PATTERNS OF REASONING: WARRANTS IN ELEMENTARY MATHEMATICS AND CODING ARGUMENTS

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Argumentation is widely used in teaching mathematics, but little research has been done on argumentation in teaching integrated mathematics and coding. As part of a larger study investigating collective argumentation in teaching mathematics, science, and coding, we classified the warrants given by elementary age students who were engaged in argumentation in mathematics and coding. Three major categories – calculation, visual, and unformalized knowledge – accounted for the majority of warrants given. Further analysis revealed differences in types of warrants when the primary focus of the argument was coding versus when the primary focus of the argument was mathematics. Our results suggest that expecting students to provide reasons for modifying their code, similar to what is expected in mathematics arguments, helps move them away from a trial-and-error to a more structured approach to coding.

Keywords: Classroom Discourse, Elementary School Education, Integrated STEM/STEAM

Background

Reasoning in Mathematics and Coding

Reasoning is important in the teaching and learning of mathematics. Research suggests students should develop an understanding of mathematics beyond a collection of facts and procedures (Cuoco et al., 1996; Goldenberg, 1996; Kilpatrick et al., 2001). Building on this research, national policy documents in the United States have highlighted the importance of reasoning in K-12 mathematics (National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Together, these policy documents suggest K-12 mathematics instruction should enable students to recognize the importance of reasoning in mathematics, make and explore mathematical conjectures, construct and critique mathematical arguments, and use various types of reasoning and proof.

Although coding/computer science/programming is a relatively new area of instruction, the K-12 Computer Science Framework ("K-12 Computer Science," 2016) recognizes communication as one of the seven core practices. This practice requires students to describe and justify their processes and solutions, promoting a more structured approach to coding rather than the trial-and-error approach commonly used by novice programmers (Lye & Koh, 2014; see recommendation by Fessakis et al., 2013). Thus, reasoning and the ability to communicate rationales are valued in both coding and mathematics.

Collective Argumentation in Mathematics and Coding

One lens that provides insight into the reasoning practices of students and teachers is that of collective argumentation. We define collective argumentation as teachers and students working together to establish or reject claims. There are multiple examples in the mathematics education literature of teachers facilitating collective argumentation to support student learning, reasoning, and sense making (e.g., Forman et al., 1998; Krummheuer, 1995, 2007; Yackel, 2002).

Although argumentation has not been widely used in teaching and learning coding, the larger project from which this study originates proposed that teaching coding through argumentation has several benefits, including (1) constructing and critiquing arguments provides a more structured approach to coding than trial-and-error and (2) teaching coding through argumentation allows teachers to use methods they already use in teaching mathematics, thus making it more likely for them to teach coding in conjunction with mathematics. One of the goals of the larger project was to provide teachers with strategies to help improve students' abilities to construct and critique arguments. The present study builds on the collective argumentation literature by focusing on one component, the warrant, of Toulmin's (1958/2003) model for argumentation.

Conceptual Framework

In mathematics education research, argumentation is often studied using Toulmin's (1958/2003) structure of an argument which includes data, claims, warrants, qualifiers, rebuttals, and backings. Toulmin argued that although what is accepted as valid for each component is discipline dependent, the structure of arguments is the same across disciplines. This structure can be expanded to include sub-arguments and the contributor(s) of each component (see Conner, 2008). Although entire arguments can provide insights into the reasoning that occurs in classrooms, warrants can provide a clearer understanding of what students and teachers use and accept as rationales. According to Toulmin (1958/2003), a warrant in argumentation serves as a bridge that explains how a person got from the data to a claim. The types of warrants provided during collective argumentation can illuminate the ideas on which teachers and students base their reasoning. Existing research classifies warrants in multiple ways (Conner, 2012; Inglis et al., 2007; Nardi et al., 2011). In this study, we adapted and expanded Conner's (2012) framework for analyzing warrants. The initial framework identified 29 types of warrants that were collapsed into ten major categories.

