



# Incorporating Fairness in Large-scale Evacuation Planning

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## ABSTRACT

Evacuation planning is an essential part of disaster management where the goal is to relocate people in a safe and orderly manner. Existing research has shown that such problems are hard to approximate and current methods are difficult to scale to real-life applications. We introduce a notion of fairness and two related objectives while studying evacuation planning, namely: minimizing maximum inconvenience and minimizing average inconvenience. We show that both problems are not just NP-hard to solve exactly, but in fact are NP-hard to approximate. On the positive side, we present a heuristic optimization method MIP-LNS, based on the well-known Large Neighborhood Search framework, that can find good approximate solutions in reasonable amount of time. We also consider a multi-objective problem where the goal is to minimize both objectives and solve it using MIP-LNS. We use real-world road network and population data from Harris County in Houston, Texas (a region that needed large-scale evacuations in the past), and apply MIP-LNS to calculate evacuation plans for the area. We compare the quality of the plans in terms of evacuation efficiency and fairness. We find that the solutions to the multi-objective problem are superior in both of these aspects. We also perform statistical tests to show that the solutions are significantly different.

## CCS CONCEPTS

• **Theory of computation** → **Mixed discrete-continuous optimization**; **Network flows**; *Random search heuristics*; Proof complexity.

## KEYWORDS

Evacuation, Fairness, Inconvenience, Routing, Scheduling, Mixed Integer Program, NP-hard, Large Neighborhood Search

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## 1 INTRODUCTION

Evacuation plans are essential to ensure the safety of people after human-initiated or natural disasters such as hurricanes, tsunamis, wildfires, bioterrorism, toxic chemical spills [5, 6, 9, 21, 32]. Emergency management organizations have often made plans on best ways to evacuate individuals during such disasters; see [5, 9, 21, 32]. During the past hurricane seasons, for example, many states such as Florida, Texas, Louisiana, and Mississippi, executed large-scale evacuations in affected regions. Examples of hurricanes when such evacuations were carried out include Katrina & Rita (2005), Ike & Gustav (2008), Irma & Harvey (2017), Laura (2020), and Ida (2021). The recent category four hurricane, Ida caused a total of \$75 billion in damages and 55 deaths in the United States alone [6]. We are also anticipating that the 2022 hurricane season will have above-normal activity [31]. To give a sense of the scale of evacuations due to such hurricanes, about 2.5 million individuals were evacuated from the coastal areas of Texas [5] before the landfall of Hurricane Rita. It is therefore crucial for cities or communities to have effective and efficient evacuation plans in place, to be sustainable. Any such plan needs to have two components: (i) Evacuation Routes, which are paths that the evacuees will take to egress out of the area under danger, and (ii) Evacuation Schedule which dictates when people should leave from different regions. Unfortunately, optimizing over both is often theoretically hard and computationally intractable in a realistic scenario. Thus, it remains open to provide an approximation algorithm with a reasonable runtime given real-life data.

In addition to the efficiency aspect of an evacuation, it is also important to reduce inequality among evacuees so that no person faces any undue burden. For instance, assigning longer routes or delaying the evacuation of certain evacuees to improve overall evacuation efficiency could place extra burden on them. Moreover, people living in low-lying areas incur a higher level of risk due to potential flooding during hurricanes. It is, therefore, necessary to evacuate them as early as possible. This might not happen if we design evacuation plans by optimizing efficiency only.

**Our Contributions** As our first contribution, we define two objective functions to capture the notion of fairness in the context of evacuation. We then formulate the optimization problems: Min-Max Fair Dynamic Confluent Flow Problem (MM-FDCFP) and Total Fair Dynamic Confluent Flow Problem (T-FDCFP). In MM-FDCFP, the



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goal is to determine a dynamic confluent flow of evacuees so that the maximum average inconvenience cost incurred by evacuees at different regions is minimized. In contrast, the aim in T-FDCFP is to minimize total inconvenience cost of all evacuees. The problems are formally defined in Section 3. We show in Section 4 that both MM-FDCFP and T-FDCFP are hard to approximate. We also define the Hybrid Fair Dynamic Confluent Flow Problem (H-FDCFP) where the goal is to simultaneously optimize for both objectives.

As our second contribution, we present a Large Neighborhood Search heuristic (MIP-LNS) for MM-FDCFP, T-FDCFP and H-FDCFP where we use Mixed Integer Program (MIP) solvers in conjunction with combinatorial methods. MIP-LNS, designed based on the Large Neighborhood Search (LNS) framework, starts with an initial feasible solution and then uses MIP solvers to solve problems that explore the neighborhood of the solution. It significantly reduces the network, providing a tractable approximation to the problems. It can also be adapted easily for other objective functions.

As our final contribution, we apply MIP-LNS on a real-world situation: evacuating the 1.5 million households of Harris County in Houston, Texas that is often affected by hurricanes (e.g. Rita, Ike, Harvey, Laura). We use road network data from HERE maps [18] and population data generated by Adiga et al. [3] to construct a realistic problem instance. MIP-LNS was able to efficiently find solutions to all three problems for this instance. We compare these solutions in terms of evacuation completion time, average evacuation time, and the fairness objectives. We observe that the MM-FDCFP objective ensures the inconvenience cost at all the regions to be small but resulting evacuation completion time is high. In contrast, the T-FDCFP objective induces a lower evacuation completion time but at the expense of high inconvenience cost at some regions. By optimizing for both in H-FDCFP, we were able to find solutions that have low inconvenience cost and low evacuation completion time.

## 2 RELATED WORK

Researchers have approached the evacuation planning problem primarily with a focus on efficiency. Hamacher and Tjandra [16] formulated it as a dynamic network flow optimization problem and used mathematical optimization methods to solve it. However, the computational cost of their proposed method was prohibitively expensive. This led to several heuristic methods [20, 23, 29] that provide unbounded suboptimal solutions. However, these methods are designed to solve the routing problem only and they either do not consider the scheduling problem at all or propose simple schemes such as letting evacuees leave at a constant rate. Even and Pillac *et al.* [10], and Romanski and Van Hentenryck *et al.* [28] formulated the problem as Mixed Integer Programs and proposed optimization techniques to find (bounded- sub)optimal solutions. However, their proposed techniques are not scalable enough to handle city or county-scale evacuation planning problems. A comprehensive review of existing works on evacuation planning and management can be found in the survey paper by Bayram [4].

