Low-Complexity Minimum Energy Beamforming in Directional Networking

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***Abstract*—We study the problem of finding the minimum energy beamforming that satisfies the users' signal-to-noise ratio requirements. Traditional transform based beamforming uses heuristics to determine the radiation pattern first, and then calculates the beamforming vector. Recent optimization based beamforming optimizes the beamforming vector directly and greatly outperforms the transform based beamforming. However, optimization based beamforming has prohibitively high complexity due to the semidefinite programming to be solved. In this paper, we propose to solve the optimal beamforming problem by the alternating direction method of multipliers (ADMM). We can achieve near optimal performance while greatly reducing the computational complexity. Simulations demonstrate the advantages of our proposed algorithm over transform based beamforming in terms of performance (e.g., 4 dB reduction in transmit power), and over optimization based beamforming in terms of computation time (e.g., 10 times reduction in computation time).**

I. Introduction

Beamforming is an important component of cellular networks. It is especially critical for directional networking in rural areas, where the mobile users are sparsely and asymmetrically distributed across the cell. In these scenarios, it is much more energy efficient to focus the beams towards the users while limiting the radiation to the other directions. In this paper, we study the problem of finding the minimum energy beamforming while satisfying the users' signal-to-noise ratio (SNR) requirements.

Traditional methods for beamforming are based on transforms between the radiation pattern in the angular domain and the beamforming vector [1]. For instance, in the inverse Fourier transform based beamforming [1, Chapter 3], we specify the radiation pattern in the angular domain, and calculate the beamforming vector as the inverse Fourier transform of the radiation pattern. While this approach has low computational complexity, it can result in beamforming that requires high transmit power. This is because the radiation pattern is usually determined heuristically based on the number of users, path losses, etc., without rigorous performance guarantees.

Recent advances in beamforming introduced optimization based approaches [2]–[4]. The optimization based approach works directly on the beamforming vectors, and formulates the beamforming problem as an optimization problem. The resulting optimization problem is usually nonconvex, and is often solved through semidefinite relaxation (SDR) [2]–[4]. While this approach can give optimal performance, the computation complexity is prohibitively high for real-time implementation. The high complexity comes from the technique that "lift" the original optimization variables in the domain of M-dimensional vectors to the domain of M-by-M matrices, where M is the number of antenna elements.

In this paper, we propose a low-complexity minimum energy beamforming algorithm that achieves optimal performance of the optimization based approach while keeping the computation complexity close to that of the transform based approach. To this end, we solve the SDR problem in the optimization approach via the alternating direction method of multipliers (ADMM) [5]. ADMM is a first-order algorithm with much lower complexity than the standard interior-point method [5]. The key contribution of our work is to reformulate the optimization problem in a way that facilitates the use of ADMM. In particular, under our reformulated problem, all the steps in the ADMM algorithm can be computed analytically, which greatly reduces the computational complexity. Through simulations, we show that our proposed algorithm achieves nearly optimal performance within much shorter computation time, compared to SDR based approaches [2]–[4].

II. System Model

We consider a cell with one base station communicating with mobile users. Without loss of generality, we focus on one sector of the cell. In each sector, the base station has a uniform linear antenna array. Our goal is to design a low-complexity beamforming algorithm that calculates the weights of each antenna element, in order to form a desirable radiation pattern. More specifically, we would like to minimize the transmit power while satisfying the SNR requirements of all the users in the sector.

For an array with M antenna elements, the gain at the direction θ (with respect to the line perpendicular to the array) can be written as $g(\theta) = \left|\sum_{m=1}^{M} w_m \cdot e^{\mathbf{j}\frac{2\pi(m-1)d\sin(\theta)}{\lambda}}\right|^2$, where $\mathbf{j} = \sqrt{-1}$, d is the distance between adjacent antenna elements, λ is the wavelength, and w_m is the complex weight applied to the m-th antenna element. It is convenient to represent the weights as a beamforming vector $\boldsymbol{w} = [w_1, \ldots, w_m]^T \in \mathbb{C}^M$, where $\mathbb{C}$ is the set of complex numbers. We also define the steering vector at direction θ as $\boldsymbol{a}(\theta) = \left[1, e^{\mathbf{j}\frac{2\pi d\sin(\theta)}{\lambda}}, \ldots, e^{\mathbf{j}\frac{2\pi(M-1)d\sin(\theta)}{\lambda}}\right]^H$. Then we can write the antenna gain compactly as $g(\theta) = \left|\boldsymbol{a}(\theta)^H \boldsymbol{w}\right|^2$.

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We assume that we know the locations (i.e., directions and distances) of the mobile users in the sector. The locations could be retrieved by a separate user discovery antenna array [6]. Suppose that there are N users. Based on each user n's direction, distance, and SNR requirement, we can impose the following constraint to ensure that the SNR requirement is met: $g(\theta_n) = \left|\boldsymbol{a}(\theta_n)^H \boldsymbol{w}\right|^2 \geq \gamma_n$, where θ_n is user n's direction, and γ_n is the minimum antenna gain needed for user n and can be calculated based on user n's SNR requirement and path loss.

Now we can formulate the minimum energy beamforming problem as

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \|\boldsymbol{w}\|_2^2 \\ \text{subject to} \quad & \left|\boldsymbol{a}(\theta_n)^H \boldsymbol{w}\right|^2 \geq \gamma_n, \ n = 1, \ldots, N. \end{aligned} \tag{1}$$

The optimization problem (1) is nonconvex due to the SNR requirement constraints. In the next section, we will propose a low-complexity algorithm to obtain the optimal solution to the problem (1).

