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Adaptive formation control architectures for a team of quadrotors with multiple performance and safety constraints

Zhongjun Hu | **Xu Jin**

Department of Mechanical and Aerospace Engineering, University of Kentucky, Lexington, Kentucky, USA

Correspondence
Xu Jin, Department of Mechanical and Aerospace Engineering, University of Kentucky, Lexington, KY, 40506, USA.
Email: xu.jin@uky.edu

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Abstract

In this work, we propose a novel adaptive formation control architecture for a group of quadrotor systems, under line-of-sight (LOS) distance and relative distance constraints as well as attitude constraints, where the constraint requirements can be both asymmetric and time-varying in nature. The LOS distance constraint consideration ensures that each quadrotor is not deviating too far away from its desired flight trajectory. The LOS relative inter-quadrotor distance constraint is to guarantee that the LOS distance between any two quadrotors in the formation is neither too large (which may result in the loss of communication between quadrotors, for example) nor too small (which may result in collision between quadrotors, for example). The attitude constraints make sure that the roll, pitch, and yaw angles of each quadrotor do not deviate too much from the desired profile. Universal barrier functions are adopted in the controller design and analysis, which is a generic framework that can address system with different types of constraints in a unified controller architecture. Furthermore, each quadrotor's mass and inertia are unknown, and the system dynamics are subjected to time-varying external disturbances. Through rigorous analysis, an exponential convergence rate can be guaranteed on the distance and attitude tracking errors, while all constraints are satisfied during the operation. A simulation example further demonstrates the efficacy of the proposed control framework.

KEYWORDS

adaptive formation control, multi-vehicle systems, performance and safety constraints, quadrotors, universal barrier functions

1 | INTRODUCTION

Formation control problems of unmanned aerial vehicles (UAVs), especially quadrotors, have received much attention from the research, civil, industrial, and military communities, including notable applications in surveillance,[1,2] search and rescue,[3] contour mapping,[4,5] object lifting and transporting,[6,7] just to name a few.

Most of the works in the literature considering quadrotor formation problems, including but not limited to References 8-17, do not address the issues of various *system constraints* during the operation. However, to ensure the precise and safe operations of the quadrotor team, several constraint requirements cannot be ignored and have to be taken into consideration. First, for the *performance constraints*, we need to ensure that the quadrotor team is tracking the desired formation trajectory closely. More specifically, the line-of-sight (LOS) distance between the quadrotor and its reference trajectory should not be too large. Failing to meet such constraint requirements would result in undesirable formation performance. Second, for *safety constraints*, we need to guarantee that the LOS relative distance between any two quadrotors cannot be either too small or too large. On one hand, if two quadrotors in the formation come too close, it can result in collisions of quadrotors. On the other hand, if the distance between two quadrotors is too large, the communication link can be lost, as many communicating devices like XBEE or WiFi-based gadgets can only work effectively within a certain range. Furthermore, for safety considerations, the attitude of each quadrotor, namely the roll, pitch, and yaw angles have to be confined within a certain range during the flight, so that to ensure accurate pointing for any onboard devices, and to make sure the quadrotor is not destabilized.

Few works in the quadrotor or UAV formation literature have addressed the above issues regarding *performance* and *safety constraints*. Some notable exceptions include,[18-26] which consider mere collision avoidance between UAVs, but ignore the upper constraints of the inter-vehicle distances, and fail to address constraints on the LOS distance tracking errors and the attitude. For a single aerial robot, position constraints of a quadrotor have been considered in Reference 27, yet attitude constraint requirements are ignored. Authors in Reference 28 propose an adaptive position/attitude tracking control algorithm, considering constant and symmetric constraints for both position and attitude tracking errors, for a class of fully actuated helicopters. Such a framework cannot be extended to address underactuated unmanned vehicles. The work[29] addresses both position and attitude constraints for a single quadrotor, which only addresses constant and symmetric constraint functions. A distributed formation control framework for underactuated quadrotors with pre-assigned constraints of the position is developed in Reference 30. The work[31] investigates the attitude synchronization problem for cooperative quadrotors subjected to unknown nonlinear dynamics and multiple actuator faults. A formation control algorithm for the leader quadrotors and a finite-time containment control for the follower quadrotors with unknown model dynamics are proposed in Reference 32. None of these works[27-32] can address formation control problems of a team of underactuated quadrotors, with time-varying and asymmetric constraint requirements on the LOS distance, relative inter-quadrotor distance, and attitude tracking errors.

In this work, we develop a novel adaptive constrained formation control architecture for a team of quadrotors. Multiple constraint requirements on *performance* and *safety* are considered during operation. For the performance considerations, we address the constraint requirements on the distance tracking error between the actual and the desired positions for each quadrotor, so that to ensure *precise* trajectory tracking and formation keeping. For the *safety constraints*, we consider the constraints on the relative inter-quadrotor distance, so that to ensure the distance between any two quadrotors is neither too large nor too small. *Safety constraints* also include constraint requirements on the attitude of each quadrotor, so that to ensure accurate pointing and avoid destabilization. All constraint functions can be time-varying and asymmetric. Universal barrier functions are adopted in the controller design and analysis, which is a unified structure that can address different types of constraints in a single control architecture. Adaptive estimation is used to handle time-varying uncertainties presented in the system dynamics and unknown system properties. Through rigorous mathematical discussions, we show that exponential convergence can be guaranteed on the LOS distance and relative distance tracking errors, as well as the attitude tracking errors, while all constraint requirements are satisfied during the operation.

The notations used in this work are fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers and I_m means the identity matrix in the space $\mathbb{R}^{m\times m}$. Moreover, $(\cdot)^{\mathrm{T}}$ implies the transpose vector, $|\cdot|$ is the absolute value for scalars, and $\|\cdot\|$ represents the Euclidean norm for vectors and induced norm for matrices. Next, $\mathrm{diag}[x_1, \ldots, x_m]$ denotes a diagonal matrix in the space $\mathbb{R}^{m\times m}$, and the only nonzero elements being $x_1, \ldots, x_m$ on the diagonal in that specific sequence. Furthermore, we use $\mathrm{c}\theta$ to denote $\cos\theta$, $\mathrm{s}\theta$ to denote $\sin\theta$, and $\mathrm{t}\theta$ to denote $\tan\theta$. We also write $\dot{(\cdot)}$ as the first order time derivative of $(\cdot)$, if $(\cdot)$ is differentiable, and $(\cdot)^{(n)}$ as the nth order time derivative of $(\cdot)$ for n being a positive integer. Next, C^2 denotes the class of functions that are two-times differentiable with respect to time, with the derivatives being in the class of C^1, which consists of all differentiable functions whose derivative is continuous. Besides, for any two vectors $v_1,\ v_2 \in \mathbb{R}^3$, the cross-product operator $\mathbb{S}(\cdot)$ gives $\mathbb{S}(v_1)v_2 = v_1 \times v_2$. It is also true that $\mathbb{S}(v_1)v_2 = -\mathbb{S}(v_2)v_1$ and $v_1^{\mathrm{T}}\mathbb{S}(v_2)v_1 = 0$. Furthermore, we use $\circ$ to denote the Hadamard product, that is, the element-wise product of two vectors/matrices with the same dimension, such that for $A, B \in \mathbb{R}^{m\times n}$, where $i = 1, \ldots, m$ and $j = 1, \ldots, n$, we have $(A \circ B)_{ij} = (A)_{ij}(B)_{ij}$. Finally, $\mathrm{SO}(3) = \{\Omega \in \mathbb{R}^{3\times 3} \mid \Omega^{\mathrm{T}}\Omega = I_3\}$ is a set of orthogonal matrices in $\mathbb{R}^{3\times 3}$, and $\mathrm{S}^2 = \{x \in \mathbb{R}^3 \mid \|x\| = 1\}$ is a set of unit vectors in $\mathbb{R}^3$.

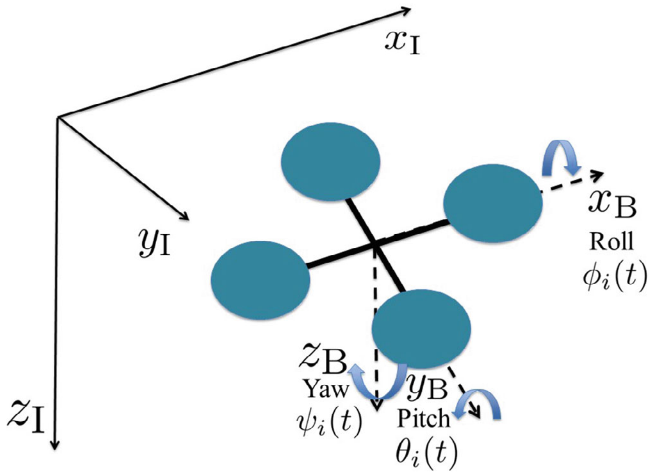

FIGURE 1 Illustration of the inertial {I} and body-fixed {B} frames.

2 | PROBLEM FORMULATION

2.1 | System dynamics

Consider the following class of multi-vehicle systems with N quadrotors, where, for the ith quadrotor ($i = 1, \dots, N$) shown in Figure 1, the position and attitude in the inertial reference frame are represented as $p_i(t) = [x_i(t),\ y_i(t),\ z_i(t)]^{\text{T}} \in \mathbb{R}^3$ and $\Theta_i(t) = [\phi_i(t),\ \theta_i(t),\ \psi_i(t)]^{\text{T}} \in \mathbb{R}^3$, respectively. The translational velocities with respect to the inertial reference frame are represented as $v_i(t) = [v_{xi}(t),\ v_{yi}(t),\ v_{zi}(t)]^{\text{T}} \in \mathbb{R}^3$. Moreover, define a body-fixed frame with the origin being at the center of mass for each quadrotor, and the rotational velocities with respect to this body-fixed frame are denoted by $\omega_i(t) = [\omega_{xi}(t),\ \omega_{yi}(t),\ \omega_{zi}(t)]^{\text{T}} \in \mathbb{R}^3$. The kinematics for the ith quadrotor ($i = 1, \dots, N$) are expressed as

$$\dot{p}_i(t) = v_i(t), \tag{1}$$

$$\dot{\Theta}_i(t) = T(\Theta_i(t))\omega_i(t), \tag{2}$$

where $p_i(0) = p_{i0} \in \mathbb{R}^3$ and $\Theta_i(0) = \Theta_{i0} \in \mathbb{R}^3$, with p_{i0} and Θ_{i0} being initial conditions. Besides, $T(\Theta_i(t))$ is the transformation matrix that relates the angular velocity in the body-fixed frame to the rate of change of the Euler angles in the inertial frame, and is given by

$$T(\Theta_i) = \begin{bmatrix} 1 & \text{s}\phi_i \text{t}\theta_i & \text{c}\phi_i \text{t}\theta_i \\ 0 & \text{c}\phi_i & -\text{s}\phi_i \\ 0 & \text{s}\phi_i/\text{c}\theta_i & \text{c}\phi_i/\text{c}\theta_i \end{bmatrix}, \tag{3}$$

where we have

$$||T(\Theta_i(t))|| \leq T_{\max}, \tag{4}$$

with $T_{\max} > 0$ being a known constant, when $-\frac{\pi}{2} < \phi_i(t) < \frac{\pi}{2}$ and $-\frac{\pi}{2} + \epsilon < \theta_i(t) < \frac{\pi}{2} - \epsilon$, for any user-defined positive ϵ. The constraints are known as the *safety constraints* that will be discussed shortly.

The dynamics for the ith quadrotor ($i = 1, \dots, N$) are expressed as

$$m_i\dot{v}_i(t) = m_i g e_z - F_i(t)R(\Theta_i(t))e_z + N_{1i}(t), \tag{5}$$

$$J_i\dot{\omega}_i(t) = \mathbb{S}(J_i\omega_i(t))\omega_i(t) + \tau_i(t) + N_{2i}(t), \tag{6}$$

where $v_i(0) = v_{i0} \in \mathbb{R}^3$ and $\omega_i(0) = \omega_{i0} \in \mathbb{R}^3$, with v_{i0} and ω_{i0} being the initial conditions. $m_i \in \mathbb{R}$, $m_i > 0$ is the mass of the ith quadrotor ($i = 1, \dots, N$), and $J_i \in \mathbb{R}^{3\times 3}$ is a symmetric positive definite matrix representing the inertia. $F_i(t) \in \mathbb{R}$ and $\tau_i(t) \in \mathbb{R}^3$ represent the thrust and torques of the ith quadrotor ($i = 1, \dots, N$), respectively. $N_{1i}(t) \in \mathbb{R}^3$ and $N_{2i}(t) \in \mathbb{R}^3$ denote the external disturbances of the ith quadrotor ($i = 1, \dots, N$). Furthermore, $g \in \mathbb{R}$ is the gravitational acceleration and $e_z = [0,\ 0,\ 1]^{\mathrm{T}} \in \mathbb{R}^3$ is the unit vector. $R(\Theta_i(t)) \in \mathrm{SO}(3)$ is the rotation matrix, with the expression

$$R(\Theta_i) = \begin{bmatrix} c\theta_i c\psi_i & s\phi_i s\theta_i c\psi_i - c\phi_i s\psi_i & c\phi_i s\theta_i c\psi_i + s\phi_i s\psi_i \\ c\theta_i s\psi_i & s\phi_i s\theta_i s\psi_i + c\phi_i c\psi_i & c\phi_i s\theta_i s\psi_i - s\phi_i c\psi_i \\ -s\theta_i & s\phi_i c\theta_i & c\phi_i c\theta_i \end{bmatrix}, \tag{7}$$

which translates the translational velocity vector in the body-fixed frame into the rate of change of the position vector in the inertial frame. It is straightforward to see that

$$\|R(\Theta_i(t))\| \le R_{\max}, \tag{8}$$

with $R_{\max} > 0$ being a known constant.

