**Proceedings of the ASME 2022
International Mechanical Engineering Congress and Exposition
IMECE2022
October 30-November 3, 2022, Columbus, Ohio**

IMECE2022-94489

DYNAMICS OF LAMINAR-TO-TURBULENT TRANSITION IN A WALL-BOUNDED CHANNEL FLOW UP TO RE = 40,000

Mohsin Al Barwani
University of Nebraska-Lincoln, Lincoln, NE

Jae Sung Park*
University of Nebraska-Lincoln, Lincoln, NE

ABSTRACT

The transition from laminar to turbulent flow is of great interest since it is one of the most difficult and unsolved problems in fluids engineering. The transition processes are significantly important because the transition has a huge impact on almost all systems that come in contact with a fluid flow by altering the mixing, transport, and drag properties of fluids even in simple pipe and channel flows. Generally, in most transportation systems, the transition to turbulence causes a significant increase in drag force, energy consumption, and, therefore, operating cost. Thus, understanding the underlying mechanisms of the laminar-to-turbulent transition can be a major benefit in many ways, especially economically. There have been substantial previous studies that focused on testing the stability of laminar flow and finding the critical amplitudes of disturbances necessary to trigger the transition in various wall-bounded systems, including circular pipes and square ducts. However, there is still no fundamental theory of transition to predict the onset of turbulence. In this study, we perform direct numerical simulations (DNS) of the transition flows from laminar to turbulence in a channel flow. Specifically, the effects of different magnitudes of perturbations on the onset of turbulence are investigated. The perturbation magnitudes vary from 0.001 (0.1%) to 0.05 (5%) of a typical turbulent velocity field, and the Reynolds number is from 5,000 to 40,000. Most importantly, the transition behavior in this study was found to be in good agreement with other reported studies performed for fluid flow in pipes and ducts. With the DNS results, a finite amplitude stability curve was obtained. The critical magnitude of perturbation required to cause transition was observed to be inversely proportional to the Reynolds number for the magnitude from 0.01 to 0.05. We also investigated the temporal behavior of the transition process, and it was found that the transition time or the time required to begin the transition process is inversely correlated with the Reynolds number only for the magnitude from 0.02 to 0.05, while different temporal behavior occurs for smaller perturbation magnitudes. In addition to the transition time, the transition dynamics were investigated by observing the time series of wall shear stress. At the onset of transition, the shear stress experiences an overshoot, then decreases toward sustained turbulence. As expected, the average values of the wall shear stress in turbulent flow increase with the Reynolds number. The change in the wall shear stress from laminar to overshoot was, of course, found to increase with the Reynolds number. More interestingly was the observed change in wall shear stress from the overshoot to turbulence. The change in magnitude appears to be almost insensitive to the Reynolds number and the perturbation magnitude. Because the change in wall shear stress is directly proportional to the pumping power, these observations could be extremely useful when determining the required pumping power in certain flow conditions. Furthermore, the stability curve and wall shear stress changes can be considered robust features for future applications, and ultimately interpreted as evidence of progress toward solving the unresolved fluids engineering problem.

Keywords: Laminar-to-turbulent transition, direct numerical simulation, channel flow

1. INTRODUCTION

Since Reynolds' phenomenal experiments in 1883, the laminar-to-turbulent transition in wall-bounded flows (i.e., pipe,

*Corresponding author: jaesung.park@unl.edu

channel, or boundary layer) has intrigued many scientists and engineers over the past 140 years [1]. The principal issue is that laminar flow subjected to a small finite perturbation eventually becomes turbulent in practice despite its linear stability in theory [2–4]. It has been observed in pipe flow that the turbulent flow arises due to separate patches emerging across the laminar flow. The transition seems to occur only when these patches invade the laminar flow. This is quite unexpected since one might expect the flow to turn turbulent at once rather than separating into distinct patches. These observations led to more detailed investigations to understand the mechanism of the turbulent transition process [5, 6].

