Reliability Assessment of Pile-Founded T-Walls using Kriging Method

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ABSTRACT

The pile-founded T-walls play a vital role in flood protection. However, most of the current studies on the stability of pile-founded T-walls are performed using a deterministic approach. However, many uncertainties exist in the soil properties and neglecting these uncertainties might overestimate the safety of the pile-founded T-walls. This paper presents a comprehensive study on the stability assessment of pile-founded T-wall systems using both deterministic analysis and probabilistic analysis. In the deterministic analysis, a finite difference model is first developed to evaluate the stability of the flood protection system following the traditional procedures, in which the factors of safety under different flooding levels are obtained. In probabilistic analysis, the uncertainties in geotechnical parameters are explicitly considered. In the probabilistic assessment, an MCS-based kriging surrogate model is formulated to predict the stability of the pile-founded T-wall under different flooding hazard levels. The kriging-based surrogate model can approximate the finite difference solutions with its input parameters, which improves the computational efficiency of the probabilistic assessment. A case study of the pile-founded T-wall design resting on layered foundation soils is used to demonstrate the effectiveness of the proposed framework. The results can lead to a critical evaluation of the probability of failure in addition to the factor of safety under different flood water levels, which can provide information for risk-informed decision-making.

INTRODUCTION

The pile-founded walls (e.g., I-walls or T-walls) are often built in the New Orleans area on the top of existing levees to meet the design requirements of the flood protection system and protect low-lying areas from flooding due to hurricane hazards. However, since the floodwall protection systems with I-walls in New Orleans were ruined by Hurricane Katrina while that with T-walls survived in several places, it has attracted much attention to the research on the re-evaluation of safety and failure mechanism of pile-founded T-wall or I-wall systems. The insufficiency of traditional design (i.e., two-dimensional limit equilibrium analysis) for the pile-founded wall was systematically evaluated by Won et al. (2011) using numerical analyses. In traditional design, the complicated pile-soil interactions at deep ground layers could not be fully considered. The unreasonable assumption of the load distribution on the battered pile was the major factor causing the instability of T-wall systems in practice. Hu et al. (2013) found that the response of I-wall systems showed a three-dimensional effect and two-dimensional analysis did not predict the displacement of I-wall systems accurately. Adhikari et al. (2014) pointed out that gap development and soil strength reduction were the two main reasons for the failure of pile-founded I-wall systems in the New Orleans area.

Although great achievements have been reached in the flood protection system due to the contribution of previous work, most previous studies are based on deterministic analysis (e.g., Won et al. 2011; Johnson et al. 2017; Kokkali et al. 2018). Since only a limited number of site investigation tests are available, the uncertainties of soil properties cannot be avoided (Yang et al. 2019). Therefore, probabilistic studies are usually conducted to investigate the effect of these uncertainties on the geotechnical system response. However, small-probability events are not rare in the geotechnical designs (e.g., the safety assessment of the pile-founded T-walls). Traditional methods (e.g., the Monte Carlo simulations) might not be applicable in this case. To reduce the computational demands, some surrogate models have been proposed in previous studies such as the kriging-based method (Zhang et al 2011; Liu and Cheng 2018), response surface methods (Li et al. 2016), deep learning methods (Wang et al. 2022), etc. The kriging-based method is easy-to-implement and much experience has been accumulated in the application of this method. The kriging-based method will be adopted for the probabilistic analysis in this paper. This paper aims to evaluate the different levels of flooding hazards (in terms of different flood water elevations) on the stability of the pile-founded T-wall system using both deterministic and probabilistic approaches. The factor of safety is obtained by the strength reduction method (Dawson et al. 1999) to assess the stability of the pile-founded T-wall system in the deterministic analysis. In probabilistic analysis, the MCS-based kriging method (Zhang et al. 2011; Liu and Cheng 2018) is adopted for the derivation of the probability of failure.

