Highly Anisotropic Even-Denominator Fractional Quantum Hall State in an Orbitally Coupled Half-Filled Landau Level

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The even-denominator fractional quantum Hall states (FQHSs) in half-filled Landau levels are generally believed to host non-Abelian quasiparticles and be of potential use in topological quantum computing. Of particular interest is the competition and interplay between the even-denominator FQHSs and other ground states, such as anisotropic phases and composite fermion Fermi seas. Here, we report the observation of an even-denominator fractional quantum Hall state with highly anisotropic in-plane transport coefficients at Landau level filling factor $\nu = 3/2$. We observe this state in an ultra-high-quality GaAs two-dimensional hole system when a large in-plane magnetic field is applied. By increasing the in-plane field, we observe a sharp transition from an isotropic composite fermion Fermi sea to an anisotropic even-denominator FQHS. Our data and calculations suggest that a unique feature of two-dimensional holes, namely the coupling between heavy-hole and light-hole states, combines different orbital components in the wave function of one Landau level, and leads to the emergence of a highly anisotropic even-denominator fractional quantum Hall state. Our results demonstrate that the GaAs two-dimensional hole system is a unique platform for the exploration of exotic, many-body ground states.

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When a strong perpendicular magnetic field ($B_\perp$) is applied to a two-dimensional electron system (2DES) at low temperatures, the thermal and kinetic energies are quenched as the electrons occupy the quantized Landau levels (LLs), and various many-body ground states emerge as the Coulomb energy dominates. Half-filled LLs are particularly interesting as they host a plethora of emergent, exotic quantum phases, including compressible composite fermion (CF) Fermi seas in the lowest orbital energy ($N = 0$) LLs, e.g., at Landau level filling factors $\nu = 1/2$ and $3/2$ in GaAs 2DESs [1,2], and even-denominator fractional quantum Hall states (FQHSs) in the first-excited ($N = 1$) LLs, e.g., at $\nu = 5/2$ and $7/2$ [3,4]. The even-denominator FQHSs have been of considerable, continued attention because they are generally expected to be Pfaffian states with non-Abelian statistics [5–8]. FQHSs have also been reported at $\nu = 3/2$ in different 2DESs. It was first observed in a wide GaAs quantum well with a bilayerlike charge distribution when the Fermi level (E_F) lies in the $N = 0$ LL [9]. More recently, $\nu = 3/2$ FQHSs were reported in different materials, such as ZnO [10], AlAs [11], WSe_2 [12], and bilayer graphene [13–15], when E_F is in an $N = 1$ LL. In contrast to these incompressible FQHSs in the $N = 0$ and $N = 1$ LLs, anisotropic phases emerge in the $N \geq 2$ LLs as the ground states at half fillings [16–24]. These are generally believed to be stripe phases which, at finite temperatures and in the presence of ubiquitous disorder, turn into nematic phases.

As highlighted in Fig. 1, here, we report a $B_\parallel$-induced, highly anisotropic, even-denominator FQHS at $\nu = 3/2$ in an ultra-high-mobility 2D hole system (2DHS) confined to a GaAs quantum well [25–28]. In a purely $B_\perp$, our 2DHS is nearly isotropic in the vicinity of $\nu = 3/2$, and its in-plane longitudinal magnetoresistance traces (R_{xx} and R_{yy}) exhibit smooth and shallow minima at $\nu = 3/2$, flanked by the standard, odd-denominator FQHSs at $\nu = 1 + p/(2p \pm 1)$, such as $\nu = 4/3, 7/5, 10/7, \ldots$, and $5/3, 8/5, 11/7, \ldots$ [Fig. 1(a)]. This is consistent with a compressible (Fermi sea) ground state of CFs at $\nu = 3/2$, similar to what has been seen previously in very high-mobility GaAs 2DESs [1,2,45,46] and 2DHSs [25,47]. When we tilt the sample in magnetic field to induce a large $B_\parallel$ component [Figs. 1(b) and 1(c)], our 2DHS breaks its rotational symmetry and becomes highly anisotropic near $\nu = 3/2$, with $R_{xx}/R_{yy} \simeq 10^4$, and yet shows deep minima in both R_{xx} and R_{yy} at $\nu = 3/2$ and $R_{xy} \simeq (2/3)(e^2/h)$ [28]. Note that R_{xx} exceeds 10 kΩ near $\nu = 3/2$, whereas $R_{yy} \lesssim 1$ Ω.

