

Data-Driven Blind Synchronization and Interference Rejection for Digital Communication Signals

Alejandro Lancho*, Amir Weiss*, Gary C.F. Lee,
 Jennifer Tang, Yuheng Bu, Yury Polyanskiy, and Gregory W. Wornell
 Massachusetts Institute of Technology, Cambridge, MA, USA
 Emails: {lancho, amirwei, glcf411, jstang, buyuheng, ypol, gww}@mit.edu

Abstract—We study the potential of data-driven deep learning methods for separation of two communication signals from an observation of their mixture. In particular, we assume knowledge on the generation process of one of the signals, dubbed signal of interest (SOI), and no knowledge on the generation process of the second signal, referred to as interference. This form of the single-channel source separation problem is also referred to as interference rejection. We show that capturing high-resolution temporal structures (nonstationarities), which enables accurate synchronization to both the SOI and the interference, leads to substantial performance gains. With this key insight, we propose a domain-informed neural network (NN) design that is able to improve upon both “off-the-shelf” NNs and classical detection and interference rejection methods, as demonstrated in our simulations. Our findings highlight the key role communication-specific domain knowledge plays in the development of data-driven approaches that hold the promise of unprecedented gains.

Index Terms—Blind synchronization, source separation, interference rejection, deep neural network, supervised learning.

I. INTRODUCTION

The proliferation of wireless devices is leading to an increasingly crowded radio spectrum, and consequently, spectrum sharing will be unavoidable [1], [2]. Thus, different wireless communication systems will coexist in the same frequency bands, thereby generating unintentional interferences among them. In order to maintain high reliability, separation of the overlapping signals from the received mixture will become an essential building block in such communication systems.

In the image and audio domains, machine learning techniques have been successfully applied for *source separation*, e.g., [3]. These methods usually exploit domain knowledge relating to the signals’ structures. For example, color features

and local dependencies are useful for separating natural images [4], whereas time-frequency spectrogram masking methods are typically adopted for separating audio signals [5].

For communication signals, if the sources are separable in time and/or frequency, one can separate them via appropriate masking and classical filtering methods (see, e.g., [6]). The key challenge in this domain is the separation of signals overlapping in *both* time and frequency when the receiver is equipped with a single antenna, which inherently implies there is no spatial diversity to be exploited. This problem is also referred to as *single-channel* source separation (SCSS). In this case, standard approaches exploiting spatial diversity for blind source separation, such as [7], [8], are irrelevant.

Various methods are available in the literature to perform SCSS of digital communication signals. A common approach is maximum likelihood sequence estimation of the target signal, for which algorithms such as particle filtering [9] and per-surviving processing algorithms [10] can be used. However, such methods require prior knowledge of the signal models, which in practice may not be known or available.

Perhaps a more realistic approach is to assume that only a dataset of the underlying communication signals is available. This can be obtained, for example, through direct/background recordings, or using high fidelity simulators (e.g., [11]), allowing for a *data-driven* approach. In this setup, deep neural networks (DNNs) arise as a natural choice. This problem has been recently promoted by the “RF Challenge” [12].

In this paper, we study the data-driven SCSS problem where two communication signals overlap in time and frequency, and the receiver is equipped with one single antenna. We consider a signal of interest (SOI) whose generation process is known, and an interference signal with cyclic statistical properties that are unknown *a priori*—as is the case in standard protocols.¹ This problem is also referred to as *interference rejection*. As a performance measure, we consider the bit error rate (BER).

Contributions: We show that temporal nonstationarities of the signals constitute strong regularities that translate to better separation conditions. In particular, when such temporal structures exist, the notion of (time-)synchronization becomes not only sensible, but advantageous for separation. Based on

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¹We only assume the cyclic period is known. In practice, provided a dataset of the respective signal, this parameter can be consistently estimated [13].

our theoretical results that bind synchronization with MMSE optimal separation, we propose a data-driven DNN approach that is BER-superior to the classical methods of demodulation with matched filtering (MF) and interference rejection with linear minimum mean-square error (LMMSE) estimation of the SOI. Our proposed DNNs architectures, which can incorporate explicit synchronization, are inspired by specific domain knowledge, relevant to digital communication signals.

Notation: We use lowercase letters with standard font and sans-serif font, e.g., x and $\mathbf{x}$, to denote deterministic and random scalars, respectively. Similarly, we use $\mathbf{x}$ and $\mathbf{x}$ for deterministic and random vectors, respectively; and $\mathbf{X}$ and $\mathbf{X}$ for deterministic and random matrices, respectively. The uniform distribution over a set $\mathcal{S}$ is denoted as $\text{Unif}(\mathcal{S})$, and for $K \in \mathbb{N}$, we denote $\mathcal{S}_K \triangleq \{1, \dots, K\}$. For brevity, we refer to the complex normal distribution as Gaussian. We denote $\mathbf{C}_{zw} \triangleq \mathbb{E}[\mathbf{z}\mathbf{w}^H] \in \mathbb{C}^{N_z \times N_w}$ as the covariance matrix of $\mathbf{z} \in \mathbb{C}^{N_z \times 1}$ and $\mathbf{w} \in \mathbb{C}^{N_w \times 1}$ (specializing to $\mathbf{C}_{zz}$ for $\mathbf{z} = \mathbf{w}$).

II. PROBLEM FORMULATION

We consider the single-channel, baseband signal model of a noisy mixture of two sources, given by

$$y[n] = s[n - k_s] + \rho_{\text{SIR}}^{-1/2} b[n - k_b] + \rho_{\text{SNR}}^{-1/2} w[n], \quad n \in \mathbb{Z}, \quad (1)$$

where $s[n], b[n] \in \mathbb{C}$ are assumed to be cyclostationary processes with known fundamental cyclic periods $K_s, K_b \in \mathbb{N}$, respectively; $w[n] \in \mathbb{C}$ denotes additive white Gaussian noise, statistically independent of $s[n]$ and $b[n]$; and $\rho_{\text{SIR}}, \rho_{\text{SNR}} \in \mathbb{R}_+$. We refer to the signal $s[n]$ as the SOI, and to $b[n]$ as interference. The variables $k_s, k_b \in \mathbb{Z}$ denote unknown (discrete) time-shifts with respect to the start of the cyclic periods of $s[n]$ and $b[n]$, respectively, where the start of the cyclic periods are chosen arbitrarily to be at $n = 0$ without loss of generality. Hence, we assume that $k_s \sim \text{Unif}(\mathcal{S}_{K_s})$ and $k_b \sim \text{Unif}(\mathcal{S}_{K_b})$.