2.2 | System performance and safety constraints

In the formation control problem, each quadrotor has its own reference trajectory to track, with the coordinate of the reference trajectory for the ith vehicle ($i = 1, \dots, N$) denoted by $p_{di}(t) \triangleq [x_{di}(t),\ y_{di}(t),\ z_{di}(t)]^{\mathrm{T}} \in \mathbb{R}^3$. Hence the line-of-sight (LOS) distance tracking error for the ith quadrotor ($i = 1, \dots, N$), which is the distance between the desired and actual position of the quadrotor, is defined as

$$d_{ei}(t) \triangleq \sqrt{(x_i(t) - x_{di}(t))^2 + (y_i(t) - y_{di}(t))^2 + (z_i(t) - z_{di}(t))^2}. \tag{9}$$

Furthermore, the desired LOS relative distance between the ith and jth ($i, j = 1, \dots, N, j \neq i$) quadrotors is given as

$$L_{ij}(t) \triangleq \sqrt{(x_{di}(t) - x_{dj}(t))^2 + (y_{di}(t) - y_{dj}(t))^2 + (z_{di}(t) - z_{dj}(t))^2}, \tag{10}$$

with the actual LOS relative distance being

$$d_{ij}(t) \triangleq \sqrt{(x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2 + (z_i(t) - z_j(t))^2}. \tag{11}$$

The configurations in the case of three quadrotors can be seen in Figure 2.

During the formation operation, there are certain **system constraint requirements** that need to be satisfied, in order to ensure the *precise* and *safe* functioning of the system. Specifically, one *performance constraint* and four *safety constraints* have to be satisfied for each quadrotor in the operation.

First, the LOS distance tracking error for the ith quadrotor $d_{ei}(t)$ ($i = 1, \dots, N$) has to satisfy the following *performance constraint*

$$d_{ei}(t) < \Omega_{dHi}(t), \tag{12}$$

where, for all $t \ge 0$, $\Omega_{dHi}(t) > 0$ is the user-defined time-varying constraint requirement for the distance tracking error $d_{ei}(t)$ and is C^3. The constraint requirement (12) means that the LOS distance tracking error for the ith quadrotor cannot be too large.

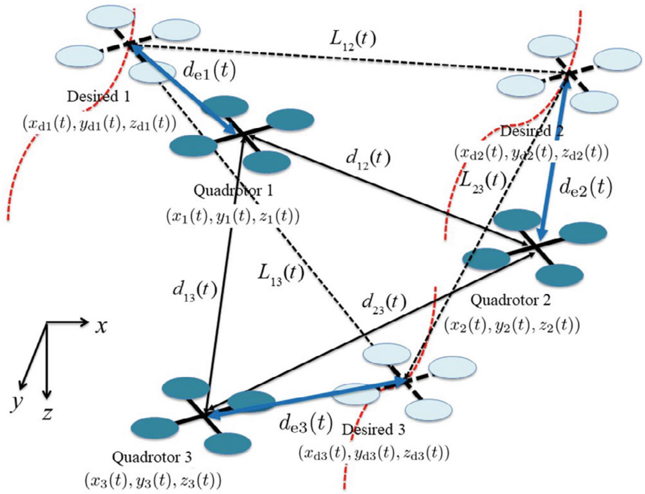

FIGURE 2 Schematics of the formation control problem for three quadrotors: for $i, j = 1, 2, 3, j \neq i$, quadrotors in dark blue and solid black represent the actual positions $(x_i(t),\ y_i(t),\ z_i(t))$, quadrotors in light blue and dashed black represent the desired positions $(x_{\mathrm{d}i}(t),\ y_{\mathrm{d}i}(t),\ z_{\mathrm{d}i}(t))$, red dashed lines represent the desired trajectories, black dashed lines represent the desired inter-quadrotor distances $L_{ij}(t)$, black solid lines represent the actual inter-quadrotor distances $d_{ij}(t)$, and blue solid lines represent the distance tracking errors $d_{\mathrm{e}i}(t)$.

Second, define the LOS relative distance tracking error between the ith and jth quadrotors $(i, j = 1, \dots, N, j \neq i)$ as $d_{\mathrm{e}ij}(t) \triangleq d_{ij}(t) - L_{ij}(t)$, which has to meet the following *safety constraint*

$$-\Omega_{\mathrm{L}ij}(t) < d_{\mathrm{e}ij}(t) < \Omega_{\mathrm{H}ij}(t), \tag{13}$$

where, for all $t \geq 0$, $\Omega_{\mathrm{H}ij}(t) > 0$ is the user-defined time-varying higher bound for the distance tracking error $d_{\mathrm{e}ij}(t)$, and $-\Omega_{\mathrm{L}ij}(t) < 0$ is the lower bound, with $L_{ij}(t) > \Omega_{\mathrm{L}ij}(t) > 0$. Both $\Omega_{\mathrm{H}ij}(t)$ and $\Omega_{\mathrm{L}ij}(t)$ are C^3. The constraint requirement (13) means that the inter-quadrotor distance cannot be either too large or too small.

Remark 1. On the one hand, the satisfaction of constraint requirement (13) ensures that

$$0 < L_{ij}(t) - \Omega_{\mathrm{L}ij}(t) < d_{ij}(t) < \Omega_{\mathrm{H}ij}(t) + L_{ij}(t),$$

for $i, j = 1, \dots, N$, $j \neq i$, which ensures collision avoidance and guarantees that the LOS relative distance between any two quadrotors is not too far. On the other hand, if we have the following inter-vehicle distance constraint requirement

$$0 < \bar{\Omega}_{\mathrm{L}ij}(t) < d_{ij}(t) < \bar{\Omega}_{\mathrm{H}ij}(t), \tag{14}$$

for all $t \geq 0$, then it can be easily transferred into the form of (13), by designing $\Omega_{\mathrm{H}ij}(t) = \bar{\Omega}_{\mathrm{H}ij}(t) - L_{ij}(t) > 0$ and $\Omega_{\mathrm{L}ij}(t) = L_{ij}(t) - \bar{\Omega}_{\mathrm{L}ij}(t) > 0$. Note that $\bar{\Omega}_{\mathrm{L}ij}(t) < L_{ij}(t) < \bar{\Omega}_{\mathrm{H}ij}(t)$ should be guaranteed for all $t \geq 0$ in order for the perfect tracking to be achievable.

Last but not least, the attitude tracking error for the ith quadrotor $(i = 1, \dots, N)$ is defined as

$$e_{\Theta i}(t) = [e_{\phi i}(t),\ e_{\theta i}(t),\ e_{\psi i}(t)]^{\mathrm{T}} = \Theta_i(t) - \Theta_{\mathrm{d}i}(t), \tag{15}$$

where $\Theta_{\mathrm{d}i}(t) = [\phi_{\mathrm{d}i}(t),\ \theta_{\mathrm{d}i}(t),\ \psi_{\mathrm{d}i}(t)]^{\mathrm{T}}$ is the desired attitude to be specified later. The attitude tracking error has to satisfy the following *safety constraint*

$$-\Omega_{\phi \mathrm{L}i}(t) < e_{\phi i}(t) < \Omega_{\phi \mathrm{H}i}(t), \tag{16}$$

$$-\Omega_{\theta \mathrm{L}i}(t) < e_{\theta i}(t) < \Omega_{\theta \mathrm{H}i}(t), \tag{17}$$

$$-\Omega_{\psi \mathrm{L}i}(t) < e_{\psi i}(t) < \Omega_{\psi \mathrm{H}i}(t), \tag{18}$$

where, for all $t \geq 0$, the constraint functions $\Omega_{\phi \mathrm{L}i}(t)$, $\Omega_{\phi \mathrm{H}i}(t)$, $\Omega_{\theta \mathrm{L}i}(t)$, $\Omega_{\theta \mathrm{H}i}(t)$, $\Omega_{\psi \mathrm{L}i}(t)$, and $\Omega_{\psi \mathrm{H}i}(t)$ are C^2 and

$$\begin{aligned}
&0 < \Omega_{\phi \mathrm{L}i}(t) \leq \frac{\pi}{2} + \phi_{\mathrm{d}i}(t), && 0 < \Omega_{\phi \mathrm{H}i}(t) \leq \frac{\pi}{2} - \phi_{\mathrm{d}i}(t), \\
&0 < \Omega_{\theta \mathrm{L}i}(t) \leq \frac{\pi}{2} - \epsilon + \theta_{\mathrm{d}i}(t), && 0 < \Omega_{\theta \mathrm{H}i}(t) \leq \frac{\pi}{2} - \epsilon - \theta_{\mathrm{d}i}(t), \\
&0 < \Omega_{\psi \mathrm{L}i}(t) \leq \pi + \psi_{\mathrm{d}i}(t), && 0 < \Omega_{\psi \mathrm{H}i}(t) \leq \pi - \psi_{\mathrm{d}i}(t).
\end{aligned}$$

Remark 2. The safety constraints (16)–(18) require that the attitude tracking errors to be confined within a user-defined range. Note that the constraint functions on the roll, pitch, and yaw angles can be different. These constraint functions can also be time-varying and asymmetric. Furthermore, (16)–(18) ensure that the transformation matrix $T(\Theta_i(t))$ is invertible. Violation of the constraint requirements (16)–(18) will not only affect the formation performance of the system, but can also destabilize the quadrotor dynamics and result in system failure.

2.3 | Control objective

The **control objective** for the formation control problem is to design a control framework such that:

(1) The LOS distance tracking error $d_{\mathrm{e}i}(t)$ for the ith quadrotor ($i = 1, \ldots, N$) can converge into an arbitrary small neighborhood of zero;
(2) The relative distance tracking error $d_{\mathrm{e}ij}(t)$ between the ith and jth ($i, j = 1, \ldots, N,\ j \neq i$) quadrotors can converge into an arbitrarily small neighborhood of zero;
(3) For the ith quadrotor ($i = 1, \ldots, N$), the attitude tracking error $e_{\Theta i}(t) = [e_{\phi i}(t),\ e_{\theta i}(t),\ e_{\psi i}(t)]^{\mathrm{T}}$ can converge into an arbitrarily small neighborhood of zero;
(4) The performance and safety constraint requirements (12), (13), (16)–(18) are satisfied during the operation.

The following assumptions are used to facilitate the discussion and analysis of the main result.

Assumption 1. The reference trajectory coordinates for the ith quadrotor ($i = 1, \ldots, N$) $x_{\mathrm{d}i}(t)$, $y_{\mathrm{d}i}(t)$, and $z_{\mathrm{d}i}(t)$, are all C^3. The reference attitudes $\phi_{\mathrm{d}i}(t)$, $\theta_{\mathrm{d}i}(t)$, and $\psi_{\mathrm{d}i}(t)$, are all C^2. Furthermore, for the reference attitude we require that

$$-\frac{\pi}{2} < \phi_{\mathrm{d}i}(t) < \frac{\pi}{2},\ -\frac{\pi}{2} + \epsilon < \theta_{\mathrm{d}i}(t) < \frac{\pi}{2} - \epsilon,\ -\pi \leq \psi_{\mathrm{d}i}(t) \leq \pi.$$

Assumption 2. The thrust $F_i(t)$ and disturbances $N_{1i}(t)$ and $N_{2i}(t)$ for the ith quadrotor ($i = 1, \ldots, N$) are uniformly bounded with *unknown* bounds.

Assumption 3. The mass m_i and inertia J_i for the ith quadrotor ($i = 1, \ldots, N$) are *unknown*, but the inverses of J_i are assumed to be both upper and lower bounded, such that for any $z \in \mathbb{R}^3$, $\underline{b}_{Ji} z^{\mathrm{T}} z < z^{\mathrm{T}} J_i^{-1} z < \overline{b}_{Ji} z^{\mathrm{T}} z$, where $\overline{b}_{Ji}$, and $\underline{b}_{Ji}$ are *unknown* positive constants.

