Probabilistic Assessment of Earthen Levees Considering Soil Spatial Variability

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ABSTRACT

Earthen levees are important earthen structures built along waterways for flood protection. Many of the levees have lived out their design life. The reliability of these levees under flooding hazard conditions is critical since the failure can be catastrophic and cause serious damage to the community. The paper aims to present a probabilistic scheme to evaluate the reliability of earthen levees considering the soil spatial variability using the subdomain sampling and finite difference methods. The effect of spatial variability of soil parameters will be explicitly incorporated based on a discretization of the finite difference model and the random field theory. The samples will be generated in a set of subdomains partitioned in the space of uncertain input variables to improve computational efficiency. A case study is utilized to demonstrate the effectiveness of the proposed framework for probabilistic assessment of earthen levees. The effects of different scales of fluctuation and flood water elevations on the reliability of earthen levees are also studied. The results from this paper can provide useful insights for practitioners to evaluate the reliability of the earthen levees and make a risk-informed engineering decision.

INTRODUCTION

Earthen levees as one type of critical civil engineering infrastructure to resist flood hazards have been widely used in the United States. To improve the performance of this flood protection system, a variety of studies have been carried out to investigate the potential problems in the design life of the earthen levees using numerical, experimental, and field monitoring techniques. Zeghal et al. (2013) formulated a cost-effective sensor-based model-aided health assessment system that integrates multiscale monitoring to assess the performance of earthen levee infrastructure. Saran and Viswanadham (2018) adopted centrifuge tests to investigate the deformation characteristics and stability of levees subjected to flooding using a large beam centrifuge. Digital image analysis was used to elucidate the mechanism of flood-induced levee failure. Firas et al. (2017, 2020) studied the effects of extreme events (e.g., extreme precipitations, compound flooding) on the probability of failure of earthen levees with numerical simulations and pointed out that extreme events were vital triggering factors of the failure of the earthen levees. Balistrocchi et al. (2019) proposed a copula-based model to describe the critical multiple flood characteristics when overtopping occurred in the earthen levees.

Although many contributions have been made to the levee design, the uncertainties of the design parameters (e.g., the spatial variability of soil properties) are not sufficiently considered in the existing studies. It has been indicated that the inherent spatial variability of soil properties can considerably impact the stability of the slopes, tunnels, and foundations (Cho 2012; Huang et al. 2015; Xiao et al. 2016; Gong et al. 2018; Jiang et al. 2020; Zhao et al. 2021). In this paper, a parametric study is performed to investigate the influence of spatial variability of soil properties on the stability of the earthen levees. The derived results can provide useful insights for practitioners to evaluate the reliability of the earthen levees and make a risk-informed engineering decision.

DETERMINISTIC ANALYSIS OF EARTHEN LEVEE STABILITY

The stability of the earthen levees can be evaluated by the equilibrium method or numerical method (Jadid et al. 2020; Ceccato et al. 2022). In this study, the 3-D explicit finite difference program FLAC3D version 7.0 (Itasca 2022) is adopted for the stability analysis of the earthen levees, where the strength reduction method is built for the stability analysis. With the strength reduction method, the shear strength parameters in terms of cohesion (c) and friction angle (φ) of each soil layer are progressively reduced (or increased) by trial values of FS_i to bring the flood protection system to a state of limiting equilibrium, which can be described as

$$c_{ri} = \frac{c}{FS_i} \tag{1a}$$

$$\tan\varphi_{ri} = \frac{\tan\varphi}{FS_i} \tag{1b}$$

where c_{ri} and φ_i are reduced cohesion and friction angle, respectively. Then the factor of safety (FS) can be obtained to describe the stability of the earthen levees.