Most of the previous studies have investigated the laminar-to-turbulent transition in circular ducts or pipes because of its extensive applications. Thus, we begin by discussing the characteristics of the transition process in pipe flows. Reynolds found that the onset of turbulence directly depends on the dimensionless parameter called the Reynolds number (*Re*). This parameter represents the ratio of inertia force to viscous force. He observed that the transition process in pipe flows naturally occurs at $Re \approx 2300$. However, he also reported that laminar flow could be maintained up to $Re \approx 13,000$ [1]. Other studies afterwards managed to achieve laminar flows up to $Re \approx 100,000$ [7]. A laminar state can be maintained at such high Reynolds numbers only by minimizing external factors, such as heat transfer, surface roughness, vibration, noise, and disturbances. Laminar flow can also be achieved at such high Reynolds numbers by allowing settling times until the disturbance decays [8]. In related studies, the effect of inlet disturbances on the transition was investigated both experimentally and numerically to understand the mechanism of the transition process. Darbyshire and Mullin experimentally introduced inlet disturbance via injection and suction to determine a critical magnitude of the disturbance required to cause the turbulent transition in pipe flows. The transition process was found to be directly dependent on both the Reynolds number and the magnitude of disturbance [9]. This study showed that laminar flow is very sensitive to perturbation magnitudes. The subsequent study showed that the physical length of the perturbation flow field also played a role in the transition [10]. These findings allowed Mullin and Peixinho to experimentally uncover a scaling law, indicating that the critical perturbation magnitude required to trigger the laminar-to-turbulent transition scales as $1/Re$ for a limited range of Reynolds numbers [11]. The flow is found to be globally stable below this range, while it is very sensitive to inlet disturbance above the critical magnitude. It is also numerically observed for pipe flows [12].

There are two types of localized states, which are puffs (transition) and slugs (turbulent) [13]. Puffs are formed due to large perturbation magnitudes, while slugs are formed due to unstable boundary layers. Puffs are observed right below the critical or transitional Reynolds number, while slugs could be observed at higher Reynolds numbers [9]. For pipe flows, puffs are found to replicate without any changes at the critical Reynolds number [14–17], which are known as equilibrium puffs [13]. Recently, the puffs and slugs were investigated numerically for square ducts, showing characteristics similar to pipe flows [18].

For channel flows, the transition appears naturally at $Re \approx 5,772$ [19, 20]. It has been reported that the forming mechanism of the localized turbulent patches is similar to that of circular and rectangular ducts [21, 22]. Interestingly, although the Reynolds number of the channel flow is much smaller than the critical Reynolds number ($Re \approx 5,772$), the localized turbulent patches generated by the inlet disturbance could grow separately and spread into extended spatial regions, leading to the global instability [23]. However, despite the great efforts of the previous studies, the transition dynamics in a wall-bounded channel flow have not yet been fully characterized.

In this study, we perform a direct numerical simulation (DNS) to study the transition process in a wall-bounded channel flow. We aim to uncover a scaling law, such as $1/Re$, for wall-bounded channel flows by identifying the critical perturbation magnitude required to cause the turbulent transition for a certain range of the Reynolds number. We will also address the temporal and dynamical behaviors of the transition process. The paper is organized as follows: the problem formulation, results and discussion, and conclusions.

2. PROBLEM FORMULATION

We consider an incompressible Newtonian fluid flow. The flow geometry is a wall-bounded channel domain. The x, y and z coordinates are aligned with the streamwise, wall-normal and spanwise directions, respectively. A no-slip boundary condition is applied at both top and bottom walls, and periodic boundary conditions are applied in the x and z directions. The periods are same as the box dimensions in the corresponding directions, i.e. L_x and L_z respectively. Half-channel height, i.e., $L = L_y$, is chosen as the length scale for the non-dimensionalization of all lengths in the geometry. The velocity scale is the laminar centerline velocity $U = 3/2U_b$, where U_b is the bulk velocity in the laminar flow. We applied a constant U_b equal to $2/3U$. Time t is scaled with L/U, and pressure p with ρU^2, where ρ is the fluid density. The non-dimensional governing equations are given as:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}. \tag{2}$$

Here, the Reynolds number $Re = UL/\nu$, where ν is the kinematic viscosity of the fluid.