The use of convergent evacuation routes has been explored in the literature [10, 15, 17, 28], where all evacuees coming to an intersection follow the same path afterwards. This is also known as confluent flow [7]. Golin *et al.* [12] investigated the single-sink confluent quickest flow problem where the goal is minimizing the

time required to send supplies from sources to a single sink. They showed that the problem cannot be approximated in polynomial time within a logarithmic approximation factor.

Compared to the existing research works on efficient evacuation planning, there are few works that have considered fairness. Aalami and Kattan [1, 2] approached the problem of fair trip planning in the context of short-notice and transit-based emergency evacuation. Their proposed method aims to allocate resources such as fleet of transit vehicles and shelters to different regions in a fair manner. In contrast, we consider the planning problem where evacuees use their own personal vehicle and our goal is to prescribe them evacuation routes and schedule. Evacuation by personal vehicles was considered by Yan and Liu *et al.* [33], and Oh and Yu *et al.* [25]. Yan and Liu *et al.* [33] defined a risk associated with each location by looking at its distance from a hazard and combined it with the efficiency objective of total evacuation time. Oh and Yu *et al.* [25] also considered the same notion of risk and defined the ‘suffering’ of each evacuee based on their risk level and required time to reach safety. The authors evaluated the fairness of an evacuation plan by looking at how equally the evacuees suffered and quantified it using the Gini Coefficient. In contrast to these works, we define an inconvenience cost for each evacuee that considers the risk level and the inconvenience caused by the length and congestion on the route. Minimizing inconvenience, instead of time to reach safety, is important because an evacuee may be situated far away from safe locations and therefore naturally need longer time to reach safety. We formally define the inconvenience of an evacuee in Section 3.

Heuristic search methods are generally applied to problems that are computationally intractable, to find good solutions in a reasonable amount of time. The Large Neighborhood Search (LNS) framework [30] has been successfully applied to various hard combinatorial optimization problems in the literature [26]. Very recently, Li *et al.* [22] proposed MAPF-LNS, where the LNS framework was used to find solutions for the Multi-Agent Path Finding Problem. Due to the hardness of the evacuation planning problem (Section 4), we have also designed our algorithm based on the LNS framework.

## 3 PROBLEM FORMULATION

In this section, we introduce some preliminary terms that we use in our problem formulation. Then we define two objective functions to capture the notion of fairness in the context of evacuations. This allows us to formally define the three optimization problems: MM-FDCFP, T-FDCFP, and H-FDCFP. Next, we describe how we construct and use time expanded graphs to model the flow of evacuees over time using a sample problem instance. Finally, we present Mixed Integer Program (MIP) formulations of MM-FDCFP and T-FDCFP.

**Definition 3.1.** A *road network* is a directed graph  $\mathcal{G} = (\mathcal{N}, \mathcal{A})$  where every edge  $e \in \mathcal{A}$  has (i) a capacity  $c_e$ , representing the number of vehicles that can enter the edge at a given time and (ii) a travel time  $T_e$  representing the time it takes to traverse the edge.

**Definition 3.2.** Given a road network, a *single dynamic flow* is a flow  $f$  along a single path and timestamps  $a_v$ , representing the arrival time of the flow at vertex  $v$ , that obeys the travel times. In other words,  $a_v - a_u \geq T_{uv}$ . A *valid dynamic flow* is a collection of single dynamic flows where no edge at any point in time exceeds its edge capacity.

**Definition 3.3.** An *evacuation network* is a road network that specifies  $\mathcal{E}, \mathcal{S}, \mathcal{T} \subset \mathcal{N}$ , representing a set of source, safe and transit nodes respectively. Furthermore, for each source node  $k \in \mathcal{E}$ , let  $W(k)$  and  $d_k$  represent the set of evacuees and the number of evacuees at source  $k$  respectively.

For the purpose of scheduling an evacuation, we observe that once an evacuee has left their home, it is difficult for them to pause until they reach their desired destination. We also assume that people from the same location evacuate to the same destination. Similarly, we assume that if two evacuation routes meet, they should both be directed to continue to the same location.

**Definition 3.4.** Given an evacuation network, we say a valid dynamic flow is an *evacuation schedule* if the following are satisfied:

- all evacuees end up at some safe node,
- no single dynamic flow has any intermediary wait-time (i.e.  $a_v - a_u = T_{uv}$  and,
- the underlying flow (without considering time) is confluent, where if two single dynamic flows use the same vertex (possibly at different times), their underlying path afterwards is identical.

In order to define the objective functions, we introduce a notion of fairness based on the idea of risk and inconvenience.

Due to various natural factors such as proximity to shelter, elevation, etc., it is very natural for different locations to have different levels of risk and thus different urgency to evacuate. Thus, it is reasonable to incorporate the risk  $r_k$  for source node  $k$ .

Now, let  $\tau_k$  be the average evacuation time of evacuees at source  $k$ , if they were the only evacuees in the network and they followed the quickest path [8] to safety. Let  $t_i$  be the actual evacuation time of evacuee  $i \in W(k)$  for a given evacuation schedule. Then, the *inconvenience cost* ( $s_i$ ) of evacuee  $i$ , in this schedule, is defined as:

$$s_i = r_k(t_i - \tau_k) \quad (1)$$

Since all evacuees from the same location needs to evacuate to the same safe spot, it is natural to consider them as a collective and use their average inconvenience as a measure of the effectiveness of the schedule for that location. Then, two natural objectives arise, (i) minimizing the worst average inconvenience:  $\min \left\{ \max_{k \in \mathcal{E}} \frac{1}{d_k} \sum_{i \in W(k)} s_i \right\}$  or (ii) minimizing the total inconvenience:  $\min \left\{ \sum_{k \in \mathcal{E}} \sum_{i \in W(k)} s_i \right\}$ .

Minimizing the max average inconvenience is equivalent to:

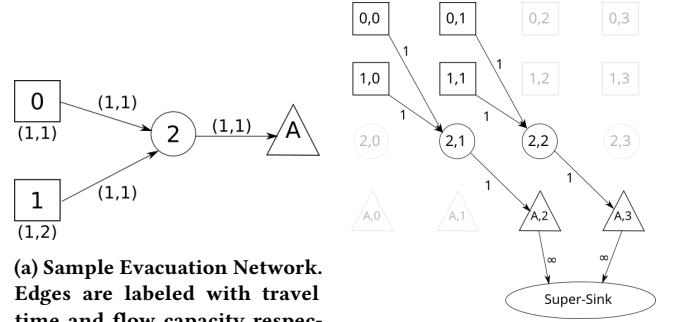
$$\min \left\{ \max_{k \in \mathcal{E}} \frac{r_k}{d_k} \left( \sum_{i \in W(k)} t_i \right) - r_k \tau_k \right\} \quad (2)$$

Minimizing the total inconvenience is equivalent to:

$$\min \sum_{k \in \mathcal{E}} r_k \sum_{i \in W(k)} t_i \quad (3)$$

Detailed derivations are provided in Appendix A. Now, we can formally define our two problems.

