PHYSICS

Universal theory of strange metals from spatially random interactions

Aavishkar A. Patel[1,2], Haoyu Guo[3,4,5], Ilya Esterlis[4,6], Subir Sachdev[4,7]*

Strange metals—ubiquitous in correlated quantum materials—transport electrical charge at low temperatures but not by the individual electronic quasiparticle excitations, which carry charge in ordinary metals. In this work, we consider two-dimensional metals of fermions coupled to quantum critical scalars, the latter representing order parameters or fractionalized particles. We show that at low temperatures (T), such metals generically exhibit strange metal behavior with a T-linear resistivity arising from spatially random fluctuations in the fermion-scalar Yukawa couplings about a nonzero spatial average. We also find a $T \ln(1/T)$ specific heat and a rationale for the Planckian bound on the transport scattering time. These results are in agreement with observations and are obtained in the large N expansion of an ensemble of critical metals with N fermion flavors.

A major theme in the study of correlated metals has been their strange metal behavior at low temperatures—i.e., a linear-in-temperature resistivity smaller than the quantum unit of resistivity (h/e^2 in two dimensions), which appears to be controlled by a dissipative Planckian relaxation time of order $\hbar/(k_B T)$ (where h is Planck's constant, $\hbar = h/(2\pi)$, e is the electron charge, k_B is Boltzmann's constant, and T is the absolute temperature) (*1–8*). This behavior is in sharp contrast to T^2 dependence of the resistivity and the $1/T^2$ relaxation time, invariably observed in conventional metals described by Fermi liquid theory. Moreover, the anomalous resistivity of strange metals is accompanied by a logarithmic enhancement of the Sommerfeld metallic specific heat to $T \ln(1/T)$ (*1*) from the ~T behavior of conventional metals.

Starting with the seminal work by Hertz (*9*), there has been extensive research on the properties of electronic Fermi surfaces at quantum phase transitions (*10*). The quantum critical fluctuations are represented by a scalar field, which is usually a symmetry-breaking order parameter but could also be a fractionalized particle at phase transitions without an order parameter (*11*). This scalar field has a Yukawa coupling to the electrons, by which the electrons scatter by emitting or absorbing a scalar field excitation (the Yukawa coupling is similar to the electron-phonon coupling but without a suppression by the gradient of the scalar field). It is now known that such a Fermi surface coupled to a quantum critical scalar leads to a breakdown of the electronic quasiparticle excitations in two spatial dimensions (*10*, *12*). But the Fermi surface survives as a sharp boundary in momentum space, separating particle- and hole-like excitations, which are diffuse in energy space. In the presence of random impurities that scatter the electrons (*13–17*), there are cases where the quasiparticles are at the boundary of stability, which leads to marginal Fermi liquid behavior (*18*) in single-particle observables, such as those observed in photoemission experiments.

However, despite these advances, theory has so far found limited success in explaining all of the defining transport properties (such as the linear-in-T resistivity) of strange metals. Conservation of momentum in the low-energy theory of a clean metal implies that the dc and optical conductivities are not affected by the anomalous self-energy of the excitations near the Fermi surface (*15–17*, *19–22*). In other words, the strong coupling between the Fermi surface and the scalar field places the system in the limit of strong scalar drag, and this clean metal theory cannot describe strange metal behavior. This is in contrast to the electron-phonon system, where the weak electron-phonon coupling makes phonon drag a factor only in ultrapure samples (*23*). Umklapp scattering can lead to nonzero resistance, and its influence in quantum critical metals has been investigated in other works (*16*, *24*). However, umklapp is suppressed at low T, its predictions for transport are not universal and depend upon specific Fermi surface details, and there is no corresponding $T \ln(1/T)$ specific heat.