METHODOLOGIES ADOPTED FOR THE STABILITY OF THE PILE-FOUNDED T-WALL SYSTEM

The stability of the pile-founded T-wall can be evaluated by either a limit equilibrium approach or a numerical approach (Rajabalinejad et al. 2010; USACE 2012). In this study, the 3-D explicit finite difference program FLAC3D version 7.0 (Itasca 2022) is selected as the solution model, where the built-in strength reduction method is implemented. The strength of soil properties (i.e., the cohesion c and the friction angle φ) is progressively reduced (or increased) to bring the pile-founded T-wall system to a state of limiting equilibrium. The factor of safety of the reduced strength of soil properties is determined by the equations as follows:

$$c_{ri} = \frac{c}{FS_i} \tag{1a}$$

$$\tan\varphi_{ri} = \frac{\tan\varphi}{FS_i} \tag{1b}$$

where c_{ri} and φ_i represent the reduced cohesion and friction angle, respectively.

For the probabilistic analysis to evaluate the failure probability of flooding walls, the direct Monte-Carlo simulations (MCS) can be computationally challenging for low-probability scenarios (Jiang and Huang 2016). To improve the computational efficiency, the Monte-Carlo simulation-based kriging method is employed to address the issue through a kriging-based surrogate model (Zhang et al 2011; Liu and Cheng 2018). With the adopted method, a sufficient number N_{ini} of Monto-Carlo simulations are performed to obtain the equal number of *FS* of the pile-founded T-wall system first. Then these Monte-Carlo simulation results are taken as the initial samples for establishing the kriging model. Based on the constructed kriging model, the *FS* can be derived for a large number of new soil property samples. The probability of failure can be calculated as:

$$P_f \approx \frac{1}{N}\sum_{i=1}^{N} I[FS(X_i) \le FS_T] \tag{2}$$

where N is the total number of newly generated soil property samples; X_i is the i^{th} sample vector of input random soil parameters (e.g., c_i and φ_i); $FS(X_i)$ is the factor of safety with the i^{th} sample; FS_T is the target factor of safety, which is determined by the practical design; I [·] is an indicator function equal to the unit if the calculated *FS* is less than demanded FS_T and equal to zero otherwise.

The kriging method assumes that the performance function $G(X)$ can be estimated as:

$$G(X) = f(X)^T\beta + z(X) \tag{3}$$

where $f(X) = [f_1(\mathrm{X}), f_2(\mathrm{X}), \ldots, f_p(\mathrm{X})]^T$ is a vector of p basis function; $\beta = [\beta_1, \beta_2, \ldots, \beta_p]^T$ is a vector of regression of coefficients; $z(X)$ is the fluctuation of the regression part $f(X)^T\beta$, which is generally assumed as a Gaussian process with the mean of zero and the variance of σ^2. Suppose the initial input soil property samples $X = [X_1, X_2, \ldots, X_{N\mathrm{ini}}]$ and the calculated factor of safety $Y = [FS_1, FS_2, \ldots, FS_{N\mathrm{ini}}]$. The unknown coefficients β and σ^2 can be calibrated by the maximum likelihood estimation as:

$$\beta = (F^T R^{-1} F)^{-1} F^T R^{-1} Y \tag{4}$$

$$\sigma^2 = \frac{1}{N_{ini}}(Y - F\beta)^T R^{-1}(Y - F\beta) \tag{5}$$

where F is a vector of $f(X)$; R is the correlation matrix, which is calculated as:

$$R = \begin{pmatrix} R(X_1, X_1) & \cdots & R(X_1, X_{N_{ini}}) \\ \vdots & \ddots & \vdots \\ R(X_{N_{ini}}, X_1) & \cdots & R(X_{N_{ini}}, X_{N_{ini}}) \end{pmatrix} \tag{6}$$

where $R(X_i, X_j)$ is the spatial correlation between the sample X_i and X_j, and is generally computed using the Gaussian correlation function (Liu and Cheng 2018) as:

