

Search for Extraterrestrial Intelligence as One-Shot Hypothesis Testing

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Abstract—Both the search for extraterrestrial intelligence (SETI) and messaging extraterrestrial intelligence (METI) struggle with a strong indeterminacy in what data to look for and when to do so. This has led to attempts at finding both fundamental mathematical limits for SETI as well as benchmarks regarding specific signals. Due to the natural correspondence, previous information-theoretic work has been formulated in terms of communication between extraterrestrial and human civilizations. In this work, we instead formalize SETI as a detection problem, specifically (quantum) one-shot asymmetric hypothesis testing. This framework holds for all detection scenarios—in particular, it is relevant for detection of any technosignature, including quantum mechanical signals. To the best of our knowledge, this is the first work to consider the applicability of SETI for quantum signals. Using this formalism, we are able to unify the analysis of fundamental limits and benchmarking specific signals. To show a distinction between METI and SETI, we show that significantly weaker signals may be useful in detection in comparison to communication. Furthermore, the framework is computationally efficient, so it can be implemented by practicing astrobiologists.

Index Terms—Quantum Information Theory, Search for Extraterrestrial Intelligence, Asymmetric Hypothesis Testing, One-Shot Information Theory

I. INTRODUCTION & SUMMARY OF RESULTS

SETI, the Search for Extraterrestrial Intelligence (ETI), is an ongoing international scientific project to study the detection of ETI civilizations. These civilizations may either be our contemporaries or have left remains of their civilization that we may detect, referred to as *technosignatures*. This is a difficult task because scientists do not know what signals to expect nor when to expect them. It is therefore useful to constrain the problem either through information-theoretic limits or by benchmarking specific types of signals [1]–[3]. Previous information-theoretic considerations have presumed communication with the ETI [4], [5], thereby conflating SETI and METI (messaging with ETI). In this work we avoid this conflation by viewing SETI as a detection problem rather than a communication problem and explore both the fundamental implications of the framework as well as its practicality in analyzing specific signals.

This material is based upon work supported in part by the National Science Foundation under Grant 2112890.

First, we argue why SETI should be viewed as asymmetric one-shot hypothesis testing, which is captured by a semidefinite program (SDP). We do this in sufficient generality to capture quantum mechanical as well as classical signals. This establishes the limit of detection of independent and identically distributed signals (i.i.d.) as being determined by Stein's lemma and its generalizations [6], [7]. Given the nature of SETI, it is likely that signals would not be i.i.d. nor sufficiently large block length for Stein's lemma to hold, and so we derive a generalization of the one-shot hypothesis testing SDP for finite sets of null and alternative hypotheses. We then use this to show for a finite set of i.i.d. null hypotheses and finite-length messages, an optimal decision function exists and can be computationally determined. We discuss what this would imply for ETIs to design signals if they wish to prove their existence. We also consider the possibility of an advantage to using quantum signals.

Having established the general framework and the information-theoretic limits, we then move on to exploring what this framework tells us about specific choices of signals. First, we show that weak signals may be sufficient for detection in contrast to communication. We then consider the case of near earth projectiles which have had some recent interest and provide a numerical example to demonstrate the practicality of this framework for benchmarking. Finally, we discuss previously unconsidered implications for inscribed matter and artificial transit signals given this framework.

II. SETI AND ONE-SHOT HYPOTHESIS TESTING

As the name suggests, SETI is looking to detect an ETI civilization. Without loss of generality, this may be expressed as the question “is the signal coming from a given spacetime region generated by an ETI?” This question is clearly a yes or no question, i.e. at most one bit of information. Then we can view the signal as the input and the answer as a decoder, or equivalently a decision function, which outputs either 1 (“yes”) or 0 (“no”). We can denote the probability of false positives and false negatives as α, β respectively. As the detection of ETI is an exciting result, it is important that we avoid false positives. As such, we should choose an upper bound on our false positive and minimize β under this constraint. Letting

$\alpha \leq \varepsilon \in (0, 1)$ denote our upper bound, we talk of β^ε as it's a function of this choice.