Given the ubiquity of strange metal transport across numerous correlated electron materials (from the cuprates and the pnictides to recently discovered twisted bilayer graphene), a simple and universal mechanism may be at play. We propose that spatial disorder in the fermion-scalar Yukawa coupling, about a nonzero spatial average, provides just such a universal mechanism. Such disorder is ubiquitous in models of correlated electron materials; for example, in a Hubbard model with on-site repulsion U and an impurity-induced disorder in the electron hopping t_{ij}, the Schrieffer-Wolff transformation generates disorder in the exchange interaction $J_{ij} = 4t_{ij}^2/U$, and this disorder then feeds into the Yukawa coupling after a standard decoupling procedure (*9*) that introduces the scalar field. Moreover, our mechanism applies universally across different classes of quantum critical metals, with scalars that are either fractionalized particles or order parameters at zero or nonzero momentum, which have distinct critical behaviors in the clean limit. In the limit of a large number of fermion flavors, N, we find a universal phenomenology that matches observations, including the T-linear resistivity, the Planckian relaxation time, and the $T \ln(1/T)$ specific heat.

A key observation of our analysis is that although the fermion inelastic self-energy corrections can be dominated by the spatially uniform coupling, the transport is nevertheless dominated by the spatially random coupling, and this leads to our main results. Our work follows other recent works with random Yukawa interactions (*25–34*) inspired by the Sachdev-Ye-Kitaev (SYK) model (*35*, *36*) along with studies that found linear-in-T resistivity with random interactions but with vanishing spatial average (*32*, *34*, *37–39*).

Spatially uniform quantum critical metal

We begin by recalling the SYK-inspired large N theory of the two-dimensional quantum critical metal (*32*, *34*) for the case where the order parameter has zero momentum. The imaginary time (τ) action for the fermion field ψ_i and scalar field φ_i (with $i = 1 \ldots N$ a flavor label carried by the fermions introduced only to enable a controlled large N limit) is (*34*)

$$\mathcal{S}_g = \int d\tau \sum_{\mathbf{k}} \sum_{i=1}^{N} \psi_{i\mathbf{k}}^\dagger(\tau)[\partial_\tau + \varepsilon(\mathbf{k})]\psi_{i\mathbf{k}}(\tau) + \frac{1}{2}\int d\tau \sum_{\mathbf{q}} \sum_{i=1}^{N} \phi_{i\mathbf{q}}(\tau)(-\partial_\tau^2 + K\mathbf{q}^2 + m_b^2)\phi_{i,-\mathbf{q}}(\tau) + \frac{g_{ijl}}{N}\int d\tau d^2 r \sum_{i,j,l=1}^{N} \psi_i^\dagger(\mathbf{r},\tau)\psi_j(\mathbf{r},\tau)\phi_l(\mathbf{r},\tau) \quad (1)$$

where $\mathbf{k}$ and $\mathbf{q}$ are spatial momenta, the fermion dispersion $\varepsilon(\mathbf{k})$ determines the Fermi surface, the scalar mass m_b must be tuned to criticality and is needed for infrared regularization but does not appear in final results, and the Yukawa fermion-scalar coupling g_{ijl} is space independent but random in flavor

[1]Center for Computational Quantum Physics, Flatiron Institute, New York, NY 10010, USA. [2]Department of Physics, University of California Berkeley, Berkeley, CA 94720, USA. [3]Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, NY 14853, USA. [4]Department of Physics, Harvard University, Cambridge, MA 02138, USA. [5]Kavli Institute for Theoretical Physics, University of California Santa Barbara, Santa Barbara, CA 93106, USA. [6]Department of Physics, University of Wisconsin–Madison, Madison, WI 53706, USA. [7]School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA.
*Corresponding author. Email: sachdev@g.harvard.edu

space with

$$\overline{g_{ijl}} = 0, \overline{g^*_{ijl} g_{abc}} = g^2 \delta_{ia}\delta_{jb}\delta_{lc} \quad (2)$$

where the overline represents average over flavor space. The hypothesis is that a large domain of flavor couplings all flow to the same universal low-energy theory (as in the SYK model), so we can safely examine the average of an ensemble of theories. Momentum is conserved in each member of the ensemble, and the flavor-space randomness does not lead to any essential difference from nonrandom theories. This is in contrast to position-space randomness, which we consider later and which does relax momentum and modify physical properties.