$$R(X_i, X_j) = \exp(-\sum_{l=1}^{n} \theta_l \,|\, X_{il} - X_{jl} \,|^2) \tag{7}$$

where θ_l is the correlation parameter, which is determined by minimizing the following expression:

$$\Psi(\theta_l) = \frac{1}{2}(N_{ini} \ln \hat{\sigma}^2 + \ln |R|) \tag{8}$$

DETERMINISTIC ANALYSIS FOR THE STABILITY ASSESSMENT OF THE PILE-FOUNDED T-WALL SYSTEM

A modified case from Won et al. (2011) is adopted to evaluate the stability of the pile-founded T-wall system in the face of flooding hazards. As shown in Figure 1, the pile-founded T-wall system is composed of a concrete T-wall, three rows of batter H-piles (HP 14×73), a row of sheet piles (PZ22), and an earth levee. The T-wall with a height of 5.15 m and a base width of 4.1 m is located at the elevation of EL. +3.65 m. Three rows of the batter H-piles are inclined with the same ratio of 1H:3V to the vertical plane. The spacing between batter piles is 1.5 m in the longitudinal direction (out of plane), which is also the longitudinal length of the numerical model adopted in this study. The sheet pile is embedded between the first row and the second row of H-piles to control the effect of the seepage and reduce the soil extrusion between the two rows of H-piles. The earth levee has a height of 1m and a slope of 3H:1V.

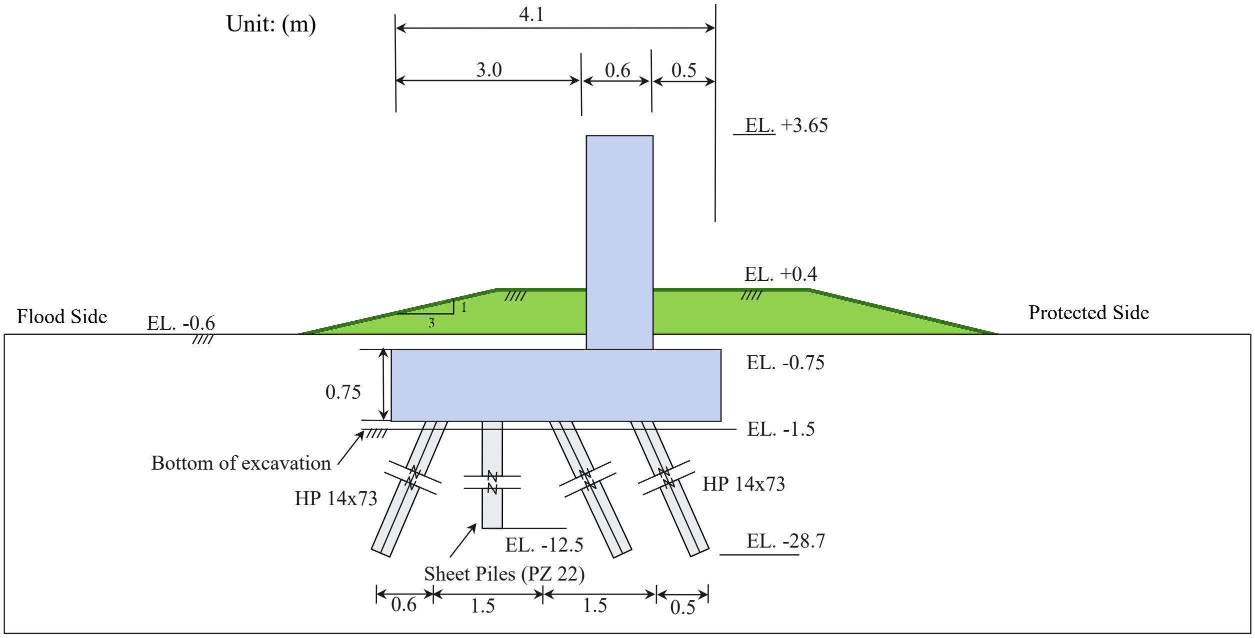

Figure 1. Illustration of Pile-Founded T-Wall System (after Won et al. 2011).