PROBABILITIC ANALYSIS OF EARTHEN LEVEE STABILITY

To improve the computational efficiency, the subdomain sampling method (SSM) (Gong et al. 2016) is adopted for the random field simulations in this study. In the SSM, the possible domain of input uncertain variables is discretized into a series of continuous subdomains. Then a distance index (d) is used to partition the domain based on Hasofer–Lind reliability index (Hasofer and Lind 1974), which is formulated as

$$d = \sqrt{[u]^T [R][u]} \tag{2}$$

where $u = [u_1, u_2, u_3, \ldots, u_n]^T$ is a vector of transformed standard normal variables from the original space of the input parameters and n is the number of input uncertain variables X; R is the correlation matrix among the standard normal variables u. The relations between the component u_i of u and component x_i of the X is described as

$$u_i = \Phi^{-1}(F(x_i)) \tag{3}$$

where $F(x_i)$ is the cumulative distribution function (CDF) of uncertain variable x_i; the $\Phi(\cdot)$ is the CDF of the standard normal variables. The domain of input uncertain variables can be segmented into a series of subdomains $[0, d_1)$, $[d_1, d_2)$, $[d_2, d_3)$, etc. With the sampled domain $[0, d_{max})$, d^2 follows the chi-square distribution with n degree of freedom. The possibility of the samples located in and outside this domain is $(1\text{-}\varepsilon)$ and ε, which can be formulated as

$$\chi_n^2(d_{\max}^2) = \varepsilon \tag{4}$$

where $\chi_n^2(\cdot)$ is the chi-square CDF with n degrees of freedom. Then the probability of each subdomain can be calculated as

$$p_{A_i} = \Pr[d_{i-1}^2 \leq [u]^T [R][u] < d_i^2] = \chi_n^2(d_i^2) - \chi_n^2(d_{i-1}^2) = q^i \tag{5}$$

According to previous studies, q = 1/3 is adopted in this study. The detailed procedures of sampling in each subdomain can be referred to Gong et al. (2016). Then the conditional failure probability p_{fi} in the subdomain $[d_{i-1}, d_i)$ is calculated as

$$p_{fi} = \frac{t_{fi}}{t} \tag{6}$$

where t is the total number of samples generated in the subdomain $[d_{i-1}, d_i)$ and t_{fi} is the number of failure samples in this subdomain. The total probability of failure P_f can be obtained by the summation of all the conditional probabilities of failure, which can be calculated as

$$P_f = \sum_{i=1}^{ns} (p_{A_i} \cdot p_{f_i}) \tag{7}$$

where ns is the total number of subdomains.

EXAMPLE APPLICATION

A case study was conducted using the proposed probabilistic method combined with the finite difference modeling code to demonstrate its efficiency in the earthen levee stability analyses. The example levee consists of clay soil as the embankment material and sand soil as the foundation soil underneath the levee. The slope of the levee is 2H: 1V. The height and top width of the design levee are both 8 m. Figure 1 shows the geometry of the earthen levee. The design flood water height is 6 m.