We then wish to express this as an optimization program in sufficient generality to handle quantum mechanical signals. Recall that for any finite alphabet Σ , we can talk of the complex Euclidean space (CES) $\mathcal{X} := \mathbb{C}^\Sigma$ [8]. The space of quantum probability distributions for a given CES, referred to as density matrices, is defined as $D(\mathcal{X}) := \{\rho \in L(\mathcal{X}) : \rho \succeq 0, \text{Tr}(\rho) = 1\}$ where $L(\mathcal{X})$ is the set of endomorphisms for CES $\mathcal{X}$ and $\succeq$ denotes the Löwner order. This is a subset of the positive operators, denoted $\text{Pos}(\mathcal{X})$. It is easy to see that the space of classical probability distributions over Σ are the diagonal matrices contained in $D(\mathcal{X})$. Also recall that a positive-operator-valued-measure (POVM) is a finite set of positive semidefinite operators $\{A_x\}_{x \in \Lambda}$ that sum to identity, $\sum_{x \in \Lambda} A_x = \mathbb{I}$. It may be shown that all maps from quantum distributions to classical outcomes may be written in the form $\sum_{x \in \Lambda} \text{Tr}(\rho A_x) |x\rangle\langle x|$.

Combining our goals with this quantum information-theoretic formalism, we obtain the following optimization program:

$$\beta^\varepsilon(\rho_0 || \rho_1) := \min_{\substack{0 \leq A \leq \mathbb{I} \\ \text{Tr}(\rho_0 A) \geq 1 - \varepsilon}} \text{Tr}(\rho_1 A), \quad (1)$$

where A is the POVM element corresponding to deciding the input was ρ_0 , and $\text{Tr}(\rho_0 A) \geq 1 - \varepsilon$ guarantees $\alpha < \varepsilon$. Thus we have determined the essential formalism of the SETI problem mathematically.

Note that all constraints in (1) are semidefinite constraints, and so we have an SDP. As SDPs are efficient, for any choice of null and alternative hypotheses and $\varepsilon \in (0, 1)$, this may be efficiently determined. Moreover, one can see that the negative of the logarithm of the right hand side is the one-shot hypothesis testing entropy [9] (See [10] for an in-depth review of this entropy). It follows by the (quantum) Stein's lemma that if the null hypotheses are of the form $\rho_i = \tilde{\sigma}_i^{\otimes n}$ for $i \in \{0, 1\}$, then for any $\varepsilon \in (0, 1)$, $\lim_{n \rightarrow \infty} \beta_n^\varepsilon = \exp(-nD(\sigma_0 || \sigma_1))$, where $D(\cdot || \cdot)$ is the quantum relative entropy. In other words, for i.i.d. null and alternative hypotheses, one recovers the Stein's lemma as the fundamental limit of SETI for i.i.d. signals.

A. Generalizing Beyond Stein's Lemma

While Stein's lemma may serve as the fundamental limit of SETI for i.i.d. signals, it certainly is not universally relevant. Beyond the fact that the second-order expansion for hypothesis testing shows us that only long i.i.d. signals are even close to the Shannon limit [11], we might also expect some technosignatures to have short message lengths or non-i.i.d. structure. As such, it would be useful to characterize this setting as well. To go beyond this setting one considers variations of quantum Stein's lemma. Such cases that have been considered are the adversarial [6] and (a special case of) composite [7]

settings.¹ While these aforementioned results are useful in proving fundamental limits, they only apply under certain conditions. First, both the adversarial and composite testing results only characterize the asymptotic limit, but it's not clear the signals will be long enough to achieve this Shannon limit. Furthermore, the adversarial setting, to date, only applies to quantum states under restricted measurements [6]. Similarly, [7] only applies when the null and alternative hypotheses are mixtures of i.i.d. states from some closed, convex set. In other words, our concern is while these are more general, the results don't characterize short, unstructured signals as might arise in the SETI setting. For this reason, we instead generalize (1) to consider finite sets of hypotheses.

Definition 1: Let $\mathcal{P}, \mathcal{Q} \subset \text{Pos}(\mathcal{X})$ be finite sets with index alphabets Λ, Σ respectively. Then we define the generalized hypothesis testing optimization problem as

$$\begin{aligned} \beta^\varepsilon(\mathcal{P} || \mathcal{Q}) := \min \quad & \gamma \\ \text{s.t.} \quad & \text{Tr}(X P_0^i) \geq 1 - \varepsilon \quad \forall i \in \Lambda \\ & \text{Tr}(X P_1^j) \leq \gamma \quad \forall j \in \Sigma \\ & 0 \preceq X \preceq \mathbb{I}, \gamma \geq 0. \end{aligned} \quad (2)$$