**Problem 1.** Min-Max Fair Dynamic Confluent Flow Problem (MM-FDCFP). Given an evacuation network, let  $T_{max}$  represent an upper bound on evacuation time. Find an evacuation schedule such that all evacuees arrive at some safe node before time  $T_{max}$  while minimizing  $\max_{k \in \mathcal{E}} \frac{r_k}{d_k} \left( \sum_{i \in W(k)} t_i \right) - r_k \tau_k$ .



(a) Sample Evacuation Network. Edges are labeled with travel time and flow capacity respectively. Source, safe and transit nodes are denoted by squares, triangles, and circles respectively. Source nodes are labeled with number of evacuees and risk level respectively.

(b) Time Expanded Graph (TEG) for the Sample Network. Edges are labeled with capacity. Construction of this TEG sets an upper bound of 3 time units for evacuation completion.

Figure 1: Sample Problem Instance

**Problem 2.** Total Fair Dynamic Confluent Flow Problem (T-FDCFP). Given an evacuation network, let  $T_{max}$  represent an upper bound on evacuation time. Find an evacuation schedule such that all evacuees arrive at some safe node before time  $T_{max}$  while minimizing  $\sum_{k \in \mathcal{E}} r_k \sum_{i \in W(k)} t_i$ .

Additionally, we define the following problem where we optimize for both objectives:

**Problem 3.** Hybrid Fair Dynamic Confluent Flow Problem (H-FDCFP). Given an evacuation network, let  $T_{max}$  represent an upper bound on evacuation time. Find an evacuation schedule such that all evacuees arrive at some safe node before time  $T_{max}$  while minimizing both  $\max_{k \in \mathcal{E}} \frac{r_k}{d_k} \left( \sum_{i \in W(k)} t_i \right) - r_k \tau_k$  and  $\sum_{k \in \mathcal{E}} r_k \sum_{i \in W(k)} t_i$ .

To capture the flow of the evacuees over time, we use a time expanded graph (TEG) denoted by  $\mathcal{G}^x = (\mathcal{N}^x = \mathcal{E}^x \cup \mathcal{T}^x \cup \mathcal{S}^x, \mathcal{A}^x)$ . To build it, we first fix a time horizon  $\mathcal{H}$  and discretize the temporal domain into discrete timesteps of equal length. Then we create copies of each node at each timestep within  $\mathcal{H}$ . After that, for each edge  $e(u, v)$  in the road network, we create edges in the TEG as  $e_t(u_t, v_{t+T_e})$  for each  $t \leq \mathcal{H} - T_e$  where the edges  $e_t$  have the same flow capacity as  $e$ . Finally, we add a super sink node  $v_t$  that connects to the nodes  $u_t$  for each  $u \in \mathcal{S}$  and each  $t \leq \mathcal{H}$ .

A sample evacuation network and its corresponding TEG with time horizon  $\mathcal{H} = 3$  are shown in Figure (1a-1b). The source, safe and transit nodes are denoted by squares, triangles, and circles respectively. In the TEG, there may be some nodes that are (i) not reachable from the source nodes, or (ii) no safe node can be reached from these nodes within the time horizon. These nodes are greyed out in Figure 1b.

The optimal solution of MM-FDCFP for this sample problem instance is to use the routes  $0 \rightarrow 2 \rightarrow A$  from source node 0 and  $1 \rightarrow 2 \rightarrow A$  from source node 1, where the evacuee at source node 0 and 1 leave at timestep 1 and 0 respectively. Letting the evacuee from source node 1 leave first is better because they face a higher level of risk than the evacuee at source node 0. T-FDCFP and H-FDCFP have the same optimal solution.

### 3.1 Mixed Integer Program Model

We formulate MM-FDCFP and T-FDCFP as Mixed Integer Programs (MIPs). In our formulation, we use two types of variables: (i) Binary variable  $x_e, \forall e \in \mathcal{A}$ , which will be equal to one if and only if the edge  $e$  is used for evacuation. Otherwise, it will be zero. (ii) Continuous variable  $\phi_{k,e}, \forall k \in \mathcal{E}, e \in \mathcal{A}^x$ , which denotes the flow of evacuees on edge  $e$  from source node  $k$ .

$$\text{MM-FDCFP or T-FDCFP Obj} \quad (4)$$

$$\text{s.t. } \sum_{e \in \delta^+(k)} x_e = 1 \quad \forall k \in \mathcal{E} \quad (5)$$

$$\sum_{e \in \delta^+(i)} x_e \leq 1 \quad i \in \mathcal{T} \quad (6)$$

$$\sum_{e \in \delta^+(k)} \sum_{t \leq \mathcal{H}} \phi_{k,e,t} = d_k \quad \forall k \in \mathcal{E} \quad (7)$$

$$\sum_{e \in \delta^-(i)} \phi_{k,e} = \sum_{e \in \delta^+(i)} \phi_{k,e} \quad \forall i \in \mathcal{N}^x \setminus \{v_t\} \quad (8)$$

$$\sum_{k \in \mathcal{E}} \phi_{k,e,t} \leq x_e c_{e,t} \quad \forall e \in \mathcal{A}, t \leq \mathcal{H} \quad (9)$$

$$\phi_{k,e} \geq 0 \quad \forall k \in \mathcal{E}, e \in \mathcal{A}^x \quad (10)$$

$$x_e \in \{0, 1\} \quad \forall e \in \mathcal{A} \quad (11)$$

The MIP formulation of MM-FDCFP as well as T-FDCFP is given by (4–11) when used with the corresponding objective function. To solve H-FDCFP we use a hierarchical approach that is described in Section 5.2. The objective and constraints of the above MIP model are explained in Table 1. The constraint that evacuation completion time needs to be less than the given upper bound is implicit in the model, as we set the time horizon of the TEG to this upper bound.