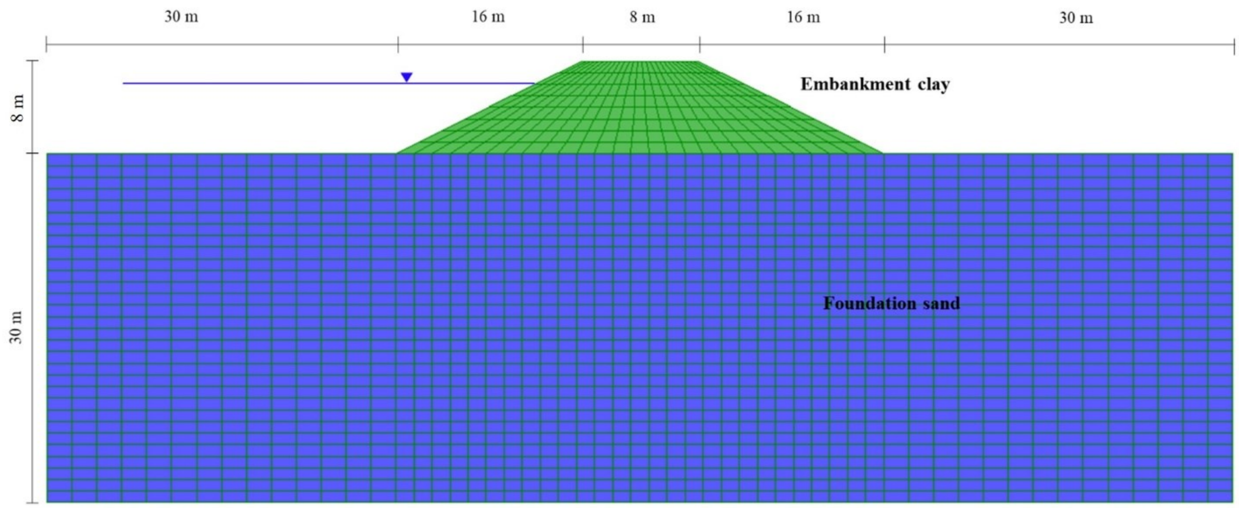

Figure 1. Sketch of the earth levee example used in this study

The strength parameters of geotechnical material are treated as uncertain soil parameters in evaluating the stability of the levee. The mean values and standard deviations of these parameters are obtained from Wolff et al. (2004), which are characterized from an extensive geotechnical survey and investigation. The statistics for the key geotechnical parameters of two soil layers are shown in Table 1.

Table 1. Statistical characterization of soil properties

Layer	Parameter (unit)	Distribution	Mean	Standard deviation	Coefficient of variation	Correlation coefficient
Embankment clay	c (kN/m^2)	Lognormal	41.18	28.73	69.8%	-0.5*
	φ (^{o})	Lognormal	18 ^{o}	4.5 ^{o}	25%	-
	γ (kN/m^3)	-	19.636	-	-	-
Foundation sand	c (kN/m^2)	-	0	0	-	-
	φ (^{o})	Lognormal	32	2	6.25%	-
	γ (kN/m^3)	-	19.636	-	-	-

*Note: Pearson's correlation coefficient between c and φ for the embankment clay.

Using the subdomain method, we generate in total 400 realizations (10 subdomains and 40 realizations per subdomain) of the random field of soil properties. The scale of fluctuation in X and Y directions are set as 10 m and 1 m, respectively. One example realization is shown in Figure ***2***. The spatial range of generating random fields is constrained by the material type. In other words, we generate random fields of soil properties within the embankment clay and foundation sand separately. It is noted that the cohesion is zero for the foundation sand, which does not exhibit any spatial variability within the foundation as shown in Figure ***2***(b).

