded material properties, b) downward sweep sub problem where the top half space matches with the top of the layer while bottom half-space matches with the top of the layer below, and c) the upward sweep where the bottom half-space matches with the bottom of the layer, while the top half-space matches with the bottom of the layer above.

Figure 2. Schematic illustrating the calculation of the incident wave. (a): The reflection $u_{ref}$ is the difference between the incoming wave $u_{d,b}^{j}$ from the upper subdomain and transmitted wavefield $u_{d,t}^{j+1}$ during downward sweep, (b): The incident wave for the upward sweep is the reflection $u_{ref}$ plus incoming wave $u_{u,t}^{j+1}$ from the lower subdomain.

Figure 3. (a) Velocity model. Comparison of the modeled wavefields at t = 0.6 sec. (b) full-wave simulation, (c) downward sweeping with 10-element thick subdomains, (d) double sweeping with 10-element thick subdomains, (e) downward sweeping with one-element thick subdomains, and (f) double sweeping with one-element thick subdomains. Star represents source and triangle represents trace location. All wavefields are plotted with the same color scale and clipping values.

Figure 4. Traces of the wavefield, for (a) and (b) thick slabs and (c) and (d) thin slabs.

Figure 5. (a) The true velocity model of the 30° reflector. (b) background velocity model, and (c) true reflectivity.

Figure 6. The migrated images of the 30° reflector at different frequency groups using (a) and (b) full-wave LSM, (c) and (d) proposed migration method, (e) and (f) one-way.

Figure 7. Comparison of convergence behavior the full-wave LSM, the proposed method, and one-way for the 30° reflector at different frequency groups.

Figure 8. (a) The true velocity model, (b) background velocity model, (c) true reflectivity, and the migrated images using (d) full-wave LSM, (e) proposed method, and (f) one-way.

Figure 9. (a) The true velocity model of the Marmousi model. (b) background velocity model, and (c) true reflectivity.

Figure 10. The migrated images of Marmousi model at different frequency groups using (a) and (b) full-wave LSM, (c) and (d) proposed migration method, (e) and (f) one-way.

Figure 11. Comparison of convergence behavior the full-wave LSM, the proposed method, and one-way for the Marmousi model at different frequency groups.