It's easy to see if $\mathcal{P} = \{\rho_0\}$, $\mathcal{Q} = \{\rho_1\}$, the problem simplifies to $\beta^\varepsilon(\rho_0 || \rho_1)$. More generally, the solution is the minimal probability of false negative for the entire set $\mathcal{Q}$ while the false positive probability for the set $\mathcal{P}$ is less than ε . One can then derive the dual problem of (2):

$$\begin{aligned} \text{maximize} \quad & (1 - \varepsilon) \|\mathbf{z}\|_1 - \text{Tr}(Z) \\ \text{subject to} \quad & \sum_{i=1}^{|\Lambda|} z_i P_0^i - \sum_{j=1}^{|\Sigma|} v_j P_1^j \preceq Z \\ & \sum_{j=1}^{|\Sigma|} v_j \leq 1, 0 \leq v_j \quad \forall j \\ & 0 \preceq \mathbf{z}, Z. \end{aligned} \quad (3)$$

It is straightforward to use Slater's criterion to prove strong duality and then use this to prove the following lemma (See the long version of the paper [14] for the full derivation).

Lemma 1: Let $\mathcal{P}, \mathcal{Q} \subset \text{Pos}(\mathcal{X})$ be finite sets and $\varepsilon \in [0, 1]$. It holds

$$\beta_H^\varepsilon(\mathcal{P} || \mathcal{Q}) = \beta_H^\varepsilon(\mathcal{P} \otimes \omega || \mathcal{Q} \otimes \omega),$$

where $\mathcal{R} \otimes \omega := \{R_k \otimes \omega : R_k \in \mathcal{R}\}$ and $\omega \in D(\mathcal{W})$.

This result is not particularly surprising in the following sense. Imagine we describe the signal over a time steps δt as a state over CES $\mathcal{X}$. Consider two signals $\rho, \sigma \in D(\mathcal{X}^{\otimes n})$ of the form $\rho = \rho_1 \otimes \omega$ and $\sigma = \sigma_1 \otimes \omega$ where $\rho_1, \sigma_1 \in D(\mathcal{X}^{\otimes k})$, i.e. the signals stop differing after time $k\delta t$ has elapsed. Then the above says you cannot distinguish them any better by observing the last $(n - k)\delta t$ time steps, which is intuitive. Regardless, while the result itself seems straightforward, it is sufficient for proving the following theorem.

¹We note "the generalized quantum Stein's lemma" [12] has recently been shown to have a gap in its proof [13], and so we do not discuss it in this manuscript.

Theorem 2: Let Σ be a finite alphabet and $\mathcal{X} := \mathbb{C}^\Sigma$ be the corresponding CES. For all $\varepsilon \in [0, 1]$, given a finite set of finite length signals $\mathcal{Q} \subset \mathcal{D}(\mathcal{X}^{\otimes k})$, a finite set of possible i.i.d. null hypotheses, $\mathcal{P}_n := \{\rho^{\otimes n} : \rho \in \mathcal{D}(\mathcal{X})\}$, the minimal false negative is achieved in finite time and characterized by $\beta^\varepsilon(\mathcal{P}_k || \mathcal{Q})$.

The intuition of this result is that at any signal length $n' \geq k$, one can truncate the signal at length k and do just as well given Lemma 1. This means the optimal decision function is converged to at signal length k and that the fundamental limit in this setting is characterized by (2). Moreover, and crucially in terms of practicality, so long as the dimension of $\mathcal{X}^{\otimes k}$ is manageable, the optimal decoder and false negative probability may be computationally determined using (2) as it is an SDP. We note that as noise from space is often i.i.d., Theorem 2 and the generalizations of the Stein's lemma [6], [7] cover the relevant case for SETI. While more general signals might asymptotically converge, given our expectation that signals would be either short or highly structured, we view these sufficient for determining the limits of SETI specifically. We leave further nuanced analysis to subsequent work.

B. Signal Design

Having established fundamental limits of SETI by viewing it as one-shot asymmetric hypothesis testing, we note what this would imply in the case that an ETI wishes to signal their existence. In this case they have some constraints on their device, for instance power, and they wish to maximize the distinguishability of their signal from their assumed natural noise model. In the case that they wish to design an i.i.d. signal, given Stein's lemma, they should measure distinguishability using relative entropy. Therefore, they should consider the problem

$$\sup_{P_D \in \mathcal{C}} D(\mathcal{N}(\rho_\emptyset) || \mathcal{N}(P_D)), \quad (4)$$

where $\rho_\emptyset$ denotes the 'no signal' input, $\mathcal{N}$ is the (assumed memoryless) noise model and $\mathcal{C}$ is the set of distributions that can be generated by devices allowed under the constraints. We note even if $\mathcal{C}$ is a convex set, this would result in a concave optimization problem, but in specific cases it may still be computationally manageable.

