# Multi-Body Hydrodynamic Interactions in Fish-Like Swimming

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*Many animals in nature travel in groups either for protection, survival, or endurance. Among these, certain species do so under the burden of aero/hydrodynamic loads, which incites questions as to the significance of the multibody fluid-mediated interactions that are inherent to collective flying/swimming. Prime examples of such creatures are fish, which are commonly seen traveling in highly organized groups of large numbers. Indeed, over the years, there have been numerous attempts to examine hydrodynamic interactions among self-propelled fish-like swimmers. Though many have studied this phenomenon, their motivations have varied from understanding animal behavior to extracting universal fluid dynamical principles and transplanting them into engineering applications. The approaches utilized to carry out these investigations include theoretical and computational analyses, field observations, and experiments using various abstractions of biological fish. Here, we compile representative investigations focused on the collective hydrodynamics of fish-like swimmers. The selected body of works are reviewed in the context of their methodologies and findings, so as to draw parallels, contrast differences, and highlight open questions. Overall, the results of the surveyed studies provide foundational insights into the conditions (such as the relative positioning and synchronization between the members, as well as their swimming kinematics and speed) under which hydrodynamic interactions can lead to efficiency gains and/or group cohesion in two- and three-dimensional scenarios. They also shed some light on the mechanisms responsible for such energetic and stability enhancements in the context of wake-body, wake-wake, and body-body interactions.* [DOI: 10.1115/1.4062219]

## 1 Introduction

In systems where multiple objects move freely in a fluid, the motion of each body determines, and is determined by, the movement of its neighbors, through the overall flow generated and encountered by the entire ensemble. Interactions of this kind often lead to deviant behaviors with important consequences, as seen in many natural and man-made contexts [1,2]. Arguably, one of the most familiar manifestations of such collective dynamics is the schooling of fish [3]. A phenomenon that has been associated with enhanced locomotion efficiency, which is hypothesized to stem from constructive hydrodynamic couplings [4–7]. For this reason, understanding the fluid-mediated interactions among self-propelled fish-like swimmers has been the subject of numerous investigations by scientists and engineers.

Biologists and ecologists, for example, have long sought to gain further insights into the interactive behavior, group dynamics, and habits of fish, which, when fully understood, could lead to predictive models for tracking the quantity and health of fish populations, their migration patterns, and even detecting abnormal environmental conditions [3,6,8–39]. On the other hand, from a flow physics perspective, fluid dynamicists have been in pursuit of the underlying hydrodynamic principles that govern the collective locomotion of swimmers similar to (or inspired by) fish [40–105]. Moreover, engineers have been determined to employ the ideas and knowledge acquired from the more fundamental studies toward advancing state-of-the-art technologies on multiple fronts, ranging from collaborative autonomous systems (e.g., swarms of aquatic robots [106–110]) to renewable energy (e.g., arrays of vertical axis wind turbines [111]).

Investigative methods pertaining to the collective motion of active swimmers can be generally divided into those that treat the dynamics of the fluid implicitly and those that explicitly take into account the fluid-mediated interactions. Approaches built around the idea that the collective behavior emerges due to individual group members following certain behavioral rules (e.g., separation, alignment, cohesion) fit into the first category [112–120]. Belonging to the same class are the methods that consider the collection of bodies under consideration as a soft matter system and apply the tools of nonequilibrium thermodynamics and statistical physics to describe its properties [1,2,121]. The second category includes strategies that prioritize fluid dynamics as a major influence in bringing about a sense of collectivity. Reduced-order modeling techniques [40–51], high-fidelity computational fluid dynamics (CFD) simulations [52–80,92], and physical experiments [81–105] are examples of such approaches. Within each method of analysis, the fish-like movement is modeled with varying degrees of sophistication, from point-like particles and oscillating airfoils to robotic and biological fish. Utilizing these abstractions, the problem of grouping behavior is simplified into a set of discreet parameters whose effects can be studied independently.

In this article, we survey the literature surrounding the hydrodynamic interactions between multiple fish-like swimmers. Our work is a natural extension of the existing reviews that are focused on the fundamentals of fish-like swimming rather than collective effects [122–136]. The studies examined here (which are admittedly noncomprehensive) are motivated primarily by questions such as: (i) How do multiple swimmers perform when traveling as a group relative to swimming in isolation, assuming that hydrodynamics is the main cause of collective behavior? (ii) What grouping formations and synchronizations among the members positively impact the group dynamics in a swimming collective? (iii) What are the fluid dynamical principles potentially responsible for reducing the energetic cost of locomotion in a collective? (iv) Can hydrodynamic interactions compel swimmers traveling as a group to passively self-organize in certain formations? If yes, under what conditions and through what mechanisms?

In what follows, first, we categorize the studies according to their specific method of analysis and the complexity of their modeling of fish movement. Then, we discuss the themes investigated and

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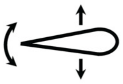
highlight the results. Lastly, we summarize the major findings, look into the commonalities between the studies, and underscore outstanding open questions. By curating this selected body of knowledge, we hope to provide the readers with a distilled version of the literature that can be used as a guide for future investigations.

## 2 Methods

The dynamics of hydrodynamically coupled multibody systems is challenging to study at the best of times, even more so when the problem is derived from the collective swimming of fish. The factors making this problem inherently complex are the intricacies

**Table 1 Categorization of studies on collective fish-like swimming according to their representation of individual swimmers and method(s) of analysis**

| | Experiments | reduced-order models | Detailed CFD simulations |
|---|---|---|---|
| **Point particle or finite dipole**<br>● | | Weihs (1973,1975) [40,41]<br>Lighthill (1975) [42]<br>Whittlesey et al. (2010) [111]<br>Tsang and Kanso (2013) [44]<br>Gazzola et al. (2016) [45]<br>Filella et al. (2018) [46] | |
| **Zero-thickness filament or sheet**<br> | Ristroph and Zhang (2008) [87] | Alben (2009) [43]<br>Heydari and Kanso (2021) [50]<br>Baddoo et al. (2021) [51] | Zhu (2009) [53]<br>Kim et al. (2010) [54]<br>Zhu et al. (2014) [56]<br>Dai et al. (2018) [64]<br>Park and Sung (2018) [65]<br>Peng et al. (2018) [66]<br>Alben (2021,2021) [72,73]<br>Jeong et al. (2021) [74]<br>Arranz et al. (2022) [78] |
| **Oscillating hydrofoil**<br> | Gursul and Rockwell (1990) [81]<br>Gopalkrishnan et al. (1994) [82]<br>Streitlien et al. (1995,1996) [83,84]<br>Triantafyllou et al. (2002) [85]<br>Spagnolie et al. (2010) [88]<br>Lee (2011) [89]<br>Boschitsch et al. (2014) [90,91]<br>Becker et al. (2015) [92]<br>Ramananarivo et al. (2016) [93]<br>Im et al. (2018) [96]<br>Kurt et al. (2018,2020,2021) [97,99,102]<br>Newbolt et al. (2019,2022) [98,103]<br>Thandiackal and Lauder (2023) [104] | Oza et al. (2019) [47]<br>Alaminos-Quesada and Fernandez-Feria [48,49] | Akhtar et al. (2007) [52]<br>Becker et al. (2015) [92]<br>Muscutt et al. (2017) [60]<br>Bao et al. (2017) [61]<br>Lin et al. (2019–2022) [69–71,76]<br>Saadat et al. (2021) [75]<br>Gungor et al. (2022) [77] |
| **Generic model fish**<br> | | | Gazzola et al. (2011) [55]<br>Chen et al. (2016) [59]<br>Novati et al. (2017) [62]<br>Zhu et al. (2022) [79] |
| **Robotic fish**<br> | Aureli et al. (2012) [29]<br>Butail et al. (2013) [31]<br>Landgraf et al. (2014) [33]<br>Bonnet et al. (2018) [36]<br>Li et al. (2020,2021) [100,101]<br>Pandhare et al. (2022) [105] | | |
| **Biological fish**<br> | Cullen et al. (1965) [8]<br>Breder (1967) [9]<br>Pitcher (1973) [10]<br>Partridge et al. (1979,1980,1983) [11–13]<br>Fréon et al. (1992) [14]<br>Herskin and Steffensen (1998) [15]<br>Svendsen et al. (2003) [17]<br>Liao et al. (2003,2003,2004) [18–20]<br>Makris et al. (2006,2009) [21,26]<br>Beal et al. (2006) [86]<br>Ward et al. (2008) [23]<br>Newlands and Porcelli (2008) [24]<br>Jagannathan et al. (2009) [25]<br>Abaid and Porfiri (2010) [27]<br>Katz et al. (2011) [28]<br>Handegard et al. (2012) [30]<br>Aureli et al. (2012) [29]<br>Tunstrøm et al. (2013) [32]<br>Butail et al. (2013) [31]<br>Landgraf et al. (2014) [33]<br>Swain et al. (2015) [34]<br>Marras et al. (2015) [6]<br>Ashraf et al. (2016,2017) [94,95]<br>Jolles et al. (2017) [35]<br>Bonnet et al. (2018) [36]<br>Murphy et al. (2019) [37]<br>Li et al. (2020) [100]<br>de Bie et al. (2020) [38]<br>Suriyampola et al. (2021) [39]<br>Thandiackal and Lauder (2023) [104] | | Hemelrijk et al. (2015) [57]<br>Daghooghi and Borazjani (2015) [58]<br>Maertens et al. (2017) [63]<br>Verma et al. (2018) [67]<br>Li et al. (2019) [68]<br>Seo and Mittal (2022) [80] |

associated with the shape, movement, and number of entities involved, as well as the interactions among them. These difficulties have been dealt with using a variety of approaches, all focusing on reducing the overall complexity of the system (while staying true to the original problem) such that an insightful scientific investigation can be conducted with reasonable resources. For instance, with varying degrees of abstraction, fish-like geometry and locomotion have been simplified to the movement of point particles and finite-length dipoles, undulatory traveling of zero-thickness flags and flexible plates, heaving and pitching of hydrofoils, and constrained swimming of mechanical fish (see Table 1).