Methods

The larger study from which these data were analyzed included two phases of data collection with 32 elementary school teachers. During the first phase, teachers participated in a semester-long professional development course that included block-based coding content across multiple platforms and discussions about using collective argumentation across multiple disciplines. In the second phase, ten teachers were selected for classroom observations and coaching sessions. This paper is focused on analysis of video recordings of classroom observations. Participating teachers selected the topics for the observed lessons, focusing on integrating multiple disciplines and using argumentation during their teaching. Videos of classroom observations were reviewed and episodes of argumentation were identified via identifying main claims and associated argument components. Episodes of argumentation from each teacher's class were selected for analysis through a random sampling process, diagrammed by pairs of researchers using Conner's (2008) modified diagram structure, and then compared until consensus was reached. A total of 222 arguments were diagrammed across ten participants. We labeled the primary and secondary focus of each argument as mathematics, coding, science, literature, or social studies. For this study, we analyzed 108 warrants from 35 arguments with a primary focus of mathematics (secondary focus coding) and a primary focus of coding (secondary focus mathematics) from four teachers' classrooms. We inserted all of the information from each argument into a spreadsheet, noting whether the warrant was implicit or explicit. Implicit warrants were identified when a warrant was not explicitly stated or written but seemed to be understood by at least part of the group. We categorized the types of warrants provided by students and teachers according to the framework developed by Conner (2012). However, due to the context of our data – mathematics and coding arguments in elementary classrooms – we made adaptations to this framework, which was originally developed from high school algebra and geometry arguments. Using this adapted framework, we examined the types of explicit and implicit warrants contributed by students and teachers to make sense of the kinds of reasoning that were evident in mathematics and coding contexts. Analysis of these data is ongoing.

Results

In our initial analysis, we identified 21 different types of warrants with 15 of these having been identified in Conner's (2012) initial framework and six being newly identified from our data. By examining our data using Conner's framework as a starting point, we collapsed the 21 types of warrants into 11 major categories (Table 1).

Table 1: Relative Frequencies and Types of Warrants

Categories	Types of Warrants	Coding Focus	Math Focus	Total
Authority	External Authority, Given, Mathematical Convention	2 (7.4%)	1 (1.2%)	3 (2.8%)
Given	Given	0 (0.0%)	1 (1.2%)	1 (0.9%)
Interpretation	Interpretation of Problem*, Interpretation of Written Code*	4 (14.8%)	0 (0.0%)	4 (3.7%)
Method	Procedure-General	2 (7.4%)	4 (4.9%)	6 (5.6%)
Patterns	Patterning, Pattern Noticing	1 (3.7%)	4 (4.9%)	5 (4.6%)
Preference	Personal Preference*	0 (0.0%)	3 (3.7%)	3 (2.8%)
Visual	Appearance, Observation*, Observation with Quantification*, Visualization	8 (29.6%)	14 (17.3%)	22 (20.4%)
Calculation	Procedure-Calculation	0 (0.0%)	29 (35.8%)	29 (26.9%)
Unformalized knowledge	Informal Understanding, Number Sense, Previous Experience	5 (18.5%)	19 (23.5%)	24 (22.2%)
Knowledge	Definition, Prior Knowledge	3 (11.1%)	0 (0.0%)	3 (2.8%)
Reasoning	Interpretation of Definition, Calculation-Why*	2 (7.4%)	6 (7.4%)	8 (7.4%)
Total		27	81	108

An asterisk (*) indicates a newly identified type of warrant.

We first examined all of the warrants aggregated across both foci: mathematics and coding. Of the 108 warrants, approximately 70% of the warrants were classified into one of three categories: *calculation*, *unformalized knowledge*, or *reasoning*. More than a quarter (26.9%) of all warrants analyzed were classified as *calculation*; these were warrants in which a student or teacher provided a mathematical process or set of steps that produced a solution to a specific problem. For example, a student offered the warrant "Because 6 x 4 = 24" to justify the claim that a square with side length 6 would have a perimeter of 24. It is unsurprising that the largest category of warrants was *calculation* because each of these arguments included mathematical operations familiar to elementary students. Warrants classified as *unformalized knowledge* made up the second largest category (22.2%). For example, when investigating the relationship between time and distance, one student explained, "When I go to the gas station, it's really close to my house so [I] have a shorter time to go. But when I go to [the grocery store], it's like in the city, it takes a way longer time because it's more farther." The large proportion of these warrants that were *unformalized knowledge* suggests that students reasoned intuitively or based on ideas that had not yet been formalized in class. The third largest group of warrants (20.4%) were classified as *visual*, as students based their reasoning on physical representations they could see. For instance, when justifying why the robot needed to travel a longer distance along one side of a rectangle that was taped on the tile floor, a student offered, “This side looks longer. This [side] is 4 squares width and then this [side] has 3 squares width.”