Figures 1(b) and 1(c) traces are truly intriguing. On the one hand, the deep minima in both R_{xx} and R_{yy} at $\nu = 3/2$, and an $R_{xy} \simeq (2/3)(e^2/h)$ are consistent with a developing FQHS at this filling. On the other hand, the enormous transport anisotropy near $\nu = 3/2$ is reminiscent of the $B_\parallel$-induced anisotropic states that are observed in GaAs 2DESs in the $N = 1$ LL, e.g., at $\nu = 5/2$, when the sample is tilted to a large angle θ [48–53]. Note, however, that the

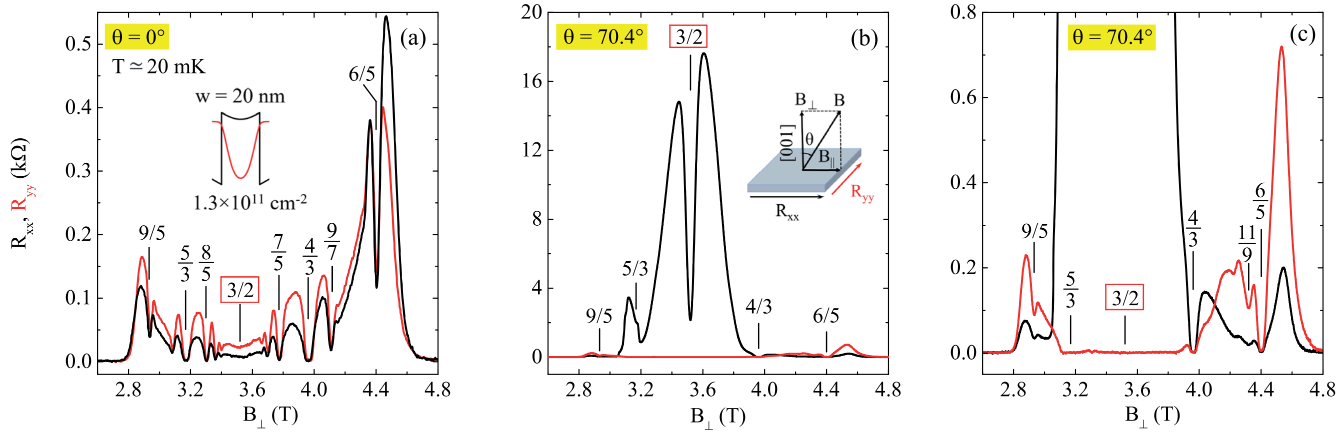

FIG. 1. Longitudinal resistances (R_{xx} in black, and R_{yy} in red) vs perpendicular magnetic field $B_\perp$ at tilt angles: (a) $\theta = 0°$ and (b) $\theta = 70.4°$. (c) Enlarged version of (b). Inset in (a): The self-consistently calculated hole charge distribution (red) and potential (black). We emphasize that our 2DHS is confined to a narrow quantum well and has a single-layer charge distribution. Inset in (b): A schematic of the experimental setup. The sample is mounted on a rotating stage to support *in situ* tilt; θ is the angle between B and $B_\perp$, and R_{xx} and R_{yy} denote resistances measured along and perpendicular to $B_\parallel$.

anisotropic states observed at $\nu = 5/2$ at large θ replace the FQHS which is present at small θ [48–53]. In contrast, in our 2DHS, at $\nu = 3/2$, an even-denominator FQHS with highly anisotropic transport coefficients replaces a compressible (CF Fermi sea) ground state at small θ [54–57]. Also remarkable is the presence of numerous FQHSs at odd-denominator filings such as $4/3$, $6/5$, $9/5$, and $11/9$ in Fig. 1(c) at large θ. These FQHSs are generally seen in 2DESs in an $N = 0$ LL [58–60]. The even-denominator FQHS at $\nu = 3/2$ and its anisotropic behavior, however, are telltale signs that E_F lies in an $N > 0$ LL. In this Letter, we present the evolution of this very unusual, anisotropic FQHS at $\nu = 3/2$ with temperature and $B_\parallel$. Our data and calculations suggest that a unique feature of 2D holes, namely, the coupling between heavy-hole (HH) and light-hole (LH) states, combines $N = 0$ and $N > 0$ components in the wave function of one LL, and leads to the emergence of the anisotropic FQHS at $\nu = 3/2$.