Let $\mathbf{y} \triangleq [y[1] \dots y[N]]^T$, $\mathbf{s}(k_s) \triangleq [s[1 - k_s] \dots s[N - k_s]]^T$, $\mathbf{b}(k_b) \triangleq [b[1 - k_b] \dots b[N - k_b]]^T$, and $\mathbf{w} \triangleq [w[1] \dots w[N]]^T$. Then, we may compactly write (1) for N samples as

$$\mathbf{y} = \mathbf{s}(k_s) + \rho_{\text{SIR}}^{-1/2} \mathbf{b}(k_b) + \rho_{\text{SNR}}^{-1/2} \mathbf{w} \in \mathbb{C}^{N \times 1}. \quad (2)$$

We further assume that $\mathbf{s}(k_s)$ and $\mathbf{b}(k_b)$ are statistically independent, which is a reasonable assumption in scenarios of unintentional interference, for which each source is not actively jamming or adapting to the other signals present in the environment. For simplicity of the exposition, we assume that $\mathbf{s}(k_s)$ and $\mathbf{b}(k_b)$ are zero-mean, unit-average-power, i.e., their (possibly time-varying) variance averages to 1. In this case, the parameters $\rho_{\text{SIR}}, \rho_{\text{SNR}}$ represent the signal-to-interference ratio (SIR) and signal-to-noise ratio (SNR) at the receiver, respectively.

The goal is to produce an estimate of $\mathbf{s}(k_s)$ from $\mathbf{y}$, denoted by $\hat{\mathbf{s}}$, so that given some metric ℓ , the cost $\mathbb{E}[\ell(\hat{\mathbf{s}}, \mathbf{s}(k_s))]$ is minimized. This problem is referred to as SCSS.

As mentioned in Section I, we assume we do not have precise knowledge of the underlying distributions of the SOI and the interference. However, we assume the availability

of a dataset of the signals and their respective time-shifts $(\mathbf{s}(k_s), k_s)$ and $(\mathbf{b}(k_b), k_b)$, allowing for a data-driven approach. Examples of such datasets can be found in [12], [14].

III. THE GAIN IN SYNCHRONIZATION TO INTERFERENCE

Before we present our approach to the SCSS problem formulated in Section II, we provide an analysis of an asymptotically optimal estimator of $\mathbf{s}(k_s)$ for the metric $\ell(\mathbf{x}, \mathbf{z}) \triangleq \|\mathbf{x} - \mathbf{z}\|_2^2$, which will shed light on key aspects in optimal separation and the role of *synchronization to interference*.

In this section, we assume that $s[n]$ and $b[n]$ are Gaussian processes, which is a reasonable assumption to model some communication signals, e.g., [15]. In this case, we define²

$$\mathbf{v}[n - k_b] \triangleq \rho_{\text{SIR}}^{-1/2} \mathbf{b}[n - k_b] + \rho_{\text{SNR}}^{-1/2} \mathbf{w}[n], \quad n \in \mathbb{Z}, \quad (3)$$

such that $\mathbf{v}(k_b) \triangleq [\mathbf{v}[1 - k_b] \dots \mathbf{v}[N - k_b]]^T \in \mathbb{C}^{N \times 1}$ is the “equivalent noise”, which, given k_b , is distributed as $\mathcal{CN}(\mathbf{0}, \mathbf{C}_{vv})$. Thus, (2) simplifies to

$$\mathbf{y} = \mathbf{s}(k_s) + \mathbf{v}(k_b) \in \mathbb{C}^{N \times 1}. \quad (4)$$

Note that, generally, the equivalent noise term $\mathbf{v}(k_b)$ is not temporally white (as opposed to $\mathbf{w}$), and exhibits a potentially informative statistical structure (e.g., in the form of $\mathbf{C}_{vv}$) that can be exploited for enhanced separation performance.

A. Linear minimum mean-square error (MMSE) Estimation

A computationally attractive approach, which already exploits (some of) the underlying statistics of both of the components of the mixture (4), is optimal *linear* estimation. The LMMSE estimator [16], given by (assuming $\det(\mathbf{C}_{yy}) \neq 0$)

$$\hat{\mathbf{s}}_{\text{LMMSE}} \triangleq \mathbf{C}_{sy} \mathbf{C}_{yy}^{-1} \mathbf{y} = \mathbf{C}_{ss} (\mathbf{C}_{ss} + \mathbf{C}_{vv})^{-1} \mathbf{y} \in \mathbb{C}^{N \times 1}, \quad (5)$$

is constructed using the statistics of the mixture that inherently takes into account the potentially non-trivial structure of $\mathbf{C}_{vv}$, i.e., some form of deviation from a scaled identity matrix.

However, while (5) coincides with the MMSE estimator for jointly Gaussian processes, it is generally suboptimal due to the linearity constraint. Specifically, in our case, although the processes $s[n], v[n]$ are jointly Gaussian, $\mathbf{s}(k_s)$ and $\mathbf{v}(k_b)$ are *not* even marginally Gaussian. Indeed, $\mathbf{s}(k_s)$ and $\mathbf{v}(k_b)$ are Gaussian mixtures due to the random time-shifts k_s, k_b . It then follows that (5) is in fact not optimal, as shown next.

B. MMSE Estimation

The optimal estimator in the MMSE sense is known to be the conditional expectation,

$$\hat{\mathbf{s}}_{\text{MMSE}} \triangleq \mathbb{E}[\mathbf{s}(k_s) | \mathbf{y}] \in \mathbb{C}^{N \times 1}, \quad (6)$$

whose mean-squared error (MSE) is an achievable lower bound of the MSE of *any* estimator of $\mathbf{s}(k_s)$. However, in most practical cases, (6) is hard to obtain analytically and computationally. In our case, by using the law of total

²Since $w[n]$ is white (and therefore stationary), $w[n - k_b]$ is also white, hence without loss of generality we may indeed define (3) with the shift k_b .

