The Impact of a Coalition: Assessing the Likelihood of Voter Influence in Large Elections

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For centuries, it has been widely believed that the influence of a small coalition of voters is negligible in a large election. Consequently, there is a large body of literature on characterizing the likelihood for an election to be influenced when the votes follow certain distributions, especially the likelihood of being manipulable by a single voter under the i.i.d. uniform distribution, known as the Impartial Culture (IC).

In this paper, we extend previous studies in three aspects: (1) we propose a more general semi-random model that combines the semi-random model for social choice [Xia, 2021] and the contamination model [Diakonikolas and Kane, 2021, Huber, 1964], where a distribution adversary chooses a worst-case distribution and then a contamination adversary modifies up to ψ portion of the data, (2) we consider many coalitional influence problems, including coalitional manipulation, margin of victory, and various vote controls and bribery, and (3) we consider arbitrary and variable coalition size B. Our main theorem provides asymptotically tight bounds on the semi-random likelihood of the existence of a size-B coalition that can successfully influence the election under a wide range of voting rules. Applications of the main theorem and its proof techniques resolve long-standing open questions about the likelihood of coalitional manipulability under IC, by showing that the likelihood is $\Theta\left(\min\left\{\frac{B}{\sqrt{n}}, 1\right\}\right)$ for many commonly-studied voting rules. We also propose a coalitional influence problem called *coalitional manipulation for the loser*, and prove that its semi-random likelihood for m alternatives is $\Theta\left(\min\left\{\frac{B}{\sqrt{n}}, 1\right\}^{m-1}\right)$ under a large class of models, showing that the likelihood can be non-linear in B, the degree of polynomial can depend on m, and each additional budget is marginally more powerful.

The main technical contribution is a characterization of the semi-random likelihood for a Poisson multinomial variable (PMV) to be unstable, which we believe to be a general and useful technique with independent interest.

The full version of this paper can be found at https://arxiv.org/abs/2202.06411

CCS Concepts: • **Theory of computation** → **Algorithmic game theory**; • **Computing methodologies** → **Multi-agent systems**.

ACM Reference Format:
Lirong Xia. 2023. The Impact of a Coalition: Assessing the Likelihood of Voter Influence in Large Elections. In *Proceedings of the 24th ACM Conference on Economics and Computation (EC '23), July 9–12, 2023, London, United Kingdom.* ACM, New York, NY, USA, 1 page. https://doi.org/10.1145/3580507.3597731

EC '23, July 9–12, 2023, London, United Kingdom

ACM ISBN 979-8-4007-0104-7/23/07.
https://doi.org/10.1145/3580507.3597731