The disorder average of the partition function of $\mathcal{S}_g$ leads to a so-called G-Σ theory, whose large N saddle point of Eq. 1 has singular fermion (Σ) and boson (Π) self-energies at T = 0 (where ω is frequency) (*34*)

$$\Pi(i\omega, \mathbf{q}) = -c_b \frac{|\omega|}{|\mathbf{q}|}, \Sigma(i\omega, \mathbf{k}) = -ic_f \,\mathrm{sgn}(\omega)|\omega|^{2/3}$$
$$c_b = \frac{g^2}{2\pi\kappa v_F}, c_f = \frac{g^2}{2\pi v_F \sqrt{3}} \left(\frac{2\pi v_F \kappa}{K^2 g^2}\right)^{1/3} \quad (3)$$

These results are obtained on a circular Fermi surface with curvature $\kappa = 1/m$, where m is the effective mass of the fermions. The result is consistent with the theory of two antipodal patches around $\pm\mathbf{k}_0$ on the Fermi surface to which $\mathbf{q}$ is tangent, with axes chosen so that $\mathbf{q} = (0, q)$ and fermionic dispersion $\varepsilon(\pm\mathbf{k}_0 + \mathbf{k}) = \pm v_F k_x + \kappa k_y^2/2$.

The large N computation of the conductivity (*17*, *22*) yields only the clean Drude result $\mathrm{Re}[\sigma(\omega)]/N = \pi \mathcal{N} v_F^2 \delta(\omega)/2$, where $\mathcal{N} = m/(2\pi)$ is the fermion density of states at the Fermi level. This is in contrast to the results of previous studies (*12*, *40*), in which it has been found that dc resistivity is $\sim T^{4/3}$ and optical conductivity is $\sim|\omega|^{-2/3}$.

Potential disorder

We now add to the model a spatially random fermion potential

$$\mathcal{S}_v = \frac{1}{\sqrt{N}} \int d^2r d\tau v_{ij}(\mathbf{r}) \psi_i^\dagger(\mathbf{r}, \tau)\psi_j(\mathbf{r}, \tau)$$
$$\overline{v_{ij}(\mathbf{r})} = 0, \overline{v^*_{ij}(\mathbf{r}) v_{lm}(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')\delta_{il}\delta_{jm} \quad (4)$$

Here, the overline is an average over both spatial coordinates and flavor space. The large N limit of the G-Σ theory (*17*) yields a saddle point that has statistical translational invariance and is similar to that found in earlier studies (*13*, *15*, *16*). The low-frequency boson propagator now has the diffusive form $\sim(q^2 + c_d|\omega|)^{-1}$ with dynamic critical exponent z = 2, whereas the fermion self-energy has an elastic scattering term along with a marginal Fermi liquid (*18*) inelastic term at low frequencies

$$\Pi(i\omega, \mathbf{q}) = -\frac{\mathcal{N} g^2 |\omega|}{\Gamma}, \Gamma = 2\pi v^2 \mathcal{N},$$
$$\Sigma\left(i\omega, \mathbf{k} = k_F \hat{k}\right) = -i\frac{\Gamma}{2}\mathrm{sgn}(\omega) - \frac{ig^2\omega}{2\pi^2\Gamma} \ln\left(\frac{e\Gamma^3}{\mathcal{N} g^2 v_F^2 |\omega|}\right) \quad (5)$$

at T = 0. However, the marginal Fermi liquid self-energy, although leading to a $T\ln(1/T)$ specific heat, does not (*17*) lead to the claimed (*18*) linear-T term in the dc resistivity because it arises from forward scattering of electrons off the $\mathbf{q} \sim 0$ bosons. These forward-scattering processes are unable to relax either current or momentum because of the small wavevector of the bosons involved and the momentum conservation of the g interactions. As a result, even a perturbative computation of the conductivity at $\mathcal{O}(g^2)$ (Fig. 1) shows a cancellation between the interaction-induced self-energy contributions and the interaction-induced vertex correction, leading to a dc conductivity that is just a constant set by the elastic potential disorder scattering rate Γ. The leading frequency dependence of the optical conductivity at frequencies $\omega \ll \Gamma$ is just a constant, and there is no linear in-frequency correction (*17*). Correspondingly, in the dc limit, there is no linear in-T correction, and a conventional T^2 correction is expected.