expectation in (6), the MMSE estimator is given by the more explicit and convenient form

$$\begin{aligned}\hat{\mathbf{s}}_{\text{MMSE}} &= \mathbb{E}[\mathbb{E}[\mathbf{s}(k_s)|\mathbf{y}, k_s, k_b]|\mathbf{y}] \stackrel{(*)}{=} \mathbb{E}[\hat{\mathbf{s}}_{\text{LMMSE}}(k_s, k_b)|\mathbf{y}] \\ &= \sum_{m_s=1}^{K_s} \sum_{m_b=1}^{K_b} \mathbb{P}[k_s = m_s, k_b = m_b|\mathbf{y}] \hat{\mathbf{s}}_{\text{LMMSE}}(m_s, m_b),\end{aligned}\quad (7)$$

where in $(*)$ we have used the fact that, given the time-shifts, $\mathbf{s}(k_s)$ and $\mathbf{y}$ are jointly Gaussian, and where $\hat{\mathbf{s}}_{\text{LMMSE}}(m_s, m_b) \triangleq \mathbf{C}_{ss}(m_s) [\mathbf{C}_{ss}(m_s) + \mathbf{C}_{vv}(m_b)]^{-1} \mathbf{y}$, with

$$\mathbf{C}_{ss}(m) \triangleq \mathbb{E}[\mathbf{s}\mathbf{s}^H | k_s = m], \quad \mathbf{C}_{vv}(m) \triangleq \mathbb{E}[\mathbf{v}\mathbf{v}^H | k_b = m]. \quad (8)$$

Put simply, (7) is a weighted average of $K_s \times K_b$ linear estimators, with the posterior probabilities—which are *non-linear* functions of the data $\mathbf{y}$ —serving as the normalized weights. Even before taking into account the computation of the posteriors, the sum in (7) scales with the product of possible time-shifts $K_s \times K_b$, rendering $\hat{\mathbf{s}}_{\text{MMSE}}$ often impractical.

As can be seen from (7), synchronization (i.e., knowledge of the time-shifts) already substantially simplifies the computation, since, in that case, only the (conditional) linear estimator $\hat{\mathbf{s}}_{\text{LMMSE}}(m_s, m_b)$ is required. In other words, eliminating this type of randomness from the mixture $\mathbf{y}$ grants us lower computational complexity and a simple form of a linear estimator. Fortunately, a two-step “synchronization-separation” estimator can approach the MMSE estimator, thus enjoying asymptotic optimality at a substantially reduced computational burden.

To show this rigorously, for simplicity of the exposition, we assume hereafter (unless stated otherwise) that the receiver is synchronized to the SOI,³ namely, $k_s = 0$ and known. However, the result below can be generalized to the case where the SOI’s time-shift k_s is random and unknown. Let

$$\hat{k}_b^{\text{MAP}} \triangleq \arg \max_{m \in \mathcal{S}_{K_b}} \mathbb{P}[k_b = m|\mathbf{y}] \quad (9)$$

be the maximum *a posteriori* (MAP) estimator of k_b , and define the (suboptimal) “plug-in”, MAP-based *quasi-linear* MMSE estimator

$$\hat{\mathbf{s}}_{\text{MAP-QLMMSE}} \triangleq \hat{\mathbf{s}}_{\text{LMMSE}}(\hat{k}_b^{\text{MAP}}) \in \mathbb{C}^{N \times 1}, \quad (10)$$

where, for brevity, we use $\hat{\mathbf{s}}_{\text{LMMSE}}(m)$ to denote $\hat{\mathbf{s}}_{\text{LMMSE}}(0, m)$. Furthermore, we define the MSEs, as a function of N , as

$$\varepsilon_{\text{MMSE}}^2(N) \triangleq \mathbb{E}[\|\hat{\mathbf{s}}_{\text{MMSE}} - \mathbf{s}\|_2^2] \in \mathbb{R}_+, \quad (11)$$

$$\varepsilon_{\text{MAP-QLMMSE}}^2(N) \triangleq \mathbb{E}[\|\hat{\mathbf{s}}_{\text{MAP-QLMMSE}} - \mathbf{s}\|_2^2] \in \mathbb{R}_+. \quad (12)$$

We now introduce a “temporal-diversity” condition (TDC) under which optimal synchronization is increasingly accurate.

Definition 1 (TDC): Let $\psi_N(\mathbf{y}, k) \triangleq \frac{1}{N} \mathbf{y}^H \mathbf{C}_{yy}^{-1}(k) \mathbf{y} - 1$. The (sufficient) TDC is satisfied if there does not exist $k \in \mathcal{S}_{K_b} \setminus k_b$ such that $\lim_{N \rightarrow \infty} |\psi_N(\mathbf{y}, k)| = 0$.

Lemma 1: Under the TDC, for any finite $\alpha \in \mathbb{R}_+$,

$$\mathbb{P}[\hat{k}_b^{\text{MAP}} \neq k_b] = o\left(\frac{1}{N^\alpha}\right). \quad (13)$$

³This is a reasonable assumption in most communication systems [17].

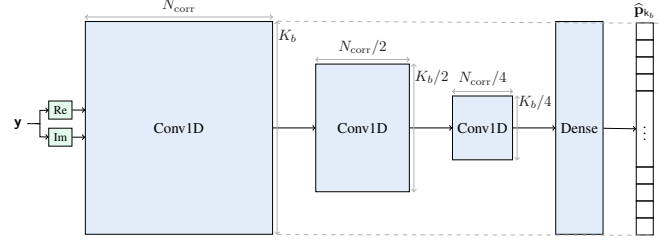


Fig. 1: Architecture of the proposed CNN for synchronization.

Proof: See Appendix A. ■

The theorem below shows that the two-step synchronization-separation approach (10) is asymptotically optimal.

Theorem 1: Under the TDC, we have

$$\lim_{N \rightarrow \infty} \frac{\varepsilon_{\text{MMSE}}^2(N)}{\varepsilon_{\text{MAP-QLMMSE}}^2(N)} = 1. \quad (14)$$

Proof: See Appendix B. ■

In words, Theorem 1 tells us that, when the time-shift can be uniquely detectable, first optimally synchronizing to the interference, and then using a suboptimal, quasi-linear estimator, is asymptotically equivalent to MMSE estimation. Further intuition to this type of behaviour, for maximum-likelihood-based MMSE estimation, is given in [8, Fig. 1].

C. Synchronization via convolutional neural networks (CNNs)

Although the estimator (10) is attractive in terms of its MSE performance, it nevertheless requires—both for synchronization and separation—*precise* knowledge of the underlying statistics, including the SIR and SNR, which can be hard to obtain in practice. In particular, without these statistics, it is impossible to obtain $\hat{k}_b^{\text{MAP}}$. However, when by measurement or generation, sufficiently large datasets with realizations of $\mathbf{s}(k_s)$ and $\mathbf{b}(k_b)$ are available, a data-driven approach can be taken.