Simulations were performed using the open source-code ChannelFlow, a spectral code for an incompressible Navier-Stokes flow in channel geometries, which is written and maintained by John Gibson [24]. We focus on the domain $L_x \times L_y \times L_z$ = $2\pi \times 2 \times \pi$. A numerical grid system was generated on $N_x \times N_y \times N_z$ (in x, y, and z) meshes, where a Fourier-Chebyshev-Fourier spectral spatial discretization was applied to all variables. For a mesh convergence study, a typical resolution used was (N_x, N_y, N_z) = (64, 81, 76). A constant time step Δt was chosen to satisfy the Courant-Friedrichs-Lewy (CFL) stability condition. A typical $\Delta t = 0.01$. Simulations have been successfully performed for low Reynolds numbers as well as high Reynolds numbers [25–27]. In this study, we introduce different magnitudes of a perturbation field (random field) to laminar flow (parabolic profile) and then use it as an initial flow field. This divergence-free random field was readily generated by ChannelFlow. The perturbation magnitudes ranged from 0.001 (0.1%) to 0.05 (5%) of a typical turbulent velocity field. Reynolds number was considered from $3,000$ to $40,000$.

3. RESULTS AND DISCUSSION

We first consider the temporal and dynamical behaviors of the transition process based on the wall shear stress. Fig. 1(a) shows the temporal evolution of the area-averaged wall shear stress τ_w at Reynolds numbers from 5,000 to 40,000, where the perturbation magnitude of 0.02 was applied to the laminar flow ($\tau_w = 2$). The inset zooms in at the beginning of the process. In an early stage of the process, flows tend to stay near the laminar state. Because of the strong perturbation magnitude, the flows then start to experience a transition as the wall shear stress sharply increases and then reaches an overshoot, which is usually called a strong burst followed by decreasing toward sustained turbulence [25]. It is clearly seen that the transition dynamics depend on the Reynolds number, where the change in the wall shear stress from laminar to overshoot increases with the Reynolds number. Prior to investigating the dependence of the transition process on the Reynolds number, we investigated the dependency of the wall shear stress of sustained turbulence on the Reynolds number. Fig. 1(b) shows the time-averaged values of the wall shear stress during sustained turbulent flows (i.e. $t > 2,000$) as a function of the Reynolds number. The turbulent wall shear stress $\tau_{w,turb}$ increases with the Reynolds number, which is quantatively in good agreement with the previous simulation results [26].

To investigate the overshoot or strong burst during the transition process, Fig. 2(a) shows the burst wall shear stress, $\tau_{w,burst}$, which is the change in the wall shear stress from laminar to overshoot, as a function of the Reynolds number for various perturbation magnitudes. The change in the wall shear stress from laminar to overshoot is found to increase with the Reynolds number. Interestingly, the burst wall shear stress values appear to be independent of the perturbation magnitude. To better illustrate this intriguing behavior, Fig. 2(b) shows the burst wall shear stress $\tau_{w,burst}$ as a function of the perturbation magnitude for various