Interaction disorder

Our main results are obtained with additional spatially random interactions. In principle, such terms will be generated under a renormalization analysis from $\mathcal{S}_v$. However, such a renormalization is not part of our large N limit, so we account for such interactions by adding an explicit term

$$\mathcal{S}_{g'} = \frac{1}{N} \int d^2r d\tau g'_{ijl}(\mathbf{r}) \psi_i^\dagger(\mathbf{r}, \tau)\psi_j(\mathbf{r}, \tau)\phi_l(\mathbf{r}, \tau)$$
$$\overline{g'_{ijl}(\mathbf{r})} = 0, \overline{g'^*_{ijl}(\mathbf{r}) g'_{abc}(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')\delta_{ia}\delta_{jb}\delta_{lc} \quad (6)$$

Note, v, g, and g' are all independent flavor-random variables. Earlier works have considered the limiting case of $g = 0$, $v = 0$, and $g' \neq 0$ (*32*, *34*). We will instead describe the more physically relevant regime where spatial disorder is a weaker perturbation to a clean quantum critical system, with g the largest interaction coupling. We therefore now have g, v, and g' all nonzero. The theory of $\mathcal{S}_g + \mathcal{S}_v + \mathcal{S}_{g'}$ is described in the supplementary materials (*41*).

As with $\mathcal{S}_g + \mathcal{S}_v$ above, we find a statistical translational invariance at large N, with a low-frequency boson propagator characterized by z = 2 and the low-frequency fermion self-energy with an elastic scattering term along with a marginal Fermi liquid inelastic term (*42*)

$$\Pi(i\omega, \mathbf{q}) = -\frac{\mathcal{N} g^2 |\omega|}{\Gamma} - \frac{\pi}{2}\mathcal{N}^2 g'^2 |\omega| \equiv -c_d|\omega|,$$
$$\Sigma\left(i\omega, \mathbf{k} = k_F \hat{k}\right) = -i\frac{\Gamma}{2}\mathrm{sgn}(\omega) - \frac{ig^2\omega}{2\pi^2\Gamma} \ln\left(\frac{e\Gamma^2}{v_F^2 c_d |\omega|}\right) - \frac{i\mathcal{N} g'^2 \omega}{4\pi} \ln\left(\frac{e\Lambda_d^2}{c_d|\omega|}\right) (T = 0) \quad (7)$$

where $\Lambda_d \sim \Gamma/v_F$. This self-energy leads to a $T\ln(1/T)$ specific heat, as for the large g' case (*34*). However, there is now an important difference with respect to the previous case where g' = 0, which leads to markedly different charge transport properties: The marginal Fermi liquid self-energy now contains a term (last term in Eq. 7) that does not arise solely from the forward scattering of electrons. This term is produced by the disordered part of the interactions in Eq. 6. Thus, this part of the self-energy represents scattering that relaxes both current and momentum carried by the electron fluid, and therefore its imaginary part on the real frequency axis determines the inelastic transport scattering rate.

We can show this as follows by computing the conductivity using the Kubo formula. If we work perturbatively in g and g', then the

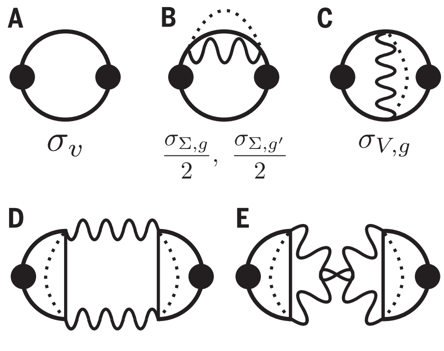

Fig. 1. Interaction corrections to the conductivity in the large *N* limit. (**A** to **E**) The current operators are denoted by solid circles, and the wavy lines denote boson propagators. Dashed lines denote random flavor averaging of the interaction couplings. The fermion Green's functions (solid lines) include the effects of the disordered potential (v), and the quantum critical boson propagators include the effects of damping due to interactions. Vertex corrections [(C) to (E)] contain only g interactions because the contributions from g' interactions vanish as a result of the decoupling of the momentum integrals in the loops containing the external current operators. The sum of the two Aslamazov-Larkin diagrams [(D) and (E)] vanishes exactly in the limit of large Fermi energy, and the perturbative result (Eq. 9) is therefore valid to all orders in the interaction strength (*41*).