In the remainder of this section, we discuss various approaches successfully utilized to analyze the multibody hydrodynamics of fish-like swimming. The covered methods include experiments and field observations involving biological fish (Sec. 2.1.1) and experiments using mechanical fish (Sec. 2.1.2), as well as numerical calculations based on reduced-order models (Sec. 2.2.1) and CFD simulations (Sec. 2.2.2). These approaches have advantages and disadvantages as related to the time it takes to develop them, how finely tunable they are, their equipment and facility needs, and their capability of measuring load and power, visualizing the fluid flow, and realizing self-propulsion and self-organization. Hence, each method can potentially be an effective research tool depending on the information desired to be collected, expected outcomes, and available resources. This methodology overview strives to inform the audience about the state-of-the-art, and to serve as a source of inspiration for the development of even more novel and elegant approaches, leading to a deeper understanding of the subject at hand.

**2.1 Experiments and Field Observations.** Systematic inquiries using experiments (be it in a laboratory or natural environment) have a long-lived history, with an ever-evolving toolbox of techniques [3,6,8–39,81–105]. Beginning from simple observations, researchers have progressively endeavored to quantify and understand the interactions of fish and fish-like swimmers with increasingly higher degrees of accuracy and sophistication. In what follows, we elaborate on some of the approaches broken into those involving biological and mechanical fish, with an emphasis on discussing nuances.

*2.1.1 Measurements Involving Biological Fish.* With the intent to study flow-induced interactions among fish-like swimmers, the first idea that might spring to mind is to start by examining biological fish. After all, these creatures have evolved over millions of years to optimize their movements while swimming, making them ideal candidates for observation, and ready-made to be used in experiments. In this vein, numerous species of fish have been studied, with a greater focus on the ones that are known to cluster together, such as bluefin tuna [13,24], cod [11,12], herring [11,12,26], saithe [11,12], tetra [94,95], and zebrafish [27,31,36,39], among others [6,8,10,14,15,17–20,23,25,28–30,32–35,37,38,86,100,104]. Of the utilized research methods, field observations offer visualization of large numbers of fish (with a wide diversity) in their natural environments, and are primarily conducted through the use of sonar [14,21,30] and aerial scans [8,13,14,24], see Figs. 1(*a*) and 1(*b*) for illustrative examples of the former.

While aerial observations offer more qualitative assessments, modern sonar techniques (e.g., the multibeam sonar [14], ocean acoustic waveguide remote sensing [21,25,26], and dual-frequency identification sonar or DIDSON [30]) can provide the size, form, and even the instantaneous density of fish schools. These observational methods have led to new revelations regarding the assembly and migration of herring while spawning [25,26] and the evasive motions of fish undergoing predation [30]. Regarding the former, for example, it was observed that the instantaneous distribution of the school has a fractal or power law behavior, which is indicative of structural similarity across a wide range of length scales.

The uncontrolled and unpredictable conditions inherent to these natural contexts, however, make it extremely difficult to isolate key features in fish schooling so as to study them (and their effects) independently. For this reason, experiments involving these organisms are often carried out in laboratory environments such as enclosed tanks [11,12,27–29,31–37] and water tunnels [6,10,15,17,18,38,39,94,95,104], where specific flow conditions can be imposed (see, e.g., Fig. 1(*c*)). A dominant technique used to examine their interactions in such circumstances (both with the surrounding fluid and with each other), is to record the motions of multiple fish together, sometimes using multiple cameras to gain three-dimensional (3D) tracking information. The resulting movements are then assessed quantitatively via manual (e.g., conventional pixel tracking for two-dimensional (2D) [17,104] or stereo and shadow methods [8], and stereophotogrammetry [37] for 3D) or automated (e.g., using Pro-Analyst [27,29] or a customized code [28,32,34,35, 94,95]) video postprocessing. The structural characteristics unveiled by the collected data are subsequently compiled to create statistical models [119,120] that complement and enhance numerical and analytical analyses. There have also been attempts to evaluate energy consumption within a school by monitoring the tailbeat motion of the fish and their oxygen intake (via intermittent-flow respirometry) [15]. By comparing the average metabolic rate of individuals, it has been suggested that swimming together can lead to energy savings. However, it has been shown that the metabolic rate of individuals in a school can also decrease due to the reduced stress experienced when swimming in the company of others. Therefore, it is not possible to attribute (with reasonable certainty) the observed energy savings to a single factor.

It is worth noting that experiments involving biological fish can also include those carried out with euthanized fish. These studies can shed light on the passive reaction of fish to the wake vortices generated by the neighbors [18–20,22,86]. For instance, when a

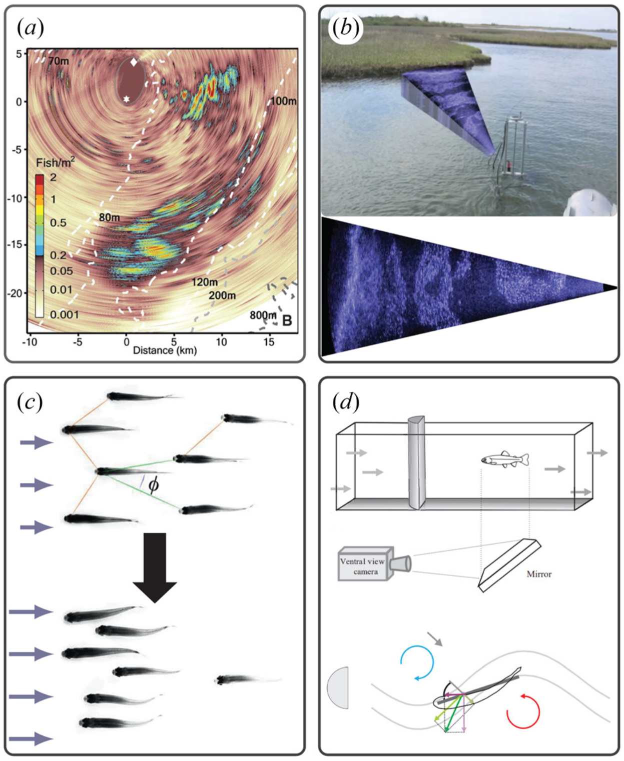


**Fig. 1 Representative examples of field observations and experiments involving biological fish. Instantaneous structure of fish schools measured using (*a*) ocean acoustic waveguide remote sensing and (*b*) high-resolution sonar imaging. (*c*) Video tracking of fish groupings in a water tunnel at low (top) and high (bottom) flow speeds. (*d*) A euthanized fish suspended in a water tunnel downstream from a rigid bluff body, passively reacting to the oncoming vortex street. Subfigures (*a*)–(*d*) are adapted with permission from Makris et al. [21], Handegard et al. [30], Ashraf et al. [95], and Liao et al. [18], respectively.**

dead fish is suspended behind a bluff body (say a cylinder) in a water tunnel, the limp form reacts to the Karman vortex street shed from the upstream body analogous to its living counterpart (see, e.g., Fig. 1(*d*)). The reactions to and interactions with the dead fish can be analyzed using electromyography to measure muscular movements and particle image velocimetry (PIV) to visualize the surrounding flow field. Additionally, the motion of biological fish may be investigated in the presence of their robotic clones [29,31,33,36]. In most cases, however, such setups are more suitable for behavioral studies focusing on cognitive, rather than hydrodynamic, effects.

In summary, experimental studies involving biological elements have provided a wealth of insights into the structure and dynamics of fish schools. Some of the limitations associated with these investigations, however, are lack of repeatability and precision control over the number and locomotion characteristics of fish, and restricted flow visualization and power measurement capabilities.

*2.1.2 Measurements Using Mechanical Fish.* Among experimental techniques, the use of mechanical/robotic fish is particularly valued as it allows for predictable and repeatable trials, while also enabling incremental adjustments. The employed mechanisms can vary widely in their forms and functions, from more abstract designs like heaving and pitching hydrofoils to fish-like and self-powered robots. Perhaps, the most basic implementation of this idea involves arranging a pair of passively flapping filaments in a soap-film tunnel, which is meant to capture 2D flow phenomena [87], see Fig. 2(*a*) for an illustrative example. This abstraction of fish-like motion facilitates the assessment and visualization of fundamental swimmer interactions at different positions; however, it cannot address other important aspects of collective swimming including variations in the propulsion mechanism and three-dimensional effects.