	Explanation
Objective (4)	The MM-FDCFP or the T-FDCFP objective.
Constraint (5)	Ensures that there is only one outgoing edge from each evacuation node.
Constraint (6)	Ensures that at each transit node, there is at most one outgoing edge. This is necessary for convergent routes.
Constraint (7)	Ensures that the total flow coming out of every evacuation node is equal to the number of evacuees at the corresponding node.
Constraint (8)	Flow conservation constraint, incoming flow equals outgoing flow. $\delta^-(i)$ and $\delta^+(i)$ denote the set of incoming and outgoing edges to/from node $i$ , respectively.
Constraint (9)	Flow capacity constraint: total flow on an edge will not exceed its capacity. Also, flows are only allowed on assigned edges.
Constraint (10)	Flow variables are continuous and non-negative.
Constraint (11)	Edge assignment variables are binary.

**Table 1: Model (4–11) Explanation**

### 4 INAPPROXIMABILITY RESULTS

In this section, we show that our problems, even with one safe node is hard to approximate.

**THEOREM 1.** *Given an arbitrarily large evacuee size of  $M$ , it is NP-hard to approximate MM-FDCFP and T-FDCFP to a factor of  $O(M/n)$  where  $M$  is the size of the total population, even when there are only two sources and one safe node.*

Since these problems are closely related to the single-sink Confluent Quickest Flow Problem in [12], the proofs of their hardness are also very similar. The main difference is in its analysis since the objective of the problems differ. The hardness result relies on the NP-hardness of the capacitated version Two Distinct Path Problem.

**Problem 4** (Two Distinct Path Problem). Let  $G$  be a graph with two sources  $x_1, x_2$  and two sinks  $y_1, y_2$ . Every edge is labelled with either 1 or 2. Determine if there exists two edge-disjoint paths  $P_1, P_2$  such that  $P_i$  is a path from  $x_i$  to  $y_i$  for  $i = 1, 2$  and  $P_2$  only uses edges with label 2 ( $P_1$  can use any edge).

The above problem is known to be NP-hard [14]. Other variations of the problem such as uncapacitated, undirected/directed, edge/node-disjoint paths are also known to be hard (see e.g. [11], [24] and [27]). The proof of hardness for our two problems are very similar. Thus, we only provide the proof in the context of MM-FDCFP.

**PROOF OF THEOREM 1.** Given an instance  $\mathcal{I}$  of the Two Disjoint Path Problem, consider constructing the following graph  $G$  where we attach safe node  $t$  to  $y_1, y_2$  with an edge of capacity 1, 2 respectively. For  $i = 1, 2$ , we also add a source  $s_i$  and attach it to  $x_i$  with an edge of capacity of  $i$ . Every edge with label  $i$  also has capacity  $i$ . Sources  $s_i$  has  $M * i$  evacuees for some large value of  $M$ , resulting in a total of  $3M$  evacuees. Both sources have a risk factor of 1. Each edge has a travel time of 1. The upperbound completion time  $T_{max}$  is set to be  $M^2 n$ . This ensures that the problem has a feasible solution, otherwise, it is futile to provide any approximations. It also follows from construction that for certain choices of  $T_{max}$ , the problem is equivalent to the Two-Disjoint Path Problem, and thus hard to determine feasibility.

First, consider the case where there exists two disjoint paths in  $\mathcal{I}$ . To upperbound the inconvenience cost at source  $i$ , we need to upperbound its average evacuation time in an optimal schedule and lowerbound the average evacuation time if the source  $i$  is the only source in the network. For the first upperbound, consider a valid schedule that sends  $i$  evacuees at every time step, where the last group of people leaves their sources at time  $M$ . Since each path has length at most  $M + n \approx M$ , the  $i$  evacuee that left source  $s_i$  at time  $k$  is guaranteed to arrive by time  $k + M$ . On average, the arrival time of the evacuees from source  $s_i$  is at most  $(M + n)/2$ . If source  $i$  is the only source in the network, since the first edge incident to  $s_i$  has capacity  $i$ , it takes at least  $M$  rounds to finish evacuating the people at its location. This results in an average of at least  $M/2$ . Therefore, the maximum inconvenience cost is at most  $(M + n)/2 - M/2 \leq n/2$ .

Now, consider the instance where  $\mathcal{I}$  does not have two disjoint paths. To lowerbound the average inconvenience at source  $i$ , we need to lowerbound its average evacuation time in a schedule and upperbound the average time if it was the only source. Consider the two paths  $P_1, P_2$  in a solution to MM-FDCFP in  $G$ . If  $P_1, P_2$  intersects



before  $t$ , since it is a confluent flow, the edge following their node of intersection is a single-edge cut that separates the sources from the sink. If the two paths only intersect at  $t$ , since we are in a NO-instance of  $\mathcal{I}$ ,  $P_2$  must have used an edge of capacity 1. Then, deleting that edge along with  $s_1x_1$  also separates the sources from the sink. Note that in both cases, the cut has capacity at most 2. Then, at every time step, at most 2 evacuees can cross the cut. Thus, for all  $3M$  evacuees to cross the cut, it takes at least  $3M/2$  time steps. Due to this bottleneck, it follows that the smallest total evacuation time is at least  $\sum_{k=1}^{3M/2} k \geq 9M^2/4$ , giving an average evacuation time of at least  $3M/4$ . Let  $T_1, T_2$  be the average evacuation time for source  $s_1, s_2$  respectively. It follows that the overall average of all evacuees is at most  $\max(T_1, T_2)$ , providing a lowerbound to their average evacuation time in the schedule. If source  $i$  is the only source in the network, note that there exists a path from  $s_2$  to  $t$  that only uses path of capacity 2 otherwise  $\mathcal{I}$  is a hard instance. Therefore,  $s_2$  can evacuate its population along the path in  $M$  rounds, achieving an average completion time of at most  $(M+n)/2$ . Similarly for  $s_1$ , there exist a path (of any label) to  $t$ , giving it an average evacuation time of at most  $(M_n)/2$ . Since  $T_1, T_2$  averages to  $3M/4$ , the one with a larger value has an inconvenience cost of at least  $3M/4 - M/2 = M/4$ .

Since the YES and NO instances cause the objective value of MM-FDCFP to have a gap of  $O(M/n)$  our result follows.  $\square$

## 5 HEURISTIC OPTIMIZATION

As shown in Section 4, solving MM-FDCFP and the T-FDCFP are both computationally hard. For this reason, we present the heuristic search method MIP-LNS where we use MIP solvers in conjunction with combinatorial methods.