Figure 2 shows the mesh size of the numerical model. As shown in Figure 2, four foundation layers (i.e., a high compressible peat layer, a soft clay layer, a medium dense sand layer, and a hard clay layer) are located at the bottom of the levee fill. In the numerical simulations, the Mohr-Coulomb model is used to model the behavior of all soil layers. The elevation depth and soil properties of each soil layer are tabulated in Table 1. The T-wall is modeled by the soil brick elements with elastic behavior. The interface elements are applied to model the interactions between the soil and T-wall. The H-piles are simulated by the pile elements while the sheet pile is modeled by embedded linear elements with coupling springs to represent the interface between the soil and the pile (Won et al. 2011). The structural properties and the interface properties for the pile element can be referred to Won et al. (2011). The horizontal displacement of the four side boundary planes of the model is restricted in the normal direction, and the displacement at the base of the model is fixed.

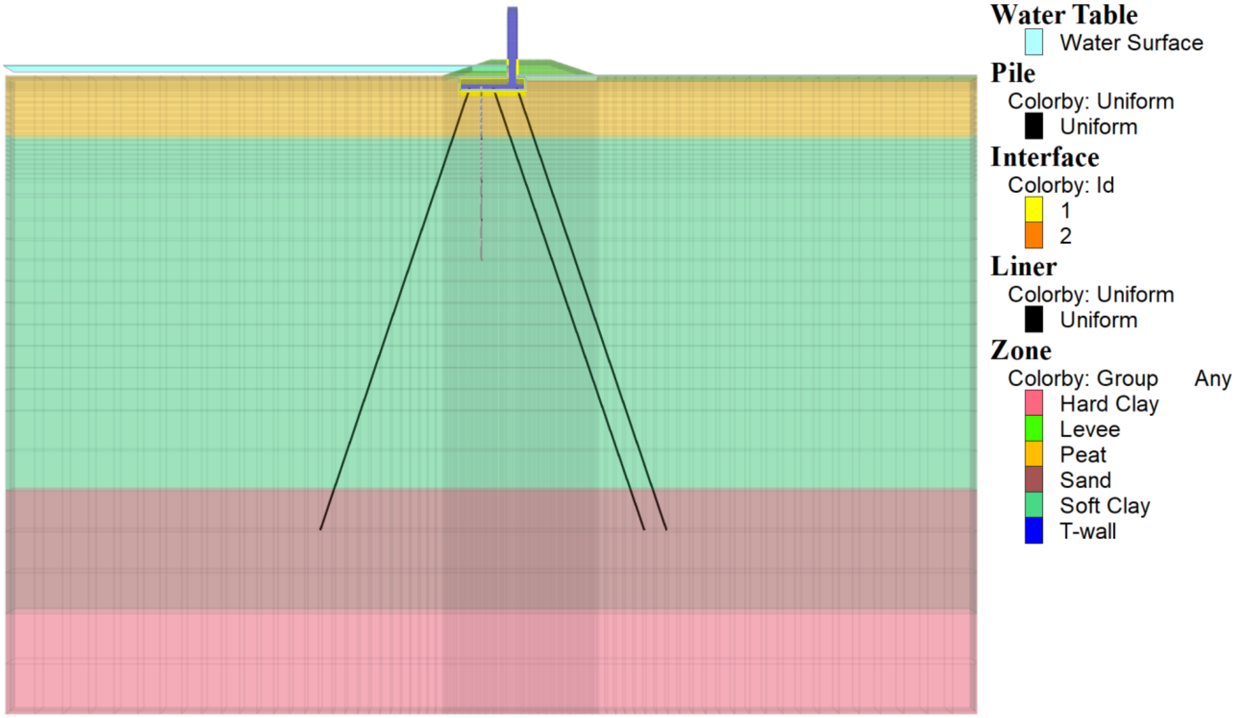

Figure 2. Mesh of Finite Difference Model for Pile-Founded T-wall System (after Won et al. 2011).