conductivity at $\mathcal{O}(g^2)$ and $\mathcal{O}(g'^2)$ in the large N limit is given by the sum of self-energy contributions and vertex corrections (Fig. 1). However, owing to the isotropy of the scattering processes arising from the g' interactions, only the vertex correction caused by the g interactions survives. The conductivity up to the first subleading frequency-dependent correction is then given by (*41*)

$$\frac{1}{N}\mathrm{Re}[\sigma(\omega \gg T)] = \sigma_v + \sigma_{\Sigma,g} + \sigma_{V,g} + \sigma_{\Sigma,g'};$$
$$\sigma_v(\omega) = \frac{\mathcal{N}v_F^2}{2\Gamma}, \sigma_{\Sigma,g}(\omega) = -\frac{\mathcal{N}v_F^2 g^2|\omega|}{8\pi\Gamma^3},$$
$$\sigma_{V,g}(\omega) = \frac{\mathcal{N}v_F^2 g^2|\omega|}{8\pi\Gamma^3}, \sigma_{\Sigma,g'}(\omega) = -\frac{\mathcal{N}^2 v_F^2 g'^2|\omega|}{16\Gamma^2} \quad (8)$$

Note that the g^2 vertex and self-energy terms cancel, and we have

$$N\mathrm{Re}\left[\frac{1}{\sigma(\omega \gg T)}\right] = \frac{1}{\mathcal{N}v_F^2}\left(2\Gamma + \frac{\mathcal{N}g'^2|\omega|}{4}\right) \quad (9)$$

The g'^2 term does not cancel and leads to a linear in-frequency correction to the constant transport scattering rate Γ. In the opposite limit $|\omega| \ll T$, this translates into a T-linear correction to the resistivity in the dc limit; computing the coefficient of the linear-T resistivity requires a self-consistent numerical analysis, which has been carried out in the large g' limit (*32*, *34*). Notably, the slope of this scattering rate with respect to $|\omega|$ (and therefore T) does not depend on Γ and hence on the residual ($\omega = T = 0$) resistivity. We show (*41*) that the perturbative result described here continues to be valid under a full resummation of all diagrams at large N in the Kubo formula because all surviving higher-order contributions are merely repetitions of the interaction insertions in Fig. 1, B and C.

We can also consider the case where $v = 0$ but $g \neq 0$ and $g' \neq 0$. In this case, we have (at $T = 0$) (*41*)

$$\Pi(i\omega, \mathbf{q}) = -c_b\frac{|\omega|}{|\mathbf{q}|} - \frac{\pi}{2}\mathcal{N}^2 g'^2|\omega|,$$
$$\Sigma(i\omega, \mathbf{k}) = -ic_f\,\mathrm{sgn}(\omega)|\omega|^{2/3} - \frac{i\mathcal{N}g'^2\omega}{6\pi}\ln\left(\frac{e\tilde{\Lambda}^3}{c_b|\omega|}\right) \quad (10)$$

where $\tilde{\Lambda} \sim g^2/(g'^2 v_F\mathcal{N})$ is an ultraviolet (UV) momentum cutoff. Notably, the disordered interactions induce a marginal Fermi liquid term in Σ, which manifests as the first higher-order correction to the translationally invariant result in Eq. 3 (*43*)

It is sufficient in this $v = 0$ but $g \neq 0$ and $g' \neq 0$ case to compute the conductivity using the theory of modes in the vicinity of antipodal points on the Fermi surface (*44*) We then find, as before, that $\sigma_{\Sigma,g}$ and $\sigma_{V,g}$ cancel and (*41*)