First, we calculate an initial feasible solution in two steps: (i) calculating an initial convergent route set, and (ii) calculating the schedule that minimizes the target objective using the just calculated initial route set (Appendix B). To calculate the initial route set, we take the shortest path from each source to its nearest safe node by road. To calculate the schedule, we use the just calculated route set to fix the binary variables  $x_e$  in model (4-11). This gives us a linear program that can be solved optimally to get the schedule.

Next, we start searching for better solutions in the neighborhood of the solution at hand using MIP-LNS (Algorithm 1). It runs for  $n$  iterations, where in each iteration, we select  $q = (100 - p)\%$  of source locations uniformly at random and keep their routes fixed. This reduces the size of the MIP as we have fixed values for a subset of the variables. We then optimize this ‘reduced’ MIP model using the Gurobi [13] MIP solver. Essentially, we are searching for a better solution in the neighborhood where the selected  $q\%$  routes are already decided. Any solution found in the process will also be a feasible solution for the original problem. If we find a better solution with an evacuation completion time  $T'$  that is less than the current time horizon,  $T$ , then we also update the model by setting the time horizon to  $T'$ . When resetting the time horizon, we (i) remove edges in the time expanded graph whose start or end node have a time stamp greater than  $T'$ , and (ii) we prune the TEG by removing nodes that are unreachable from the evacuation nodes, and nodes from which none of the safe nodes can be reached within time  $T'$ . This pruning process reduces the number of variables in the MIP model and simplifies the constraints. At the end of each iteration,

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### Algorithm 1: MIP-LNS Method

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**Input:** Initial solution:  $sol$ , Time Expanded Graph:  $TEG$ , Time Horizon:  $T$ , Model to optimize:  $model$ , Percent of routes to update in the first iteration:  $p$ , Number of Iterations:  $n$

**Output:** Solution of  $model$

```

1 for 1 to  $n$  do
2   Select  $(100-p)\%$  of the source locations uniformly at
   random. Let their set be  $S$ .
3   Fix the routes from the source locations in  $S$ . Set  $x_e = 1$ 
   if  $e$  is on any of the routes from  $S$  in  $sol$ .
4    $sol \leftarrow$  Solution of reduced  $model$  from a MIP solver
5    $T' \leftarrow$  evacuation completion time for solution  $sol$ 
6   if  $T - T' > +threshold$  then
7     Update the  $model$  by setting the time horizon to  $T'$ .
     Prune  $TEG$  and  $model$  by removing:
8     (i) nodes that are unreachable from the evacuation
     nodes within time horizon  $T'$ , and
9     (ii) nodes from which none of the safe nodes can be
     reached within time horizon  $T'$ 
10  Increase  $p$ 
11 return  $sol$ 

```

---

we increase the value of  $p$ . Note that, when  $p = 100$ , we will be solving the original optimization problem. In our experiments, we set the initial value of  $p$  to 50 and then gradually increased it to 65.

When solving the reduced problem in each iteration (line 4), we use (i) a time limit, and (ii) a parameter *threshold\_gap* to decide when to stop. MIP solvers keep track of an upper bound ( $Z_U$ ) (provided by the current best solution) and a lower bound ( $Z_L$ ) (obtained by solving relaxed LP problems) of the objective value. We stop the optimization when the relative gap between these two becomes smaller than the *threshold\_gap*. In our experiments, we set this threshold to 10%. In some iterations, it may happen that the current solution is already within this threshold, in that case, the algorithm will simply continue to the next iteration.

### 5.1 Selection of Sources

In MIP-LNS, instead of selecting source locations (to optimize over) using a uniform random distribution (Algorithm 1 line 2), we can assign weights to source locations according to the inconvenience cost of evacuees at those locations and then use a probability distribution calculated based on these values.

---

### Algorithm 2: Heuristic Selection

---

**Input:** Current solution:  $sol$ , Number of Sources to Optimize Over:  $m$

**Output:** Set of Sources to Optimize Over  $S_{opt}$

```

1  $W \leftarrow$  List containing average inconvenience of evacuees at
   the sources for  $sol$ 
2  $P \leftarrow$  Normalized  $W$  so that the values sum up to 1
3  $S_{opt} \leftarrow$  Select  $m$  sources from the distribution  $P$  at random
   without replacement
4 return  $S_{opt}$ 

```

---

Algorithm 2 shows the process in detail. First, we calculate the average inconvenience at each of the source locations for the current solution. Then, we normalize the values so that they sum up to one and represent a probability distribution. Finally, we randomly pick the required number of sources from this distribution. The motivation for using this selection process is to focus on the source locations that currently have high inconvenience cost.

## 5.2 Solving H-FDCFP with MIP-LNS

To solve H-FDCFP, we can take a linear combination of the MM-FDCFP objective and the T-FDCFP objective and then optimize the new objective using MIP-LNS. An alternative method is to use a hierarchical approach of solving multi-objective problems. Here, each objective is assigned a priority and the objectives are optimized in a decreasing order of priority.

As our goal in this paper is to focus on the fairness aspect of evacuation, we used the hierarchical approach to solve H-FDCFP. Specifically, we define the MM-FDCFP objective (Equation 2) as the primary objective (i.e. high priority) as it ensures that the maximum inconvenience over the sources is minimized. We then use the T-FDCFP objective (Equation 3) as the secondary (i.e. lower priority) objective. During the optimization process, in each iteration of MIP-LNS, we first optimize the ‘reduced’ MIP for the primary objective. Once the optimality gap for the primary objective becomes less than a threshold or a predefined time limit is exceeded, we start optimizing for the second objective. At this stage, we do not allow any degradation in the primary objective.

## 6 EXPERIMENT RESULTS

In this section, we present details of our problem instance, its MIP formulation, and a pruning technique that we used to reduce the MIP size. Then, we provide analyses of the algorithm execution and the solutions to MM-FDCFP, T-FDCFP and H-FDCFP.

### 6.1 Problem Instance

We use data from HERE maps [18] to construct a road network for our study area Harris County, Houston, Texas. The network contains roads of five (1 to 5) different function classes. It has a hierarchical structure, where lower-level roads (function class 3/4) are connected to higher-level roads (function class 1/2) through entrance and exit ramps. In our experiments, we consider the nodes (of the road network) which connect and lead from function class 3/4 roads to function class 1/2 roads as the source nodes. We then consider the problem of (i) when should evacuees enter the function class 1/2 roads and (ii) how to route them through the function class 1/2 roads to safety. We use the synthetic population data of Adiga *et al.* [3] to extract the location of each household in the study area. We consider that one vehicle is used per household for evacuation. We fix the nearest exit ramp to each household as their source. As safe location, we select eight locations at the periphery of Harris County, which are on major roads. A visualization of the problem instance is presented in Figure 2. Additional details regarding the final network and the study area are provided in Table 2.