C. Quantum Signals

As our framework is general enough to handle quantum mechanical signals, it is natural to ask if quantum mechanics can provide an advantage. A common issue in achieving advantages with quantum communication is the need for reference frame alignment [15]. For example, the x, y, z coordinates for defining the polarization of a state must agree between sender and receiver in general for utilizing polarized photons to transmit information. In our setting, this is avoided because to mathematically represent the null and alternative hypotheses we have assumed we know the reference frame of the received signal. Further possible solutions to the reference frame alignment problem could be macroscopic quantum

signals [16] or using superpositions of degrees of freedom that don't require aligned reference frames [17].

Under the assumption that quantum signals could be possible, there are various approaches to consider. One option would be for there to exist a technosignature/civilization that generates entangled states. Then under the assumption it is unlikely to detect entanglement from natural noise, this could result in an alternative hypothesis that is separated from all classical null hypotheses. This is distinct from any classical noise model which could not satisfy such a strong restriction.

Lastly, we note that for all relevant classical signals, if quantum signals could be generated with the same ease and the receiver's reference frame were correct or not relevant, we can conclude quantum signals could only help. This is effectively a corollary of the data-processing inequality for β^ε .

Proposition 1: For any classical distributions P_0, P_1 , if quantum signals are implementable, we can achieve at least the same optimal false negative, β^ε , using quantum signals. Moreover, there exist cases where the advantage is strict.

We note in the proof the strict advantage holds for classical hypotheses $P_0 = |0\rangle\langle 0|$, $P_1 = \frac{1}{2}\mathbb{I}$, and replacing the P_1 by $\rho_1 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ to achieve the advantage. This suggests an advantage from coherence for such signals.

III. EXAMPLES

Having established general fundamental limits, we now use the same framework to analyze specific signals to garner further insights as well as show the advantage of a computationally efficient general framework.

A. Electromagnetic Signals

As it is our own standard method of communication, a traditional approach to both SETI and METI is electromagnetic signals [18]. In such a setting, the standard limitation is the power of the signal. If the signal had enough power, any message could be transmitted. This is formally well known in that the capacity of a Gaussian channel is unbounded without a power restriction. Of course, as generating power consumes resources, one would expect there to be constraints. One common proposal is that if an ETI wished to signal its existence, it would pulse a laser at its target. Such pulsing would ideally be of an i.i.d. form, thereby making the Stein's lemma relevant in the large blocksize limit. It follows then that we could consider the earlier signal design discussion, particularly (4), to be refined for average and peak power constraints:

$$\max_{\substack{\mathbb{E}_P(P_{\text{init}}) \leq \bar{P}_{\text{av}} \\ f(P_{\text{init}}) \leq P_{\text{max}}}} D(\mathcal{N}_{\text{total}}(\rho_\emptyset^{\otimes n}) || \mathcal{N}_{\text{total}}(P_{\text{init}})), \quad (5)$$

where P_{init} is the initial distribution of the signal, f is a function that calculates the power cost, $\bar{P}_{\text{av}}$ (resp. P_{max}) is the upper bound on the average (resp. peak) power, $\mathcal{N}_{\text{total}} := \mathcal{N}_r \circ \mathcal{N}_t \circ \mathcal{N}_s$ is the total noise, $\mathcal{N}_r$ is the noise at the receiver's end, $\mathcal{N}_t$ is noise during the transmission, $\mathcal{N}_s$ is the noise from the sender's end, and $\mathbb{E}_P$ is the expectation of

$f(P_{\text{init}})$. If one assumes a specific noise model, this would give a good approximation of the fundamental limit of the distinguishability for a specific electromagnetic signal.