$$\frac{1}{N}\sigma(i\omega \gg T) = \frac{\mathcal{N}v_F^2}{2\omega} - \frac{\mathcal{N}^2 v_F^2 g'^2}{24\pi\omega}\ln\left(\frac{e^3\tilde{\Lambda}^6}{c_b^2\omega^2}\right)$$
$$\approx \frac{\mathcal{N}v_F^2}{2\omega + \frac{\mathcal{N}g'^2\omega}{6\pi}\ln\left(\frac{e^3\tilde{\Lambda}^6}{c_b^2\omega^2}\right)};$$
$$\frac{1}{N}\frac{\mathrm{Re}[\sigma(\omega \gg T)]}{\mathcal{N}^2 v_F^2 g'^2} = \left(6|\omega|\left\{\left[2 + \frac{\mathcal{N}g'^2}{6\pi}\ln\left(\frac{e^3\tilde{\Lambda}^6}{c_b^2\omega^2}\right)\right]^2 + \frac{\mathcal{N}^2 g'^4}{36}\right\}\right)^{-1} \quad (11)$$

The transport scattering rate is therefore still linear in $|\omega|$ (and hence T) up to logarithms, and there is no residual resistivity when $v = 0$, despite the presence of disorder in g'. This also turns out to be valid to all orders in perturbation theory in the large N limit (*41*).

Crossovers

For energy (E) scales larger than $E_{c,1} \sim \Gamma^2/(v_F^2 c_d)$ but smaller than $E_{c,2} \sim g^4/(g'^6 v_F^2\mathcal{N}^4)$ ($E_{c,1} < E_{c,2}$ because $\mathcal{N}g'^2 < g^2/\Gamma$ as disorder is a correction to the clean system), the leading frequency dependence of the inelastic part of the fermion self-energy induced by g changes from $i\omega\ln(1/|\omega|)$ to $i\mathrm{sgn}(\omega)|\omega|^{2/3}$, as in the theory with $v = 0$ described above (*41*). However, then, as shown above for $v = 0$, the $|\omega|$ or T dependence of the transport scattering rate continues to arise from g' and remains linear (up to logarithms) but with a slope that is ~⅔ of the slope in the $E < E_{c,1}$ theory. For energy scales larger than $E_{c,2}$, there is an additional crossover to the theory with $g = 0$ considered in previous studies (*32*, *34*), which also has a linear $|\omega|$ or T dependence (up to logarithms) of the transport scattering rate, but now with the same slope as in the $E < E_{c,1}$ theory (*41*).

Planckian behavior

Experimental analyses (*6*, *7*) have compared the slope of the linear-T resistivity to the renormalization of the effective mass in a proximate Fermi liquid and have so deduced a scattering time τ_{tr}^* appearing in a Drude formula for the resistivity. In our theory, we obtain

$$\frac{1}{\tau_{\mathrm{tr}}^*} = \alpha\,\frac{k_B T}{\hbar} \quad (12)$$

The dimensionless number α has been computed previously (*32*, *34*) in the limit $g' \gg g$ to be $\alpha \approx (\pi/2) \times$ (ratio of logarithms of T). For smaller g', we find (at $v \neq 0$) (*41*)

$$\alpha \approx \frac{\pi}{2}\frac{g'^2}{g'^2 L_1(T) + \frac{g^2}{\Gamma\mathcal{N}}L_2(T)},\ L_{1,2}(T) \sim -\ln T \quad (13)$$

Therefore, Planckian behavior [$\alpha \sim \mathcal{O}(1)$ and depending only slowly on T and nonuniversal parameters] only occurs in the regime of large g' considered in previous studies (*32*, *34*). Otherwise, $\alpha \ll 1$ when g is the largest interaction coupling. Our theory therefore provides a concrete realization of the often-conjectured Planckian bound of $\alpha \lesssim 1$ on the transport scattering times of quantum critical metals (*1*, *5*, *8*). It is worth noting that quantum critical T-linear resistivity with $\alpha \ll 1$ has been observed recently in experiments on heavy fermion materials (*7*). Finally, for $v = 0$ but $g \neq 0$, $\alpha \ll 1$ and has a power law dependence on T; therefore, there is manifestly no Planckian behavior in this case.