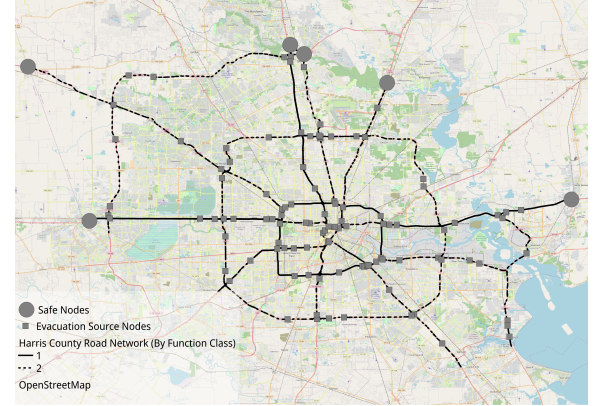


Figure 2: Harris County Problem Instance.

# Nodes in the road network	927
# Edges in the road network	1339
# Source locations	109
Number of Households in the study area	~ 1.5M

Table 2: Harris County Dataset Summary

### 6.2 Pruning the MIP Model

The Time expanded graph for our problem instance, with a time unit of five minutes and a time horizon of 15 hours, contains 135,596 nodes and 207,867 edges. This makes the problem about four times as large as previously considered problems.

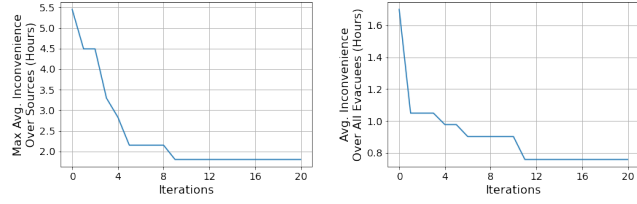
Property	Before Pruning	After Pruning
# Binary Variables	1339	1339
# Continuous Variables	~ 22.7M	~ 14.5M
# Constraints	~ 15M	~ 10M

Table 3: Properties of the MIP for our problem instance before and after pruning. The number of constraints goes down by 33.33% and the number of continuous variables by about 36%.

In our MIP formulation (Section 3.1), we define a flow variable for flow coming from each source to each edge of the TEG. However, not all edges  $e \in \mathcal{A}$  are reachable from a source node. For example, in our sample problem (Figure 1a) the edge (1, 2) is not reachable from source 0 and therefore a flow from source 0 can never reach any copies of this edge in the TEG. Based on this observation, we discard the unnecessary flow variables which reduces the number of variables and the number of constraints as shown in (Table 3).

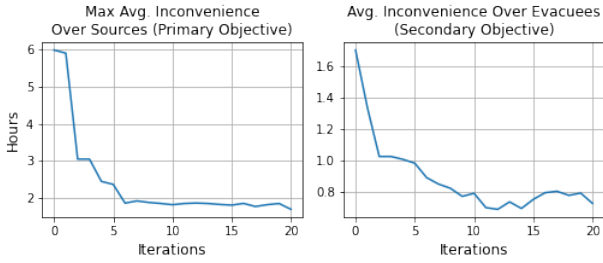
### 6.3 Analyses of Results

In our formulation of MM-FDCFP and T-FDCFP, we consider different levels of risk associated with source locations. However, estimating these risks requires accurate domain knowledge (Appendix C). For this reason, in our experiments we set the risk values of all source locations to one. This is equivalent to assuming that all the sources



(a) MM-FDCFP maximum average inconvenience cost over the sources at different iterations. (b) T-FDCFP average inconvenience cost over all evacuees at different iterations.

**Figure 3: MM-FDCFP and T-FDCFP objective values over the iterations of MIP-LNS. A decrease in value indicates improvement of the objective.**



**Figure 4: H-FDCFP primary and secondary objective values at different iterations of MIP-LNS. A decrease in value indicates improvement of the objective.**

face the same level of risk. However, our methodology is capable of using this information effectively when provided.

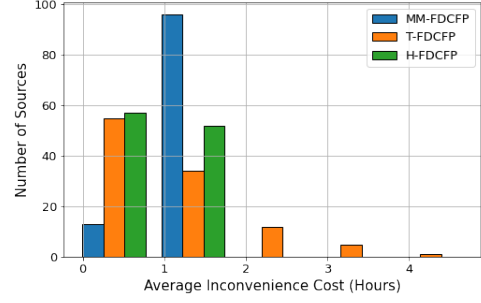
We performed all our experiments and subsequent analyses on a high-performance computing cluster, with 128GB RAM and 24 CPU cores allocated to our tasks. We tried two different methods to solve MM-FDCFP and T-FDCFP for our problem instance. These are:

- (1) Solve model (4–11) directly using the Gurobi MIP solver
- (2) Apply MIP-LNS

**6.3.1 Gurobi.** In this experiment, we used Gurobi [13] to directly solve model (4–11) for our problem instance. However, Gurobi reported that it needs about 700GB of memory to optimize the model, which exceeds our memory capacity. Note that, this large amount of memory is needed for the internal data structures Gurobi uses during the optimization process.

**6.3.2 MIP-LNS.** We ran MIP-LNS to solve MM-FDCFP, T-FDCFP and also H-FDCFP (as described in Section 5.2). We used two different selection methods for choosing the sources to optimize over in each iteration of MIP-LNS, the random selection method and the heuristic selection method (Algorithm 2). Furthermore, as we have randomness in both selection procedures, we ran ten experiment runs using different random seeds, for each problem and each selection method. In each run of MIP-LNS, we performed twenty iterations (Algorithm 1 line 1).

