

# Comparison of the full model and phase-matched model for transverse mode instability

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**Abstract:** We compare the full model and phase-matched model for the transverse mode instability. The phase-matched model, which requires less longitudinal discretization with less computational time, predicts the same refractive index gratings as the full model.

High-energy fiber amplifiers have generated great interest due to their rapid increase in output powers with improved beam quality [1]. To achieve even higher output powers, it is necessary to suppress nonlinear effects. The transverse mode instability (TMI) is one of the lowest order nonlinear effects that limits output powers in fiber amplifiers [2–4]. This effect appears when a higher-order mode (HOM) in an optical fiber couples to the fundamental mode via thermal stimulated Rayleigh scattering that is driven by defect heating. Efficiently suppressing TMI requires understanding TMI with a computationally efficient model. However, modeling TMI has been computationally intensive, because the temperature, refractive index, and mode coupling are proportional to  $\exp(i\Delta\beta z)$  that oscillate over the beat length  $L_B = 2\pi/\Delta\beta$ , where  $\Delta\beta$  is the difference in propagation constants between the fundamental and the HOM. The beat length is typically on the order of a centimeter and is far less than the length scale over which the mode amplitudes and other quantities change, which is typically on the order of meters. Resolution of these rapid beat length oscillations implies the requirement that the longitudinal discretization in the model be substantially smaller than the beat length. This requirement, along with the need to resolve the transverse temperature along the entire fiber has limited most prior studies of TMI to only short length fibers, which are less than the typical length of fiber amplifiers on a scale of 10s of meters. On the other hand, the power residing in the optical modes along the fiber varies slowly compared to the beat length  $L_B$ . The  $\exp(i\Delta\beta z)$  dependent oscillation residing in the temperature and refractive index distributions does not significantly contribute to the overall gain for the optical modes in the fiber. In this paper, we use the phase-matched model [5], which neglects the higher frequency oscillating terms in the coupled mode equations, allowing a much coarser longitudinal discretization with no loss of accuracy. We compare the refractive index in the fiber using the full model [4] and the phase-matched model when modeling TMI, which has not been done previously.

The usual coupled mode equations are given by [4]

$$\begin{aligned} \frac{dA_0}{dz} &= \frac{i\omega^2}{\beta c^2} \int d^2 \mathbf{r}_\perp n_0 \Delta n [|\mathcal{E}_0|^2 A_0 + \mathcal{E}_0^* \cdot \mathcal{E}_1 \Delta \exp(-i\Delta\beta z) A_1], \\ \frac{dA_1}{dz} &= \frac{i\omega^2}{\beta c^2} \int d^2 \mathbf{r}_\perp n_0 \Delta n [|\mathcal{E}_1|^2 A_1 + \mathcal{E}_0 \cdot \mathcal{E}_1^* \exp(i\Delta\beta z) A_0], \end{aligned} \quad (1)$$

where  $\mathcal{E}_0$  and  $\mathcal{E}_1$  are the transverse mode profiles for the fundamental mode and the HOM, while  $A_0$  and  $A_1$  are the corresponding amplitudes. The phase-matched model for TMI decomposes the changes in refractive index using [5]

$$\Delta n = \Delta n_0 + [\Delta n_+ \exp(i\Delta\beta z) + \Delta n_- \exp(-i\Delta\beta z)]/2, \quad (2)$$

where  $\Delta n_0$  is the 0<sup>th</sup> order term, and  $\Delta n_{+,-}$  are the positive and negative harmonic beating tones between fundamental mode and the first HOM. Substituting  $\Delta n$  into Eq. (1) and keeping only the phase-matched terms, we obtain [5]

$$\begin{aligned} \frac{dA_0}{dz} &= \frac{i\omega^2}{\beta c^2} \int d^2 \mathbf{r}_\perp n_0 [|\mathcal{E}_0|^2 \Delta n_0 A_0 + \frac{1}{2} \mathcal{E}_0^* \cdot \mathcal{E}_1 \Delta n_+ A_1], \\ \frac{dA_1}{dz} &= \frac{i\omega^2}{\beta c^2} \int d^2 \mathbf{r}_\perp n_0 [|\mathcal{E}_1|^2 \Delta n_0 A_1 + \frac{1}{2} \mathcal{E}_0 \cdot \mathcal{E}_1^* \Delta n_- A_0]. \end{aligned} \quad (3)$$

To compare the full model [4] for TMI with the phase-matched model [5], we consider a 10-m-long fiber with a core diameter of 50  $\mu\text{m}$ , an input pump power of 450 W, and a simulation time of 0.5 ms. For both models, we use a longitudinal discretization such that there are 90 discretized points per beat length with 1% of relative error in the full model. Further comparison [5] shows that only two discretization points per beat length are required to achieve a relative error of 1% using the phase-matched model, which shows the significant advantage in computational speed. Here, the same number of discretized points per beat length are used so that we can directly compare the Fourier transform of the refractive index grating along the longitudinal direction for both models.