Other widely adopted representations are extruded hydrofoils oscillating in the bulk of water (see, e.g., Figs. 2(*b*) and 2(*c*)) [81–85,88–93,96–99,102–104]. Conventionally, these mechanisms are tested in water-tunnels with prescribed freestream flows, where two foils (in leader–follower or side-by-side arrangements) are precisely positioned so as to examine the impact of the collectively generated vortical structures on the performance of each foil [89–91,97,99,102], see Fig. 2(*b*) for an illustrative example. In some situations, the generated wake from the leading foil is approximated by a stationary bluff-body placed upstream [81,82] (see, e.g., Fig. 1(*d*)) or by some mechanical means [85]. Many configurations are possible, from stationary elliptical foils [81], to heaving [89], pitching [90,91,97,99,102], or simultaneously heaving and pitching hydrofoils [82,104]. The flow in these configurations can be visualized using classical dye markers [81,82] or, more quantitatively, with PIV [89–91,97,99,102,104]. Moreover, load measurements can be conducted using force/torque transducers attached to each foil [90,91,97,99,102].

Despite their prevalence, a limitation of these conventional approaches is that the hydrofoils are generally not free to self-propel, i.e., their propulsive speeds are predetermined. A solution to this challenge is to work with circular, instead of linear, arrays of

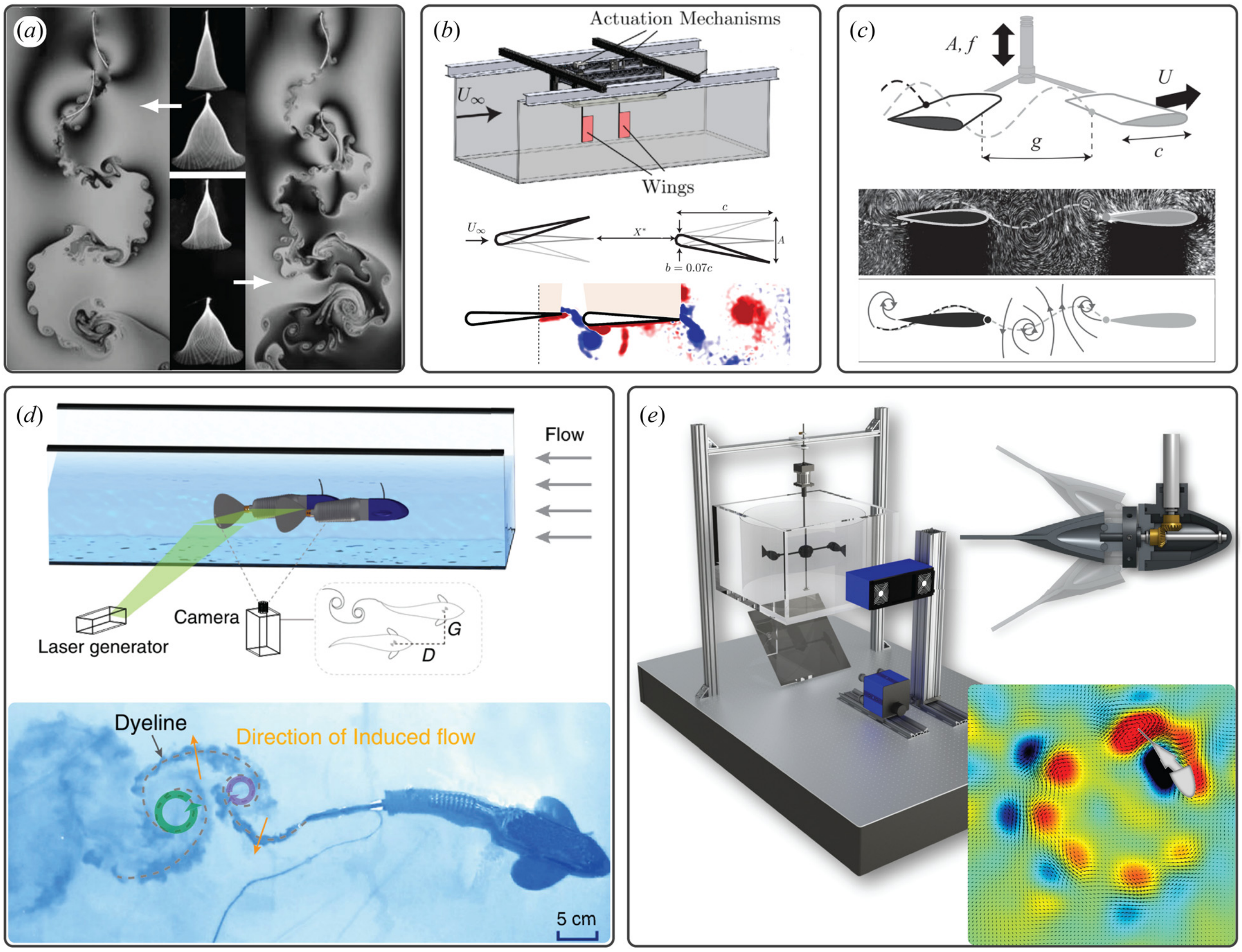


**Fig. 2 Representative examples of experiments with mechanical fish abstractions. (*a*) A pair of passively flapping filaments arranged in two different spacings in a soap-film tunnel. (*b*) A couple of actively pitching hydrofoils in a water tunnel [97]. (*c*) Two heaving hydrofoils traveling in orbits around a cylindrical water tank. (*d*) A stationary robotic fish duo beating their tails in a water tunnel. (*e*) A circular array of fish robots swimming in a cylindrical water tank. Subfigures (*a*)–(*e*) are adapted with permission from Ristroph and Zhang [87], Ramananarivo et al. [93], Kurt and Moored [97], Li et al. [100], and Pandhare et al. [105], respectively.**

oscillating foils [88,92,93,96,98,103], see Fig. 2(*c*) for an illustrative example. In this situation, the driving motion is prescribed, but the forward (rotational) motion is determined naturally from the hydrodynamic interactions. This, in effect, mimics an infinite linear array of traveling fish-like swimmers, with the added benefit of not having to stop or reset the actuation, and without the added cost and complexity of a water tunnel, while also providing constant visual access from a stationary point of view.

There can be several adaptations of this methodology depending on the desired testing conditions. For instance, the source of propulsion can be as straight forward as simple heaving, with [88] or without [92,93,96,98,103] passive pitching. Also, the spacing between the foils can be prescribed [88,92,96] or determined naturally as a consequence of hydrodynamic interactions [93,98,103]. The foils themselves may be rigid [88,92,93,98,103] or flexible [96]. As for the measurements, often, the propulsion speed of the swimmers (foils) is quantified via encoders attached to the axis of rotation, and the flow field is visualized via standard PIV.

Notwithstanding their usefulness, hydrofoils are still mainly two-dimensional and somewhat distant abstractions of real fish or comparable man-made swimmers. This disconnect in representation can be bridged with fish-like robots [29,31,100,101,105,107], see Fig. 2(*d*) for an illustrative example. In such cases, the robots are not intended to mimic specific fish, but instead, are generalized to encompass the typical form of multiple species, or to simulate a fish-like body movement (e.g., carangiform or subcarangiform). Usually these robotic fish are split into two major components, their head and tail, with some sort of flapping motion creating the fish-like movement.

These robots potentially allow for multiple individuals to swim together in different combinations. However, the precise relative positioning of freely swimming robots in specific patterns is a major obstacle. The most restrictive way to address this hurdle is to spatially fix the robots in a water tunnel and test them while allowing their tails to beat [100]. Another way to overcome the positioning problem is to use physical separators (e.g., rigid bars) to fix the distance between the robots, without presetting their propulsion speed [101,105]. This solution can be implemented in a setup where the connected fish swim together along a straight path in an enclosed body of water [101]. Such a setting requires a large container to accommodate reasonable travel distances and also, for flow analysis, demands a visualization system that is able to move with the fish assembly.