Table 1. Soil Properties for the Model (after Won et al. 2011)

Layer	Levee fill	Peat	Soft Clay	Sand	Hard Clay
Top elevation (m)	0.4	-0.6	-4.3	-26.2	-33.8
s_u (kPa)	23.9	5.7	7.2	-	46.9
φ (°)	-	-	-	30	-
γ (kN/m^3)	17.3	12.6	15.7	18.1	18.1
G (kPa)	3585	575	934	7804	7517
K (kPa)	8.86e4	1.42e4	2.30e4	1.69e4	1.85e4

To investigate the stability of the pile-founded T-wall system with different flood water elevations, initial stress and strain for the numerical model without any support of the geotechnical structures (i.e., the T-wall, H-piles, and sheet pile) are first computed. Then these structures are built to update the stress and strain before the pore water pressure is applied to the flood protection system. The initial water table elevation is set as EL. -0.6 m on protect side and the initial flood water elevation is set as EL. +0 m in the flood side. Finally, the pore pressure is updated with the flood water elevation in an increment of 0.3 m from EL +0 m to EL +3.0 m on the flood side (i.e., the left side of the model).

The factor of safety with different flood water elevations is plotted in Figure 3. With the increase in flood water level, the stability of the pile-founded T-wall system is decreased rapidly. The minimum factor of safety occurs at the highest flood water elevation (i.e., EL. +3.0m). The decrease in the factor of safety is mainly owing to the increase of the pore pressure and the load triggered by the water weight. The pore water pressure reduces the effective stress of the soils while the load normally distributed on the T-wall and left earth levee leads to the slide and rotation of the whole flood protection system. If the target factor of safety for the pile-founded T-wall is considered to be 1.5 for the highest flood elevation (EL. +3.0 m), the evaluated FS of 2.49 indicates that the pile-founded T-wall system is sufficiently safe from the deterministic view.

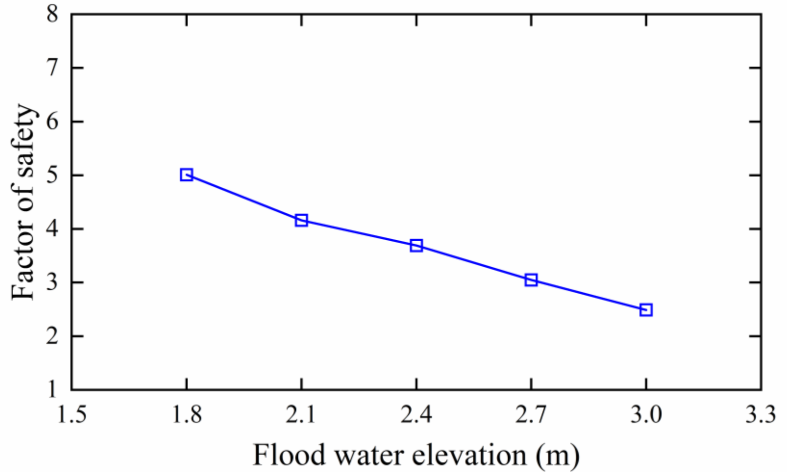

Figure 3. The Effect of the Flood Water Elevation on the Factor of Safety.