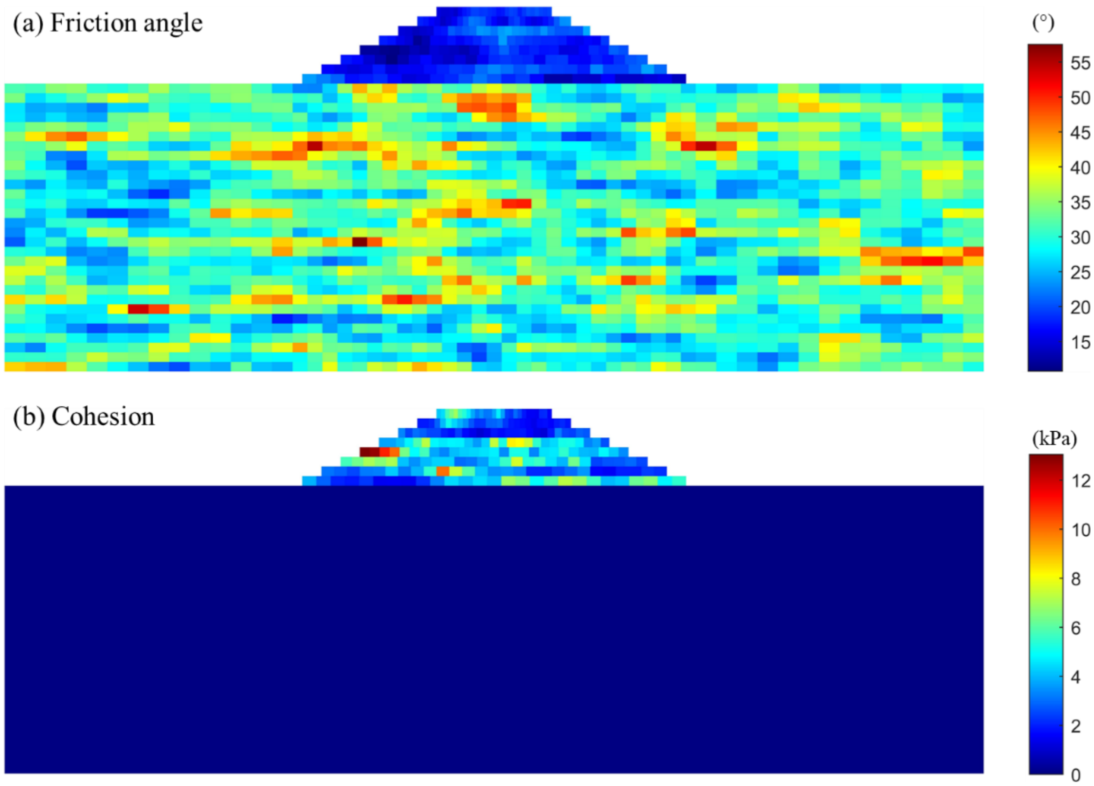

Figure 2. Random field realization of the soil parameters: (a) friction angle, and (b) cohesion.

The factors of safety of all 400 realizations are collected to probabilistically evaluate the safety of the levee. The probability that the calculated factor of safety less than a threshold value of the target factor of safety is defined as the probability of unsatisfactory. We choose 1.4 as the target factor of safety value (USACE 2000). Using the parameters mentioned above, the computed probability of unsatisfactory (performance) is 0.12 and the corresponding histogram of the factor of safety distribution is shown in Figure 3. The scale of fluctuation in Y and X directions of 1 m and 10 m is deemed as the base scenario.

We further evaluate the sensitivity of probability of unsatisfactory to different values of scale of fluctuation (e.g., the values listed in
Figure ***4***). The scale of fluctuation in Y direction (denoted as scaleY) varies from 0.1, 1, and 10 m, and that in the X direction (denoted as scaleY) varies from 5, 10, and 50 m based on the typical range reported in the literature (e.g., Phoon and Kulhawy 1999; Wang et al. 2020). According to
Figure ***4***, we observe that the probability of unsatisfactory increases along the rise of scale factors of fluctuation and there is a clear nonlinear relationship with the increase of the scale of fluctuation. The probability of unsatisfactory shows a rapid rise when the scale of fluctuation increases at small values. For example, the probability of unsatisfactory increases quickly when

the scale of fluctuation in the Y direction varies from 0.1 to 1 m than that from 1 m to 10 m as depicted in
Figure *4*.