III. Proposed Algorithm

A common approach to solve (1) is semidefinite relaxation. Defining a Hermitian matrix $\boldsymbol{W} = \boldsymbol{w} \cdot \boldsymbol{w}^H \in \mathbb{C}^{M \times M}$, the problem (1) is then equivalent to

$$\begin{aligned} \min_{\boldsymbol{w}, \boldsymbol{W}} \quad & \text{tr}(\boldsymbol{W}) \\ \text{subject to} \quad & \text{tr}(\boldsymbol{A}_n \boldsymbol{W}) \geq \gamma_n, \ n = 1, \ldots, N, \\ & \boldsymbol{W} = \boldsymbol{w} \cdot \boldsymbol{w}^H, \end{aligned} \tag{2}$$

where tr$(\cdot)$ is the trace of a matrix, and $\boldsymbol{A}_n \triangleq \boldsymbol{a}(\theta_n) \cdot \boldsymbol{a}(\theta_n)^H$.

The problem (2) is still nonconvex due to the constraint $\boldsymbol{W} = \boldsymbol{w} \cdot \boldsymbol{w}^H$. By removing it, we obtain a relaxed convex problem

$$\begin{aligned} \min_{\boldsymbol{W}} \quad & \text{tr}(\boldsymbol{W}) \\ \text{subject to} \quad & \text{tr}(\boldsymbol{A}_n \boldsymbol{W}) \geq \gamma_n, \ n = 1, \ldots, N. \end{aligned} \tag{3}$$

It is known that the relaxed problem (4) actually has an exact solution to the original problem (1) under uniform linear arrays. However, it is expensive to solve the semidefinite program (4) using standard interior-point methods, which have worst-case complexity of $O\left((M^2)^{6.5}\right)$. The high complexity makes it hard to implement such minimum energy beamforming in low-cost hardware. Therefore, we propose to use the low-complexity ADMM algorithm to solve it.

We reformulate the problem in the following form:

$$\begin{aligned} \min_{\boldsymbol{W}, \boldsymbol{W}_1, \ldots, \boldsymbol{W}_n} \quad & \text{tr}(\boldsymbol{W}) \\ \text{subject to} \quad & \text{tr}(\boldsymbol{A}_n \boldsymbol{W}_n) \geq \gamma_n, \ n = 1, \ldots, N, \\ & \boldsymbol{W} = \boldsymbol{W}_n, \ n = 1, \ldots, N. \end{aligned} \tag{4}$$

The key here is to define a new variable $\boldsymbol{W}_n$ for each user n's SNR constraint. In this way, we have a SDP with exactly one constraint in each iteration of the ADMM algorithm, which we can solve analytically. We describe the ADMM algorithm below.

Algorithm 1 Low-Complexity Min-Energy Beamforming

Input: θ_n and γ_n for $n = 1, \ldots, N$, $\rho > 0$, # of iterations K
Output: beamforming vector $\boldsymbol{w}$

Initialization : a random $\boldsymbol{w} \in \mathbb{C}^M$

$\boldsymbol{W}^0 = \max_n \left\{\gamma_n / \left|\boldsymbol{a}(\theta_n)^H \boldsymbol{w}\right|^2\right\} \cdot (\boldsymbol{w}\boldsymbol{w}^H)$

$\boldsymbol{W}_n^0 = \boldsymbol{W}^0$ and $\boldsymbol{\Lambda}_n^0 = \boldsymbol{0} \in \mathbb{C}^{M \times M}$ for $n = 1, \ldots, N$

for $k = 1, 2, \ldots, K$ **do**

 $\boldsymbol{W}^k = \frac{1}{N}\left[\sum_{n=1}^N \boldsymbol{W}_n^{k-1} + \rho^{-1}\left(\boldsymbol{I} + \sum_{n=1}^N \boldsymbol{\Lambda}_n^{k-1}\right)\right]$

 for $n = 1, \ldots, N$ **do**

  $\boldsymbol{W}_n^k = \boldsymbol{W}^{k-1} - \rho^{-1}\boldsymbol{\Lambda}_n^{k-1}$

  $\boldsymbol{W}_n^k \leftarrow \max\left\{1, \frac{\gamma_n}{\text{tr}(\boldsymbol{A}_i \boldsymbol{W}_n^k)}\right\} \cdot \boldsymbol{W}_n^k$

  $\boldsymbol{\Lambda}_n^k = \boldsymbol{\Lambda}_n^{k-1} + \rho\left(\boldsymbol{W}_n^k - \boldsymbol{W}_n^k\right)$

 end for

end for

$\boldsymbol{w} \leftarrow$ eigenvector of the largest eigenvalue of $\boldsymbol{W}^K$

return $\boldsymbol{w}$

IV. Performance Evaluation

We compare our proposed algorithm with the transform based beamforming in [1] and SDR beamforming (i.e., the solution to the SDP (4)), in terms of performance and computation time.

We consider a base station with an 8-element array and 5 users. The SNR requirement of all the users is fixed at 20 dB. We generate 1000 samples of user location profiles, where the directions and distances are both uniformly distributed. We compute the average transmit power and computation time over these 1000 samples. We summarize the results in Table I.

TABLE I: Comparison of performance and computation time.

	transform based	SDR	proposed
transmit power	35.24 dB	30.81 dB	31.03 dB
computation time	0.02 s	2.32 s	0.18 s

We can see that both SDR beamforming and our proposed beamforming outperform the transform based beamforming significantly. However, our proposed algorithm is able to obtain a high-quality solution in much more short time.

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