To this end, we leverage the strong capabilities of CNNs for capturing intricate temporal structures, to train a synchronizer in a data-driven manner. Specifically, we propose the CNN-based architecture depicted in Fig. 1, which is trained in a supervised manner based on a labeled dataset of mixtures and the underlying interference time-shifts, $\{(\mathbf{y}^{(i)}, k_b^{(i)}) : i \in \mathcal{S}_{I_T}\}$, where I_T is the size of the training dataset. We use a sufficiently large kernel size in the convolutional layers, which is proportional to the “effective correlation length”—denoted as N_{corr} in Fig. 1—so as to be able to capture the strongest, most informative temporal structures for estimation. Since the cyclic period K_b is assumed to be known, we train a model using the cross-entropy loss, which receives as its input the mixture $\mathbf{y}$ and outputs a vector of probabilities, denoted by $\hat{\mathbf{p}}_{k_b} \in [0, 1]^{K_b \times 1}$. At inference time, we synchronize to the interference via $\hat{k}_b^{\text{CNN}} \triangleq \arg \max_{m \in \mathcal{S}_{K_b}} \mathbf{e}_m^T \hat{\mathbf{p}}_{k_b}$ (cf. $\hat{k}_b^{\text{MAP}}$ in (9)), where $\mathbf{e}_m \in \mathbb{R}^{K_b \times 1}$ denotes the m -th standard basis vector.

In Fig. 2, we show the MSE for $\hat{\mathbf{s}}_{\text{LMMSE}}$, $\hat{\mathbf{s}}_{\text{MMSE}}$ and

$$\hat{\mathbf{s}}_{\text{CNN-QLMMSE}} \triangleq \hat{\mathbf{s}}_{\text{LMMSE}}(\hat{k}_b^{\text{CNN}}), \quad (15)$$

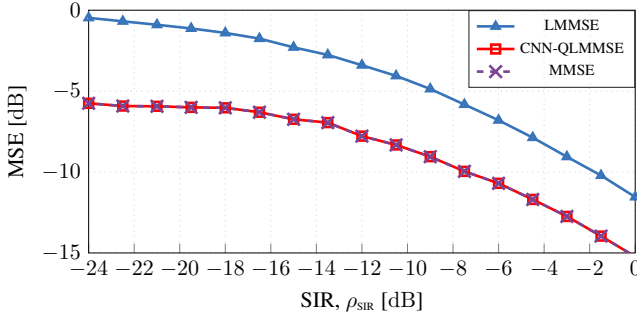


Fig. 2: MSE as a function of the SIR (ρ_{SIR}) for a fixed SNR (ρ_{SNR}) of 20 dB.

for the same communication waveforms described in detail in Section V, but considering here Gaussian alphabets instead of the discrete and finite alphabets used in Section V. As seen, the linearity restriction (5) costs a considerable price in terms of the compromised performance relative to the lower bound, given by the MMSE. It is also evident that the MSE of the CNN-based quasilinear MMSE (QLMMSE) estimator $\hat{\mathbf{s}}_{\text{CNN-QLMMSE}}$ coincides with (11), which asymptotically coincides with the MAP-QLMMSE (12) by virtue of Theorem 1.

All the above motivates our solution approach, and provides the theoretical foundations (as well as intuition) based on which we develop our system architecture, presented next.

IV. INTERFERENCE REJECTION VIA DNNs

We now present two supervised learning approaches for SCSS, used in this work as interference rejection methods. The first DNN architecture, depicted in Fig. 3, consists of two main building blocks: (i) CNN to perform synchronization to the interference, (ii) DNN (U-Net) to perform SCSS.⁴ The key motivation to perform explicit synchronization is twofold. First, as explained in Section III-B, due to Theorem 1, explicit consistent synchronization decoupled from separation, although suboptimal, can asymptotically (as $N \rightarrow \infty$) lead to optimal separation with reduced complexity. Second, although a sufficiently rich DNN might be able to perform the synchronization and separation tasks jointly, for a given architecture, acquiring synchronization knowledge explicitly helps by reducing the complexity of the separation task. In Section V-A, we show that this decoupled approach can indeed lead to performance gains. However, Lemma 1 shows that there exists a realizable synchronization method that becomes increasingly accurate as the input size grows. While this can be exploited for explicit synchronization (e.g., Fig. 3), it could also imply that, under certain conditions, a DNN architecture would be able to “implicitly synchronize” and separate, namely superior performance would be achieved *without* explicit synchronization. This is shown in Section V-B.

⁴To separate the communication signals used in this paper, other DNNs were implemented, yielding worse performance. Details can be found in the Github repository: https://github.com/RFChallenge/SCSS_DNN_Comparison.

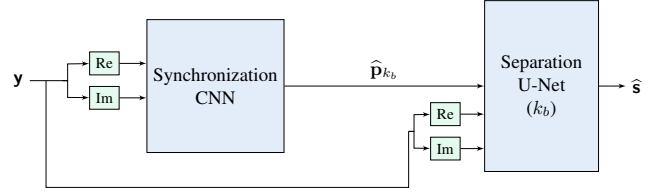


Fig. 3: System architecture of the DNN-based approach with an explicit CNN-based synchronization block prior to the separation block (U-Net).

The synchronization block is based on the CNN described in Section III-C (Fig. 1). The DNN for separation is based on the so-called U-Net (see Fig. 4) [18], which has some properties that makes it suitable to the specific informative features of digital communication signals. In particular, its CNN building blocks allow us to input and process long time intervals (e.g., $N > 10^4$), which cannot be processed using classical methods. In turn, processing such long signals allows for exploitation of temporal structures on a different scale, which can (and does) lead to substantial performance gains.

As shown in Fig. 4, our DNN approach departs from standard implementations intended to deal with images (2D signals). To handle 1D complex-valued, time-series communication signals, we use 1D convolutional layers. Furthermore, differently from standard CNN-based architectures that are designed to deal with images and hence use short kernels of size ~ 3 in all layers, our U-Net architecture utilizes a sufficiently long kernel in the first convolutional layer (denoted by κ in Fig. 4). This enables to capture the most influential temporal structures of the SOI and interference, which can lead to an order of magnitude gains, as demonstrated below.

For training, we input the stacked real and imaginary parts of $\mathbf{y}$ as separate channels to both the synchronization-to-interference CNN and the separation U-Net. For separation, if explicit synchronization is performed, we mimic a non-linear version of (10) by using an instance of the DNN architecture depicted in Fig. 4 for each possible output of the synchronization-to-interference CNN block. In other words, we implement a “conditional separation” block for each possible time-shift of the interference. If explicit synchronization-to-interference is not used, the raw unprocessed mixture is (always) fed into to the same DNN separation block.