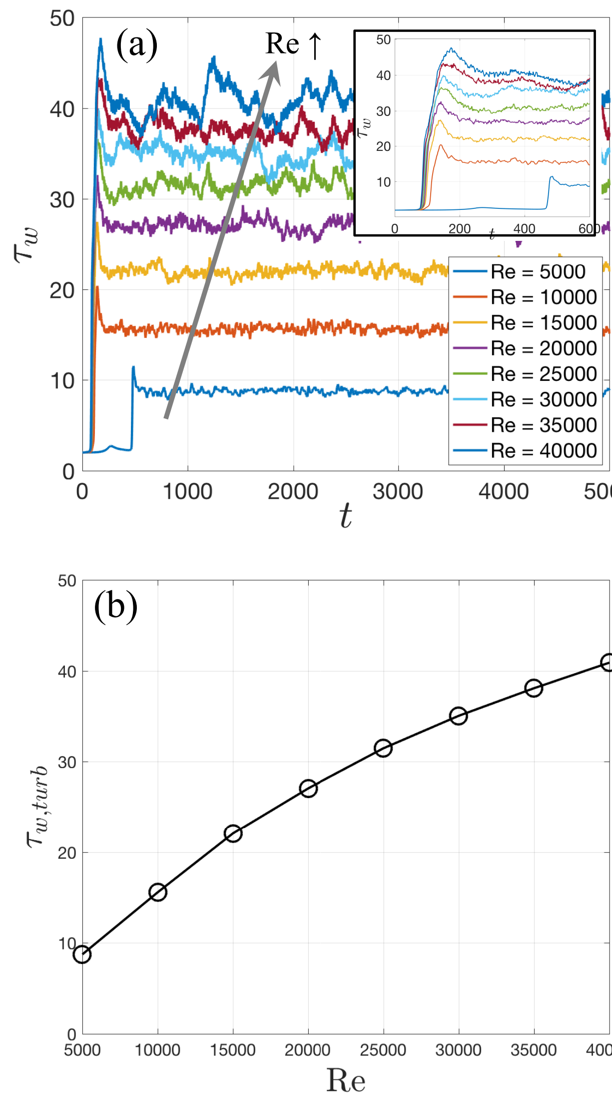

FIGURE 1. (a) Time evolution of the wall shear stress τ_w when introducing the perturbation magnitude of 0.02 to the laminar flow ($\tau_w = 2$) at different Reynolds numbers ranging from 5,000 to 40,000. The inset zooms in at the beginning of the transition process. (b) The time-average values of the wall shear stress during turbulent flows $\tau_{w,turb}$ (i.e., for $t > 2,000$ in (a)) at different Reynolds numbers.

Reynolds numbers. It is clearly observed that the burst wall shear stress is almost insensitive to the perturbation magnitude.

More interestingly, we find another intriguing behavior of the wall shear stress from the overshoot to turbulence. Fig. 3 shows the difference between the burst wall shear stress $\tau_{w,burst}$ and the turbulent wall shear stress $\tau_{w,turb}$ as a function of the Reynolds number for various perturbation magnitudes. The difference $\tau_{w,burst} - \tau_{w,turb}$ seems to be almost insensitive to both