The ultra-high-quality 2DHS studied here is confined to a 20-nm-wide GaAs quantum well grown on a GaAs (001) substrate. The 2DHS has a density of 1.3×10^{11} cm^{-2} and a low-temperature (0.3 K) record-high mobility of 5.8×10^6 cm^2/Vs [25]. We performed our experiments on a 4×4 mm^2 van der Pauw geometry sample, with alloyed In:Zn contacts at the four corners and side midpoints. The sample was then mounted on a rotating stage and cooled in a dilution refrigerator. We measured the longitudinal resistances, R_{xx} and R_{yy}, along and perpendicular to $B_\parallel$ [see Fig. 1(b) inset], using the lock-in amplifier technique.

In Figs. 2(a) and 2(b), we show R_{xx} and R_{yy} vs $B_\perp$ traces taken at $\theta \simeq 70°$ at different temperatures. With decreasing temperature, R_{xx} near $\nu = 3/2$ grows whereas R_{yy} becomes

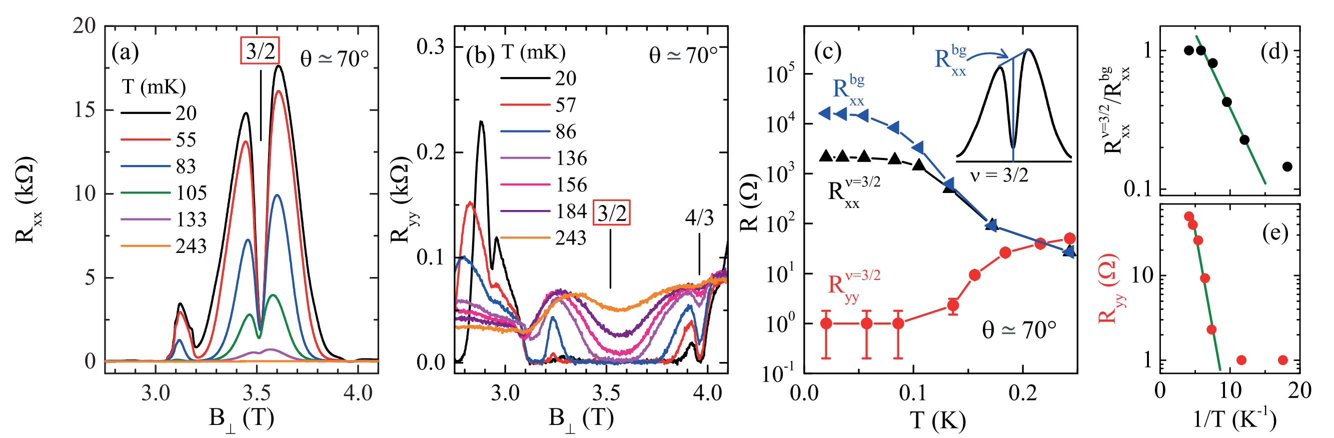

FIG. 2. Temperature dependence of R_{xx} and R_{yy}. (a),(b) R_{xx} and R_{yy} vs $B_\perp$ traces near $\nu = 3/2$ taken at $\theta \simeq 70°$ and at different temperatures. (c) $R_{xx}^{\nu=3/2}$, $R_{yy}^{\nu=3/2}$, and R_{xx}^{bg} vs T. Inset shows an R_{xx} vs $B_\perp$ trace near $\nu = 3/2$, showing how R_{xx}^{bg} at $\nu = 3/2$ is defined. (d), (e) Arrhenius plots of $R_{xx}^{\nu=3/2}/R_{xx}^{bg}$ and R_{yy} vs $1/T$. The lines are linear fits to the data at intermediate temperatures, and their slopes give energy gaps $\simeq$0.5 and $\simeq$2.1 K for (d) and (e), respectively.