The training set is processed as such to yield a labeled dataset (mixture $\mathbf{y}$ and ground-truth reference signal $\mathbf{s}$). As a loss function, we use the empirical MSE. For full implementation details, see our Github repository.⁵

V. NUMERICAL RESULTS

We generate synthetic mixtures $\mathbf{y}$ where the SOI bears quaternary phase shift keying (QPSK) symbols using a root-raised cosine pulse-shaping filter with roll-off factor 0.5, spanning 8 QPSK symbols, and with an oversampling factor 16. The interference is an orthogonal frequency-division multiplexing

⁵https://github.com/RFChallenge/SCSS_Sync

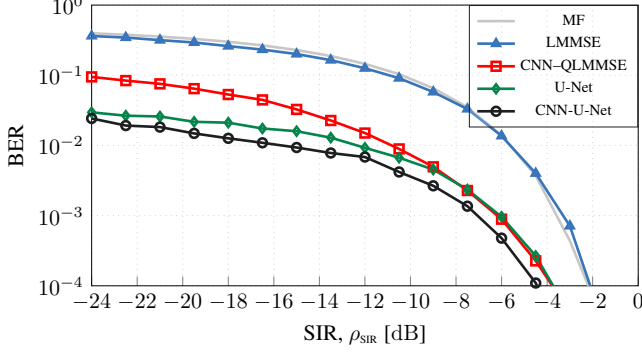


Fig. 5: BER as a function of the SIR for MF detection; LMMSE and QLMMSE interference rejection (blocks of length 320); and the data-driven U-Net approach with and without synchronization to the interference.

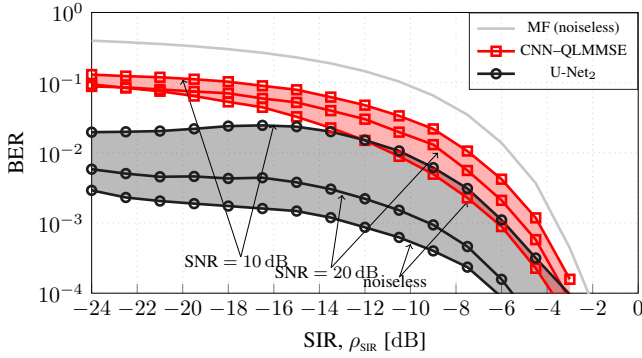


Fig. 6: BER as a function of the SIR for the noiseless case and SNR = {10, 20} dB, for MF (noiseless only), QLMMSE interference rejection (blocks of length 320), and the CNN-U-Net approach described in Fig. 3.

mixture $\mathbf{y}$, which provides the first layer with more diversity. We consider three different SNR levels of white Gaussian noise: $\rho_{\text{SNR}} \in \{10, 20, \infty\}$ dB. Specifically, we compare the performance of the DNN solution with the performance of the CNN-QLMMSE approach (computed using blocks of length 320) and the MF approach, which is only plotted for the noiseless case for the sake of clarity. Clearly, for all SNR levels, the U-Net₂ approach outperforms the CNN-QLMMSE and MF approaches. However, as expected, the smaller the SNR, the smaller the gap between them. For example, for a BER of 10^{-3} , the gain of the U-Net₂ approach compared to the CNN-QLMMSE is roughly 7 dB in the noiseless case, 4 dB for an SNR = 20 dB, and 1.8 dB for an SNR = 10 dB.

VI. CONCLUSIONS AND OUTLOOK

We study the SCSS problem with a focus on its application to interference rejection in digital communication. For Gaussian signals, we prove that a decoupled system architecture of synchronization followed by separation is asymptotically optimal in the MMSE sense. Consequently, since the optimal system can be impractical for implementation purposes, we propose a computationally attractive alternative with negligible performance loss relative to the optimal system. For (non-

Gaussian) signals, we demonstrate in simulations that the proposed DNN-based data-driven approach can exploit the underlying temporal structures of the signals, thus leading to significant gains in terms of BER, and in particular, outperforms classical methods. Extensions of this work should focus on understanding how and when to use explicit synchronization in the context of SCSS with DNNs.

APPENDIX A PROOF OF LEMMA 1

To prove Lemma 1, we shall use the following lemma.
Lemma 2: For $\psi_N(\mathbf{y}, k)$, in Definition 1 (TDC), we have,

$$\mathbb{E} \left[e^{\tau \psi_N(\mathbf{y}, k_b)} \right] = \left(1 - \frac{\tau}{N} \right)^{-N} \cdot e^{-\tau}, \quad \forall \tau < N. \quad (16)$$

Proof of Lemma 2: First, recall $\mathbf{y}|k_b \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_{yy}(k_b))$, where $\mathbf{C}_{yy}(k_b) = \mathbf{C}_{ss}(0) + \mathbf{C}_{vv}(k_b)$. Using the Cholesky decomposition, we write $\mathbf{C}_{yy}(k_b) \triangleq \mathbf{\Gamma}_y(k_b) \mathbf{\Gamma}_y^H(k_b)$, where $\mathbf{\Gamma}_y(k_b) \in \mathbb{C}^{N \times N}$. Then, conditioned on k_b , we have

$$\psi_N(\mathbf{y}, k_b) + 1 = \frac{1}{N} \mathbf{y}^H \mathbf{C}_{yy}^{-1}(k_b) \mathbf{y} \quad (17)$$

$$= \frac{1}{N} \mathbf{y}^H \mathbf{\Gamma}_y^{-H}(k_b) \mathbf{\Gamma}_y^{-1}(k_b) \mathbf{y} \quad (18)$$

$$= \frac{1}{N} \underbrace{\left(\mathbf{\Gamma}_y^{-1}(k_b) \mathbf{y} \right)^H}_{\triangleq \mathbf{u}(k_b)} \underbrace{\mathbf{\Gamma}_y^{-1}(k_b) \mathbf{y}}_{=\mathbf{u}(k_b)} \quad (19)$$

$$= \frac{1}{N} \|\mathbf{u}(k_b)\|_2^2, \quad (20)$$

where $\mathbf{u}(k_b)|k_b \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is a white Gaussian vector. Thus,

$$\mathbb{E} \left[e^{\tau \psi_N(\mathbf{y}, k_b)} \right] = \mathbb{E} \left[\mathbb{E} \left[e^{\tau \psi_N(\mathbf{y}, k_b)} | k_b \right] \right] \quad (21)$$