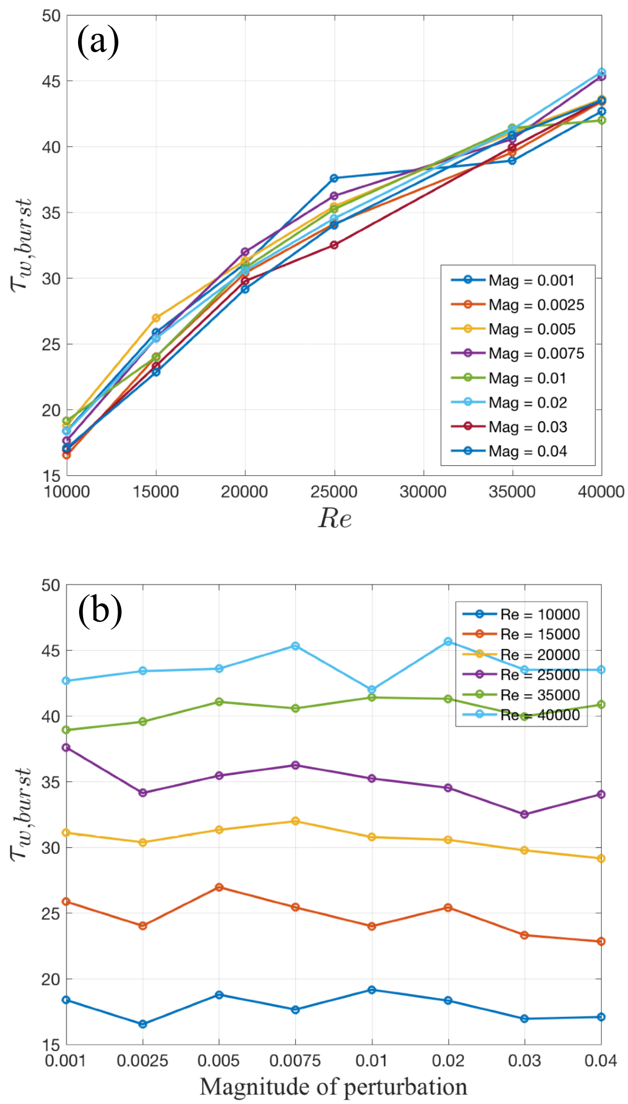

FIGURE 2. The burst wall-shear stress, $\tau_{w,burst}$, is the change in the wall shear stress from laminar to overshoot. (a) $\tau_{w,burst}$ as a function of the Reynolds number for various perturbation magnitudes and (b) $\tau_{w,burst}$ as a function of the perturbation magnitude for various Reynolds numbers.

the Reynolds number and the perturbation magnitude. Because the change in the wall shear stress is directly proportional to the pumping power of the flow system, these observations could be useful when determining the required pumping power in fluid flow systems involving the laminar-to-turbulent transition.

To provide a deeper understanding of the transition process, we investigated the transition time as a function of the Reynolds number and the perturbation magnitude. The transition time was defined as a time when the wall shear stress becomes 10% larger than the laminar value, i.e. when $\tau_w > 2.2$. The choice of the cutoff wall shear stress for the transition time was tested with 5% and 15%, giving an almost identical trend. Fig. 4(a) shows the transition time for magnitudes from 0.001 to 0.01. It was found that the transition time in this magnitude range behaves inconsistently as the Reynolds number increases; particularly for very small magnitudes, such as 0.001 and 0.0025. This could result from the fact that the perturbation magnitudes are too small to cause a consistent transition trend to the laminar state at high Reynolds numbers. Fig. 4(b) shows the transition time for magnitudes from 0.02 to 0.05, where there is a clear, consistent trend as the Reynolds number increases. The transition time in this magnitude range monotonically decreases as the Reynolds number. In addition, the transition time also decreases with increasing the perturbation magnitude. It is expected as the laminar-to-turbulent transition is very sensitive to the perturbation magnitude.

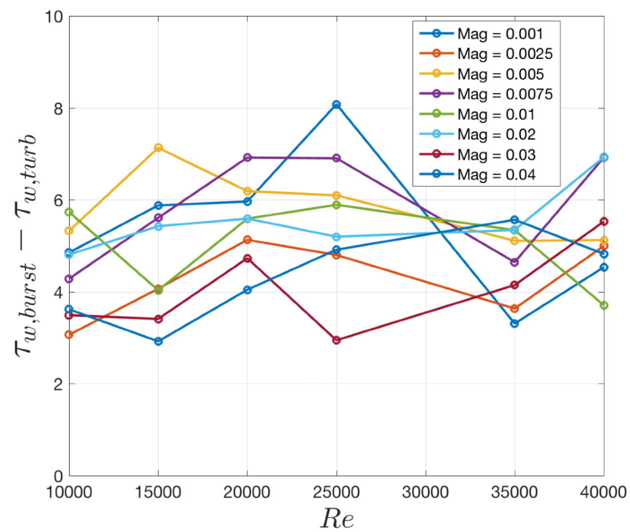

FIGURE 3. The difference between the burst wall shear stress $\tau_{w,burst}$ and the turbulent wall shear stress $\tau_{w,turb}$ as a function of the Reynolds number for various perturbation magnitudes.