1) *Weak Signals Can Be Sufficient for Detection:* As the power is a major limitation, it has been proposed in previous works that electromagnetic signals are not the optimal manner for *communicating* with ETI [4]. However, we argue this is less straightforward in the detection setting where less power may be sufficient. This can be seen in the following simple example.²

We consider an ideal square pulsed laser as our source. For clarity, we discretize the total signal as a sequence of length $n = 5$. Assuming a power cutoff for the device and discretizing the power, we assume each element of the sequence is an element of the interval $[0, g] \subset \mathbb{N}$. It follows the expected optimal choice of the initial distribution could be written as $|0, P, 0, 0, 0\rangle$, i.e. a delta distribution. We can imagine that while there is no noise at the source, there is memoryless jitter in the laser which with probability $q/2$ shifts the sequence forward or backward one time bin. We therefore define the distribution

$$P_{\text{init}} = (1-q)|0, 0, P, 0, 0\rangle + \frac{q}{2}(|0, P, 0, 0, 0\rangle + |0, 0, 0, P, 0\rangle)$$

We assume the noise during travel $\mathcal{N}_t$ is loss-only, so that for each time bin the map $y \rightarrow \max(y - c, 0)$ is applied, where c is a function of the distance travelled and possibly the conditions over the travel path. Finally, we assume the noise at the receiver is composed of two maps. First we assume the data is taken over a short enough time (as lasers can pulse reasonably quickly) that the Sun is additive power so that for each time bin the Sun is modeled by the map $y \rightarrow \min(y + s, g)$. The second map assumes with some probability there is any given possible sequence, which allows us to guarantee absolute continuity of the relative entropy. This is modeled by a linear map on distributions, $q \mapsto (1 - \delta)q + \frac{\delta}{|\mathcal{Y}^{\times n}|} \vec{1}_{\mathcal{Y}^{\times n}}$, where $\vec{1}$ is the all-ones vector, q is any probability distribution, and $\delta \in (0, 1)$. Given these maps, one can determine P_0, P_1 from P_{init} . Under the assumption $c < P < g - s + c$, one finds that so long as $c \neq P$, $D(P_0||P_1)$ is a non-zero constant (See the long version of the paper [14] for the full expression). As the assumption $c < P$ implies $c \neq P$, the asymptotic error rate for all powers in this range is the same.

In other words, the above example tells us that almost any power will work. Of course, this is a simplistic model, but the message is clear and significantly more general: a detectable signal only requires distinguishability from the set of null hypotheses, which is significantly less demanding than distinguishing many messages. Moreover, while this has been presented with a pulsed laser which implies an ETI signalling its existence, this applies equally well for detecting electromagnetic technosignatures. That is, this is an optimistic takeaway for the success of SETI.

²See [14] for a more involved derivation.

B. Near Earth Projectiles

Having discussed the traditional method of electromagnetic signals, we move to alternative methods of SETI. We stress that some of these methods have been discussed under the assumption that the signal was intentional, however this is not relevant from the perspective of the mathematical framework. This is crucial as much of SETI is interested in detecting technosignatures that may not be intentional. What is important is the distinguishability between the technosignature and the null hypothesis. For example, we first consider near Earth projectiles, which we take to be any sequence of macroscopic objects that passes near the Earth and could be a technosignature. Under this definition, the recent 'Oumuamua [19], [20] would qualify as a near Earth projectile. While it has been concluded to not be a technosignature [20], even if it had been, it would not have mattered whether it was intentional or not, and our framework captures this.

1) *Simple Numerical Example:* One purported advantage of our framework is computational efficiency. Here we present this with a simple example considering near Earth projectiles by comparing distributions of meteors burning up in the atmosphere, known as “meteor bursts.” A reason to consider this example is that there are existing systems for monitoring meteor bursts as we use their ionization trails for telecommunication systems [21], and so this data would be analyzed anyways.

Mathematically, meteor bursts are found to be Poisson processes: for any interval Δt , $\mathbb{P}[n \text{ meteors}] = \frac{(\lambda \Delta t)^n}{n!} e^{-\lambda \Delta t}$, where λ may depend on many things, such as the time of day and year [21]. For simplicity, we assume a scaling such that $\Delta t = 1$ and assume the ETI message begins in this time interval. Using (1), we look at the probability of missing an ETI signal β^ε , denoted β_n , as a function of how many (additive) ETI meteors appear over the time interval for two choices of λ and three choices of ε which corresponds to the maximum allowed error probability α_n . We note that our numerical analysis must be finite, whereas the Poisson process has a countably infinite number of outcomes. This can be rigorously handled by truncating the tail of the distribution, given the tail property of the distribution and the data-processing inequality for β^ε . We provide the results of our example determined numerically using the CVXPY package in Fig. 1.

The major takeaway of Figure 1 is that the error probability β^ε does not scale linearly in α . Clearly the ability to numerically investigate the system is an advantage for benchmarking signals regardless of whether they are deliberate or not. Moreover, it is one method for determining which technosignatures are most likely to be detected, allowing the scientific community to lower their resource to risk consumption tradeoff.