Scalar mass disorder

Finally, we consider spatial disorder in the scalar mass m_b and argue that it does not modify our results over substantial intermediate scales. Such a term is not allowed for emergent gauge fields, but it can appear as a fluctuation in the position of the quantum critical point for the cases where ϕ is a symmetry-breaking order parameter

$$\mathcal{S}_w = \int d\tau\frac{1}{2\sqrt{N}}\int d^2r\sum_{ij=1}^{N} w_{ij}(\mathbf{r})\phi_i(\mathbf{r},\tau)\phi_j(\mathbf{r},\tau) \quad (14)$$

with

$$\overline{w_{ij}(\mathbf{r})w_{lm}(\mathbf{r}')} = \frac{w^2}{2}\delta(\mathbf{r}-\mathbf{r}')(\delta_{il}\delta_{jm} + \delta_{im}\delta_{jl}) \quad (15)$$

The large N analysis shows that $\mathcal{S}_w$ is strongly relevant, so w may well be a substantial source of spatial disorder in experimental systems. Consequently, it is appropriate to account for $\mathcal{S}_w$ first by transforming to the bases of eigenmodes of ϕ, which are eigenstates of the harmonic terms for ϕ in a given disorder realization. On this basis, we obtain a theory that has the same form as $\mathcal{S}_g + \mathcal{S}_v + \mathcal{S}_{g'}$ with additional spatial disorder in the couplings, including in K. However, it is not difficult to show that spatial disorder in K is unimportant. So, $\mathcal{S}_w$ can be absorbed in a renormalization of the values of v and g', and we can continue to use our results for $\mathcal{S}_g + \mathcal{S}_v + \mathcal{S}_{g'}$. A more thorough analysis of disorder fluctuation effects is required to determine whether this transformation remains valid at the longest scales near the quantum critical point.

Discussion and outlook

A phenomenologically attractive feature of our theory is that the residual resistivity and the slope of the linear-T resistivity are determined by different types of disorder—the potential disorder v (which determines the elastic scattering rate Γ) and the interaction disorder g' (which determines the inelastic self-energy in the last term of Eq. 7), respectively. We obtain a marginal Fermi liquid electron self-energy (*18*) as is often observed in quantum critical metals (*45*).

Because the coupling g' is spatially random, momentum is not conserved at its Yukawa interaction vertex. The physical properties therefore remain unchanged for order parameters at nonzero momentum and for theories with multiple Fermi surfaces.

Our calculations were done in the large N limit; we argue that computing all diagrams directly at $N = 1$ would have led to the same crucial cancellations. The large N mainly serves to systematically select a consistent set of diagrams to resum from the saddle point of an effective action. Furthermore, the large N or Eliashberg theory agrees well with quantum Monte Carlo (QMC) studies in the clean limit (carried out with the number of fermion or boson flavors of order one) (*46*–*49*) and does not have a potentially destabilizing Schwarzian zero mode (*34*). A comparison with QMC for the disordered case requires substantially more advanced computational techniques and is the subject of ongoing work (*50*). The mechanism in the work of Shi *et al.* (*22*) for the noncommutativity of the large N and small ω limits applies only for order parameters with the same symmetry as the momentum in the spatially uniform case and does not apply for the spatially disordered case, which has no conserved momentum and for which the patches do not decouple.

Our theory of the influence of spatial disorder includes some disorder terms to all orders, and this yields the $z = 2$ diffusive scalar propagator. This is in contrast to the perturbative disorder analysis of earlier memory function treatments (*16*, *20*).

Unlike earlier approaches (*2*) to constructing controlled theories of strongly correlated metals with low-temperature T-linear resistivity, there is no local criticality in our theory. The quantum critical scalar fluctuations live in two—and not zero—spatial dimensions.

When the values of the interaction couplings and T are large enough to make the fermion self-energy Σ comparable to the Fermi energy, we expect the theories described in this work to cross over into a so-called bad metal regime (*51*). It would be interesting to examine the transport properties of such a regime.

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SUPPLEMENTARY MATERIALS

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Universal theory of strange metals from spatially random interactions

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, and Subir Sachdev

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Editor's summary
Many correlated electron systems, such as cuprates and heavy fermion materials, host an unusual type of metallic state called the strange metal. Strange metals have transport and thermodynamic properties with temperature dependences that differ from those of ordinary metals. Devising a theory that describes all of these properties correctly remains challenging. Patel *et al*. achieved this goal by introducing disorder in the coupling constants of a model of strongly interacting systems. —Jelena Stajic

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