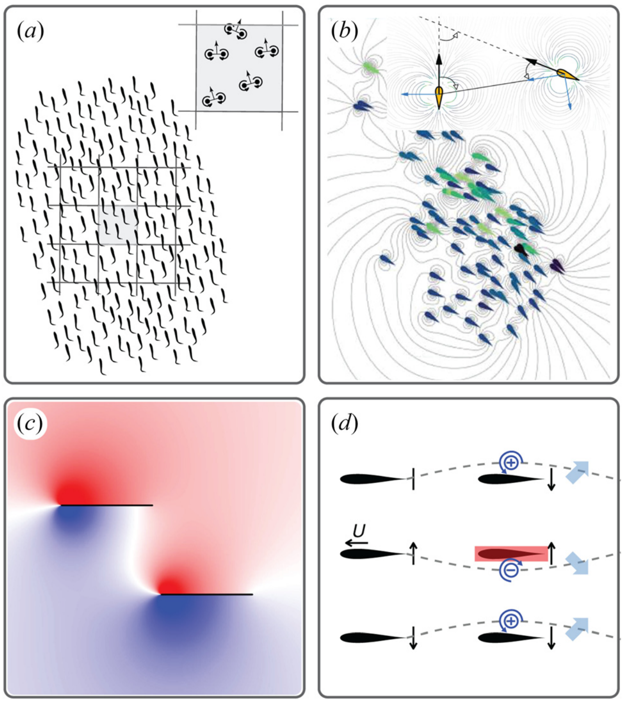


**Fig. 3 Representative examples of reduced-order models. Schools of finite-length vortex dipoles interacting (*a*) freely and (*b*) subject to behavioral rules. (*c*) A pair of zero-thickness heaving plates. (*d*) A lattice of flapping hydrofoils. Subfigures (*a*)–(*d*) are adapted with permission from Tsang and Kanso [44], Filella et al. [46], Baddoo et al. [51], and Oza et al. [47], respectively.**

Alternatively, the robotic fish can be incorporated in the rotational setup introduced earlier (as shown in Fig. 2(*e*)), where multiple robots are driven mechanically and attached rigidly to a central axis, enabling precise positioning and synchronization between the swimmers [105]. This adaption benefits from an easy and continual access to a stationary interrogation volume for flow visualization (using techniques such as dye marking, PIV, or even particle tracking velocimetry [105]), and, at the same time, circumvents the need for an excessively large tank size. Of course, with all of its advantages, the circular array is still an approximation for a linear group of swimmers; a fact that ought to be accounted for when interpreting the results.

### 2.2 Numerical Calculations and Theoretical Analyses.

Developed in parallel to (and in combination with) experimental techniques, mathematical modeling has served as another powerful tool in the ongoing efforts to probe the flow physics associated with the collective locomotion of fish-like swimmers. The utilized approaches vary widely in their fidelity and complexity, but, here, we categorize them into reduced-order models [40–51] and detailed CFD simulations [52–80,92]. In this section, we outline some of the more broadly used methods.

*2.2.1 Reduced-Order Models.* The origins of theoretical analysis have their roots often in reduced-order models, based on inviscid, 2D representations of the flow field generated and encountered by the swimmers. Arguably, the simplest approach involves treating each swimmer as a point particle moving at a constant speed in a fixed configuration. In this context, the flow produced by the collective is approximated as a superposition of the wake of individuals, which themselves are modeled as an array of point vortices in the form of reverse Karman vortex streets [40,41,111]. This idealization of the emerging flow field enables straightforward comparison between the resultant hydrodynamic forces experienced by each swimmer in a group and in isolation.

Complementary to this approach is an approximation technique that models the swimmers as finite-width vortex dipoles that are free to interact with each other [44–46], see Fig. 3(*a*) for an illustrative example. This method prioritizes long-range hydrodynamic interactions over the details of the flow structure in the vicinity of the swimmers, and, as such, is better suited for investigations into the stability and overall group dynamics of swimming collectives. Supplemented by behavioral rules, simulations based on this model have predicted the existence of robust and/or efficient schooling patterns [45,46], see Fig. 3(*b*) for an illustrative example.

A more advanced alternative to the above-mentioned approaches directly solves the linearized Euler equations for the flow field around the swimmers, while explicitly accounting for their geometry (see Table 1) and vortices shed from them [43,48–51], see Fig. 3(*c*) for an illustrative example. The solution in this method can be expressed in the velocity form as the summation of a background flow and the flow resulting from vortex sheets (both those bound to the swimmers and those shed from their trailing edges) [43,48–50]. Aided by the use of conformal mapping, the solution can also be expressed in the form of the pressure field corresponding to the collectively generated flow [51]. Once the flow field is resolved (one way or another), then the hydrodynamic forces exerted on each swimmer are calculated. It is also possible to creatively construct other reduce-order models by combining and adapting the aforementioned ideas [47], see Fig. 3(*d*) for an illustrative example.

Overall, numerical approaches based on reduce-order models are attractive because they are computationally economical (if

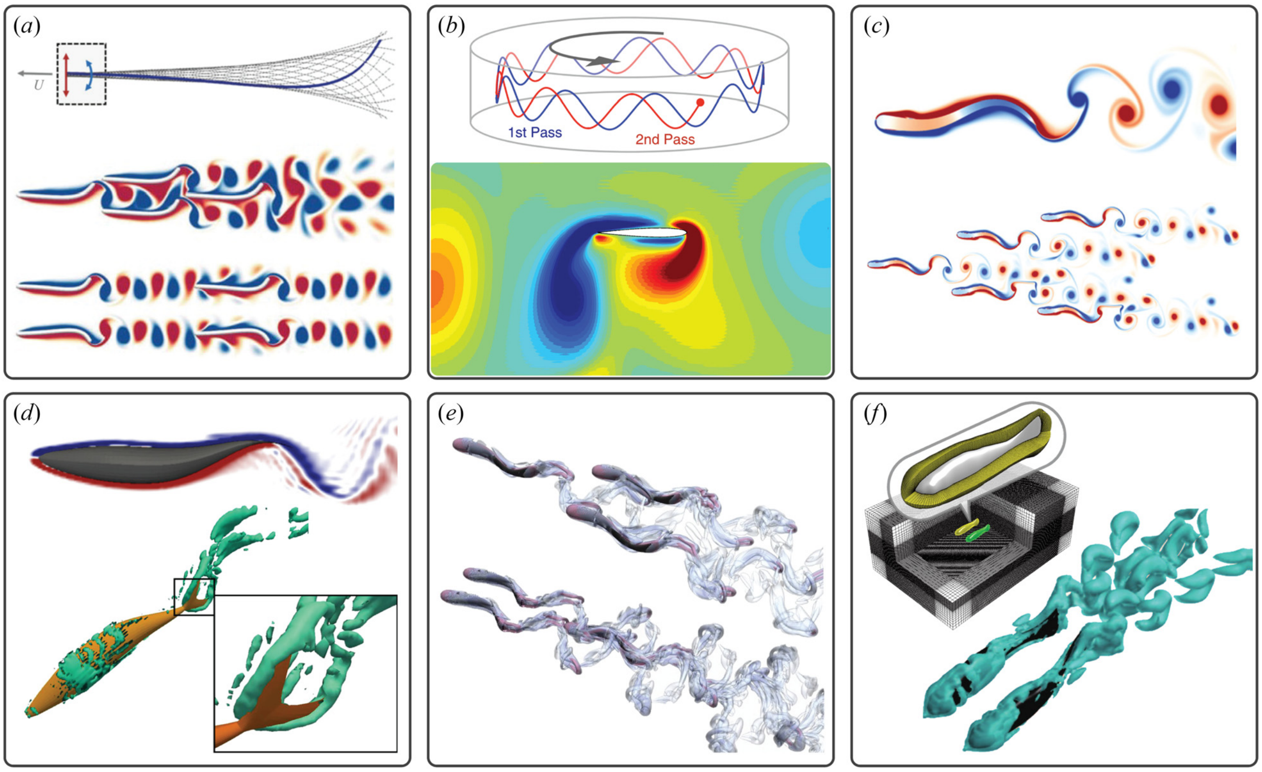


**Fig. 4 Representative examples of high-fidelity CFD simulations. (*a*) Multiple undulating plates driven by their heaving and pitching heads. (*b*) A hydrofoil undergoing prescribed heaving in a periodic domain. (*c*) A generic fish-like swimmer traveling in isolation and in a group. Collective swimming of emulated (*d*) mackerel, (*e*) zebrafish, and (*f*) red nose tetra. Subfigures (*a*)–(*f*) are adapted with permission from Dai et al. [64], Becker et al. [92], Gazzola et al. [55], Daghooghi and Borazjani [58], Verma et al. [67], and Li et al. [68], respectively.**

implemented efficiently) and can capture the leading-order hydrodynamic interactions. However, they are primarily limited to 2D scenarios and simple geometries, and also only partially account for viscous effects. The latter can introduce inaccuracies in force and power measurements.

*2.2.2 Detailed Computational Fluid Dynamics Simulations.* Recent advances in high-performance computing combined with the continuous improvement of numerical schemes has led to the development of high-fidelity simulation techniques pertinent to the hydrodynamics of collective fish-like swimming. Common among these approaches is that they are all centered on the detailed solution of the incompressible Navier–Stokes (NS) equations (or their equivalent forms) in either two [52–54,56,57,59–62,64–66,69–77,79,92] or three [55,58,63,67,68,78,80] dimensions. What distinguishes them from each other is the geometry and arrangement of swimmers, the degree to which self-propulsion and self-organization are realized, the specific scheme used to discretize the governing equations, and, finally, the manner in which the coupling between the motion of a swimmer and the flow around it is handled numerically.