Assumption 4 [33]. Denote the approximation of the time derivative of a continuous function $\dot{\vartheta}(t)$ as $\hat{\dot{\vartheta}}(t)$, where

$$\hat{\dot{\vartheta}}(t) = \frac{\vartheta(t) - \vartheta(t - T)}{T}, \tag{19}$$

for some small $T > 0$. Then

$$|\hat{\dot{\vartheta}}(t) - \dot{\vartheta}(t)| \leq \varepsilon_{\vartheta} \approx o(T). \tag{20}$$

To facilitate the analysis, we present the following lemma from the literature.

Lemma 1. For any constant $\varepsilon > 0$ and any variable $z \in \mathbb{R}$, we have $0 \leq |z| - \frac{z^2}{\sqrt{z^2+\varepsilon^2}} < \varepsilon$.

From this point onwards, to simplify the notation, the time and state dependence of the system will be omitted whenever no confusion would arise.

3 | UNIVERSAL BARRIER FUNCTION

Here we introduce the structure of universal barrier function to be used later in the analysis, which is adopted but modified from our earlier work.[34] Specifically, to address the constraint requirements (12) and (13), which are on the LOS distance tracking error d_{ei} and relative inter-quadrotor distance tracking error d_{eij} $(i, j = 1, \ldots, N, j \neq i)$, we first introduce the transformed error variables as follows

$$\eta_{ei} = \frac{\Omega_{dHi} d_{ei}}{\Omega_{dHi} - d_{ei}}, \tag{21}$$

and

$$\eta_{ij} = \frac{\Omega_{Hij}\Omega_{Lij} d_{eij}}{(\Omega_{Hij} - d_{eij})(\Omega_{Lij} + d_{eij})}. \tag{22}$$

The universal barrier functions used to deal with the constraint requirements (12) and (13) for the ith quadrotor $(i = 1, \ldots, N)$ are then defined as

$$V_{ei} = \frac{1}{2}\eta_{ei}^2, \quad V_{ij} = \frac{1}{2}\eta_{ij}^2. \tag{23}$$

Take V_{ij} for an example. It is easy to see that $\eta_{ij} = 0$ if and only if $d_{eij} = 0$. Besides, when $d_{eij} \to \Omega_{Hij}$, we have $\eta_{ij} \to +\infty$, hence $V_{ij} \to +\infty$. Alternatively, when $d_{eij} \to -\Omega_{Lij}$, we have $\eta_{ij} \to -\infty$, therefore $V_{ij} \to +\infty$.

Remark 3. For the universal barrier function V_{ij}, note that if the constraint functions are symmetric, namely if $\Omega_{Hij} = \Omega_{Lij} = \Omega_{ij}$, then the barrier function V_{ij} becomes

$$V_{ij} = \frac{1}{2}\eta_{ij}^2, \quad \eta_{ij} = \frac{\Omega_{ij}^2 d_{eij}}{\Omega_{ij}^2 - d_{eij}^2}. \tag{24}$$

When there are no constraint requirements on d_{eij}, which can equivalently be seen as $\Omega_{Hij} = \Omega_{Lij} = \Omega_{ij} \to +\infty$, we have

$$\lim_{\Omega_{ij}\to+\infty} \eta_{ij} = d_{eij}, \quad \lim_{\Omega_{ij}\to+\infty} V_{ij} = \frac{1}{2}d_{eij}^2, \tag{25}$$

which means systems without output constraint requirements can in fact be regarded as a special case of the generic discussion on asymmetric constraint requirements.

Second, regarding the constraint requirements (16)–(18), which are on the attitude tracking errors $e_{\Theta i}(t)$ of the ith quadrotor $(i = 1, \ldots, N)$, we first introduce the transformed error variables as follows

$$\eta_{\phi i} = \frac{\Omega_{\phi Hi}\Omega_{\phi Li} e_{\phi i}}{(\Omega_{\phi Hi} - e_{\phi i})(\Omega_{\phi Li} + e_{\phi i})}, \tag{26}$$

$$\eta_{\theta i} = \frac{\Omega_{\theta \mathrm{H}i}\Omega_{\theta \mathrm{L}i}e_{\theta i}}{(\Omega_{\theta \mathrm{H}i} - e_{\theta i})(\Omega_{\theta \mathrm{L}i} + e_{\theta i})}, \tag{27}$$

$$\eta_{\psi i} = \frac{\Omega_{\psi \mathrm{H}i}\Omega_{\psi \mathrm{L}i}e_{\psi i}}{(\Omega_{\psi \mathrm{H}i} - e_{\psi i})(\Omega_{\psi \mathrm{L}i} + e_{\psi i})}, \tag{28}$$

and the universal barrier function used to deal with the constraint requirements (16)–(18) for the ith quadrotor ($i = 1, \dots, N$) is designed as

$$V_{\Theta i} = \frac{1}{2}\eta_{\Theta i}^{\mathrm{T}}\eta_{\Theta i} = \frac{1}{2}\eta_{\phi i}^2 + \frac{1}{2}\eta_{\theta i}^2 + \frac{1}{2}\eta_{\psi i}^2, \tag{29}$$

where $\eta_{\Theta i} = [\eta_{\phi i},\ \eta_{\theta i},\ \eta_{\psi i}]^{\mathrm{T}} \in \mathbb{R}^3$.

Remark 4. In order to address asymmetric constraint functions, one barrier Lyapunov function that has been discussed in the literature[35,36] has the following structure

$$V_{\mathrm{b}} = \frac{q(e)}{p}\log\frac{\Omega_{\mathrm{bH}}^p}{\Omega_{\mathrm{bH}}^p - e^p} + \frac{1-q(e)}{p}\log\frac{\Omega_{\mathrm{bL}}^p}{\Omega_{\mathrm{bL}}^p - e^p}, \tag{30}$$

where e is the output tracking error to be constrained, Ω_{bH} and Ω_{bL} are the higher and lower bounds, p is an even number such that $p > n$, with n being the order of the systems, and

$$q(\cdot) = \begin{cases} 1, & \text{if } \cdot > 0; \\ 0, & \text{otherwise.} \end{cases} \tag{31}$$

Note that (30) does not have the generic property stated in Remark 3. Besides, since $q(\cdot)$ is a discontinuous function, the form (30) requires that the error variable e is raised to the pth power in order to avoid discontinuity when $e = 0$ for the derivatives of V_{b}, which may put a higher demand than necessary on the control signal when e is large.

Remark 5. Another widely used BLF in the literature utilizes the tangent form proposed in our previous works[37-39]

$$V_{\mathrm{b}} = \frac{\Omega_{\mathrm{b}}^2}{\pi}\tan\left(\frac{\pi e^2}{2\Omega_{\mathrm{b}}^2}\right), \quad |e(0)| < \Omega_{\mathrm{b}}(0). \tag{32}$$

By the L' Hopital's rule we get $\lim_{\Omega_{\mathrm{b}}\to+\infty} V_{\mathrm{b}} = \frac{1}{2}e^2$, hence this BLF can be used to address general systems without constraint requirements. However such a form cannot be extended to address asymmetric constraints without raising the error variable to higher powers as in (30). From this perspective, the barrier function (23) and (29) are more general than both (30) and (32).

4 | CONTROL DESIGN AND ANALYSIS

In this section, we present the backstepping design procedure that will lead to our controller design and main theorem.

4.1 | Distance control design

Step 1:

At this step, we consider the position kinematics of the quadrotors. Design the barrier function as

$$V_1 = \sum_{i=1}^{N}\left(V_{\mathrm{e}i} + \sum_{j=1, j\neq i}^{N} V_{ij}\right), \tag{33}$$

and its derivative with respect to time leads to

$$\dot{V}_1 = \sum_{i=1}^{N}\left(\dot{V}_{\mathrm{e}i} + \sum_{j=1,j\neq i}^{N}\dot{V}_{ij}\right) = \sum_{i=1}^{N}\left(\eta_{\mathrm{e}i}\dot{\eta}_{\mathrm{e}i} + \sum_{j=1,j\neq i}^{N}\eta_{ij}\dot{\eta}_{ij}\right). \tag{34}$$

First we examine the dynamics for $\eta_{\mathrm{e}i}$ $(i = 1, \dots, N)$. From (21), we have

$$\dot{\eta}_{\mathrm{e}i} = \frac{\partial\eta_{\mathrm{e}i}}{\partial\Omega_{\mathrm{dH}i}}\dot{\Omega}_{\mathrm{dH}i} + \vartheta_{di}\dot{d}_{\mathrm{e}i} = \Delta_{\mathrm{H}i} + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(x_i - x_{\mathrm{d}i})\dot{x}_i + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(y_i - y_{\mathrm{d}i})\dot{y}_i + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(z_i - z_{\mathrm{d}i})\dot{z}_i - \xi_i, \tag{35}$$

where

$$\Delta_{\mathrm{H}i} \triangleq \frac{\partial\eta_{\mathrm{e}i}}{\partial\Omega_{\mathrm{dH}i}}\dot{\Omega}_{\mathrm{dH}i}, \quad \vartheta_{di} \triangleq \frac{\partial\eta_{\mathrm{e}i}}{\partial d_{\mathrm{e}i}} = \frac{\Omega_{\mathrm{dH}i}^2}{(\Omega_{\mathrm{dH}i} - d_{\mathrm{e}i})^2},$$
$$\xi_i \triangleq \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(x_i - x_{\mathrm{d}i})\dot{x}_{\mathrm{d}i} + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(y_i - y_{\mathrm{d}i})\dot{y}_{\mathrm{d}i} + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(z_i - z_{\mathrm{d}i})\dot{z}_{\mathrm{d}i}.$$

Hence for $\dot{V}_{\mathrm{e}i}$ $(i = 1, \dots, N)$ we have

$$\dot{V}_{\mathrm{e}i} = \eta_{\mathrm{e}i}\vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(x_i - x_{\mathrm{d}i})\dot{x}_i + \eta_{\mathrm{e}i}\vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(y_i - y_{\mathrm{d}i})\dot{y}_i + \eta_{\mathrm{e}i}\vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(z_i - z_{\mathrm{d}i})\dot{z}_i + \eta_{\mathrm{e}i}\Delta_{\mathrm{H}i} - \eta_{\mathrm{e}i}\xi_i. \tag{36}$$

Similarly, for $\dot{V}_{ij}$ $(i,j = 1, \dots, N, j \neq i)$ we have

$$\begin{aligned}\dot{V}_{ij} = {} & \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(x_i - x_j)\dot{x}_i + \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(y_i - y_j)\dot{y}_i + \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(z_i - z_j)\dot{z}_i \\ & - \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(x_i - x_j)\dot{x}_j - \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(y_i - y_j)\dot{y}_j - \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(z_i - z_j)\dot{z}_j + \eta_{ij}\Delta_{ij} - \eta_{ij}\xi_{ij},\end{aligned} \tag{37}$$

where

$$\Delta_{ij} \triangleq \frac{\partial\eta_{ij}}{\partial\Omega_{\mathrm{H}ij}}\dot{\Omega}_{\mathrm{H}ij} + \frac{\partial\eta_{ij}}{\partial\Omega_{\mathrm{L}ij}}\dot{\Omega}_{\mathrm{L}ij}, \quad \vartheta_{ij} \triangleq \frac{\partial\eta_{ij}}{\partial d_{\mathrm{e}ij}} = \frac{\Omega_{\mathrm{H}ij}\Omega_{\mathrm{L}ij}(d_{\mathrm{e}ij}^2 + \Omega_{\mathrm{H}ij}\Omega_{\mathrm{L}ij})}{(\Omega_{\mathrm{H}ij} - d_{\mathrm{e}ij})^2(\Omega_{\mathrm{L}ij} + d_{\mathrm{e}ij})^2},$$
$$\xi_{ij} \triangleq \vartheta_{ij}\dot{L}_{ij} = \vartheta_{ij}\frac{1}{L_{ij}}(x_{\mathrm{d}i} - x_{\mathrm{d}j})(\dot{x}_{\mathrm{d}i} - \dot{x}_{\mathrm{d}j}) + \vartheta_{ij}\frac{1}{L_{ij}}(y_{\mathrm{d}i} - y_{\mathrm{d}j})(\dot{y}_{\mathrm{d}i} - \dot{y}_{\mathrm{d}j}) + \vartheta_{ij}\frac{1}{L_{ij}}(z_{\mathrm{d}i} - z_{\mathrm{d}j})(\dot{z}_{\mathrm{d}i} - \dot{z}_{\mathrm{d}j}).$$