$$= \mathbb{E} \left[\mathbb{E} \left[e^{\tau \left(\frac{1}{N} \|\mathbf{u}(k_b)\|_2^2 - 1 \right)} | k_b \right] \right] \quad (22)$$

$$= \mathbb{E} \left[\mathbb{E} \left[e^{\frac{\tau}{N} \sum_{n=1}^N |u_n(k_b)|^2} | k_b \right] \right] e^{-\tau} \quad (23)$$

$$= \mathbb{E} \left[\prod_{n=1}^N \mathbb{E} \left[e^{\frac{\tau}{2N} |\sqrt{2} u_n(k_b)|^2} | k_b \right] \right] e^{-\tau} \quad (24)$$

$$\stackrel{\forall \tau < N}{=} \mathbb{E} \left[\prod_{n=1}^N \left(1 - \frac{\tau}{N} \right)^{-1} \right] e^{-\tau} \quad (25)$$

$$= \left(1 - \frac{\tau}{N} \right)^{-N} \cdot e^{-\tau}, \quad (26)$$

where we have used the law of total expectation in (21); the conditional statistical independence of the elements of $\mathbf{u}(k_b)$ (given k_b) in (24); the fact that $\{|\sqrt{2} u_n(k_b)|^2\}_{n=1}^N \sim \chi_2^2$, namely all the squared absolute-valued elements of $\mathbf{u}(k_b)$, given k_b , are chi-squared random variables with two degrees of freedom; and, accordingly, that the moment generating function of a random variable $q \sim \chi_2^2$ is $\mathbb{E}[e^{\tilde{\tau} q}] = (1 - 2\tilde{\tau})^{-1}$, for all $\tilde{\tau} < \frac{1}{2}$, in (25), where in our case $\tilde{\tau} = \tau/2N$, hence the condition on τ in (24) ■

Equipped with Lemma 2, we now prove Lemma 1.

By definition, the MAP estimator has the lowest error probability. Therefore, to show (13), it is sufficient to show that

there exists another estimator of k_b , whose error probability is $o(N^{-\alpha})$ for any finite $\alpha \in \mathbb{R}_+$, independent of N . For this, let us consider the estimator,

$$\hat{k}_b \triangleq \arg \min_{m \in \mathcal{S}_{K_b}} |\psi_N(\mathbf{y}, m)|. \quad (27)$$

In words, as $N \rightarrow \infty$, the error probability of (27) is governed by how far is $|\psi_N(\mathbf{y}, k_b)|$ from zero, since from the TDC, $\nexists k \in \mathcal{S}_{K_b} \setminus k_b : \lim_{N \rightarrow \infty} |\psi_N(\mathbf{y}, k)| = 0$, whereas

$$\lim_{N \rightarrow \infty} \psi_N(\mathbf{y}, k_b) = \mathbb{E}[\psi_N(\mathbf{y}, k_b)] \quad (28)$$

$$= \mathbb{E}[\mathbb{E}[\psi_N(\mathbf{y}, k_b) | k_b]] \quad (29)$$

$$= \frac{1}{N} \mathbb{E}[\mathbb{E}[\|\mathbf{u}(k_b)\|_2^2 | k_b]] - 1 = 0, \quad (30)$$

where we have used (20), $\mathbf{u}(k_b) | k_b \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$, and (28) follows from the fact that $\mathbb{V}\text{ar}(\psi_N(\mathbf{y}, k_b)) = 1/N$, which can be shown in a similar fashion to (28)–(30).

Formally, the error probability of this estimator is given by,

$$\mathbb{P}[\hat{k}_b \neq k_b] = \mathbb{P}\left[|\psi_N(\mathbf{y}, k_b)| > \min_{m \in \mathcal{S}_{K_b} \setminus k_b} |\psi_N(\mathbf{y}, m)|\right]. \quad (31)$$

We now show that the probability that $\psi_N(\mathbf{y}, k_b)$ is bounded away from zero decreases in the desired rate. Clearly, for any $a > 0$, we have

$$\mathbb{P}[|\psi_N(\mathbf{y}, k_b)| > a] = \mathbb{P}[\psi_N(\mathbf{y}, k_b) > a] \quad (32)$$

$$+ \mathbb{P}[\psi_N(\mathbf{y}, k_b) < -a]. \quad (33)$$

Using the Chernoff bound, we have

$$\mathbb{P}[\psi_N(\mathbf{y}, k_b) > a] \leq \mathbb{E}\left[e^{t\psi_N(\mathbf{y}, k_b)}\right] e^{-ta} \triangleq B_1(t, a), \quad (34)$$

$$\mathbb{P}[\psi_N(\mathbf{y}, k_b) < -a] \leq \mathbb{E}\left[e^{-t\psi_N(\mathbf{y}, k_b)}\right] e^{-ta} \triangleq B_2(t, a). \quad (35)$$

Using Lemma 2, it follows that

$$B_1(t, a) = \left(1 - \frac{t}{N}\right)^{-N} \cdot e^{-t(1+a)}, \quad \forall t < N, \quad (36)$$

$$B_2(t, a) = \left(1 + \frac{t}{N}\right)^{-N} \cdot e^{t(1-a)}, \quad \forall t > -N. \quad (37)$$

Minimizing $B_1(t, a)$ and $B_2(t, a)$ with respect to t and choosing $a = N^{-(0.5-\epsilon)}$ for some $0 < \epsilon < 0.5$, we obtain

$$\min_{t < N} B_1(t, N^{-(0.5-\epsilon)}) = \left(1 + \frac{1}{N^{(0.5-\epsilon)}}\right)^N e^{-N^{0.5+\epsilon}} \quad (38)$$

$$\triangleq B_1^*[N], \quad (39)$$

$$\min_{t > -N} B_2(t, N^{-(0.5-\epsilon)}) = \left(1 - \frac{1}{N^{(0.5-\epsilon)}}\right)^N e^{N^{0.5+\epsilon}} \quad (40)$$

$$\triangleq B_2^*[N]. \quad (41)$$

Finally, as for any $\alpha \in \mathbb{R}_+$ and any $\delta > 0$ independent of N ,

$$\lim_{N \rightarrow \infty} N^{\alpha+\delta} B_1^*[N] = \lim_{N \rightarrow \infty} N^{\alpha+\delta} B_2^*[N] = 0, \quad (42)$$

it follows that for any $\alpha \in \mathbb{R}_+$ independent of N ,

$$\mathbb{P}\left[|\psi_N(\mathbf{y}, k_b)| > \frac{1}{N^{0.5-\epsilon}}\right] = o\left(\frac{1}{N^\alpha}\right) \quad (43)$$

$$\implies \mathbb{P}[\hat{k}_b \neq k_b] = o\left(\frac{1}{N^\alpha}\right). \quad (44)$$

APPENDIX B PROOF OF THEOREM 1

From Lemma 1, we have the following corollary.