As shown in Table 1, fish-like geometry and body motion is typically represented by waving plates (Fig. 4(*a*)), oscillating hydrofoils (Fig. 4(*b*)), undulating slender bodies mimicking the generic shape of fish (Fig. 4(c)), and even replicas of actual fish (Figs. 4(*d*)–4(*f*)). As for numerical schemes, conventional choices are discretizing the Navier–Stokes equations using finite difference/volume/element approaches, while applying the fractional step method for the velocity–pressure coupling. The Navier–Stokes equations may be solved equivalently in the vorticity form using a Fourier pseudo-spectral [92] or remeshed vortex method [55,62,67]. There are also several particle-based approaches such as the lattice Boltzmann method [66,79] and multiparticle collision dynamics [57] that are viable alternatives to the more traditional NS-based methods. Lastly, with respect to the handling of the fluid-structure interaction, the immersed boundary method stands out as the most popular among the available options, including (but not limited to) the penalty method [55,62,67,92] and overset gridding [68], see Fig. 4(*f*) for an illustrative example.

The main drawback of high-fidelity CFD calculations is their excessive computational costs. To cope with this challenge, these simulations are generally restricted to groups of few members, except for cases where an infinite array is simulated through the application of periodic boundary conditions. Furthermore, the flow is usually considered laminar and turbulent regimes are rarely explored [58]. Despite being resource intensive, the more economical CFD simulations can be integrated with optimization and machine learning algorithms to find swimming strategies with superior collective efficiency and cohesion [62,67,79].

## 3 Topics of Analysis

Having explored the expansive array of methodologies for examining flow-mediated interactions among fish-like swimmers, we now discuss the topics of practical interest, which primarily are (i) the collective locomotion efficiency of swimmers and (ii) their stability and group cohesion. In this section, we report on selective case studies and elaborate on them in the context of the strategies implemented to identify the role of hydrodynamics.

**3.1 Locomotion Efficiency.** From early on, it has been hypothesized that fish travel together in specific patterns to reduce their hydrodynamic burden and, thereby, increase their locomotion efficiency [4–7,40]. As a prelude to this subject, we first discuss common definitions of efficiency, beginning with the Froude efficiency, which is the ratio of the useful to input power [136,137]. Here, the input power is the actual power spent by the swimmer (i.e., the dot product of the body velocity and fluid stresses integrated over the surface of the swimmer), whereas the useful power is the component of the total power that results in forward motion (i.e., the product of the swimming speed and thrust force). Another measure of efficiency is the cost of transport (COT), which is the energy consumption per unit distance traveled or power-to-speed ratio in the instantaneous sense [136,137]. In both definitions, powers are usually averaged over one cycle of tailbeat oscillations. For biological swimmers, the spent power is usually represented by the metabolic rate of the organism [138]. In the realm of collective locomotion, efficiency is also sometimes characterized as a comparative metric between the desired functional quantities (such as the generated thrust or swimming speed) of an isolated individual versus that of the same individual in a collective (or an average of the collective itself).

Since the energy consumed by swimmers is to compensate for the work done by hydrodynamic forces opposing their motion, it is natural to examine schooling scenarios wherein the members are surmised to experience reduced hydrodynamic hindrance. With this in mind, a logical place to start is the formation of swimmers. As one might imagine, when swimmers travel in groups, individuals in the rear interact with the vortex-rich wakes of those ahead, altering the fluid load exerted on them during their motion. Commonly inquired patterns are rectangular and diamond (see Fig. 5), which are formed with at least four swimmers. There are also variants of these formations with usually two individuals, namely, in-line, side-by-side (also referred to as phalanx), and staggered (see Fig. 5) configurations. Considered less frequently, random formations are among the tested arrangements as well [45].

A defining feature of fish-like swimmers is their mechanism of propulsion, i.e., thrust generation via oscillatory body motion. It is these movements that create the vortical flows swimmers must navigate when traveling in groups. Therefore, it only makes sense to examine how the characteristics of the propulsion mechanism employed by swimmers influence the said flows and, thereby, the dynamics of the collective itself. Relevant traits of fish-like propulsion include the gestural intensity (e.g., heaving amplitude or pitching angle) and, specifically for group locomotion, the kinematic synchronicity among the members (e.g., the phase difference in the propulsion-generating body movement of neighboring swimmers).

It can be deduced from the preceding discussion that the key dimensionless parameters in collective swimming are the Reynolds Re $= U\ell/\nu$ and Strouhal St $= fA/U$ numbers, as well as the phase shift $\phi$ and the ratios $A/\ell$, $L_\parallel/\ell$, and $L_\perp/\ell$. Here, $U$ and $\ell$ are the propulsion speed and characteristic length of a representative swimmer, and $f$ and $A$ are the frequency and peak-to-peak amplitude of oscillations associated with its propulsion mechanism, respectively. Also, $\nu$ is the kinematic viscosity of the host fluid, and $L_\parallel$ and $L_\perp$ are the characteristic distances between the neighboring swimmers in the streamwise and lateral directions, respectively (see Fig. 5). A couple of useful combinations of the above parameters are Sw $= 2\pi fA\ell/\nu$ and $S = fL_\parallel/U$, which are commonly referred to as the swimming number (or the oscillatory Reynolds number) and schooling number, respectively. In what follows, we summarize the primary outcomes of a selection of recent studies centered on exploring the effects of the above-mentioned dimensionless quantities on the locomotion efficiency of swimming collectives in various schooling formations.

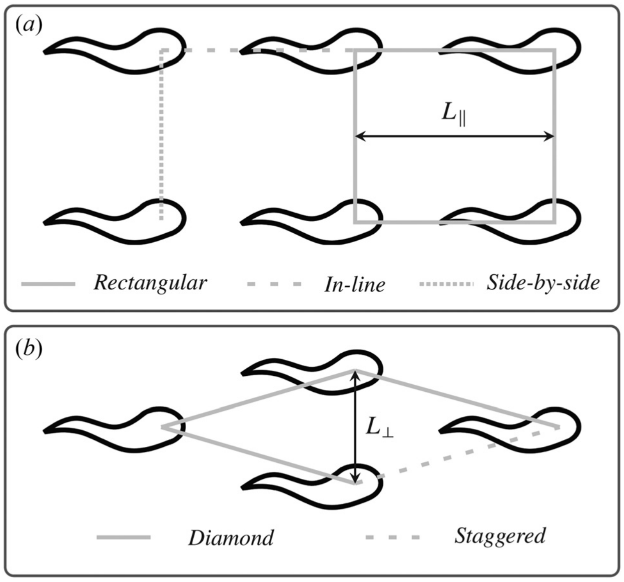


**Fig. 5 Schematics outlining common formations considered in collective swimming studies. (*a*) and (*b*) depict rectangular and diamond configurations, and their respective derivatives.**

Within systematic studies on the locomotion efficiency of fish-like collectives, the theoretical work of Weihs [40,41] is one of the earliest, often cited for linking swimmers' formation to reduced energy consumption. Using the point-particle approach mentioned in Sec. 2.2.1, he concluded that the vortices shed immediately behind leading swimmers create a vortex street that would hinder movement in rectangular formations, but would increase efficiency in diamond configurations. This early contribution has inspired more rigorous inquiries conducted with advanced tools, such as those reviewed in Sec. 2. In the rest of this subsection, we briefly discuss some of those studies starting with the experimental investigations using a pair of (i) pitching hydrofoils anchored in a water tunnel [90,91,97,99,102] and (ii) heaving hydrofoils self-propelling around a circular path in a water tank [92,93,98,103]. We then review several experimental works involving biological [95] and robotic [100,101,105] fish. Finally, we discuss a number of computational studies, beginning with those that considered prescribed swimmer arrangements [57,58,68] followed by the ones where some degree of self-organization was allowed [50,64,70,78].

Among the experimental investigations in the past decade, Boschitsch et al. [90,91] and Kurt et al. [97,99,102] used a pair of pitching hydrofoils in a water tunnel to study the effects of relative positioning and synchronicity between the swimmers (see Fig. 2(*b*)). In the case of Boschitsch et al. [90,91], the prescribed separation and relative phase between the foils were varied from $L_\parallel/\ell = 1.25$ to 5.25 (in-line arrangement [90]), $L_\perp/\ell = 0.5$ to 2 (side-by-side arrangement [91]), and $\phi = 0$ to $2\pi$, where $\ell$ represents the chord length of each foil. Also, the freestream flow speed was set to achieve Re $= 4.7 \times 10^3$ and St $= 0.25$. In this parameter space, Boschitsch et al. observed that, when placed in line [90], the performance of the leading foil was marginally affected by the presence of its trailing partner unless the spacing between the two was very tight. On the other hand, they found that the thrust production and efficiency of the downstream foil could be enhanced or attenuated by up to 50% (relative to an isolated foil) depending on the values of $L_\parallel$ and $\phi$. Moreover, when operating side by side [91], Boschitsch et al. reported that, at $\phi = 0$ (i.e., for in-phase oscillations), the efficiency increased, but this enhancement was associated with a reduction in the generated thrust. However, at $\phi = \pi$ (i.e., for out-of-phase oscillations), the thrust was seen to increase while efficiency stayed practically unchanged. For the intermediate values of the phase shift, an enhancement in the thrust and efficiency of one foil was shown to always come at the cost of a decline in the performance of the other. Boschitsch et al. employed PIV flow visualizations to make connections between the wake interactions and the measured efficiency characteristics of the tandem foils.