PROBABILISTIC ANALYSIS FOR THE STABILITY ASSESSMENT OF THE PILE-FOUNDED T-WALL SYSTEM

It is well known that significant uncertainties are existing in the soil properties. To address these uncertainties, the undrained shear strength (s_u) of clay layers and the friction angle (φ) of the sand layer are treated as lognormal random variables. The mean of the soil properties is equal to the values in the deterministic analysis. The coefficient of variation (COV) of s_u for the clay layers is assumed as 0.3 while that of φ for the sand layer is set as 0.2 (Phoon and Kulhawy 1999). The designed factor of safety FS_T is 1.5. In the probabilistic analysis, the MCS-based kriging method is adopted to calculate the probability of failure. To build the kriging model, 200 Monte-Carlo simulations, which are sufficiently large according to the previous studies (Zhang et al. 2011; Liu and Cheng 2018), are first performed to generate the initial samples. 80% of initial samples are taken as the training samples while the remaining 20% are employed as the testing samples. The accuracy of the constructed kriging model is validated by the root-mean-square error (RMSE), which is described as:

$$RMSE = \sqrt{\frac{1}{m_t}\sum_{i=1}^{m_t}(y_i - \hat{y}_i)^2} \tag{9}$$

where m_t is the number of testing samples; y_i and $\hat{y}_i$ are the actual and predicted value of the factor of safety of the i^{th} sample, respectively.

The mentioned kriging method is built in the MATLAB toolbox DACE developed by Lophaven et al. (2002). In this case, the toolbox can be easily implemented to construct the kriging model in this study. Figure 4 shows a strong linear correlation with a Pearson correlation coefficient of 0.9917 between the *FS* directly calculated from numerical simulations and the predicted *FS* from the kriging model for the case where the flood water elevation is EL +3.0 m. The RMSE for this comparison results is 0.0677, which implies a high accuracy of the built kriging model. Based on the kriging model, another 100,000 new input samples are generated and the corresponding factor of safety can be easily obtained.

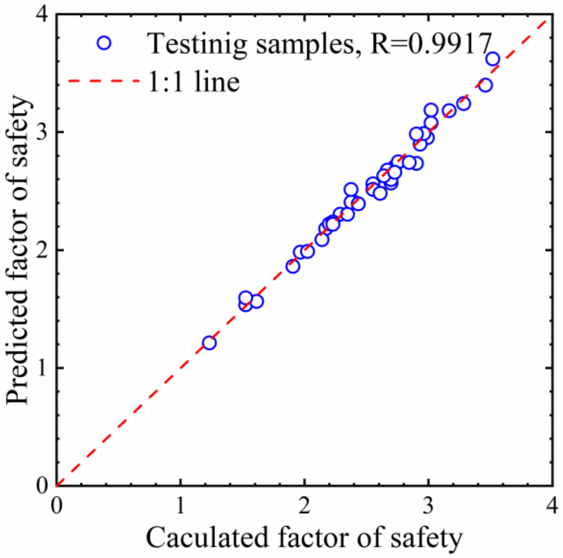

Figure 4. Validation of the Kriging Model for Predicting Factor of Safety of the Pile-Founded T-Wall System.

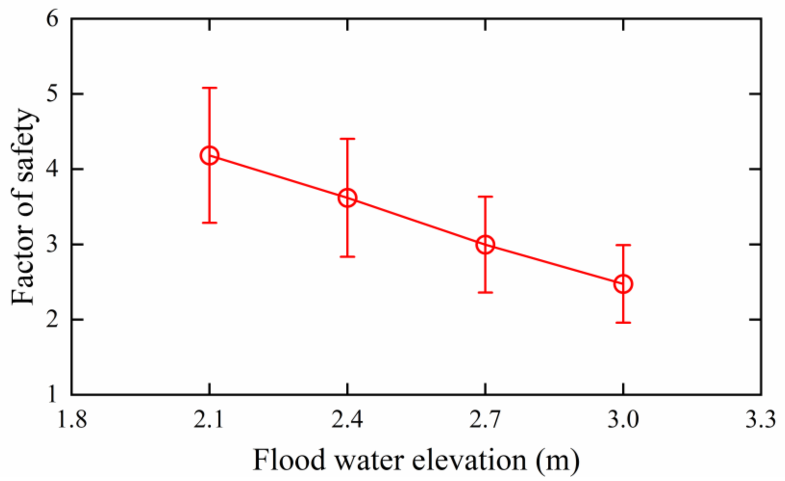

Figure 5. The Variation of the Factor of Safety with the Increase of the Flood Water Elevation.