Corollary 1: Using (13), we have

$$\mathbb{E}\left[\mathbb{P}[\hat{k}_b^{\text{MAP}} \neq k_b | \mathbf{y}]\right] = o\left(\frac{1}{N^\alpha}\right), \quad (45)$$

$$\mathbb{E}\left[\mathbb{P}[\hat{k}_b^{\text{MAP}} = k | \mathbf{y}]\right] = o\left(\frac{1}{N^\alpha}\right), \quad \forall k \in \mathcal{S}_{K_b} \setminus k_b. \quad (46)$$

The roadmap for the proof of the theorem is as follows:

- **Step 1:** Express the optimality gap between the MMSE and MAP-based QLMMSSE estimators as a function of the error probability of the MAP synchronizer $\hat{k}_b^{\text{MAP}}$.
- **Step 2:** Express the MMSE (11) as a sum of the MAP-based QLMMSSE (12) and the expected squared norm of the optimality gap, also known as the “regret”.
- **Step 3:** Show that the regret is upper bounded by terms that decay polynomially fast, for any fixed polynomial rate (using Lemma 1).

We now prove Theorem 1. Let us write the the MMSE estimator (6), explicitly, using (7), in terms of the MAP-based QLMMSSE estimator (10), as (recall $k_s = 0$, by assumption),

$$\begin{aligned} \hat{\mathbf{s}}_{\text{MMSE}} &= \sum_{m_b=1}^{K_b} \mathbb{P}[k_b = m_b | \mathbf{y}] \hat{\mathbf{s}}_{\text{QLMMSSE}}(m_b) \\ &= \underbrace{\sum_{\substack{m_b=1 \\ m_b \neq \hat{k}_b^{\text{MAP}}}}^{K_b} \mathbb{P}[k_b = m_b | \mathbf{y}] \hat{\mathbf{s}}_{\text{QLMMSSE}}(m_b)}_{\triangleq \delta(\mathbf{y})} \\ &\quad + \mathbb{P}[k_b = \hat{k}_b^{\text{MAP}} | \mathbf{y}] \hat{\mathbf{s}}_{\text{QLMMSSE}}(\hat{k}_b^{\text{MAP}}). \end{aligned} \quad (47)$$

Using (47), we define the optimality gap (vector),

$$\Delta(\mathbf{y}) \triangleq \hat{\mathbf{s}}_{\text{MMSE}} - \hat{\mathbf{s}}_{\text{QLMMSSE}}(\hat{k}_b^{\text{MAP}}) = \hat{\mathbf{s}}_{\text{MMSE}} - \hat{\mathbf{s}}_{\text{MAP-QLMMSSE}} \quad (48)$$

$$= \delta(\mathbf{y}) - \mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}] \hat{\mathbf{s}}_{\text{MAP-QLMMSSE}}. \quad (49)$$

Let us proceed to the second step of the proof. For shorthand, let $\mathbf{e}_{\text{MAP-QLMMSSE}} \triangleq \hat{\mathbf{s}}_{\text{MAP-QLMMSSE}} - \mathbf{s}$, and let us first write the MMSE in terms of the estimation error $\mathbf{e}_{\text{MAP-QLMMSSE}}$ and the optimality gap $\Delta(\mathbf{y})$ as,

$$\mathbb{E}[\|\hat{\mathbf{s}}_{\text{MMSE}} - \mathbf{s}\|_2^2] = \mathbb{E}[\|\hat{\mathbf{s}}_{\text{MMSE}} - \hat{\mathbf{s}}_{\text{MAP-QLMMSSE}} + \hat{\mathbf{s}}_{\text{MAP-QLMMSSE}} - \mathbf{s}\|_2^2] \quad (50)$$

$$= \varepsilon_{\text{MAP-QLMMSSE}}^2(N) - \mathbb{E}[\|\Delta(\mathbf{y})\|_2^2], \quad (51)$$

where we have used (48) in (50), and the well-known orthogonality property of the estimation error in MMSE estimation

to any function of the measurements in (51). Expanding the first term, we have,

$$\begin{aligned} \mathbb{E} [\|\Delta(\mathbf{y})\|_2^2] = & \mathbb{E} [\|\delta(\mathbf{y})\|_2^2] + \mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}]^2 \|\hat{\mathbf{s}}_{\text{MAP-QLMMSE}}\|_2^2 \right] \\ & - 2\Re \left\{ \mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}] \delta^H(\mathbf{y}) \hat{\mathbf{s}}_{\text{MAP-QLMMSE}} \right] \right\}. \end{aligned} \quad (52)$$

We now show that (the magnitude of) each of the terms in (52) is bounded. It will then follow that the expected squared norm of the optimality gap, $\mathbb{E} [\|\Delta(\mathbf{y})\|_2^2]$, is also bounded.

Starting with the first term in (52), we have,

$$\mathbb{E} [\|\delta(\mathbf{y})\|_2^2] = \sum_{n=1}^N \mathbb{E} [\delta_n^2(\mathbf{y})] = \quad (53)$$

$$\sum_{n=1}^N \mathbb{E} \left[\left(\sum_{\substack{m_b=1 \\ m_b \neq \hat{k}_b^{\text{MAP}}}}^{K_b} \mathbb{P}[k_b = m_b | \mathbf{y}] \hat{\mathbf{s}}_{\text{LMMSE},n}(m_b) \right)^2 \right]. \quad (54)$$