Complementing the above-mentioned investigations, Kurt et al. [97] examined the effects of dimensionality (2D versus 3D) on the performance of a pair of in-line pitching hydrofoils. The frequency and amplitude of pitching were held constant in their study such that Re $= 7.5 \times 10^3$, St $= 0.25$, and $A/\ell = 0.26$. Also, the relative phase between foils was varied from $\phi = 0$ to $2\pi$ and prescribed separations of $1.25 \le L_\parallel/\ell \le 2.25$ were tested. Under these conditions, Kurt et al. found that, in 2D, the maximum efficiency and thrust gains of 42% and 38%, respectively, occurred at $L_{\parallel}/\ell = 1.75$ and $\phi \approx 2\pi/3$, whereas, in 3D (where the foils did not span the entire flow channel), maximum efficiency and thrust enhancements of 25% and 15%, respectively, took place at $L/\ell = 1.25$ and $\phi \approx 7\pi/6$.

Kurt et al. later extended their work by considering interactions between a pair of low-aspect-ratio ($\mathrm{AR} = 0.5$) pitching hydrofoils at $\mathrm{Re} = 4.9 \times 10^3$, $\mathrm{St} = 0.8$, and $A/\ell = 0.26$ [99]. In this case, flow visualizations using tomographic PIV suggested that the wake structure surrounding the foils could be approximated as the superposition of the flow fields generated by each foil in isolation. Overall, their measurements indicated that the foils could benefit from thrust and efficiency augmentations in the range of 63 to 81% provided that the follower was strategically placed in the middle of the diverging momentum jets in the wake of the leader or located such that the momentum jet from the leader impinged on it.

Building on their previous efforts, Kurt et al. [102] also studied the free swimming of a pair of pitching hydrofoils oscillating with the phase shift of $\phi = \pi$. They examined in-line, staggered, and side-by-side arrangements of the foils at $\mathrm{Re} \approx 10^4$, $\mathrm{St} \approx 0.3$, and $A/\ell = 0.25$, and found that freely swimming foils experienced less than 10% efficiency improvements at their stable states. This was in contrast to the gains in the collective efficiency and thrust for constrained foils, which were measured to be as high as 40% and 100%, respectively, for the side-by-side, and 87% and 94% for the staggered arrangements.

In a series of studies, Becker et al. [92], Ramananarivo et al. [93], and Newbolt et al. [98,103] examined the collective locomotion of a pair of heaving hydrofoils traveling freely around a central axis (see Fig. 2(*c*)), with the Reynolds and Strouhal numbers in the range of $10^3 \lesssim \mathrm{Re} \lesssim 10^4$ and $0.1 \lesssim \mathrm{St} \lesssim 0.25$. They considered in-line [92,93,98], staggered [103], and side-by-side [103] arrangements of the foils, while fixing the streamwise distance between them in Ref. [92] and letting $L_{\parallel}$ evolve naturally to a stable state in Refs. [93], [98], and [103]. Note that when $L_{\parallel}$ is not prescribed, the equilibrium spacings (if they exist) emerge spontaneously and remain stable thanks to the hydrodynamic couplings (see also Sec. 3.2). They also tested the in-phase [92,93] and phase-shifted [98,103] heaving of the foils with the same [92,93,103] and independent [98] frequencies and amplitudes, where $0.25 \leq A/\ell \leq 1.67$ in Ref. [92], $0.13 \leq A/\ell \leq 0.79$ in Ref. [93], $0 \leq A/\ell \leq 1$ in Ref. [98], and $0.2 \leq A/\ell \leq 0.4$ in Ref. [103]. The conducted measurements, overall, indicated that the foils could swim up to 60% faster together than they would alone when placed next to each other side by side with $\phi = \pi$. The collective speed boost was found to be less pronounced for wide side-by-side configurations and for in-line and staggered arrangements.

That swimming in packed side-by-side (phalanx) formations leads to substantial efficiency gains is further supported by the experiments of Ashraf et al. [95] using a collection of red nose tetra fish (*Hemigrammus bleheri*) swimming in a shallow-water tunnel (see Fig. 1(*c*)), and also by the measurements of Li et al. [101] using a pair of robotic fish swimming forward freely, while laterally connected together via a rigid bar with $L_{\perp}/\ell = 0.33$. Specifically, at $\mathrm{Re} \approx 10^5$, $\mathrm{St} \approx 0.8$, and $A/\ell = 0.36$, Li et al. [101] observed considerable improvements in the speed and efficiency of the robotic swimmers independent of the phase difference between their tail beats. Quantitatively, they reported a maximum collective efficiency gain of nearly 60% and speed enhancement of roughly 10% for in-phase swimming, whereas, for out-of-phase swimming, they showed similar increases of about 25% for both speed and efficiency. Moreover, it was demonstrated that, depending on the value of $\phi$, the gains in the efficiency for the two swimmers could differ by as much as 100% or even more. In a separate investigation [100], Li et al. tested the collective performance of their tandem robotic fish in staggered arrangements with $1.22 \leq L_{\parallel}/\ell \leq 2$, $0.27 \leq L_{\perp}/\ell \leq 0.33$, $A/\ell = 0.09$, and $0 \leq \phi \leq 2\pi$ (see Fig. 2(*d*)). In these experiments, the robots were tethered in a water tunnel with a freestream speed that resulted in $\mathrm{Re} \approx 10^5$ and $\mathrm{St} \approx 0.35$. Analyzing the gathered data, they realized that, in the leader–follower format, the follower could energetically take advantage of the leader's wake provided that $\phi$ follows a linear relationship with $L_{\parallel}$, a strategy that they called vortex phase matching. Li et al. also experimented with pairs of freely swimming goldfish (*Carassius auratus*) and found that the followers do indeed utilize this strategy, without relying on their visual or lateral line sensory systems. Northern bald ibises (*Geronticus eremita*) were also previously reported to harness a similar strategy when flying in a V formation [139].

Collective hydrodynamics of fish-like robots was also scrutinized by Pandhare et al. [105] in an experimental setting where the robots were allowed to swim in line along a circular path measuring approximately six times the length of the robots (see Fig. 2(*e*)). The mechanical fish were propelled via the prescribed flapping motion of their tails, and their swimming speed was determined naturally as a result of hydrodynamic interactions. Pandhare et al. studied the variations of the emerging collective speed as a function of the frequency and amplitude of the robots' tail beat and the phase difference between the tail motions of the neighboring robots. At $10^3 \lesssim \mathrm{Re} \lesssim 10^5$, $10^4 \lesssim \mathrm{Sw} \lesssim 10^6$, and $0.08 \leq A/\ell \leq 0.6$, they observed that two fish swimming in phase could travel up to about 2.5 faster than a single fish with identical tail kinematics. They also found that the speed gains generally trended downward, and in some cases turned into speed losses, with increasing $\phi$ from 0 to $\pi$. Particle tracking velocimetry (PTV) flow visualizations were used to link the observed behaviors with the way fish interact with the previously shed vortex rings.

As noted in Sec. 2, the collective efficiency of fish-like swimmers has also been investigated computationally. Representative examples of these studies include the works of Hemelrijk et al. [57], Daghooghi and Borazjani [58], Li et al. [68], and Seo and Mittal [80], where numerical simulations were employed to examine how fixed group formations affect efficiency in schools of undulating fish. Specifically, Hemelrijk et al. [57] considered two-dimensional schools of mullet-like fish in in-line, side-by-side, rectangular, and diamond formations, where periodic boundary conditions were applied to model infinite arrays. They fixed the frequency and amplitude of tail beating and varied the streamwise and lateral spacings between the swimmers. Their simulations showed that all configurations experienced increased locomotion efficiency (compared to an isolated swimmer) except for a phalanx configuration with a spacing of less than one body length. The in-line, side-by-side, rectangular, and diamond formations experienced maximum rises in efficiency of 9% (at $L_{\parallel}/\ell = 1$), 4% (at $L_{\perp}/\ell = 2$), 22% (at $L_{\parallel}/\ell = 1$ and $L_{\perp}/\ell = 1.5$), and 24% (at $L_{\parallel}/\ell = 1$ and $L_{\perp}/\ell = 1.5$), respectively. The Reynolds and Strouhal numbers and the dimensionless amplitude in the simulations were about $\mathrm{Re} = 1.2 \times 10^3$, $\mathrm{St} = 0.55$, and $A/\ell = 0.2$, respectively.