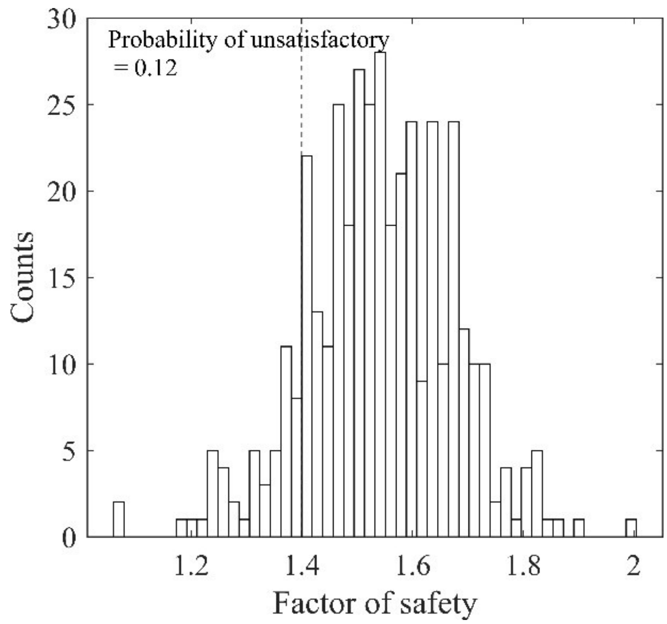

Figure 3. Distribution of factor of safety for the base scenario

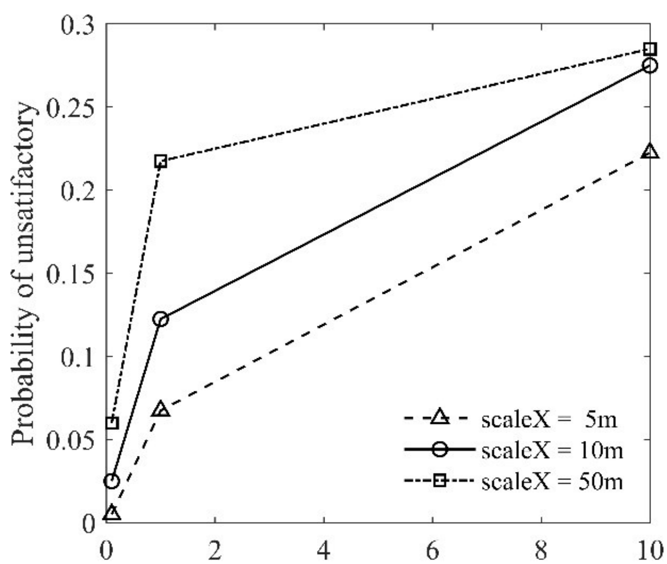

Figure 4. Sensitivity of probability of unsatisfactory to scale of fluctuation in X and Y directions.

We further evaluate the effects of water height on the probability of unsatisfactory. Using scaleX and scaleY of 10 and 1 meters, respectively, we compute the conditional probability of unsatisfactory under water height of 2, 4, 6, and 8 meters, respectively. The computed results are shown in Figure 5. Figure 5 shows that the conditional probability of unsatisfactory exponentially increases with the rise of water height, which shows the strong sensitivity of levee stability to an increase in the flood water level.

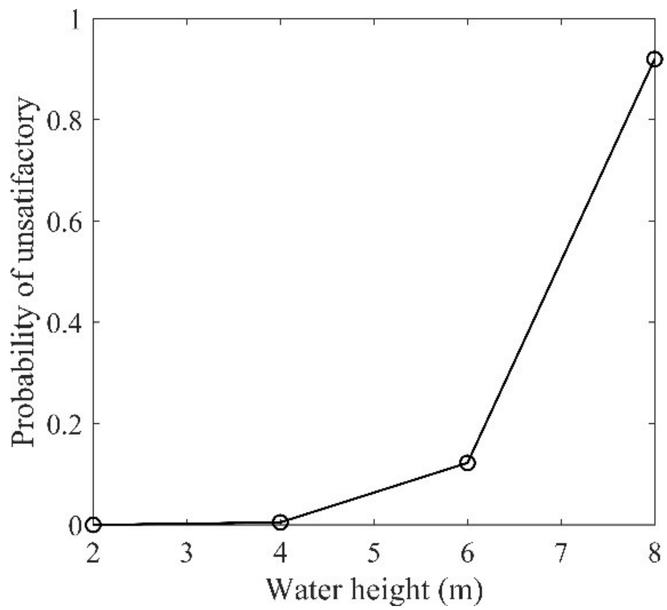

Figure 5. Relationship between probability of unsatisfactory and water heights.

CONCLUSION

This paper presents a probabilistic framework to evaluate the reliability of earthen levees under flooding hazards considering the effects of soil spatial variability. The proposed framework is realized through the subdomain sampling and random field theory combining with the finite difference method. An earthen levee case study is used to illustrate the effectiveness of the proposed framework. Based on the sensitivity studies on the effects of scale of fluctuation, it is found the characterization of scale of fluctuation in both directions has a significant on the reliability assessment. The resulting probability of unsatisfactory has a clear nonlinear relationship with the increase of the scale of fluctuation. Furthermore, the probability of unsatisfactory is very sensitive to the increase in the flood water level. The proposed framework can provide useful references for stakeholders to make a more informed decision for earthen levees in the face of uncertainties.

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