First, we look at the execution of a single run of MIP-LNS for MM-FDCFP, T-FDCFP and H-FDCFP where we used the heuristic selection method. Figure (3a) shows the MM-FDCFP objective value of the



**Figure 5: Histogram showing the distribution of average inconvenience cost at sources for the final solutions of MM-FDCFP, T-FDCFP and H-FDCFP. Some sources in T-FDCFP solution have high average inconvenience cost (last three orange bars on the right). However, the maximum average inconvenience cost in MM-FDCFP and H-FDCFP solutions is nicely bounded (second blue and green bar from the left).**

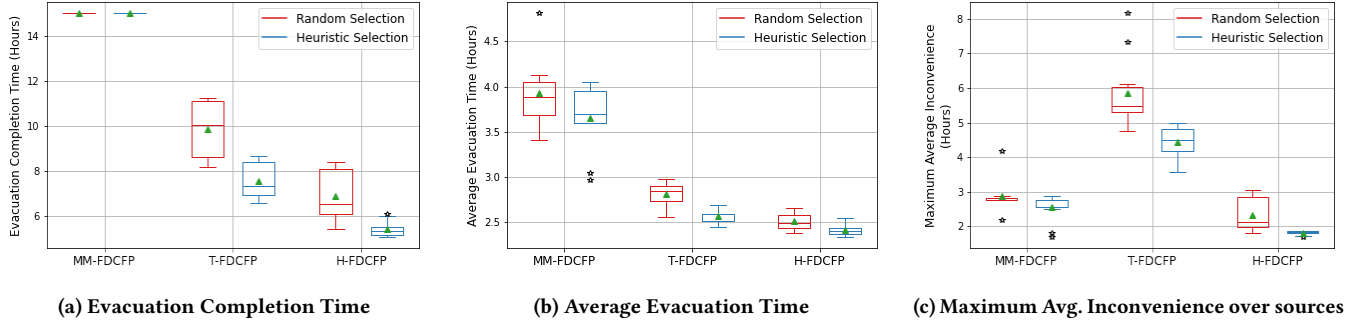
solution in hand at different iterations of MIP-LNS. We observe improvements of the objective value over the iterations starting from 5.45 hours for the initial solution and ending at a final value of 1.8 hours. Figure (3b) shows the average inconvenience cost of all evacuees (which is proportional to the T-FDCFP objective) over the iterations. Here, we start with a solution with average inconvenience of 1.7 hours and at the end find a solution with average inconvenience of 0.76 hours.

Figure 4 shows the primary and secondary objective values of H-FDCFP at different iterations of MIP-LNS. As we use a hierarchical approach, we see that the primary objective value steadily decreases over the iterations. However, as seen in some iterations, the secondary objective value can degrade. This is because, in the hierarchical approach, even if a new solution is worse than the old one in terms of the secondary objective, we accept it if it is better than the old one in terms of the primary objective.

Figure (5) shows the histogram of average inconvenience cost at the sources for the final solution of MM-FDCFP, T-FDCFP and H-FDCFP. We observe from this figure that the average inconvenience cost at several source locations is quite high (highest value of 4.83 hours) for the T-FDCFP solution. For the MM-FDCFP and H-FDCFP solutions, on the other hand, the maximum value is nicely bounded at 1.8 and 1.69 hours respectively. This indicates that in terms of minimizing maximum average inconvenience, MM-FDCFP and H-FDCFP solutions clearly outperform the T-FDCFP solution.

So far, we have only looked at one experiment run of the three problems. To do a complete comparison, we now look at all the solutions found from the ten experiment runs for each problem and each source selection procedure. To assess the quality of the solutions, we use the following three metrics: (i) Evacuation Completion Time, (ii) Average Evacuation Time over all evacuees, and (iii) Maximum Average Inconvenience over all sources. Figure 6 shows a comparison of the solutions in terms of the three metrics using boxplots. Our observations from these results are:

- (1) From Figure (6a), all ten MM-FDCFP solutions have an evacuation completion time of 15 hours which is the time horizon we have



**Figure 6: Box-plots showing comparison of MM-FDCFP, T-FDCFP and H-FDCFP solutions when using Random and Heuristic selection. Ten experiment runs were performed in each configuration. The performance metrics used are evacuation completion time, average evacuation time over all evacuees and maximum average inconvenience over all sources. H-FDCFP solutions generally outperform the MM-FDCFP and the T-FDCFP solutions in terms of all three metrics. The Heuristic selection method, in general, finds better solutions than the Random selection method in terms of the three metrics.**

	MM-FDCFP	T-FDCFP	H-FDCFP
Random Selection	2.1	1.28	1.16
Heuristic Selection	2.98	2.21	3.69

**Table 4: Average run-time (hours) of MIP-LNS for each problem and source selection procedure over ten experiment runs.**

used. This indicates that optimizing MM-FDCFP objective does not help in improving evacuation completion time.

- (2) Although T-FDCFP solutions have lower evacuation completion time and average evacuation time than MM-FDCFP solutions (Figure 6a, 6b), the maximum average inconvenience cost over the sources for these solutions can be very high (Figure 6c).
- (3) H-FDCFP solutions have the good properties of both MM-FDCFP and T-FDCFP solutions, outperforming both of them in general.
- (4) Comparing the red boxplots to the blue boxplots in Figures 6a, 6b, 6c, we observe that the solutions found by performing heuristic selection generally have lower values for the three metrics, compared to solutions found by random selection. However, as shown in Table 4, the better solutions found by using heuristic selection comes at a cost of additional run time.

Finally, we performed statistical tests to examine if the solutions found by solving the problems MM-FDCFP, T-FDCFP and H-FDCFP, using MIP-LNS and heuristic selection, are significantly different in terms of the above three performance metrics. The motivation for this experiment is to investigate whether optimizing for different objective functions (or their combination) provides us significantly different solutions. To do the test accurately, we did twenty more experiment runs (thirty in total), using the heuristic selection method, for all three problems. Then we performed the Welch's t-test, where the null hypothesis is that the solutions of the three problems have equal mean values for the different performance metrics. Table 5 shows the p-values from the test for each pair of problems and the three metrics. We observe that all the p-values are quite small i.e. smaller than 0.05. We therefore reject the null hypothesis, which means the solutions of the different problems have significantly different means for the three metrics.

	Evacuation Completion Time	Average Evacuation Time	Maximum Avg. Inconvenience over Sources
MM-FDCFP VS T-FDCFP	$1.24e - 26$	$3.45e - 15$	$1.46e - 20$
MM-FDCFP VS H-FDCFP	$6.06e - 29$	$1.71e - 17$	$1.87e - 09$
T-FDCFP VS H-FDCFP	$3.79e - 13$	0.0111	$4.15e - 27$

**Table 5: p-values from Welch's t-test for each pair of problems and three metrics. All the p-values are smaller than 0.05, which implies that the solutions of the different problems have significantly different means for the three metrics.**

## 7 CONCLUSION AND FUTURE PLANS

In this paper, we present an approach that uses the Large Neighborhood Search framework in conjunction with mathematical programming to find confluent routes and schedule for individuals from multiple sources to multiple safe nodes in an urban region. Our problem formulation accounts for fairness of the routes assigned as well as the overall time needed to evacuate. A notion of risk is also considered in the objective function to acknowledge that individual source nodes might have different levels of risk. Using realistic networks and a hurricane scenario, we illustrate the scalability and utility of our methods. In future works, we plan to consider time varying risks associated with different locations. A preliminary model for this purpose was proposed by Islam *et al.* [19], we plan to incorporate it into our current formulation. We also have plans to consider evacuation using public transportation along with private vehicles and investigate notions of fairness in such scenarios.