C. Inscribed Matter

A similar method to near Earth projectiles is inscribed matter [4]. This was originally proposed as a more energy-efficient means of METI than electromagnetic signals. The

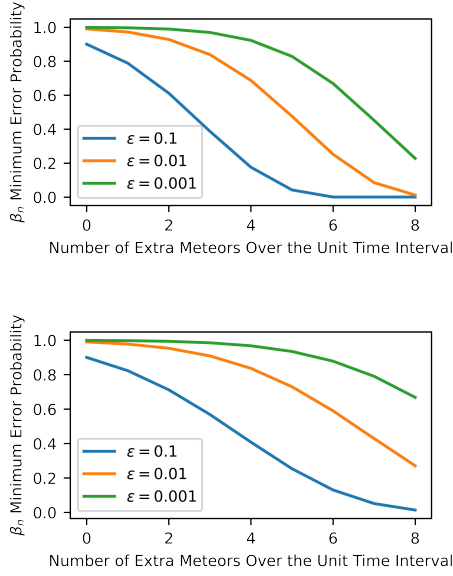


Fig. 1: Error probability of the optimal decision function for determining if there were extra detected meteors in a unit interval as a function of the extra meteors. (a) is for natural meteors distributed according to $\text{Pois}(3)$, and (b) is for $\text{Pois}(6)$ (i.e. larger variance). Each plot is for three different tolerances of false positive probability $\alpha_n < \varepsilon$.

idea is to encode massive amounts of information on a small piece of matter and then send that as the message. However, this and subsequent work ignored that the message must be noticed before any communication can be achieved. In other words, achieving SETI is a necessary precondition for METI, and it follows that there must be energy costs to the detectability of the matter. By the data-processing inequality, one would expect that the piece of matter should have macroscopic properties that give rise to distinguishability from the local surroundings. One would expect this would lead to an increase in energy costs, and our framework would be useful for such investigations. In particular, we propose the following (non-linear) variation of one-shot hypothesis testing optimization problem could be used to study specific proposals:

$$\begin{aligned}
 & \underset{0 \leq A \leq I, Q \in \mathcal{D}(\mathcal{X})}{\text{minimize}} && \langle P_1, A \rangle \\
 & && \langle P_0, A \rangle \geq 1 - \varepsilon \\
 & && P_1 = \mathcal{N}_n(Q) \\
 & && E(Q) \leq B,
 \end{aligned} \tag{6}$$

where, $\mathcal{X}$ is the Euclidean space the designed signal Q is defined over, $E(\cdot)$ is an energy cost function, B is a constraint on the total energy, and $\mathcal{N}_n$ is a linear map representing the noise introduced to the design during transmission.

D. Transits

As a final method of signaling, we consider artificial transits. Artificial transits are (ETI-constructed) objects which orbit

around stars. It has been shown that these could be detected while searching for exoplanets as they result in strange detected flux from the star [22]. While suggested as a means for an ETI to deliberately signal its existence [22]–[24], it is not unreasonable to imagine there would be reasons to build artificial transits for other reasons. One advantage of deliberate signals proposed in the previously mentioned works is that they can cause the flux of the star to pulse for a much longer time than a laser can be pulsed. In particular, the transit can generate pulses long after the civilization has ceased to exist. This would result in significantly larger block lengths. By our discussion of the Stein’s lemma and it’s second-order expansion, we can view our framework as giving a further formal justification to the advantage of both the use of and attempt to detect artificial transits.

IV. CONCLUSION

In this work we have made a strong distinction between *messaging* and *detecting* extraterrestrial intelligent civilizations. By focusing on SETI as a detection problem, we introduced a new general framework for analyzing SETI in terms of one-shot hypothesis testing. This framework is general enough to handle arbitrary technosignatures regardless of if they are deliberately sent. We generalize this framework to being able to handle finitely many null and alternative hypotheses. This result combined with the generalized Stein’s lemma captures all standard settings for SETI. We then use this framework to explore the limits of deliberate signaling design and the possibility of an advantage using quantum signals. We then analyzed specific classes of signals. We showed that in contrast with METI, signals with low power may be sufficient for SETI. We presented an example of the advantage of a computationally efficient framework. Lastly we gave qualitative discussions on where our framework could be applied quantitatively to analyze inscribed matter and artificial transits. Our hope is this computationally efficient framework may be of use to the general SETI community.

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