Hence, for $\dot{V}_1$ we have

$$\dot{V}_1 = \sum_{i=1}^{N}\left(\eta_{\mathrm{e}i}\Delta_{\mathrm{H}i} - \eta_{\mathrm{e}i}\xi_i + \sum_{j=1,j\neq i}^{N}(\eta_{ij}\Delta_{ij} - \eta_{ij}\xi_{ij}) + E_{xi}\dot{x}_i + E_{yi}\dot{y}_i + E_{zi}\dot{z}_i\right), \tag{38}$$

where

$$E_{xi} = \eta_{\mathrm{e}i}\vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(x_i - x_{\mathrm{d}i}) + \sum_{j=1,j\neq i}^{N}2\eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(x_i - x_j),$$
$$E_{yi} = \eta_{\mathrm{e}i}\vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(y_i - y_{\mathrm{d}i}) + \sum_{j=1,j\neq i}^{N}2\eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(y_i - y_j),$$
$$E_{zi} = \eta_{\mathrm{e}i}\vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(z_i - z_{\mathrm{d}i}) + \sum_{j=1,j\neq i}^{N}2\eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(z_i - z_j).$$

Now, define $E_i = [E_{xi},\ E_{yi},\ E_{zi}]^{\mathrm{T}} \in \mathbb{R}^3$. For the term $E_{xi}\dot{x}_i + E_{yi}\dot{y}_i + E_{zi}\dot{z}_i$ in (38), from (1) we can get

$$E_{xi}\dot{x}_i + E_{yi}\dot{y}_i + E_{zi}\dot{z}_i = E_i^{\mathrm{T}} v_i. \tag{39}$$

Next, define the fictitious velocity tracking error as $e_{vi} = v_i - \alpha_{vi}$, with the stabilizing function $\alpha_{vi} \in \mathbb{R}^3$ $(i = 1, \dots, N)$ designed as

$$\alpha_{vi} = \frac{E_i}{E_i^{\mathrm{T}} E_i}\left(-K_{\mathrm{e}i}\eta_{\mathrm{e}i}^2 - \sum_{j=1, j\neq i}^{N} K_{ij}\eta_{ij}^2 - \eta_{\mathrm{e}i}\Delta_{\mathrm{H}i} + \eta_{\mathrm{e}i}\xi_i - \sum_{j=1, j\neq i}^{N}(\eta_{ij}\Delta_{ij} - \eta_{ij}\xi_{ij})\right), \tag{40}$$

where $K_{\mathrm{e}i} > 0$ and $K_{ij} > 0$ are the control gains.

Remark 6. In (40), singularity can occur when $\|E_i\| = 0$. Since $\|E_i\| = 0$ if and only if $E_{xi} = 0, E_{yi} = 0$, and $E_{zi} = 0$ at the same time, there are two cases when this can happen. First, $\|E_i\| = 0$ when both $d_{\mathrm{e}i} = 0$ and $d_{\mathrm{e}ij} = 0$. In this case, note that all the terms in the bracket on the right-hand-side of (40) are also zero, and we simply have $\alpha_{vi} = 0$. Second, $E_{xi} = 0$, $E_{yi} = 0$, and $E_{zi} = 0$ can happen at the same time when the reference direction vector for tracking is opposite to and same in magnitude with the direction vector for collision avoidance. This is usually referred to as the "deadlock situation"[33] in the literature, which can be resolved by modifying the reference trajectories or the time-varying constraint functions to allow the vehicle to move out of the deadlock. For the rest of the analysis we assume $\|E_i\| > 0$ is guaranteed.

Therefore, (38) leads to

$$\dot{V}_1 = \sum_{i=1}^{N}\left(E_i^{\mathrm{T}} e_{vi} - K_{\mathrm{e}i}\eta_{\mathrm{e}i}^2 - \sum_{j=1, j\neq i}^{N} K_{ij}\eta_{ij}^2\right). \tag{41}$$

Step 2:

At this step, we consider the translational dynamics of the quadrotors. Design the Lyapunov function candidate at this step as

$$V_2 = \sum_{i=1}^{N}\frac{1}{2}e_{vi}^{\mathrm{T}} e_{vi}, \tag{42}$$

and its time derivative gives

$$\dot{V}_2 = \sum_{i=1}^{N} e_{vi}^{\mathrm{T}}\left(g e_z - \frac{1}{m_i}u_i - \frac{1}{m_i}F_i(R_i - R_{\mathrm{d}i})e_z + \frac{1}{m_i}N_{1i} - \dot{\alpha}_{vi}\right), \tag{43}$$

where we denote $u_i = F_i R_{\mathrm{d}i} e_z$. Now, for the ith quadrotor $(i = 1, \dots, N)$, the control law $u_i \in \mathbb{R}^3$ is designed as

$$u_i = \hat{m}_i \bar{u}_i, \tag{44}$$

$$\bar{u}_i = E_i + g e_z + (K_{vi} + \nu_i)e_{vi} + \hat{\mu}_{mi}\frac{e_{vi}}{\sqrt{e_{vi}^{\mathrm{T}} e_{vi} + \varepsilon_i^2}} - \hat{\alpha}_{vi}, \tag{45}$$

where $K_{vi} > 0$ and $\nu_i > 0$ are control gains, $\varepsilon_i > 0$ is a small design constant introduced in view of Lemma 1, $\hat{m}_i$ is the estimation of the *unknown* constant m_i, and $\hat{\mu}_{mi}$ is the estimation of the *unknown* constant μ_{mi} such that

$$\left\| -\frac{1}{m_i}F_i(R_i - R_{\mathrm{d}i})e_z + \frac{1}{m_i}N_{1i}\right\| \leq \mu_{mi}.$$

Next, we substitute the control design (44) back into (43), which yields

$$-\frac{1}{m_i}e_{vi}^{\mathrm{T}}u_i = -\frac{\hat{m}_i}{m_i}e_{vi}^{\mathrm{T}}\bar{u}_i = -e_{vi}^{\mathrm{T}}\bar{u}_i - \frac{\tilde{m}_i}{m_i}e_{vi}^{\mathrm{T}}\bar{u}_i, \tag{46}$$

where $\tilde{m}_i = \hat{m}_i - m_i$ $(i = 1, \dots, N)$.

Hence, (41) and (43) lead to

$$\begin{aligned}\dot{V}_1 + \dot{V}_2 \leq \sum_{i=1}^{N}\Big(&E_i^{\mathrm{T}}e_{vi} - K_{\mathrm{e}i}\eta_{\mathrm{e}i}^2 - \sum_{j=1,j\neq i}^{N}K_{ij}\eta_{ij}^2 + e_{vi}^{\mathrm{T}}ge_z - e_{vi}^{\mathrm{T}}\bar{u}_i - \frac{\tilde{m}_i}{m_i}e_{vi}^{\mathrm{T}}\bar{u}_i \\ &- \frac{1}{m_i}e_{vi}^{\mathrm{T}}F_i(R_i - R_{\mathrm{d}i})e_z + \frac{1}{m_i}e_{vi}^{\mathrm{T}}N_{1i} - e_{vi}^{\mathrm{T}}\dot{\alpha}_{vi}\Big).\end{aligned} \tag{47}$$

Note also that

$$e_{vi}^{\mathrm{T}}(\dot{\hat{\alpha}}_{vi} - \dot{\alpha}_{vi}) \leq \|e_{vi}\|\varepsilon_{\alpha_{vi}} < \nu_i\|e_{vi}\|^2 + \frac{1}{\nu_i}\varepsilon_{\alpha_{vi}}^2, \tag{48}$$

and

$$-\frac{1}{m_i}e_{vi}^{\mathrm{T}}F_i(R_i - R_{\mathrm{d}i})e_z + \frac{1}{m_i}e_{vi}^{\mathrm{T}}N_{1i} \leq \|e_{vi}\|\mu_{mi} < \varepsilon_i\mu_{mi} + \mu_{mi}\frac{e_{vi}^{\mathrm{T}}e_{vi}}{\sqrt{e_{vi}^{\mathrm{T}}e_{vi} + \varepsilon_i^2}}. \tag{49}$$

Therefore, it follows from (47) that

$$\dot{V}_1 + \dot{V}_2 < \sum_{i=1}^{N}\left(-K_{\mathrm{e}i}\eta_{\mathrm{e}i}^2 - \sum_{j=1,j\neq i}^{N}K_{ij}\eta_{ij}^2 - K_{vi}e_{vi}^{\mathrm{T}}e_{vi} - \frac{\tilde{m}_i}{m_i}e_{vi}^{\mathrm{T}}\bar{u}_i - \tilde{\mu}_{mi}\frac{e_{vi}^{\mathrm{T}}e_{vi}}{\sqrt{e_{vi}^{\mathrm{T}}e_{vi} + \varepsilon_i^2}} + \mu_{mi}\varepsilon_i + \frac{1}{\nu_i}\varepsilon_{\alpha_{vi}}^2\right), \tag{50}$$

where $\tilde{\mu}_{mi} = \hat{\mu}_{mi} - \mu_{mi}$ $(i = 1, \dots, N)$.

Next, design the adaptive laws for the estimators $\hat{m}_i$ and $\hat{\mu}_{mi}$ $(i = 1, \dots, N)$ as the following

$$\dot{\hat{m}}_i = n_{mi}e_{vi}^{\mathrm{T}}\bar{u}_i - \sigma_{mi}\hat{m}_i, \tag{51}$$

$$\dot{\hat{\mu}}_{mi} = n_{\mu_{mi}}\frac{e_{vi}^{\mathrm{T}}e_{vi}}{\sqrt{e_{vi}^{\mathrm{T}}e_{vi} + \varepsilon_i^2}} - \sigma_{\mu_{mi}}\hat{\mu}_{mi}, \tag{52}$$

with $\hat{m}_i(0) = \hat{m}_{i0}$ and $\hat{\mu}_{mi}(0) = \hat{\mu}_{mi0}$, where $\hat{m}_{i0}$ and $\hat{\mu}_{mi0}$ are the initial conditions, n_{mi}, $n_{\mu_{mi}}$, σ_{mi}, and $\sigma_{\mu_{mi}}$ $(i = 1, \dots, N)$ are positive design constants. Design the Lyapunov function candidates for the estimators of the quadrotors as

$$V_m = \sum_{i=1}^{N}\frac{1}{2n_{mi}m_i}\tilde{m}_i^2, \quad V_{\mu_m} = \sum_{i=1}^{N}\frac{1}{2n_{\mu_{mi}}}\tilde{\mu}_{mi}^2. \tag{53}$$

Denote $V_{\mathrm{pos}} = V_1 + V_2 + V_m + V_{\mu_m}$, after some algebraic manipulation, we can arrive at

$$\dot{V}_{\mathrm{pos}} < \sum_{i=1}^{N}\left(-K_{\mathrm{e}i}\eta_{\mathrm{e}i}^2 - \sum_{j=1,j\neq i}^{N}K_{ij}\eta_{ij}^2 - K_{vi}e_{vi}^{\mathrm{T}}e_{vi} - \frac{\sigma_{mi}}{2n_{mi}m_i}\tilde{m}_i^2 - \frac{\sigma_{\mu_{mi}}}{2n_{\mu_{mi}}}\tilde{\mu}_{mi}^2 + C_{1i}\right), \tag{54}$$

where

$$C_{1i} = \frac{\sigma_{mi}}{2n_{mi}} m_i + \frac{\sigma_{\mu_{mi}}}{2n_{\mu_{mi}}} \mu_{mi}^2 + \mu_{mi}\varepsilon_i + \frac{1}{v_i}\varepsilon_{\alpha_{vi}}^2.$$

4.2 | Attitude control design

Step 3:

At this step, we consider the attitude kinematics of the quadrotors. First, we need to extract the reference attitude from the position control design. Recall that $u_i = F_i R_{\mathrm{d}i} e_z$, and from (44) we have

$$u_i = \hat{m}_i \bar{u}_i = F_i R_{\mathrm{d}i} e_z = F_i \begin{bmatrix} \mathrm{c}\phi_{\mathrm{d}i}\mathrm{s}\theta_{\mathrm{d}i}\mathrm{c}\psi_{\mathrm{d}i} + \mathrm{s}\phi_{\mathrm{d}i}\mathrm{s}\psi_{\mathrm{d}i} \\ \mathrm{c}\phi_{\mathrm{d}i}\mathrm{s}\theta_{\mathrm{d}i}\mathrm{s}\psi_{\mathrm{d}i} - \mathrm{s}\phi_{\mathrm{d}i}\mathrm{c}\psi_{\mathrm{d}i} \\ \mathrm{c}\phi_{\mathrm{d}i}\mathrm{c}\theta_{\mathrm{d}i} \end{bmatrix}, \tag{55}$$

in which we recall that F_i is the thrust of the ith quadrotor. Here, for any designated reference yaw signal $\psi_{\mathrm{d}i}$ satisfying Assumption 1, we define

$$F_i = \|u_i\|, \tag{56}$$

$$\phi_{\mathrm{d}i} = \arcsin\left(\frac{u_{i1}\mathrm{s}\psi_{\mathrm{d}i} - u_{i2}\mathrm{c}\psi_{\mathrm{d}i}}{\|u_i\|}\right), \tag{57}$$

$$\theta_{\mathrm{d}i} = \arctan\left(\frac{u_{i1}\mathrm{c}\psi_{\mathrm{d}i} + u_{i2}\mathrm{s}\psi_{\mathrm{d}i}}{u_{i3}}\right), \tag{58}$$

where $u_i = [u_{i1},\ u_{i2},\ u_{i3}]^{\mathrm{T}} \in \mathbb{R}^3$, with $\phi_{\mathrm{d}i}$ and $\theta_{\mathrm{d}i}$ satisfying Assumption 1.