Focusing on one element of the sum in (54), we have,

$$\mathbb{E} \left[\left(\sum_{\substack{m_b=1 \\ m_b \neq \hat{k}_b^{\text{MAP}}}}^{K_b} \mathbb{P}[k_b = m_b | \mathbf{y}] \hat{\mathbf{s}}_{\text{LMMSE},n}(m_b) \right)^2 \right] \leq \quad (55)$$

$$\sum_{\substack{m_1=1 \\ m_1 \neq \hat{k}_b^{\text{MAP}}}}^{K_b} \sum_{\substack{m_2=1 \\ m_2 \neq \hat{k}_b^{\text{MAP}}}}^{K_b} \mathbb{E} [\mathbb{P}[k_b = m_1 | \mathbf{y}]^2 \mathbb{P}[k_b = m_2 | \mathbf{y}]^2]^{\frac{1}{2}}. \quad (56)$$

$$\sum_{\substack{m_1=1 \\ m_1 \neq \hat{k}_b^{\text{MAP}}}}^{K_b} \sum_{\substack{m_2=1 \\ m_2 \neq \hat{k}_b^{\text{MAP}}}}^{K_b} \mathbb{E} [\mathbb{P}[k_b = m_1 | \mathbf{y}]^4]^{\frac{1}{4}} \mathbb{E} [\mathbb{P}[k_b = m_2 | \mathbf{y}]^4]^{\frac{1}{4}}. \quad (57)$$

$$\sum_{\substack{m_1=1 \\ m_1 \neq \hat{k}_b^{\text{MAP}}}}^{K_b} \sum_{\substack{m_2=1 \\ m_2 \neq \hat{k}_b^{\text{MAP}}}}^{K_b} \underbrace{\mathbb{E} [\mathbb{P}[k_b = m_1 | \mathbf{y}]^4]^{\frac{1}{4}}}_{o(\frac{1}{N^{4\alpha}})} \underbrace{\mathbb{E} [\mathbb{P}[k_b = m_2 | \mathbf{y}]^4]^{\frac{1}{4}}}_{o(\frac{1}{N^{4\alpha}})}. \quad (58)$$

$$\mathbb{E} [\hat{\mathbf{s}}_{\text{LMMSE},n}^2(m_1) \hat{\mathbf{s}}_{\text{LMMSE},n}^2(m_2)]^{\frac{1}{2}} = \quad (59)$$

$$\underbrace{\mathbb{E} [\hat{\mathbf{s}}_{\text{LMMSE},n}^2(m_1) \hat{\mathbf{s}}_{\text{LMMSE},n}^2(m_2)]^{\frac{1}{2}}}_{\mathcal{O}(1)} = \quad (60)$$

where we have used the Cauchy-Schwarz inequality repeatedly in (55) and (56), the following (almost trivial) observation,

$$\mathbb{P}[\mathbf{z} = \mathbf{z}]^\beta \leq \mathbb{P}[\mathbf{z} = \mathbf{z}], \quad \forall \beta \geq 1, \quad (61)$$

in (57), and (46) in (58). Since (54) is a sum of N terms as in (55), we obtain

$$\mathbb{E} [\|\delta(\mathbf{y})\|_2^2] = o\left(\frac{1}{N^{\alpha-1}}\right). \quad (62)$$

Moving to the second term in (52), we have,

$$\mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}]^2 \|\hat{\mathbf{s}}_{\text{MAP-QLMMSE}}\|_2^2 \right] \leq \quad (63)$$

$$\mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}] \|\hat{\mathbf{s}}_{\text{MAP-QLMMSE}}\|_2^2 \right] \leq \quad (64)$$

$$\mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}]^2 \right]^{\frac{1}{2}} \mathbb{E} [\|\hat{\mathbf{s}}_{\text{MAP-QLMMSE}}\|_2^4]^{\frac{1}{2}} \leq \quad (65)$$

$$\underbrace{\mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}]^2 \right]^{\frac{1}{2}}}_{o(\frac{1}{N^{2\alpha}})} \underbrace{\mathbb{E} [\|\hat{\mathbf{s}}_{\text{MAP-QLMMSE}}\|_2^4]^{\frac{1}{2}}}_{\mathcal{O}(N)} = o\left(\frac{1}{N^{\alpha-1}}\right), \quad (66)$$

where we have used (61) in (63) and (65), the Cauchy-Schwarz inequality in (64), and (45) in (66). As for the magnitude of the last term in (52), we similarly obtain,

$$\left| \Re \left\{ \mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}] \delta^H(\mathbf{y}) \hat{\mathbf{s}}_{\text{MAP-QLMMSE}} \right] \right\} \right| \leq \quad (67)$$

$$\left| \mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}] \delta^H(\mathbf{y}) \hat{\mathbf{s}}_{\text{MAP-QLMMSE}} \right] \right| \leq \quad (68)$$

$$\mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}]^2 \right]^{\frac{1}{2}} \mathbb{E} [\|\delta^H(\mathbf{y}) \hat{\mathbf{s}}_{\text{MAP-QLMMSE}}\|^2]^{\frac{1}{2}} \leq \quad (69)$$

$$\underbrace{\mathbb{E} \left[\mathbb{P}[k_b \neq \hat{k}_b^{\text{MAP}} | \mathbf{y}]^2 \right]^{\frac{1}{2}}}_{o(\frac{1}{N^{2\alpha}})} \underbrace{\mathbb{E} [\|\delta^H(\mathbf{y}) \hat{\mathbf{s}}_{\text{MAP-QLMMSE}}\|^2]^{\frac{1}{2}}}_{\mathcal{O}(N)} = \quad (70)$$

$$o\left(\frac{1}{N^{\alpha-1}}\right), \quad (71)$$

where we have used, again, the Cauchy-Schwarz inequality in (68), (61) in (69), and (45) in (70). We note in passing that the term on the right in (70) may be bound more tightly, but this is not necessary for the following steps of this proof.

We have established upper bounds on the magnitudes of the terms in (52). Hence, using (62), (66) and (71), we now have

$$\mathbb{E} [\|\Delta(\mathbf{y})\|_2^2] = o\left(\frac{1}{N^{\alpha-1}}\right), \quad (72)$$

which, together with (51), yields

$$\varepsilon_{\text{MMSE}}^2(N) = \varepsilon_{\text{MAP-QLMMSE}}^2(N) + o\left(\frac{1}{N^{\alpha-1}}\right). \quad (73)$$

By the definition of the MMSE estimator, the (trivial) upper bound

$$\varepsilon_{\text{MMSE}}^2(N) \leq \varepsilon_{\text{MAP-QLMMSE}}^2(N) \implies \frac{\varepsilon_{\text{MMSE}}^2(N)}{\varepsilon_{\text{MAP-QLMMSE}}^2(N)} \leq 1 \quad (74)$$

holds for any $N \in \mathbb{N}_+$. Therefore, and since (73) hold for any $\alpha \in \mathbb{R}_+$, we can always choose some α to have

$$\frac{\varepsilon_{\text{MMSE}}^2(N)}{\varepsilon_{\text{MAP-QLMMSE}}^2(N)} = 1 - o\left(\frac{1}{N^{\alpha}}\right), \quad (75)$$

where we used $\varepsilon_{\text{MAP-QLMMSE}}^2(N) = \mathcal{O}(N)$, proving the theorem.

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