Which L_p norm is the fairest? Approximations for fair facility location across all "p"

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Given a set of *facilities* and *clients*, and costs to open facilities, the classic facility location problem seeks to open a set of facilities and assign each client to one open facility to minimize the cost of opening the chosen facilities and the total distance of the clients to their assigned open facilities. Such an objective may induce an unequal cost over certain socioeconomic groups of clients (i.e., total distance traveled by clients in such a group). This is important when planning the location of socially relevant facilities such as emergency rooms.

In this work, we consider a *fair* version of the problem where we are given r *clients groups* that partition the set of clients, and the distance of a given group is defined as the average distance of clients in the group to their respective open facilities. The objective is to minimize the Minkowski p-norm of vector of group distances, to penalize high access costs to open facilities across r groups of clients. This generalizes classic facility location ($p = 1$) and the minimization of the maximum group distance ($p = \infty$). However, in practice, fairness criteria may not be explicit or even known to a decision maker, and it is often unclear how to select a specific "p" to model the cost of unfairness. To get around this, we study the notion of *solution portfolios* where for a fixed problem instance, we seek a small portfolio of solutions such that for any Minkowski norm p, one of these solutions is an $O(1)$-approximation. Using the geometric relationship between various p-norms, we show the existence of a portfolio of cardinality $O(\log r)$, and a lower bound of $\tilde{\Omega}(\sqrt{\log r})$.

There may not be common structure across different solutions in this portfolio, which can make planning difficult if the notion of fairness changes over time or if the budget to open facilities is disbursed over time. For example, small changes in p could lead to a completely different set of open facilities in the portfolio. Inspired by this, we introduce the notion of *refinement*, which is a family of solutions for each p-norm satisfying a combinatorial property. This property requires that (1) the set of facilities open for a higher p-norm must be a subset of the facilities open for a lower p-norm, and (2) all clients assigned to an open facility for a lower p-norm must be assigned to the same open facility for any higher p-norm. A refinement is α-approximate if the solution for each p-norm problem is an α-approximation for it. We show that it is sufficient to consider only $O(\log r)$ norms instead of all p-norms, $p \in [1, \infty]$ to construct refinements. A natural greedy algorithm for the problem gives a poly(r)-approximate refinement, which we improve to poly$(r^{1/\sqrt{\log r}})$-approximate using a recursive algorithm. We improve this ratio to $O(\log r)$ for the special case of tree metric for uniform facility open cost. Our recursive algorithm extends to other settings, including to a hierarchical facility location problem that models facility location problems at several levels, such as public works departments and schools.

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