Lastly and most importantly, Fig. 5 shows the finite-amplitude stability curve for a wall-bounded channel flow, which suggests the critical perturbation magnitude at a given Reynolds number. The critical magnitude is the minimum perturbation magnitude required to cause the laminar-to-turbulent transition. These results were obtained by increasing the perturbation magnitude at a fixed Reynolds number until the transition was observed. It is found that the critical magnitude is inversely correlated to the Reynolds number up to $Re = 10,000$, while all the flows at higher Reynolds numbers exhibit the transition even with the smallest magnitude (0.001) considered in this study. To uncover a scaling law, the inset shows the log-log scale of the

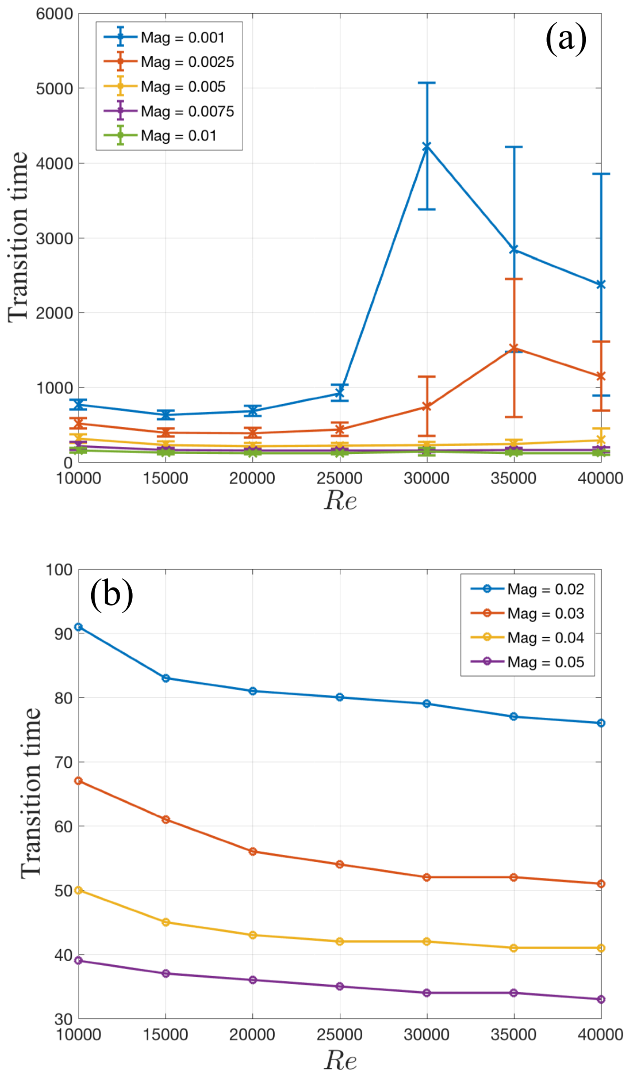

FIGURE 4. Transition time as a function of Reynolds number for the perturbation magnitude (a) from 0.001 to 0.01 and (b) from 0.02 to 0.05.

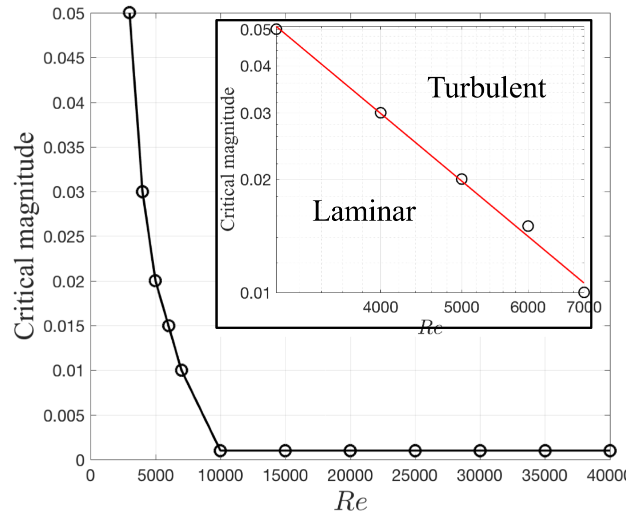

FIGURE 5. The critical perturbation magnitude required to trigger the laminar-to-turbulent transition at a given Reynolds number. The inset is for the range of the Reynolds number from 3,000 to 7,000 on the log-log scale to clearly show the scaling of $1/Re$. The laminar region lies below the red line, while the turbulent region lies above.

critical magnitude in the range of the Reynolds number from $Re = 3,000$ to $Re = 7,000$. It is clearly observed that the critical perturbation magnitude scales as $1/Re$ for this range, which is the same scaling law of $1/Re$ for a typical pipe flow [9]. This scaling law can introduce a clear stability line that separates the laminar and turbulent regions for this range of Reynolds number. This stability curve can also be considered a robust feature for future applications.