Figure 1(a) shows  $\Delta n$  at 10  $\mu\text{m}$  away from the fiber center as a function of  $z$  with  $t = 0.5$  ms. The changes in the refractive index are due to the temperature changes, which in turn are due to the quantum defect. The solid blue and dotted red curves in Fig. 1(a) represent the results from the phase-matched model and full model, respectively. The inset in Fig. 1(a) shows a magnified plot of  $\Delta n$  along the fiber between 1.2 and 1.3 m, so that the agreement between

the two models in the refractive index grating may be clearly seen. Figure 1(b) shows the absolute value of the Fourier transform of  $\Delta n$  that appeared in Fig. 1(a) as a function of  $k$ . The inset in Fig. 1(b) shows a magnified plot at the first positive harmonic tone. The phase-matched model and full model capture the same content in the first harmonic tone. Figure 1(b) shows that the refractive index calculated using the phase-matched model only contains the first positive and negative harmonic tones at  $\pm\Delta\beta = \pm 528 \text{ m}^{-1}$  in spatial frequency, while the full model contains all the harmonic tones. Figures 1(a) and 1(b) show that the refractive index distribution can be predicted by ignoring the higher harmonic tones in spatial frequency, so that we can use Eq. (2) to represent the changes in refractive index and remove the non-phase-matched terms that are proportional to  $\exp(i\Delta\beta z)$  in the coupled mode equations, which leads to Eq. (3). The absence of the factor  $\exp(i\Delta\beta z)$  in Eq. (3) in contrast to Eq. (1) for the standard coupled mode equations enables a significant reduction in the number of longitudinal points.

Next, we use the phase-matched model to study the refractive index of the fiber amplifier. Figure 1(c) shows the real value of the first positive harmonic term in the changes of the refractive index,  $\text{Real}[\Delta n_+ \exp(i\Delta\beta z)]$ , as a function of the transverse  $x$  position and the longitudinal  $z$  position. The inset in Fig. 1(c) shows a magnified plot between 1.2 and 1.3 m so that the refractive index grating may be easily seen. Figure 1(d) shows the 0<sup>th</sup> order changes of the index term,  $\Delta n_0$ , in which the refractive index grating is visible. Figure 1(e) shows the changes in refractive index from the full model. The inset in Fig. 1(e) shows a magnified plot between 1.2 and 1.3 m so that the grating in the refractive index may be clearly seen. The oscillations in the refractive index that are present in the full model may be captured by only considering the sum of 0<sup>th</sup> order  $\Delta n_0$  and first order harmonic tones  $\Delta n_{+-}$ , according to Eq. (2). It has also been shown that both the phase-matched model and full model yield the same power threshold when modeling TMI [5].

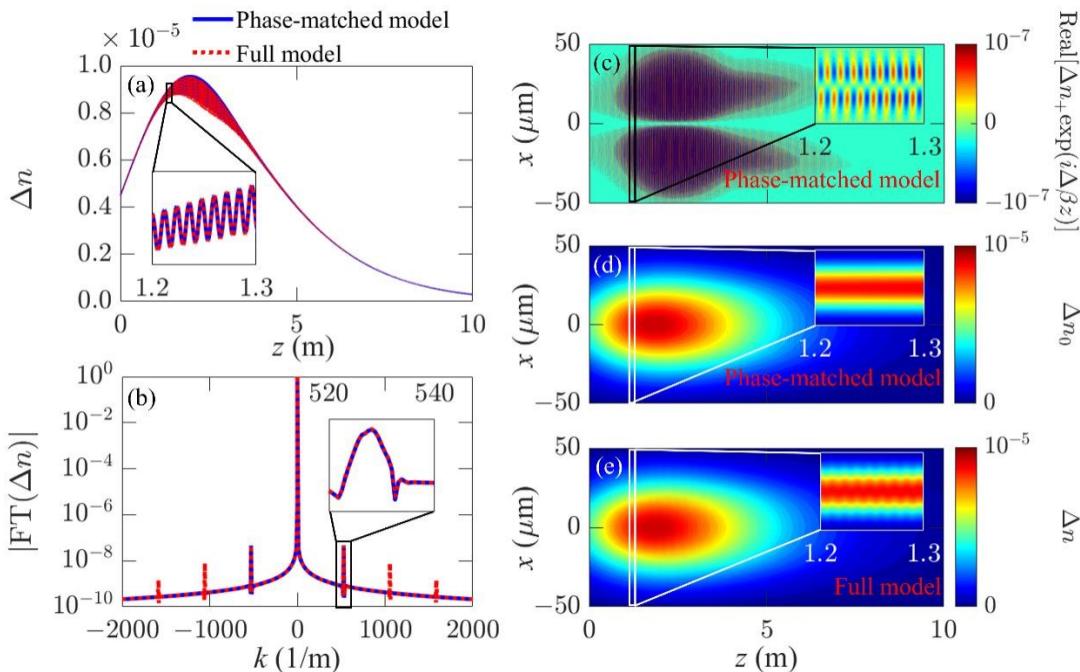


Fig. 1. (a)  $\Delta n$  as a function of  $z$ . (b) Fourier transform of  $\Delta n$  as a function of  $k$ . (c)  $\text{Real}[\Delta n_+ \exp(i\Delta\beta z)]$ , (d)  $\Delta n_0$ , from the phase-matched model, and (e)  $\Delta n$  from the full model, as a function of the transverse  $x$  position and the longitudinal  $z$  position .

In conclusion, we compared the refractive index predicted by the full model and phase-matched model with only the phase-matched terms. Both models predicted almost the same refractive index. The phase-matched model leads to a large computational speedup since the number of longitudinal points can be greatly reduced.

## References

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