Note that the dynamics for $\eta_{\Theta i} = [\eta_{\phi i},\ \eta_{\theta i},\ \eta_{\psi i}]^{\mathrm{T}}$ $(i = 1, \dots, N)$, with $\eta_{\phi i},\ \eta_{\theta i}$, and $\eta_{\psi i}$ introduced in (26)–(28), are as the following

$$\dot{\eta}_{\Theta i} = \begin{bmatrix} \Delta_{\mathrm{H}\phi i} + \Delta_{\mathrm{L}\phi i} + \vartheta_{\phi i}\dot{e}_{\phi i} \\ \Delta_{\mathrm{H}\theta i} + \Delta_{\mathrm{L}\theta i} + \vartheta_{\theta i}\dot{e}_{\theta i} \\ \Delta_{\mathrm{H}\psi i} + \Delta_{\mathrm{L}\psi i} + \vartheta_{\psi i}\dot{e}_{\psi i} \end{bmatrix} = \eta_{\Omega_{\Theta i}} + \vartheta_{\Theta i} \circ \dot{e}_{\Theta i}, \tag{59}$$

where $\eta_{\Omega_{\Theta i}} \triangleq [\Delta_{\mathrm{H}\phi i} + \Delta_{\mathrm{L}\phi i},\ \Delta_{\mathrm{H}\theta i} + \Delta_{\mathrm{L}\theta i},\ \Delta_{\mathrm{H}\psi i} + \Delta_{\mathrm{L}\psi i}]^{\mathrm{T}} \in \mathbb{R}^3$, $\vartheta_{\Theta i} \triangleq [\vartheta_{\phi i},\ \vartheta_{\theta i},\ \vartheta_{\psi i}]^{\mathrm{T}} \in \mathbb{R}^3$, and $\dot{e}_{\Theta i} = [\dot{e}_{\phi i},\ \dot{e}_{\theta i},\ \dot{e}_{\psi i}]^{\mathrm{T}} \in \mathbb{R}^3$. Furthermore,

$$\Delta_{\mathrm{H}\phi i} = \frac{\partial \eta_{\phi i}}{\partial \Omega_{\phi \mathrm{H}i}} \dot{\Omega}_{\phi \mathrm{H}i}, \quad \Delta_{\mathrm{L}\phi i} = \frac{\partial \eta_{\phi i}}{\partial \Omega_{\phi \mathrm{L}i}} \dot{\Omega}_{\phi \mathrm{L}i},$$
$$\Delta_{\mathrm{H}\theta i} = \frac{\partial \eta_{\theta i}}{\partial \Omega_{\theta \mathrm{H}i}} \dot{\Omega}_{\theta \mathrm{H}i}, \quad \Delta_{\mathrm{L}\theta i} = \frac{\partial \eta_{\theta i}}{\partial \Omega_{\theta \mathrm{L}i}} \dot{\Omega}_{\theta \mathrm{L}i},$$
$$\Delta_{\mathrm{H}\psi i} = \frac{\partial \eta_{\psi i}}{\partial \Omega_{\psi \mathrm{H}i}} \dot{\Omega}_{\psi \mathrm{H}i}, \quad \Delta_{\mathrm{L}\psi i} = \frac{\partial \eta_{\psi i}}{\partial \Omega_{\psi \mathrm{L}i}} \dot{\Omega}_{\psi \mathrm{L}i},$$

where

$$\frac{\partial \eta_{\chi i}}{\partial \Omega_{\chi \mathrm{H}i}} = -\frac{\Omega_{\chi \mathrm{L}i} e_{\chi i}^2}{(\Omega_{\chi \mathrm{H}i} - e_{\chi i})^2 (\Omega_{\chi \mathrm{L}i} + e_{\chi i})}, \quad \chi = \phi,\ \theta,\ \psi,$$
$$\frac{\partial \eta_{\chi i}}{\partial \Omega_{\chi \mathrm{L}i}} = \frac{\Omega_{\chi \mathrm{H}i} e_{\chi i}^2}{(\Omega_{\chi \mathrm{H}i} - e_{\chi i})(\Omega_{\chi \mathrm{L}i} + e_{\chi i})^2}, \quad \vartheta_{\chi i} = \frac{\partial \eta_{\chi i}}{\partial e_{\chi i}} = \frac{\Omega_{\chi \mathrm{H}i}\Omega_{\chi \mathrm{L}i}(e_{\chi i}^2 + \Omega_{\chi \mathrm{H}i}\Omega_{\chi \mathrm{L}i})}{(\Omega_{\chi \mathrm{H}i} - e_{\chi i})^2 (\Omega_{\chi \mathrm{L}i} + e_{\chi i})^2}.$$

Note that $\vartheta_{\phi i}$, $\vartheta_{\theta i}$, and $\vartheta_{\psi i}$ are always positive. Design the universal barrier function as $V_3 = \sum_{i=1}^{N} \frac{1}{2}\eta_{\Theta i}^{\mathrm{T}}\eta_{\Theta i}$, and its time derivative leads to

$$\begin{aligned}\dot{V}_3 &= \sum_{i=1}^{N}\left(\eta_{\Theta i}^{\mathrm{T}}\eta_{\Omega_{\Theta i}} + \eta_{\Theta i}^{\mathrm{T}}(\vartheta_{\Theta i}\circ\dot{e}_{\Theta i})\right) = \sum_{i=1}^{N}\left(\eta_{\Theta i}^{\mathrm{T}}\eta_{\Omega_{\Theta i}} + (\eta_{\Theta i}\circ\vartheta_{\Theta i})^{\mathrm{T}}\dot{e}_{\Theta i}\right)\\ &= \sum_{i=1}^{N}\left(\eta_{\Theta i}^{\mathrm{T}}\eta_{\Omega_{\Theta i}} + (\eta_{\Theta i}\circ\vartheta_{\Theta i})^{\mathrm{T}}(T_i(e_{\omega i}+\alpha_{\omega i}) - \dot{\Theta}_{\mathrm{d}i})\right),\end{aligned} \tag{60}$$

where we define $e_{\omega i} = \omega_i - \alpha_{\omega i}$ $(i = 1, \ldots, N)$, with the stabilizing function $\alpha_{\omega i} \in \mathbb{R}^3$ designed as

$$\alpha_{\omega i} = T_i^{-1}\left(\hat{\dot{\Theta}}_{\mathrm{d}i} - K_{\Theta i}\mathrm{diag}\left[\frac{1}{\vartheta_{\phi i}}, \frac{1}{\vartheta_{\theta i}}, \frac{1}{\vartheta_{\psi i}}\right]\eta_{\Theta i} - \nu_i(\eta_{\Theta i}\circ\vartheta_{\Theta i}) - \mathrm{diag}\left[\frac{1}{\vartheta_{\phi i}}, \frac{1}{\vartheta_{\theta i}}, \frac{1}{\vartheta_{\psi i}}\right]\eta_{\Omega_{\Theta i}}\right), \tag{61}$$

where $K_{\Theta i} > 0$ is a control gain. Note that in view of Assumption 4, we have $\|\hat{\dot{\Theta}}_{\mathrm{d}i} - \dot{\Theta}_{\mathrm{d}i}\| \leq \varepsilon_{\Theta_{\mathrm{d}i}}$, and hence

$$(\eta_{\Theta i}\circ\vartheta_{\Theta i})^{\mathrm{T}}(\hat{\dot{\Theta}}_{\mathrm{d}i} - \dot{\Theta}_{\mathrm{d}i}) \leq \|\eta_{\Theta i}\circ\vartheta_{\Theta i}\|\varepsilon_{\Theta_{\mathrm{d}i}} < \frac{1}{\nu_i}\varepsilon_{\Theta_{\mathrm{d}i}}^2 + \nu_i\|\eta_{\Theta i}\circ\vartheta_{\Theta i}\|^2.$$

Therefore, from (60) we can get

$$\dot{V}_3 \leq \sum_{i=1}^{N}\left(-K_{\Theta i}\eta_{\Theta i}^{\mathrm{T}}\eta_{\Theta i} + (\eta_{\Theta i}\circ\vartheta_{\Theta i})^{\mathrm{T}}T_i e_{\omega i} + \frac{1}{\nu_i}\varepsilon_{\Theta_{\mathrm{d}i}}^2\right). \tag{62}$$

Step 4:

At this step, design the Lyapunov function candidate as $V_4 = \sum_{i=1}^{N}\frac{1}{2}e_{\omega i}^{\mathrm{T}}e_{\omega i}$, and its rate of change is

$$\dot{V}_4 = e_{\omega i}^{\mathrm{T}}(J_i^{-1}\mathbb{S}(J_i\omega_i)\omega_i + J_i^{-1}\tau_i - \dot{\alpha}_{\omega i} + J_i^{-1}N_{2i}). \tag{63}$$

Note that we can further parameterize the term $e_{\omega i}^{\mathrm{T}}J_i^{-1}\mathbb{S}(J_i\omega_i)\omega_i$ as $e_{\omega i}^{\mathrm{T}}J_i^{-1}\mathbb{S}(J_i\omega_i)\omega_i = e_{\omega i}^{\mathrm{T}}\bar{J}_i\overline{\omega}_i$, where $\bar{J}_i$ and $\overline{\omega}_i$ for the ith quadrotor $(i = 1, \ldots, N)$ are defined as

$$\bar{J}_i = \left[\bar{J}_{1i}, \quad \bar{J}_{2i}, \quad \bar{J}_{3i}, \quad \bar{J}_{4i}, \quad \bar{J}_{5i}, \quad \bar{J}_{6i}\right], \tag{64}$$

with

$$\bar{J}_{1i} = \begin{bmatrix} \varsigma_{xyi}J_{zxi} - \varsigma_{xzi}J_{yxi} \\ \varsigma_{yyi}J_{zxi} - \varsigma_{yzi}J_{yxi} \\ \varsigma_{zyi}J_{zxi} - \varsigma_{zzi}J_{yxi} \end{bmatrix}, \quad \bar{J}_{2i} = \begin{bmatrix} -\varsigma_{xxi}J_{zxi} + \varsigma_{xyi}J_{zyi} + \varsigma_{xzi}(J_{xxi} - J_{yyi}) \\ -\varsigma_{yxi}J_{zxi} + \varsigma_{yyi}J_{zyi} + \varsigma_{yzi}(J_{xxi} - J_{yyi}) \\ -\varsigma_{zxi}J_{zxi} + \varsigma_{zyi}J_{zyi} + \varsigma_{zzi}(J_{xxi} - J_{yyi}) \end{bmatrix},$$

$$\bar{J}_{3i} = \begin{bmatrix} \varsigma_{xxi}J_{yxi} + \varsigma_{xyi}(J_{zzi} - J_{xxi}) - \varsigma_{xzi}J_{yzi} \\ \varsigma_{yxi}J_{yxi} + \varsigma_{yyi}(J_{zzi} - J_{xxi}) - \varsigma_{yzi}J_{yzi} \\ \varsigma_{zxi}J_{yxi} + \varsigma_{zyi}(J_{zzi} - J_{xxi}) - \varsigma_{zzi}J_{yzi} \end{bmatrix}, \quad \bar{J}_{4i} = \begin{bmatrix} -\varsigma_{xxi}J_{zyi} + \varsigma_{xzi}J_{xyi} \\ -\varsigma_{yxi}J_{zyi} + \varsigma_{yzi}J_{xyi} \\ -\varsigma_{zxi}J_{zyi} + \varsigma_{zzi}J_{xyi} \end{bmatrix},$$

$$\bar{J}_{5i} = \begin{bmatrix} \varsigma_{xxi}(J_{yyi} - J_{zzi}) - \varsigma_{xyi}J_{xyi} + \varsigma_{xzi}J_{xzi} \\ \varsigma_{yxi}(J_{yyi} - J_{zzi}) - \varsigma_{yyi}J_{xyi} + \varsigma_{yzi}J_{xzi} \\ \varsigma_{zxi}(J_{yyi} - J_{zzi}) - \varsigma_{zyi}J_{xyi} + \varsigma_{zzi}J_{xzi} \end{bmatrix}, \quad \bar{J}_{6i} = \begin{bmatrix} \varsigma_{xxi}J_{yzi} - \varsigma_{xzi}J_{xzi} \\ \varsigma_{yxi}J_{yzi} - \varsigma_{yyi}J_{xzi} \\ \varsigma_{zxi}J_{yzi} - \varsigma_{zyi}J_{xzi} \end{bmatrix},$$

where

$$J_i = \begin{bmatrix} J_{xxi} & J_{xyi} & J_{xzi} \\ J_{yxi} & J_{yyi} & J_{yzi} \\ J_{zxi} & J_{zyi} & J_{zzi} \end{bmatrix}, \quad J_i^{-1} = \begin{bmatrix} \varsigma_{xxi} & \varsigma_{xyi} & \varsigma_{xzi} \\ \varsigma_{yxi} & \varsigma_{yyi} & \varsigma_{yzi} \\ \varsigma_{zxi} & \varsigma_{zyi} & \varsigma_{zzi} \end{bmatrix},$$

and

$$\overline{\omega}_i = \left[\omega_{xi}^2, \quad \omega_{xi}\omega_{yi}, \quad \omega_{xi}\omega_{zi}, \quad \omega_{yi}^2, \quad \omega_{yi}\omega_{zi}, \quad \omega_{zi}^2\right]^{\mathrm{T}}. \tag{65}$$