Figure 5 shows the mean of the *FS* with a range of standard deviation versus the flood water elevation. It seems that similar trends of the mean of the *FS* are obtained when compared to the results of the deterministic analysis. The variation of the *FS* also decreases with the flood water elevation. The results of the probability of failure (i.e., the probability of the calculated factor of safety less than the target factor of safety FS_T = 1.5) with different flood water elevation is illustrated in Figure 6. The probability of failure increases with the flood water level. The pile-founded T-wall system with P_f of 2.72×10^{-7} at the flood water level of EL +2.1 m indicates that there is a negligible probability that the floodwall will not meet the targeted factor of safety requirement. If the flood water elevation is increased to the highest elevation (EL +3.0 m), which results in a P_f of 0.010, the results indicate the pile-founded T-wall system still has a low probability (around 1%) of unsatisfactory performance in meeting the targeted factor of safety required for the stability even at the highest flood water level. Compared to the results from the deterministic analysis, the effect of the uncertainties is considered in the probabilistic analysis and is reflected by a probability of failure. Based on the derived probability of failure and the corresponding failure consequences, the risk assessment can be performed for risk-informed decision-making. Low failure consequences of the pile-founded T-walls might not make much difference between the deterministic analysis and probabilistic analysis, since the probability of failure is generally increased with the factor of safety. However, high failure consequences of the pile-founded T-walls will lead to an over-estimate of the risk assessment of this infrastructure with a deterministic view, i.e., a high factor of safety obtained from the deterministic analysis does not always guarantee a higher level of safety (Duncan 2000; Nadim et al. 2015).

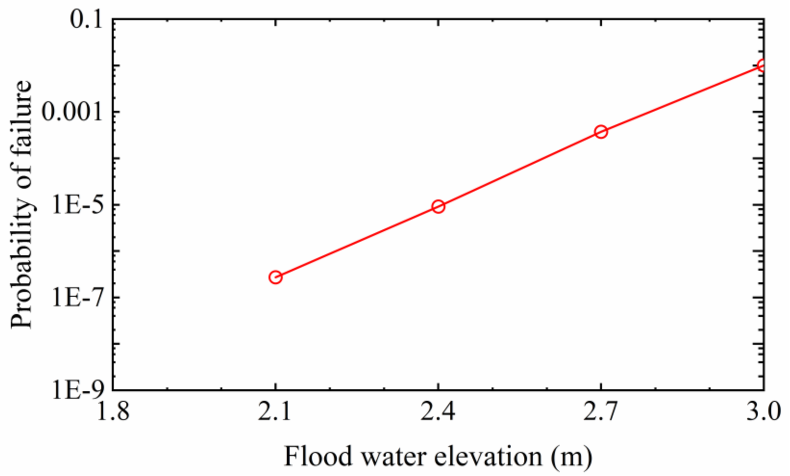

Figure 6. The Probability of Failure of the Pile-Founded T-Wall System versus the Flood Water Elevation (FS_T = 1.5).

CONCLUSION

This paper presents a comprehensive study on the stability of the pile-founded T-wall system under different flood water levels. In the deterministic analysis, the *FS* is decreased with the flood water elevation, indicating that the worst scenario occurs in the highest water level of the flood. In probabilistic analysis, a kriging-based surrogate model is used to consider the uncertainty of the soil strength properties in the probabilistic characterization of *FS*. The results show that both the mean and variation of *FS* decrease with the flood water elevation. The probability of failure increases with the flood water elevation when the target FS_T is taken as 1.5. The results of this study can provide insights for making risk-informed decisions in the practical design and management of pile-founded T-walls under flooding hazards.

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