smaller and approaches zero. Figure 2(c) presents $R_{xx}^{\nu=3/2}$, $R_{yy}^{\nu=3/2}$, and the background resistance R_{xx}^{bg} [61] on the flanks of $\nu = 3/2$ vs T. The data show that the 2DHS is highly anisotropic at low temperatures but becomes nearly isotropic at $T \gtrsim 0.2$ K. To quantify the strength of the $\nu = 3/2$ resistance minimum and extract an estimate for the FQHS energy gap, we make Arrhenius plots of R_{yy} and the ratio $R_{xx}^{\nu=3/2}/R_{xx}^{bg}$ as a function of $1/T$ [Figs. 2(d) and 2(e)]. Note that, at $\nu = 3/2$, R_{xx} does not decrease as temperature is lowered, likely because of the strong rise in R_{xx} on the flanks of $\nu = 3/2$. Therefore, we plot $R_{xx}^{\nu=3/2}/R_{xx}^{bg}$ to obtain an estimate for the $\nu = 3/2$ FQHS energy gap [62]. The lines in Figs. 2(d) and 2(e) are linear fits to the data points at intermediate temperatures, and their slopes give an estimate of the $\nu = 3/2$ FQHS gap, ~1 K.

Measuring the Hall resistance, R_{xy}, for a highly anisotropic 2D system is very challenging and seldom reported [11,63]. Near $\nu = 3/2$ our 2DHS becomes very resistive along the hard-axis direction ($R_{xx} \simeq 20$ kΩ), which inevitably induces nonuniformities in the current distribution in the sample and can also cause severe R_{xx} mixing to R_{xy}. The effect of R_{xx} mixing can be minimized by averaging R_{xy} taken for opposite polarities of B [64,65]. We did such averaging and, although we do not see a quantized Hall plateau at $\nu = 3/2$, we find that R_{xy} approaches $(2/3)(h/e^2)$ to within 1%; see Supplemental Material (SM) [28] for details. The fact that R_{xy} does not exhibit a well-developed plateau is consistent with the deep, but nonzero, R_{xx} minimum.

Now, we turn to the evolution of the different phases at and near $\nu = 3/2$ with tilt angle θ. Figure 3 shows R_{xx} vs ν traces at different θ [66]. With increasing θ, the FQHSs go through transitions, starting at the lowest ν (highest $B_\perp$). For example, the R_{xx} minimum at $\nu = 9/7$ turns into a maximum at $\theta = 65.2°$ and disappears at higher θ. At larger θ, R_{xx} rises very abruptly above a $B_\perp$ which depends on θ (see Fig. 3 inset), and attains very high values in a range of fields near $\nu = 3/2$ (main Fig. 3). The ground state at $\nu = 3/2$ exhibits an abrupt transition from a CF Fermi sea at $\theta \leq 65.2°$ to a highly anisotropic FQHS when $\theta \geq 68.7°$. The strongest $\nu = 3/2$ FQHS is seen at $\theta \simeq 70°$. With further increase in θ, R_{xx} remains large, and the $\nu = 3/2$ R_{xx} minimum gradually gets weaker, while all the FQHSs between $\nu = 4/3$ and $5/3$ disappear. We also observe qualitatively similar transitions, including the sudden appearance of a $\nu = 3/2$ FQHS, in another GaAs 2DHS with a higher density ($\simeq 2.2 \times 10^{11}$ cm^{-2}); however, the transitions occur at smaller θ ($\simeq 60°$ instead of $\simeq 69°$); see SM [28,67].

The evolution of the FQHSs near $\nu = 3/2$ presented in Fig. 3 is consistent with a crossing of the 2DHS LLs induced by increasing $B_\parallel$. Such a crossing has been previously reported [47,68,69], and can be understood based on the energy (E) vs $B_\perp$ LL diagram shown in