Here $\overline{J}_i$ is the unknown constant matrix such that there exists a constant $\overline{h}_{\overline{J}i}$ satisfying $z^{\mathrm{T}}\overline{J}_i z \le \overline{h}_{\overline{J}i} z^{\mathrm{T}} z$ for any $z \in \mathbb{R}^3$. Therefore, we have

$$e_{\omega i}^{\mathrm{T}} J_i^{-1} \mathbb{S}(J_i\omega_i)\omega_i \le \|e_{\omega i}\| \overline{h}_{\overline{J}i} \|\overline{\omega}_i\| < \varepsilon_i \overline{h}_{\overline{J}i} + \overline{h}_{\overline{J}i} \frac{e_{\omega i}^{\mathrm{T}} e_{\omega i} \overline{\omega}_i^{\mathrm{T}} \overline{\omega}_i}{\sqrt{e_{\omega i}^{\mathrm{T}} e_{\omega i} \overline{\omega}_i^{\mathrm{T}} \overline{\omega}_i + \varepsilon_i^2}}. \tag{66}$$

Besides,

$$e_{\omega i}^{\mathrm{T}} J_i^{-1} N_{2i} \le \|e_{\omega i}\| \mu_{J_i} < \varepsilon_i \mu_{J_i} + \mu_{J_i} \frac{e_{\omega i}^{\mathrm{T}} e_{\omega i}}{\sqrt{e_{\omega i}^{\mathrm{T}} e_{\omega i} + \varepsilon_i^2}}, \tag{67}$$

where μ_{J_i} is an *unknown* constant such that $\|J_i^{-1} N_{2i}\| \le \mu_{J_i}$.

For the ith quadrotor ($i = 1, \dots, N$), we design the control torque $\tau_i \in \mathbb{R}^3$ as the following

$$\tau_i = -\frac{e_{\omega i} \overline{\tau}_i^{\mathrm{T}} \overline{\tau}_i \hat{\rho}_{Ji}^2}{\sqrt{e_{\omega i}^{\mathrm{T}} e_{\omega i} \overline{\tau}_i^{\mathrm{T}} \overline{\tau}_i \hat{\rho}_{Ji}^2 + \varepsilon_i^2}}, \tag{68}$$

$$\overline{\tau}_i = -\hat{\dot{\alpha}}_{\omega i} + (K_{\omega i} + \nu_i) e_{\omega i} + T_i^{\mathrm{T}} (\eta_{\Theta i} \circ \vartheta_{\Theta i}) + \hat{\mu}_{Ji} \frac{e_{\omega i}}{\sqrt{e_{\omega i}^{\mathrm{T}} e_{\omega i} + \varepsilon_i^2}} + \hat{h}_{\overline{J}i} \frac{e_{\omega i} \overline{\omega}_i^{\mathrm{T}} \overline{\omega}_i}{\sqrt{e_{\omega i}^{\mathrm{T}} e_{\omega i} \overline{\omega}_i^{\mathrm{T}} \overline{\omega}_i + \varepsilon_i^2}}, \tag{69}$$

where $\hat{\mu}_{Ji}$ is the estimator of the *unknown* constant μ_{J_i}, $\hat{h}_{\overline{J}i}$ is the estimator of the *unknown* constant $\overline{h}_{\overline{J}i}$, and $\hat{\rho}_{Ji}$ is the estimator of the *unknown* constant $\rho_{Ji} = \frac{1}{\underline{b}_{Ji}}$.

Hence, we have

$$\begin{aligned} \dot{V}_3 + \dot{V}_4 < \sum_{i=1}^{N} \Bigg(& \varepsilon_i (\overline{h}_{\overline{J}i} + \mu_{J_i} + \underline{b}_{Ji}) + \frac{1}{\nu_i} \varepsilon_{\Theta_{di}}^2 + \frac{1}{\nu_i} \varepsilon_{\alpha_{\omega i}}^2 - K_{\Theta i} \eta_{\Theta i}^{\mathrm{T}} \eta_{\Theta i} - K_{\omega i} e_{\omega i}^{\mathrm{T}} e_{\omega i} - \underline{b}_{Ji} e_{\omega i}^{\mathrm{T}} \overline{\tau}_i \tilde{\rho}_{Ji} \\ & - \tilde{\mu}_{Ji} \frac{e_{\omega i}^{\mathrm{T}} e_{\omega i}}{\sqrt{e_{\omega i}^{\mathrm{T}} e_{\omega i} + \varepsilon_i^2}} - \tilde{h}_{\overline{J}i} \frac{e_{\omega i}^{\mathrm{T}} e_{\omega i} \overline{\omega}_i^{\mathrm{T}} \overline{\omega}_i}{\sqrt{e_{\omega i}^{\mathrm{T}} e_{\omega i} \overline{\omega}_i^{\mathrm{T}} \overline{\omega}_i + \varepsilon_i^2}} \Bigg), \end{aligned} \tag{70}$$

where $\tilde{\rho}_{Ji} = \hat{\rho}_{Ji} - \rho_{Ji}$, $\tilde{\mu}_{Ji} = \hat{\mu}_{Ji} - \mu_{Ji}$, and $\tilde{h}_{\overline{J}i} = \hat{h}_{\overline{J}i} - \overline{h}_{\overline{J}i}$ ($i = 1, \dots, N$).

Now, the adaptive laws for the estimators $\hat{\rho}_{Ji}$, $\hat{h}_{\overline{J}i}$, and $\hat{\mu}_{Ji}$ ($i = 1, \dots, N$) are designed as the following

$$\dot{\hat{\rho}}_{Ji} = n_{\rho_{Ji}} e_{\omega i}^{\mathrm{T}} \overline{\tau}_i - \sigma_{\rho_{Ji}} \hat{\rho}_{Ji}, \tag{71}$$

$$\dot{\hat{h}}_{\bar{J}i} = n_{h_{\bar{J}i}} \frac{e_{\omega i}^{\mathrm{T}} e_{\omega i} \overline{\omega}_i^{\mathrm{T}} \overline{\omega}_i}{\sqrt{e_{\omega i}^{\mathrm{T}} e_{\omega i} \overline{\omega}_i^{\mathrm{T}} \overline{\omega}_i + \varepsilon_i^2}} - \sigma_{h_{\bar{J}i}} \hat{h}_{\bar{J}i}, \tag{72}$$

$$\dot{\hat{\mu}}_{Ji} = n_{\mu_{Ji}} \frac{e_{\omega i}^{\mathrm{T}} e_{\omega i}}{\sqrt{e_{\omega i}^{\mathrm{T}} e_{\omega i} + \varepsilon_i^2}} - \sigma_{\mu_{Ji}} \hat{\mu}_{Ji}, \tag{73}$$

where $\hat{\rho}_{Ji}(0) = 0$, $\hat{h}_{\bar{J}i}(0) = 0$, and $\hat{\mu}_{Ji}(0) = 0$ are the initial conditions, $n_{\rho_{Ji}}$, $\sigma_{\rho_{Ji}}$, $n_{h_{\bar{J}i}}$, $\sigma_{h_{\bar{J}i}}$, $n_{\mu_{Ji}}$, and $\sigma_{\mu_{Ji}}$ are positive design constants.

Next, design the Lyapunov function candidates for the estimators as $V_{\rho_J} = \sum_{i=1}^{N} \frac{b_{Ji}}{2n_{\rho_{Ji}}} \tilde{\rho}_{Ji}^2$, $V_{h_{\bar{J}}} = \sum_{i=1}^{N} \frac{1}{2n_{h_{\bar{J}i}}} \tilde{h}_{\bar{J}i}^2$, $V_{\mu_J} = \sum_{i=1}^{N} \frac{1}{2n_{\mu_{Ji}}} \tilde{\mu}_{Ji}^2$. Denote $V_{\mathrm{att}} = V_3 + V_4 + V_{\rho_J} + V_{h_{\bar{J}}} + V_{\mu_J}$, after some algebraic manipulation, we can arrive at

$$\dot{V}_{\mathrm{att}} < \sum_{i=1}^{N} \left(-K_{\Theta i} \eta_{\Theta i}^{\mathrm{T}} \eta_{\Theta i} - K_{\omega i} e_{\omega i}^{\mathrm{T}} e_{\omega i} - \frac{b_{Ji} \sigma_{\rho_{Ji}}}{2n_{\rho_{Ji}}} \tilde{\rho}_{Ji}^2 - \frac{\sigma_{h_{\bar{J}i}}}{2n_{h_{\bar{J}i}}} \tilde{h}_{\bar{J}i}^2 - \frac{\sigma_{\mu_{Ji}}}{2n_{\mu_{Ji}}} \tilde{\mu}_{Ji}^2 + C_{2i} \right), \tag{74}$$

where

$$C_{2i} = \varepsilon_i (\overline{h}_{\bar{J}i} + \mu_{J_i} + \underline{b}_{Ji}) + \frac{b_{Ji} \sigma_{\rho_{Ji}}}{2n_{\rho_{Ji}}} \rho_{Ji}^2 + \frac{\sigma_{h_{\bar{J}i}}}{2n_{h_{\bar{J}i}}} \overline{h}_{\bar{J}i}^2 + \frac{\sigma_{\mu_{Ji}}}{2n_{\mu_{Ji}}} \mu_{Ji}^2 + \frac{1}{\nu_i} \varepsilon_{\Theta_{\mathrm{d}i}}^2 + \frac{1}{\nu_i} \varepsilon_{\alpha_{\omega i}}^2.$$

Hence, let the overall Lyapunov function be $V = V_{\mathrm{pos}} + V_{\mathrm{att}}$, we can get

$$\dot{V} < -\kappa V + \varrho, \tag{75}$$

where

$$\kappa \triangleq \min_{i,j} (2K_{\mathrm{e}i}, 2K_{ij}, 2K_{vi}, 2K_{\Theta i}, 2K_{\omega i}, \sigma_{\mu_{mi}}, \sigma_{mi}, \sigma_{\rho_{Ji}}, \sigma_{h_{\bar{J}i}}, \sigma_{\mu_{Ji}}), \quad \varrho \triangleq \sum_{i=1}^{N} (C_{1i} + C_{2i}).$$

The above backstepping design leads to the following theorem.