4. CONCLUSION

Direct numerical simulation of the laminar-to-turbulent transition in a wall-bounded channel flow has been carried out. The transition to turbulence was simulated by introducing a finite magnitude of perturbation velocity fields to the laminar flow and using it as an initial flow field. We consider the magnitude ranging from 0.001 (0.1%) to 0.05 (5%) of a typical turbulent velocity field and the Reynolds number up to 40,000. For the transition dynamics, it was observed that the burst wall shear stress, which is the increase in the wall shear stress from laminar to overshoot, is directly proportional to the Reynolds number but insensitive to the perturbation magnitude. More intriguingly, the change in the wall shear stress from the overshoot to turbulence is insensitive to both the Reynolds number and perturbation magnitude. Considering the direct relationship between the change in the wall shear stress and the pumping power for the flow system, these observations could be useful for determining the pumping power in fluid flow systems involving the transition. Upon further investigation of the temporal behavior of the transition, other interesting findings were observed. First, the transition time is inversely correlated with the Reynolds number only for the magnitude from 0.02 to 0.05, while inconsistent temporal behavior occurs for smaller perturbation magnitudes. Secondly and most importantly, the finite-amplitude stability curve for a channel flow was shown to uncover the scaling law for a wall-bounded channel flow, where the critical magnitude scales as $1/Re$ for $3,000 < Re < 10,000$. Therefore, there is a clear demarcation between the laminar and turbulent regions. The stability curve and wall shear stress changes can be considered robust features and ultimately interpreted as evidence of progress toward better understanding the underlying mechanisms of the transition process in a wall-bounded channel flow.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support from the University of Nebraska-Lincoln UCARE (Undergraduate Creative Activities & Research Experiences) program, the National Science Foundation (OIA-1832976 and CBET-2142916), the Nebraska EPSCoR FIRST Award grant, and the Collaboration Initiative at the University of Nebraska. The direct numerical simulation code used here was developed and distributed by John Gibson at the University of New Hampshire. This work was completed utilizing the Holland Computing Center of the University of Nebraska, which receives support from the Nebraska Research Initiative.

REFERENCES

[1] Reynolds, O., 1883. "Xxix. an experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels". *Philosophical Transactions of the Royal Society of London*(174), pp. 935–982.

[2] Grossmann, S., 2000. "The onset of shear flow turbulence". *Reviews of Modern Physics,* ***72***(2), p. 603.

[3] Manneville, P., 2015. "On the transition to turbulence of wall-bounded flows in general, and plane couette flow in particular". *European Journal of Mechanics-B/Fluids,* ***49***, pp. 345–362.

[4] Draad, A. A., Kuiken, G., and Nieuwstadt, F., 1998. "Laminar–turbulent transition in pipe flow for newtonian and non-newtonian fluids". *Journal of Fluid Mechanics,* ***377***, pp. 267–312.

[5] Mullin, T., 2011. "Experimental studies of transition to turbulence in a pipe". *Annual Review of Fluid Mechanics,* ***43***, pp. 1–24.

[6] Eckhardt, B., Schneider, T. M., Hof, B., and Westerweel, J., 2007. "Turbulence transition in pipe flow". *Annual Review of Fluid Mechanics,* ***39***, pp. 447–468.

[7] Pfeninger, W., 1961. "Transition experiments in the inlet length of tubes at high reynolds numbers". *Boundary Layer and Flow Control*, pp. 970–980.

[8] Warhaft, Z., 1997. *An introduction to thermal-fluid engineering: the engine and the atmosphere*. Cambridge University Press.