Focusing on rectangular groupings, Daghooghi and Borazjani [58] computationally investigated the energetics of Atlantic mackerel via three-dimensional large eddy simulations combined with the immersed boundary method. The streamwise spacing between the swimmers was set to one body length and the lateral separation was changed from $0.3\ell$ to $\ell$. Again, periodic boundary condition was used to mimic schools of infinite members. At $\mathrm{Re} \approx 5 \times 10^4$, $\mathrm{St} \approx 0.25$, and $A/\ell = 0.2$, they found a 2% increase in the Froude efficiency (for all configurations) compared to an isolated swimmer, and a 10 to 20% increase in the power (fuel) efficiency, favoring more compact configurations. Also using three-dimensional simulations, though with the overset mesh technique, Li et al. [68] probed the energy expenditure of a pair of modeled red-nose tetra fish. Multiple fixed configurations, at $\mathrm{Re} \approx 4 \times 10^3$, $\mathrm{St} \approx 0.4$, and $A/\ell = 0.22$, were tested from side-by-side (with $0.4 \leq L_{\perp}/\ell \leq 1$), to in-line (with $1.25 \leq L_{\parallel}/\ell \leq 2$), and staggered (with $0.2 \leq L_{\parallel}/\ell \leq 2$ and $0.2 \leq L_{\perp}/\ell \leq 1$). Moreover, four phase differences ranging from $\phi = 0$ to $3\pi/2$ were considered. In terms of the required power, phases of $\phi = 0$ and $\pi$ were shown to be less demanding than quarter phases ($\phi = \pi/2$ and $3\pi/2$) by approximately 5%. Additionally, staggered formations were found to cause up to a 3% decrease in the group COT at $L_{\parallel}/\ell = 0.45$ and $L_{\perp}/\ell = 0.75$, while in-line and side-by-side formations resulted in a possible 2% reduction at $L_{\perp}/\ell = 0.5$ and $1.25 \leq L_{\parallel}/\ell \leq 2$. 3D calculations were also carried out by Seo and Mittal [80], using a sharp-interface immersed boundary technique for a couple of common Mackerel. Applying the force partitioning analysis, it was revealed that the wake produced by the leading fish could robustly improve the thrust generation and efficiency of the follower via inducing a strong leading-edge vortex on the tail of the trailing swimmer. Gains of up to 12% were observed through this mechanism, while simulation parameters were set to $\mathrm{Re} \approx 2.5 \times 10^3$, $\mathrm{St} \approx 0.2$, $0.36 \leq L_{\parallel}/\ell \leq 0.96$, $L_{\perp}/\ell = 0.3$, $0 \leq \phi \leq 2\pi$, and $A/\ell = 0.2$.

In another set of efforts, Heydari and Kanso [50], Lin et al. [70], Dai et al. [64] and Arranz et al. [78] performed dynamically evolving simulations where the swimmers (specifically, the followers) were allowed to determine their streamwise stable position behind the leader freely. In particular, to examine the differences in heaving and pitching actions in fish-like swimming, Heydari and Kanso [50] used the vortex sheet model to numerically study the free swimming of a pair of in-line rigid plates with peak-to-peak heaving amplitudes and pitching angles ranging from $0.6\ell$ to $1.4\ell$ and 20 deg to 90 deg, respectively. They reported that a pair of heaving plates swam 5 to 10% faster than a single heaving plate, with a 15% higher cumulative COT when separated by one heaving wavelength. In the pitching mode, however, a pair separated by one wavelength was shown to travel approximately 5% slower, but with a 35% lower overall COT. In this scenario, the follower's COT could be as low as 25% of a single swimmer's COT.

Considering primarily tightly spaced ($L_{\parallel}/\ell - 1 \ll 1$) arrays of simultaneously pitching (with a peak-to-peak amplitude of $2\pi/9$) and heaving (with a peak-to-peak amplitude of $0.8\ell$) hydrofoils arranged streamwise in a line, Lin et al. [70] simulated the locomotion of five-, ten-, and fifteen-foil ensembles at $\mathrm{Re} \approx 2 \times 10^2$ and $\mathrm{St} \approx 0.16$, and with foil-to-foil phase differences of $1.3\pi \leq \phi \leq 1.8\pi$. They found that, when $\phi = 1.6\pi$ and the stable separation between the hydrofoils was between $L_{\parallel}/\ell = 0.15$ and 0.6, the locomotion efficiency increased up to 130% in the fifteen-foil array. They also observed that reducing $\phi$ and the number of swimmers led to lower efficiency gains. For example, in the five-foil arrangement with $\phi = 1.4\pi$, the efficiency increased by 60%.

Using the immersed boundary method, Dai et al. [64] studied hydrodynamic interactions among two-dimensional elastic plates as representatives of fish-like swimmers. Two-, three-, and four-swimmer groups were tested in all formations shown in Fig. 5, with the fixed lateral spacing $0.15 \leq L_{\perp}/\ell \leq 0.65$. The swimmers were driven by the prescribed heaving and pitching motions of their leading ends, with respective peak-to-peak amplitudes of $0.01\ell$ and 10 deg. Both in-phase ($\phi = 0$) and out-of-phase ($\phi = \pi$) head motions were considered for evaluating synchronicity effects. In all cases considered, only a few experienced lower collective COT compared to a single swimmer and the rest benefited energetically from the hydrodynamic interactions, with collective COT gains of up to 16%. Also, the obtained data suggested that the antiphase synchronization among the swimmers is generally linked to higher energy efficiencies.

Accounting for three-dimensional effects, Arranz et al. [78] examined the performance of a pair of undulating flexible plates in several tandem formations, with $L_{\perp}/\ell = 0$, 0.3, and 0.6, and $0 \leq \phi \leq 2\pi$. Similar to the study of Dai et al. [64], the self-propulsion of the plates was driven by the imposed heaving of their leading edges with a peak-to-peak amplitude of $\ell$. Performed at $\mathrm{Re} \approx 2 \times 10^2$, their simulations revealed that, when swimming very close to each other, both the leader and follower enjoyed slightly greater speeds compared to the isolated version of themselves, which came at the cost of higher driving power for both. However, for cases where the streamwise separation between the swimmers was comparable to their length, the performance of the leader was found to be unaffected by the presence of the follower, while the behavior of the follower was seen to be heavily influenced by the incident flow generated by the leader. Depending on the combination of parameters, the power consumed by the follower in this scenario could be up to 10% less than the power expended by the same swimmer in isolation.

**3.2 Stability and Group Cohesion.** A topic of equal relevance as efficiency (and perhaps of greater importance) when it comes to swimming collectives is stability. Stable or equilibrium states in this context are defined as those that are not easily upset or disturbed. In other words, a collective is considered to be stable (or at equilibrium) if any positional perturbation results in (cohesive) hydrodynamic forces that restore the individual(s) to their original state within the group. The stability of schooling fish-like swimmers can be studied by inspecting fixed or dynamically evolving collectives. In the former approach, group formation is prescribed, and the forces acting on each swimmer (or the entire community) are analyzed to find equilibrium arrangements. In the latter strategy, on the other hand, the hydrodynamic forces are let to dynamically persuade the swimmers into natural positions of equilibrium, and the settling distances or average group density are directly measured once a stable state is reached. As an intermediate method, collective stability can also be studied through the examination of partially constrained arrays of swimmers. Regardless of the approach, the same set of parameters that determines the collective locomotion efficiency of swimmers is expected to govern their group stability and cohesion as well.

Quite a few of the studies reviewed in Sec. 3.1 performed stability analysis, too. From those investigations, one of the early findings, that drew fresh attention to the subject of stability in relation to collective swimming, was the observation of bistable states in the experiments and numerical simulations of Becker et al. [92], where it was shown that, for a given amplitude, there exists a range of heaving frequencies over which the in-line hydrofoil array could swim in a stable manner at two distinct rotational speeds. Ramananarivo et al. [93] and Newbolt et al. [98,103] followed up on the work of Becker et al. by performing similar experiments with heaving hydrofoils traveling in a circular path. However, their tandem foil arrays were designed such that each foil was able to move independently in the streamwise direction, so as to self-dictate the interfoil spacing (see Fig. 2(*c*)). Specifically, Ramananarivo et al. found that a pair of in-line foils undergoing prescribed synchronized heaving could self-organize in multiple configurations, with the stable gap between the foils being an integer multiple of the wavelength of their trajectory denoted by $\lambda = U/f$. Through PIV flow visualizations, it was shown that the stable states occurred at points where the wake vortices encountered by the follower align with its direction of motion. Also, the cohesive hydrodynamic force that promoted stability was found to follow a springlike behavior.

Allowing the heaving motions of the two in-line hydrofoils to be controlled separately, Newbolt et al. [98] learned that the introduction of a phase shift between the heaving of a leader–follower pair swimming in line increased the stable gap between the two by about $(\phi/2\pi)\lambda$. Additionally, they discovered that having identical heaving kinematics (i.e., $f_1 = f_2$ and $A_1 = A_2$, where the properties of the leader and follower are distinguished by the subscripts 1 and 2, respectively) is not a necessary condition for a pair of in-line foils to swim together cohesively. For instance, it was shown that, for $f_2 < f_1$ and $f_1 A_1 < f_2 A_2$, the overdriven follower could swim behind the leader without colliding with or drifting apart from it. In this scenario, however, the gap between the two foils oscillated within a certain range as the follower periodically sped up and slowed down. In a later study [103], Newbolt et al. also examined the effect of lateral positioning on the stability of asynchronously heaving foils and mapped out the regions behind the leader where the follower could lock into side-by-side and staggered formations.