## ACKNOWLEDGMENTS

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## A FAIRNESS OBJECTIVES

Minimizing the maximum average inconvenience (objective of MM-FDCFP), is equivalent to:

$$\begin{aligned} & \min \left\{ \max_{k \in \mathcal{E}} \frac{1}{d_k} \sum_{i \in W(k)} s_i \right\} \\ \iff & \min \left\{ \max_{k \in \mathcal{E}} \frac{r_k}{d_k} \sum_{i \in W(k)} (t_i - \tau_k) \right\} \\ \iff & \min \left\{ \max_{k \in \mathcal{E}} \frac{r_k}{d_k} \left( \sum_{i \in W(k)} t_i \right) - r_k \tau_k \right\} \end{aligned}$$

Minimizing the total inconvenience (objective of T-FDCFP) is equivalent to:

$$\begin{aligned} & \min \sum_{k \in \mathcal{E}} \sum_{i \in W(k)} s_i \\ \iff & \min \sum_{k \in \mathcal{E}} \sum_{i \in W(k)} r_k (t_i - \tau_k) \\ \iff & \min \left\{ \sum_{k \in \mathcal{E}} r_k \left( \sum_{i \in W(k)} t_i \right) - \sum_{k \in \mathcal{E}} r_k \tau_k d_k \right\} \\ \iff & \min \sum_{k \in \mathcal{E}} r_k \sum_{i \in W(k)} t_i \end{aligned}$$

## B ALGORITHMS

We use Algorithm 3 to calculate the initial feasible solution for our heuristic search method.

---

**Algorithm 3:** Calculate Initial Convergent Route Set

---

**Input:** Road Network Graph:  $\mathcal{G}$

**Output:** A Convergent Route set

- 1 Add a sink node  $v_{sink}$  to  $\mathcal{G}$
  - 2 Add edges  $e(v, v_{sink})$  to  $\mathcal{G}, \forall v \in \mathcal{S}$  with  $T_e = 0$
  - 3 Calculate shortest paths from all nodes  $v \in \mathcal{N}$  to  $v_{sink}$
  - 4  $convergent\_routeset \leftarrow$  Shortest paths from all nodes  $v \in \mathcal{E}$
  - 5 **return**  $convergent\_routeset$
- 

We use Algorithm 4 to calculate the schedule that minimizes the target objective using the initial route set.

---

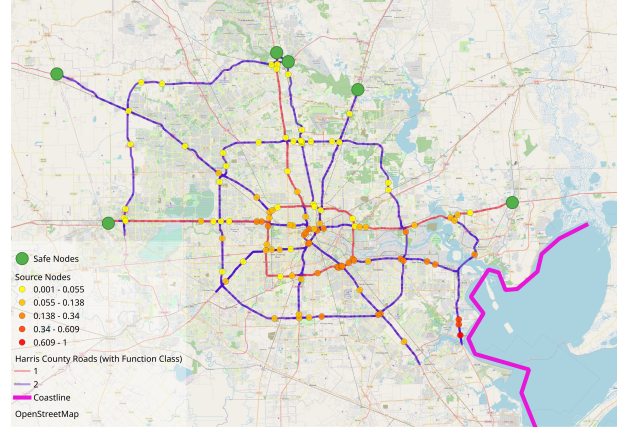
**Algorithm 4:** Calculate Best Schedule for the Initial Route Set

---

**Input:** Convergent Route set:  $routeset$ , model (4–11)

**Output:** Evacuation schedule:  $schedule$

- 1  $model_{lp} \leftarrow$  Linear Program after fixing the binary variables  $x_e$  in model (4–11) using  $routeset$
  - 2  $schedule \leftarrow$  Optimal solution of  $model_{lp}$
  - 3 **return**  $schedule$
- 



**Figure 7: Harris County Problem Instance.** Color of the source nodes indicate their associated risk level.

## C QUANTIFYING RISK

To quantify the risk  $r_k$  associated with each source node  $k \in \mathcal{E}$  due to potential hurricanes, we need accurate domain knowledge that includes but is not limited to: (i) geographical location, (ii) elevation, (iii) flooding history, (iv) available road infrastructure, (v) number of residents and their demographics etc. Here, we provide a simplified way of estimating risk level of a source node based on the the following three assumptions: (i) Locations closer to the coastline face a higher level of risk than locations situated further in-land, (ii) Locations that are closer to safe nodes face a lower level of risk, and (iii) Locations that have a history of high inundation depth face a higher level of flood risk.

To estimate the flood risk associated with each source node, we use flooding data from Hurricane Harvey (2017) that contains inundation depth at different point locations of Harris County on August 27-31 and September 2, 2017. We use the Inverse Distance Weighted (IDW) interpolation method to estimate the inundation depth at the source locations. Let  $d_{kf}$  denote the estimated depth at source  $k$ . Also, let  $d_{kc}$  and  $d_{ks}$  denote the distance to the coast line, and the distance to the nearest safe node from source  $k$  respectively.

To calculate the risk level, first we normalize each of the three features individually so that the maximal value of each feature (over the source locations) is 1. Let the normalized values of  $d_{kf}$ ,  $d_{kc}$ , and  $d_{ks}$  be denoted by  $\hat{d}_{kf}$ ,  $\hat{d}_{kc}$ , and  $\hat{d}_{ks}$  respectively. Then we use Equation (12) to calculate the risk level.

$$r_k = \frac{\hat{d}_{kf} \hat{d}_{ks}}{\hat{d}_{kc}} \quad (12)$$

Finally, we apply the same normalization process on the calculated risk values. This gives us a comparison of the source locations in terms of how much risk is associated with each of them according to our assumptions. A visualization of the risk level associated with the sources in our problem instance is shown in Figure 7. Here, Red and yellow circles denote high and low risk source nodes respectively.

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