[9] Darbyshire, A., and Mullin, T., 1995. "Transition to turbulence in constant-mass-flux pipe flow". *Journal of Fluid Mechanics,* ***289***, pp. 83–114.

[10] Hof, B., De Lozar, A., Kuik, D. J., and Westerweel, J., 2008. "Repeller or attractor? selecting the dynamical model for the onset of turbulence in pipe flow". *Physical Review Letters,* ***101***(21), p. 214501.

[11] Peixinho, J., and Mullin, T., 2006. "Decay of turbulence in pipe flow". *Physical Review Letters,* ***96***(9), p. 094501.

[12] Wu, X., Moin, P., Adrian, R. J., and Baltzer, J. R., 2015. "Osborne reynolds pipe flow: Direct simulation from laminar through gradual transition to fully developed turbulence". *Proceedings of the National Academy of Sciences,* ***112***(26), pp. 7920–7924.

[13] Wygnanski, I. J., and Champagne, F., 1973. "On transition in a pipe. part 1. the origin of puffs and slugs and the flow in a turbulent slug". *Journal of Fluid Mechanics,* ***59***(2), pp. 281–335.

[14] Nishi, M., Ünsal, B., Durst, F., and Biswas, G., 2008. "Laminar-to-turbulent transition of pipe flows through puffs and slugs". *Journal of Fluid Mechanics,* ***614***, pp. 425–446.

[15] Wygnanski, I., Sokolov, M., and Friedman, D., 1975. "On transition in a pipe. part 2. the equilibrium puff". *Journal of Fluid Mechanics,* ***69***(2), pp. 283–304.

[16] Bandyopadhyay, P. R., 1986. "Aspects of the equilibrium puff in transitional pipe flow". *Journal of Fluid Mechanics,* ***163***, pp. 439–458.

[17] Priymak, V., and Miyazaki, T., 2004. "Direct numerical simulation of equilibrium spatially localized structures in pipe flow". *Physics of Fluids,* ***16***(12), pp. 4221–4234.

[18] Khan, H. H., Anwer, S. F., Hasan, N., and Sanghi, S., 2021. "Laminar to turbulent transition in a finite length square duct subjected to inlet disturbance". *Physics of Fluids,* ***33***(6), p. 065128.

[19] Sano, M., and Tamai, K., 2016. "A universal transition to turbulence in channel flow". *Nature Physics,* ***12***(3), pp. 249–253.

[20] Orszag, S. A., 1971. "Accurate solution of the orr–sommerfeld stability equation". *Journal of Fluid Mechanics,* ***50***(4), pp. 689–703.

[21] Carlson, D. R., Widnall, S. E., and Peeters, M. F., 1982. "A flow-visualization study of transition in plane poiseuille flow". *Journal of Fluid Mechanics,* ***121***, pp. 487–505.

[22] Xiong, X., Tao, J., Chen, S., and Brandt, L., 2015. "Turbulent bands in plane-poiseuille flow at moderate reynolds numbers". *Physics of Fluids,* ***27***(4), p. 041702.

[23] He, S., and Seddighi, M., 2013. "Turbulence in transient channel flow". *Journal of Fluid Mechanics,* ***715***, pp. 60–102.

[24] Gibson, J. F., 2014. Channelflow: A spectral Navier-Stokes simulator in C++. Tech. rep., U. New Hampshire. `Channelflow.org`.

[25] Davis, E. A., and Park, J. S., 2020. "Dynamics of laminar and transitional flows over slip surfaces: effects on the laminar–turbulent separatrix". *Journal of Fluid Mechanics,* ***894***.

[26] Davis, E. A., Mirfendereski, S., and Park, J. S., 2021. "On the comparison of flow physics between minimal and extended flow units in turbulent channels". *Fluids,* ***6***(5), p. 192.
[27] Rogge, A. J., and Park, J. S., 2022. "On the underlying drag-reduction mechanisms of flow-control strategies in a transitional channel flow: temporal approach". *Flow, Turbulence and Combustion,* ***108***, pp. 1001–1016.