The above-mentioned experimental observations are supported by the results of two-dimensional computational studies using reduced-order modeling [47,50,51] and detailed CFD simulation

[56,64–67,70,71]. Therefore, for two-dimensional swimmers, the evidence is strong for the existence of discrete swimming formations that are passively stable in the streamwise direction. However, as argued by Dai et al. [64] and Kurt et al. [102], arrangements that are stable in the direction of swimming may not necessarily be stable laterally. As an example, for a pair of asynchronously pitching hydrofoils, Kurt et al. [102] created a map for the relative force exerted on the follower at various positions behind the leader. The map indicated that the pair could swim together in a two-dimensionally stable manner only when they are placed side by side. Numerical simulations of Dai et al. [64], though performed for undulating plates at a lower Reynolds number, also predicted that the side-by-side arrangement is stable in both streamwise and lateral directions.

Besides lateral stability, another issue that has received little attention is the stability of three-dimensional swimmers. Among the few investigations that did consider the collective stability of 3D swimmers, Li et al.'s simulations suggested a pair of fish could swim together stably in a side-by-side configuration [68], which is consistent with the findings of Ashraf et al. [94] for red nose tetra fish. Also, streamwise stability analysis of a pair of self-propelled flexible plates by Arranz et al. [78] revealed that, except for $\phi = 0$, for each $\phi$ and $L_\perp$ considered, there was a stable position in the wake of the leader where the follower could swim without a need for active adjustment of longitudinal distance with its peer. For synchronized swimming, on the other hand, two equilibrium positions were identified, one at $L_\parallel/\ell \approx 1$ and the other one at $L_\parallel/\ell \approx 3.4$. The coexistence of compact and wide stable configurations was likewise predicted by Zhu et al. [56] for a couple of 2D flexible plates traveling in phase. Furthermore, consistent with prior observations (see, e.g., [98,100,139]), Arranz et al. found that the equilibrium gap between the swimmers varied linearly with the prescribed phase shift between their oscillatory driving motion.

In spite of the reported similarities, due caution must be exercised when attempting to infer the stability characteristics of various schooling formations for 3D swimmers from the results of 2D studies, in particular for cases where wake-body interactions play a prominent role in establishing group cohesion. It is important to keep in mind that the wake vortices produced by a 2D swimmer are structurally different from those generated by a 3D one, with the former taking the form of a reverse von Karman vortex street aligned with the centerline of the swimmer and the latter developing into a series of single- or double-row vortex rings [55,63,67,68,78,99,129,130,140,141].

Finally, it is apt to point out that hydrodynamically-driven self-organization cases reviewed in this subsection have been considered by some as confirmations of the so-called Lighthill conjecture. What has been characterized in recent years as the Lighthill conjecture appears to be a remark by Lighthill (at the end of chapter six of his book entitled Mathematical Biofluiddynamics [42]) on the theoretical calculations of Weihs [40,41] (see also Sec. 3.1). Lighthill's statement reads in part: "There is a certain body of observational evidence that such diamond-shaped lattice patterns are in fact taken up by fish schools when moving fast, and the question arises whether *passive* forces bring the pattern into play or whether very elaborate control mechanisms act to maintain the lattice pattern. Dr. Weihs can see certain mechanisms whereby passive forces might possibly be effective. I have jokingly suggested to him that the whole phenomenon may indeed be like the way in which the disordered molecules of a liquid form into a crystal lattice when the temperature is lowered: in other words, as the speed of the fish school increases, the *energy* advantage of the fishes dropping into this diamond-shaped lattice formation is suddenly able to outweigh the loss of entropy!!!". Whether that commentary qualifies as a purposefully proposed scientific conjecture is somewhat debatable.

## 4 Summary and Conclusions

It is apparent from the (nonexhaustive) literature review of the preceding section that the scientific knowledge of collective hydrodynamic interactions in schools of fish-like swimmers has grown substantially since a decade ago. As discussed in Sec. 2, during this period, a host of new techniques and approaches have been developed to gain new insights into the role of hydrodynamics in the performance and grouping behavior of swimming collectives. Experimentally, investigators have employed diverse methods involving the utilization of biological fish (both in situ and in controlled laboratory environments), as well as the use of engineered mechanisms such as circular arrays of hydrofoils and even robotic fish. Quantitative measurements using mechanical fish have often been accompanied by flow visualization via, e.g., dye injection, PIV, or PTV. Likewise, on the computational front, researchers have adopted a variety of reduced-order and high-fidelity modeling strategies to address multibody fluid-structure interactions that is central to solving collective swimming problems. Moreover, there have been efforts to integrate fluid dynamical models with novel control and optimization algorithms to achieve or search for efficient schooling scenarios.

Using the above-mentioned tools, researchers have collectively examined various swimmer configurations (see Fig. 5) while exploring a wide parameter space that spans: $L_\parallel/\ell \sim O(1-10)$, $L_\perp/\ell \sim O(1)$, $A/\ell \sim O(10^{-1}-1)$, $0 \le \phi \le 2\pi$, St $\sim O(10^{-1}-1)$, and Re $\sim O(10^2-10^5)$. The new knowledge gained through these studies has specifically advanced the fundamental understanding of collective efficiency and stability pertaining to the group locomotion of fish-like swimmers in organized formations.

Regarding locomotion efficiency, there exists a growing body of evidence suggesting that enhancements are achievable for the right combination of spacing and synchronization in leader–follower (both in-line and staggered), as well as side-by-side, formations. In the leader–follower setting, usually, the leader behaves as if the follower is not present, whereas the follower's performance is heavily influenced by its interaction with the wake of the leader. It has been demonstrated that the follower can outperform its isolated self by matching the phase of its oscillatory body motion with the incoming quasi-periodic wake made of vortices shed by the leader. The desired phase shift, in this context, has been shown to vary nearly linearly with the streamwise gap between the leader and follower and inverse linearly with the wavelength of the leader's swimming trajectory. This follower-wake phase locking strategy, which is also referred to as vortex phase matching, appears to also be adopted by goldfish when swimming freely in pairs. Furthermore, a similar mechanism may be used by fish to optimize the interaction between their fins [142,143].

When arranged side by side (i.e., in phalanx formation), on the other hand, the interactions between the swimmers are no longer purely wake mediated. For instance, in this situation, the gain in efficiency for out-of-phase synchronization has been attributed primarily to an increase in the added mass of the swimmers, similar to what is experienced by a single swimmer traveling near a solid wall (known as the ground or channeling effect). Compared to wake phase matching, it appears that this mechanism yields lower efficiency boosts. Finally, regardless of the specific arrangement, there are some indications that performance augmentations for 3D swimmers are less pronounced than for 2D ones. This discrepancy likely stems from the difference in the flow structure surrounding the swimmers. For instance, it is known that the wake behind 3D swimmers consists of single or double-row vortex rings, in contrast with the reverse von Karman vortex street that forms after 2D swimmers. Moreover, there are more ways for vortices to break down in 3D than 2D, which can alter the effectiveness of the mechanisms responsible for efficiency improvements.

As for stability and group cohesion, experiments and numerical simulations alike seem to suggest that, for 2D swimmers arranged in leader–follower formats, there are quantized, streamwise stable configurations resulting from the same wake-mediated interactions that also improve the performance of the follower. It has been pointed out, however, that these (quasi-)equilibrium states may not necessarily be stable laterally. Moreover, it is not entirely clear yet to what extent the same trend holds for 3D swimmers. On the other hand, single-row phalanx formation has been observed to be robustly stable for both 2D and 3D swimmers, along and perpendicular to the direction of motion. This stability originates from the balance of forces due to quasi-steady body-body interactions and the deflected momentum jet in the wake of the swimmers.

Notwithstanding great strides made to date, there is still ample room to further explore the schooling dynamics of fish-like swimmers and the rich flow physics associated with it. Important lines of inquiry into this phenomenon that remain un- or under-explored include: (i) What is the relationship between stability/cohesion and locomotion efficiency in collective swimming? (ii) Are the stable formations also the most efficient ones? Or, are the most efficient arrangements usually unstable? If so, how costly is it to use active control strategies to maintain these optimal arrangements? (iii) Can the behavior of minimal collectives (e.g., a pair of swimmers in leader–follower or side-by-side arrangements) be generalized to that of larger groups with many members? (iv) Under what conditions do hard-working swimmers (those who carry higher-than-average loads) and freeloaders (those who are burdened less than average) coexist in a coherent group? Would such a mixed ensemble consisting of competing neighbors be collectively efficient?

These and other open questions of interest can be addressed most effectively using multipronged approaches involving a synergistic combination of experimental measurement and numerical calculation (see Sec. 2 for a summary of methods). In this regard, a few of the methodological challenges likely to be encountered in future investigations are: (i) dealing with excessive computational costs of (a) three-dimensional high-fidelity numerical simulations and (b) postprocessing raw imaging data for time-resolved flow visualization via, for example, tomographic PIV or PTV; (ii) designing experimental setups that (a) can accommodate desired degrees-of-freedom and are (b) manageable in size and (c) conveniently accessible for quantitative flow visualization; (iii) coupling detailed CFD simulations with the tools of machine learning for optimization and control purposes; (iv) constructing physically meaningful reduced-order models with ideally no or minimal free parameters.

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## Data Availability Statement

No data, models, or code were generated or used for this paper.

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