Theorem 1. *For the ith quadrotor ($i = 1, \ldots, N$), with the thrust laws as (44) and (45), torque laws as (68) and (69), and adaptive laws (51), (52), (71)–(73), the quadrotor formation system described by (1), (2), (5), and (6), under Assumptions 1–4 has the following properties:*

1. *The constraint requirements (12), (13), (16)–(18) will not be violated during operation.*
2. *The transformed output tracking error $\eta_{\mathrm{e}i}$, η_{ij}, and $\eta_{\Theta i}$ ($i,j = 1, \ldots, N$, $j \neq i$) will converge into the sets*

$$\left\{ x = \eta_{\mathrm{e}i},\ \eta_{ij},\ \eta_{\phi i},\ \eta_{\theta i},\ \eta_{\psi i} : |x| < \varepsilon_\eta, \quad \varepsilon_\eta = \sqrt{\frac{2\varrho}{\kappa}} \right\}, \tag{76}$$

and as a result, the output tracking error $d_{\mathrm{e}i}$, $d_{\mathrm{e}ij}$, $e_{\phi i}$, $e_{\theta i}$, and $e_{\psi i}$, will converge to the sets

$$\left\{ d_{\mathrm{e}i} : d_{\mathrm{e}i} < \varepsilon_{\chi_{\mathrm{H},i}} \right\}, \tag{77}$$

$$\left\{ d_{\mathrm{e}ij},\ e_{\phi i},\ e_{\theta i},\ e_{\psi i} : -\varepsilon_{\iota_{\mathrm{L},i}} < \varpi < \varepsilon_{\iota_{\mathrm{H},i}} \right\}, \tag{78}$$

where $\varpi = d_{\mathrm{e}ij},\ e_{\phi i},\ e_{\theta i}$, or $e_{\psi i}$. For $\varepsilon_{\chi_{\mathrm{H},i}}$ we have

$$\varepsilon_{\chi_{\mathrm{H},i}} = \frac{\varepsilon_\eta \Omega_{\mathrm{dH}i}}{\Omega_{\mathrm{dH}i} + \varepsilon_\eta}. \tag{79}$$

$\varepsilon_{\iota_{\mathrm{H},i}}$ and $\varepsilon_{\iota_{\mathrm{L},i}}$ are expressed as

$$\varepsilon_{\iota_{\mathrm{H},i}} = \frac{-(\Omega_{\mathrm{H}}\Omega_{\mathrm{L}} - \varepsilon_\eta(\Omega_{\mathrm{H}} - \Omega_{\mathrm{L}})) + \sqrt{\Omega_{\mathrm{H}}^2\Omega_{\mathrm{L}}^2 + \varepsilon_\eta^2(\Omega_{\mathrm{H}} + \Omega_{\mathrm{L}})^2 - 2\varepsilon_\eta\Omega_{\mathrm{H}}\Omega_{\mathrm{L}}(\Omega_{\mathrm{H}} - \Omega_{\mathrm{L}})}}{2\varepsilon_\eta}, \tag{80}$$

$$\varepsilon_{\iota_{\mathrm{L},i}} = \frac{-(\Omega_{\mathrm{H}}\Omega_{\mathrm{L}} + \varepsilon_\eta(\Omega_{\mathrm{H}} - \Omega_{\mathrm{L}})) + \sqrt{\Omega_{\mathrm{H}}^2\Omega_{\mathrm{L}}^2 + \varepsilon_\eta^2(\Omega_{\mathrm{H}} + \Omega_{\mathrm{L}})^2 + 2\varepsilon_\eta\Omega_{\mathrm{H}}\Omega_{\mathrm{L}}(\Omega_{\mathrm{H}} - \Omega_{\mathrm{L}})}}{2\varepsilon_\eta}, \tag{81}$$

where $\Omega_{\mathrm{H}} = \Omega_{\mathrm{H}ij}$, $\Omega_{\phi\mathrm{H}i}$, $\Omega_{\theta\mathrm{H}i}$, or $\Omega_{\psi\mathrm{H}i}$, and $\Omega_{\mathrm{L}} = \Omega_{\mathrm{L}ij}$, $\Omega_{\phi\mathrm{L}i}$, $\Omega_{\theta\mathrm{L}i}$, or $\Omega_{\psi\mathrm{L}i}$, for $i,j = 1, \dots, N$, $j \neq i$.

Proof. First, from (75), it is clear that the overall Lyapunov function V is bounded, since

$$V(t) \leq \left(V(0) - \frac{\varrho}{\kappa}\right)e^{-\kappa t} + \frac{\varrho}{\kappa}. \tag{82}$$

The boundedness of V implies boundedness of $\eta_{\mathrm{e}i}$, η_{ij}, $\eta_{\phi i}$, $\eta_{\theta i}$, and $\eta_{\psi i}$ $(i,j = 1, \dots, N,\ j \neq i)$. Hence, the constraint requirements (12), (13), (16)–(18) are satisfied during the operation.

Moreover, we have $\limsup_{t\to\infty} V = \frac{\varrho}{\kappa}$, hence $\frac{1}{2}\eta_{\mathrm{e}i}^2 \leq \frac{\varrho}{\kappa}$ when $t \to \infty$, therefore $\eta_{\mathrm{e}i}$ will converge to the set (76). Similar relationships hold for η_{ij}, $\eta_{\phi i}$, $\eta_{\theta i}$, and $\eta_{\psi i}$. Furthermore, boundedness of the adaptive estimates $\hat{m}_i$, $\hat{\mu}_{mi}$, $\hat{\rho}_{Ji}$, $\hat{h}_{\bar{J}i}$, and $\hat{\mu}_{J_i}$, as well as boundedness of the fictitious error e_{vi} and $e_{\omega i}$ $(i = 1, \dots, N)$, can be concluded from the fact that V is bounded.

Next, for $i = 1, \dots, N$, note that in the range that $d_{\mathrm{e}i} < \Omega_{\mathrm{dH}i}$, $\eta_{\mathrm{e}i}$ is a function of $d_{\mathrm{e}i}$. Hence, the range (12) gives the range for $d_{\mathrm{e}i}$ given as in (77). Besides, within the range of (13), (16)–(18), η_{ij}, $\eta_{\phi i}$, $\eta_{\theta i}$, and $\eta_{\psi i}$ are quadratically related to $d_{\mathrm{e}ij}$, $e_{\phi i}$, $e_{\theta i}$, and $e_{\psi i}$, respectively. Hence, satisfying the constraints (13), (16)–(18) means that the distance and attitude tracking errors $d_{\mathrm{e}i}$, $d_{\mathrm{e}ij}$, $e_{\phi i}$, $e_{\theta i}$, and $e_{\psi i}$ will be confined in the ranges defined by (77) and (78). ∎

Remark 7. Once the thrust and torque of the ith quadrotor $(i = 1, \dots, N)$ are determined, the propeller speeds can be calculated using the following relation

$$\begin{bmatrix} F_i \\ \tau_{\phi i} \\ \tau_{\theta i} \\ \tau_{\psi i} \end{bmatrix} = \begin{bmatrix} \Upsilon_i & \Upsilon_i & \Upsilon_i & \Upsilon_i \\ 0 & -l_i\Upsilon_i & 0 & l_i\Upsilon_i \\ -l_i\Upsilon_i & 0 & l_i\Upsilon_i & 0 \\ d_i & -d_i & d_i & -d_i \end{bmatrix} \begin{bmatrix} \omega_{\mathrm{rot}i1}^2 \\ \omega_{\mathrm{rot}i2}^2 \\ \omega_{\mathrm{rot}i3}^2 \\ \omega_{\mathrm{rot}i4}^2 \end{bmatrix},$$

where $\tau_i = [\tau_{\phi i},\ \tau_{\theta i},\ \tau_{\psi i}]^{\mathrm{T}} \in \mathbb{R}^3$, $\omega_{\mathrm{rot}i1}$, $\omega_{\mathrm{rot}i2}$, $\omega_{\mathrm{rot}i3}$, and $\omega_{\mathrm{rot}i4}$ represent the front, right, rear, and left propeller speeds of the ith quadrotor, respectively. l_i is the distance between the center of the propeller and the center of the ith quadrotor, Υ_i is a thrust factor of the ith quadrotor, and d_i is a drag factor of the ith quadrotor, $i = 1, \dots, N$.

Remark 8. By L' Hopital's rule, in Theorem 1 we have

$$\lim_{\varepsilon_\eta\to 0}\varepsilon_{\iota_{\mathrm{H},i}} = 0, \quad \lim_{\varepsilon_\eta\to 0}\varepsilon_{\iota_{\mathrm{L},i}} = 0, \quad \lim_{\varepsilon_\eta\to 0}\varepsilon_{\chi_{\mathrm{H},i}} = 0, \tag{83}$$

for $i = 1, \dots, N$, which means as the modified error variables $\eta_{\mathrm{e}i}$, η_{ij}, $\eta_{\phi i}$, $\eta_{\theta i}$, and $\eta_{\psi i}$ converge into small neighborhoods of zero, so does the tracking errors $d_{\mathrm{e}i}$, $d_{\mathrm{e}ij}$, $e_{\phi i}$, $e_{\theta i}$, and $e_{\psi i}$.

Remark 9. To reduce the size of the set in (76), we need to select large κ and small ϱ. To make κ large, we can select large control gains $K_{\mathrm{e}i}$, K_{ij}, K_{vi}, $K_{\Theta i}$, and $K_{\omega i}$, for $i,j = 1, \dots, N$, $j \neq i$, and large adaptive control parameters $\sigma_{\mu_{mi}}$, σ_{mi}, $\sigma_{\rho_{Ji}}$, $\sigma_{h_{\bar{J}i}}$, and $\sigma_{\mu_{Ji}}$, for $i = 1, \dots, N$. To make ϱ small, we can select small ε_i, large ν_i, and large adaptive control parameters n_{mi}, $n_{\mu_{mi}}$, $n_{\rho_{Ji}}$, $n_{h_{\bar{J}i}}$, and $n_{\mu_{Ji}}$.

5 | SIMULATION STUDIES

In this section, a simulation example is carried out with a team of $N = 4$ quadrotors. In this simulation, the model parameters of the quadrotors are $m_i = 2$ kg, $g = 9.81$ m/s^2, and $J_i = \text{diag}[0.109,\ 0.103,\ 0.0625]$ kg m^2, $i = 1, 2, 3, 4$. Note that the units of the position, attitude, translational and angular velocities are m, rad, m/s, and rad/s, respectively. The reference signals for the vehicles are given as $p_{\text{d}1} = [2,\ 2,\ 5]^{\text{T}}$m, $p_{\text{d}2} = [2,\ 3,\ 5]^{\text{T}}$m, $p_{\text{d}3} = [3,\ 2,\ 5]^{\text{T}}$m, and $p_{\text{d}4} = [3,\ 3,\ 5]^{\text{T}}$m. The constraint functions are selected as $\Omega_{\text{dH}i} = (10 - 0.5)e^{-0.24t} + 0.5$, $\Omega_{\text{H}ij} = (8 - 0.1)e^{-0.08t} + 0.1$, and $\Omega_{\text{L}ij} = (3 - 0.1)e^{-0.04t} + 0.1$, $i, j = 1, 2, 3, 4, i \neq j$. To implement the adaptive control framework, the design parameters are chosen as $\varepsilon_i = 0.1$, $\epsilon = 0.01$, $n_{mi} = 0.29$, $n_{\mu_{mi}} = 0.3$, $n_{\rho_{Ji}} = 2$, $n_{h_{\bar{J}i}} = 2$, $n_{\mu_{Ji}} = 5$, $\sigma_{mi} = 0.065$, $\sigma_{\mu_{mi}} = 0.1$, $\sigma_{\rho_{Ji}} = 0.01$, $\sigma_{h_{\bar{J}i}} = 0.01$, and $\sigma_{\mu_{Ji}} = 0.01$, $i = 1, 2, 3, 4$. The control gains are designed as $K_{\text{e}i} = 1.2$, $K_{ij} = 0.65$, $K_{vi} = 2$, $\nu_i = 0.5$, $K_{\Theta i} = 2$, and $K_{\omega i} = 4$, $i, j = 1, 2, 3, 4, i \neq j$. The initial positions and attitudes of the quadrotor team are $[x_1,\ y_1,\ z_1]^{\text{T}} = [0,\ 0,\ 0]^{\text{T}}$m, $[x_2,\ y_2,\ z_2]^{\text{T}} = [0,\ 5,\ 0]^{\text{T}}$m, $[x_3,\ y_3,\ z_3]^{\text{T}} = [5,\ 0,\ 0]^{\text{T}}$m, and $[x_4,\ y_4,\ z_4]^{\text{T}} = [5,\ 5,\ 0]^{\text{T}}$m. The initial attitudes of the agents are $[\phi_1,\ \theta_1,\ \psi_1]^{\text{T}} = [0.699,\ 0.9984,\ 0.5]^{\text{T}}$rad, $[\phi_2,\ \theta_2,\ \psi_2]^{\text{T}} = [-0.699,\ 0.9984,\ 0.5]^{\text{T}}$rad, $[\phi_3,\ \theta_3,\ \psi_3]^{\text{T}} = [0.699,\ -0.9984,\ 0.5]^{\text{T}}$rad, and $[\phi_4,\ \theta_4,\ \psi_4]^{\text{T}} = [-0.699,\ -0.9984,\ 0.5]^{\text{T}}$rad. The initial conditions of the translational and angular velocities of every agent are zero. The external disturbances are

$$N_{1i} = \begin{bmatrix} 0.6\sin(0.8t) + 0.005\text{rand} \\ 0.25\cos(0.6t) + 0.01\text{rand} \\ 0.33\cos(0.5t) \end{bmatrix}, \quad N_{2i} = \begin{bmatrix} 0.1\sin(0.5t) \\ 0.1\sin(0.5t) \\ 0.1\sin(0.5t) \end{bmatrix},$$

where $i = 1, 2, 3, 4$. In N_{1i}, rand represents the random noise uniformly distributed in the interval $(-1, 1)$.