Although the remaining categories made up only 30% of the warrants, there are interesting things to note in this smaller group. Warrants were classified as *reasoning* when a student or teacher provided evidence for a claim based on the *interpretation of a definition* or when they provided a rationale for performing a calculation (*calculation-why*). Even though this category makes up a small percentage of all warrants analyzed (7.4%), the idea that one should give a reason for a calculation was evident in these elementary classrooms. In addition, the category of warrants that were based on some *external authority* made up one of the smallest percentages overall (2.8%), indicating that students were not relying heavily on what the teacher said when providing evidence for claims.

When we examined the warrants according to their primary focus, we found that almost 30% of the warrants in coding-focused arguments were classified as *visual*, compared to only 17.6% of warrants in mathematics-focused arguments. The higher proportion of *visual* warrants in coding-focused arguments could be due to students working with robots that students could observe carrying out their written code. For example, the reasoning students provided for their claim “we halved one second” to adjust how far the robot should travel was “one second got us two times too far.” This warrant was classified as *observation with quantification* because students noticed the robot traveled too far and they used mathematical ideas (“two times”) to describe what they observed.

Although more than a third (35.8%) of warrants provided in mathematics-focused arguments were *calculation,* none of the warrants provided in coding-focused arguments involved only calculation. Warrants involving a calculation in coding-focused arguments included a reason for doing the calculation (*calculation-why*). When students were attempting to code a robot to go a certain distance, they often related it to a previously established distance and time: “Because the length is doubled and 12 inches is doubled so I should double the delay.” When focused on coding, it is reasonable that students include justifications related to the task in their warrants.

Discussion

Understanding the patterns of reasoning from these elementary mathematics and coding arguments provides insight into what teachers and students accept as appropriate justifications. Although some research exists on what types of warrants are acceptable in mathematics classrooms, little is known about what is considered valid reasoning in coding-focused arguments. Understanding reasoning patterns in coding contexts can help us support teachers in engaging students in argumentation in learning coding. Additionally, none of arguments in this analysis showed students used a trial-and-error approach to coding, which is commonly used by novice coders (see Lye & Koh, 2014). This is likely because the teachers insisted that students provide reasons for modifying their code, promoting a more structured approach to coding. This gives us reason to believe that argumentation is a promising approach for teaching students to code. And, the coding context, with expectations of argumentation, provided a way to access students’ reasons for their calculations.

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References

Conner, A. (2008). *Expanded Toulmin diagrams: A tool for investigating complex activity in classrooms*. Paper presented at the Joint Meeting of PME 32 and PME-NA XXX, Mexico.

Conner, A. (2012). Warrants as indications of reasoning patterns in secondary mathematics classes. In *Proceedings of the 12th International Congress on Mathematical Education (ICME-12), Topic Study Group* (Vol. 14).

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior, 15*(4), 375-402.

Fessakis, G., Gouli, E., & Mavroudi, E. (2013). Problem solving by 5–6 years old kindergarten children in a computer programming environment: A case study. *Computers & Education, 63*, 87-97.

Forman, E. A., Larreamendy-Joerns, J., Stein, M. K., & Brown, C. A. (1998). "You're going to want to find out which and prove it": Collective argumentation in a mathematics classroom. *Learning and Instruction*, *8*(6), 527-548.

Goldenberg, E. P. (1996). "Habits of Mind" as an organizer for the curriculum. *Journal of Education, 178*(1), 13-34.

Inglis, M., Mejia-Ramos, J. P., & Simpson, A. (2007). Modelling mathematical argumentation: The importance of qualification. *Educational Studies in Mathematics*, *66*(1), 3-21.

K-12 Computer Science Framework. (2016). Retrieved from http://www.k12cs.org.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229-269). Lawrence Erlbaum Associates.

Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom: Two episodes and related theoretical abductions. *The Journal of Mathematical Behavior*, *26*(1), 60-82.

Lye, S. Y., & Koh, J. H. L. (2014). Review on teaching and learning of computational thinking through programming: What is next for K-12?. *Computers in Human Behavior, 41*, 51-61.

Nardi, E., Biza, I., & Zachariades, T. (2011). 'Warrant' revisited: Integrating mathematics teachers' pedagogical and epistemological considerations into Toulmin's model for argumentation. *Educational Studies in Mathematics, 79*(2), 157-173.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Author.

National Governors Association Center for Best Practices and Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics).* Author.

Toulmin, S. E. (2003). *The uses of argument* (updated ed.). Cambridge University Press. Originally published in 1958.

Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *Journal of Mathematical Behavior, 21*(4), 423-440.