The communication topology diagram is shown in Figure 3.

The simulation results are presented in Figures 4–9. First, the 3D trajectories of four quadrotors are depicted in Figure 4. It can be observed that the quadrotors can move to small regions close to their desired fixed points $p_{\text{d}i}$, despite the large initial distance tracking error and presence of system uncertainties and disturbances. Next, the LOS distance tracking errors $d_{\text{e}i}$ under the proposed controller are shown in Figure 5 with the constraint function $\Omega_{d\text{H}i}$. From this figure, we see that $d_{\text{e}i}$ can converge to a small neighborhood of the origin without violation of the performance constraint $\Omega_{\text{dH}i}$. Here, performance constraint function $\Omega_{\text{dH}i}$ is selected as an exponentially decaying function and $\lim_{t\to\infty} \Omega_{\text{dH}i} = 0.5$. When distance tracking errors $d_{\text{e}i}$ are constrained by constraints functions, $d_{\text{e}i}$ can converge exponentially to the set in (77) which is close to zero. Thus, the transient and steady-state performance of distance tracking errors $d_{\text{e}i}$ can be guaranteed by performance functions $\Omega_{\text{dH}i}$. Figure 6 gives us the exhibition of the profile of the inter-quadrotor distance tracking errors $d_{\text{e}ij}$ under the proposed controller. It is obvious that the safety constraints are always satisfied during the operation since $d_{\text{e}ij}$ always stayed between the constraint functions $-\Omega_{\text{L}ij}$ and $\Omega_{\text{H}ij}$. Here, safety constraint functions $\Omega_{\text{H}ij}$ and $\Omega_{\text{L}ij}$ are both selected as exponentially decaying functions and $\lim_{t\to\infty} \Omega_{\text{H}ij}, \Omega_{\text{L}ij} = 0.1$. When relative distance tracking

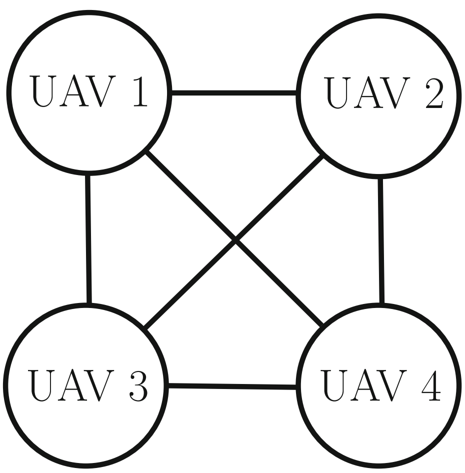

FIGURE 3 Undirected communication graph of the UAV team.

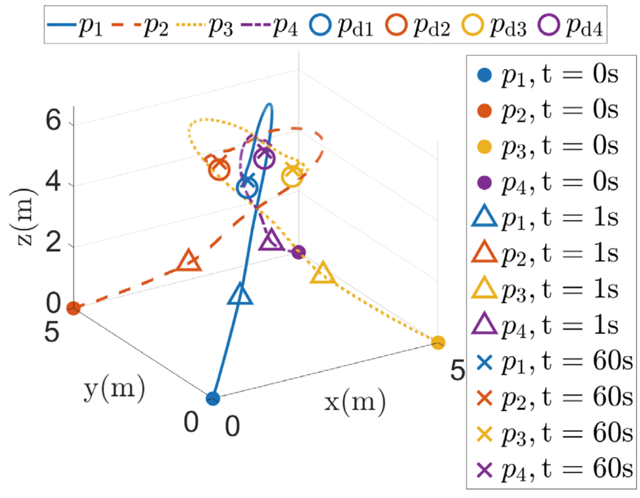

FIGURE 4 Position tracking trajectories of quadrotors.

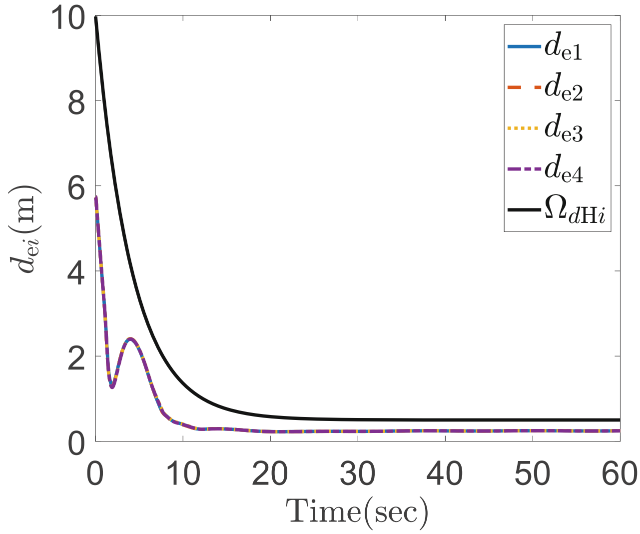

FIGURE 5 The profile of the LOS distance tracking errors d_{ei} with Ω_{dHi}, $i = 1, 2, 3, 4$.

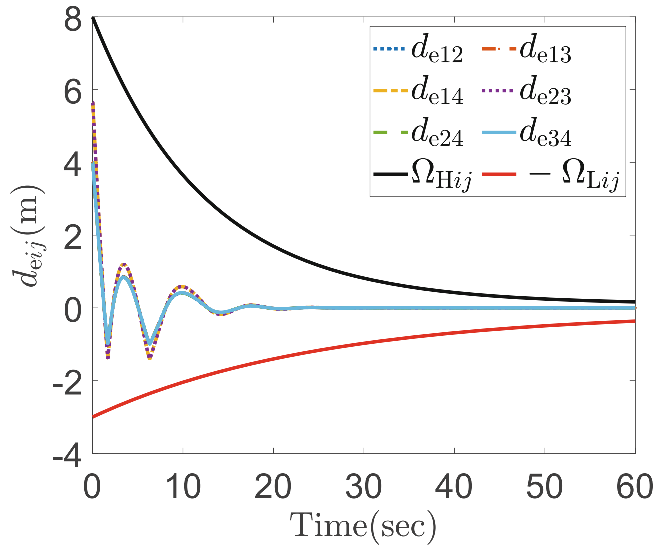

FIGURE 6 The profile of the relative inter-quadrotor distance tracking errors d_{eij} with Ω_{Hij} and $-\Omega_{Lij}$, $i, j = 1, 2, 3, 4, i \neq j$.

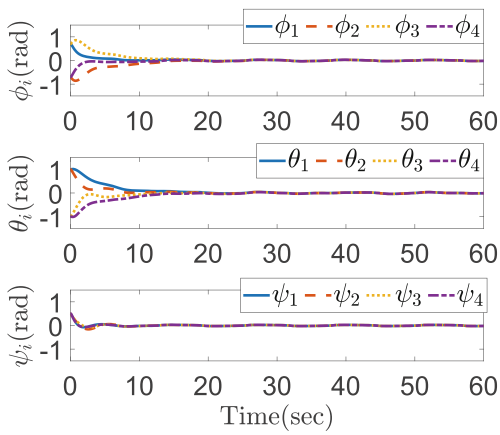

FIGURE 7 The profile of the attitudes of quadrotors, ϕ_i, θ_i, and ψ_i, $i = 1, 2, 3, 4$.

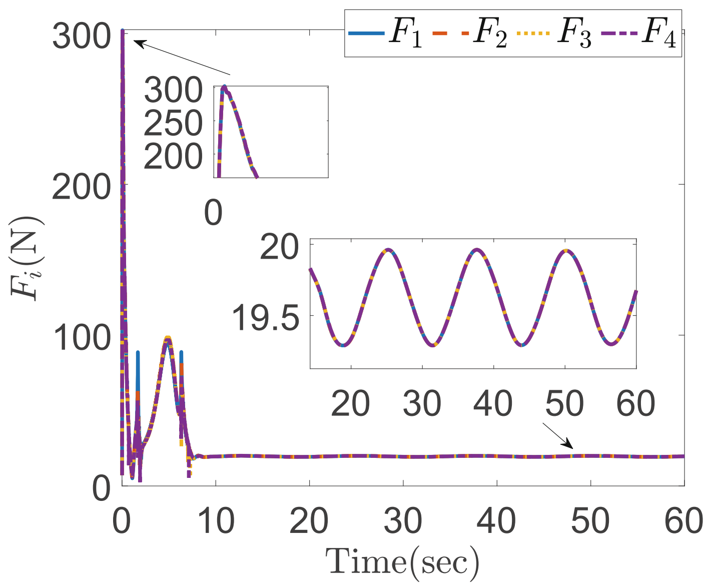

FIGURE 8 The thrust F_i of quadrotors, $i = 1, 2, 3, 4$.

errors d_{eij} are constrained by constraints functions, d_{eij} can converge exponentially to the set in (78) which is close to zero. Thus, safety requirements including collision avoidance and communication link can be guaranteed during the formation operation.

The profile of quadrotor attitudes ϕ_i, θ_i, and ψ_i presented in Figure 7 shows that the convergence of the attitudes to regions close to zero despite unknown model parameters and the influence of unknown time-varying external disturbances N_{2i}. Besides, safety constraints of the attitudes are not violated during the formation operation, that is, $\phi_i \in (-\frac{\pi}{2}, \frac{\pi}{2})$ and $\theta_i \in (-\frac{\pi}{2} + \epsilon, \frac{\pi}{2} - \epsilon)$ where $\epsilon = 0.01$, such that $T(\Theta_i(t))$ is always invertible to avoid the singularity of $\alpha_{\omega i}$ in (61). Finally, the thrust F_i and torques $\tau_{\phi i}$, $\tau_{\theta i}$, and $\tau_{\psi i}$ are plotted in Figures 8 and 9, respectively. The thrust can mitigate the influence of external disturbances N_{1i} and provide gravitational force for the quadrotor to make the vehicle hover on the neighborhood of its desired set point. The torques can accommodate unknown time-varying disturbances N_{2i} despite the lack of the accurate model parameters. The unknown time-varying external disturbances N_{2i} are selected to include sinusoidal signals such that the torques will also include sinusoidal signals to mitigate the influence of disturbances N_{2i}. Therefore, under our proposed controllers, the quadrotors have the good robustness against the disturbances. Based on the above discussion, we can conclude that the simulation results confirm the theoretic analysis shown in Theorem 1.

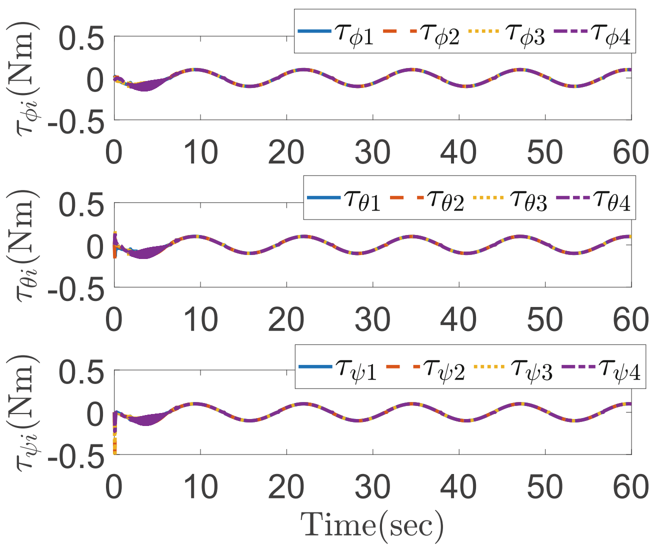

FIGURE 9 The torques $\tau_{\phi i}$, $\tau_{\theta i}$, and $\tau_{\psi i}$ of quadrotors, $i = 1, 2, 3, 4$.

6 | CONCLUSION

In this work, we address the formation control problem for a team of quadrotors with two types of constraints, namely performance constraints and safety constraints. A new adaptive formation control architecture is proposed. Specifically, we employ the universal barrier functions into the controller design and analysis, to ensure that the constraint requirements on the LOS distance tracking error, relative distance error between two quadrotors, and the attitude of each quadrotor, are all satisfied during the operation. The universal barrier function approach is also a generic framework that can address system with different types of constraints in a unified controller architecture. Exponential convergence rate can be guaranteed on the LOS distance, relative inter-quadrotor distance, and attitude tracking errors, while all constraints are satisfied during the operation. Future research includes experimental validation of the proposed formation control algorithm and extension of the analysis to constrained formation control problems for UAVs with collaborate objectives such as load lifting and transporting.

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

ORCID

Xu Jin https://orcid.org/0000-0001-9788-2051

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