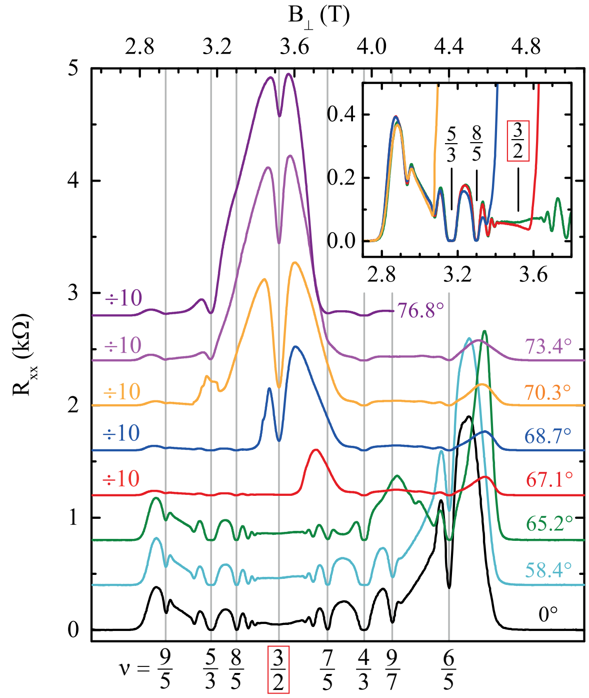

FIG. 3. Tilt-angle dependence of R_{xx} is shown vs ν at 20 mK. Each trace is vertically shifted by 0.4 kΩ for clarity. The heights of some traces are divided by 10, as marked on the left. Inset: An enlarged version of R_{xx} vs $B_\perp$ at $\theta = 65.2°$, 67.1°, 68.7°, and 70.3°.

Fig. 4. This LL diagram is from calculations performed for our 2DHS in a purely $B_\perp$ using the multiband envelope function approximation (EFA) and the 8×8 Kane Hamiltonian [70]. Because of the strong HH-LH coupling, the LLs of GaAs 2DHSs are highly nonlinear and show

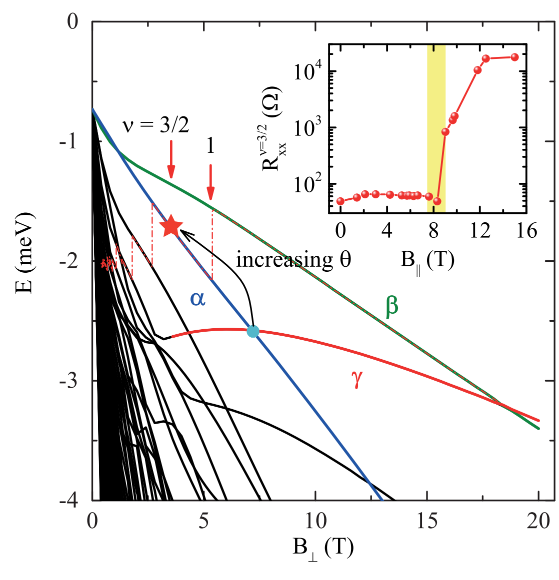

FIG. 4. Calculated LL diagram at $\theta = 0°$. The red dash-dotted line traces the Fermi energy. The position of E_F at $\nu = 3/2$ is marked with a star. Inset: R_{xx} at $\nu = 3/2$ vs $B_\parallel$.

numerous crossings [70]. There are two crossings between the top two LLs, one between the $\boldsymbol{\alpha}$ level (in blue) and $\boldsymbol{\beta}$ level (in green) at very low $B_\perp$ ($\simeq$1 T), and another between the $\boldsymbol{\beta}$ level and $\boldsymbol{\gamma}$ level (in red) at higher $B_\perp$ ($\simeq$18 T). These two crossings are, by themselves, very interesting and lead to many exotic interaction phenomena [71–74] near $\nu = 1/2$ and 1.

The relevant crossing in our study is between the $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ levels [47]. Note that, in Fig. 4, this crossing occurs at $\nu < 3/2$. While we cannot perform calculations for a 2DHS in tilted fields because of computational challenges, we expect that the addition of $B_\parallel$ would increase the effective Zeeman energy and the separation between the $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ levels. This implies that the crossing between $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ would move to higher fillings, eventually reaching $\nu = 3/2$ and even higher ν. Our data support this scenario. At $\theta = 0°$, E_F at $\nu = 3/2$ lies in the $\boldsymbol{\alpha}$ level (Fig. 4). This is essentially an $N = 0$ LL, consistent with our observation of a compressible CF Fermi sea ground state at $\nu = 3/2$, flanked by the standard, odd-denominator FQHSs at $\nu = 1 + p/(2p \pm 1)$ [Figs. 1(a) and 3]. As we increase θ and, therefore, $B_\parallel$, the $B_\perp$ position of the crossing moves toward lower $B_\perp$. At sufficiently large θ, E_F at $\nu = 3/2$ moves to the other side of the crossing, and lies in the $\boldsymbol{\gamma}$ level. This is manifested by an extremely abrupt rise of R_{xx} at $\nu = 3/2$ as a function of $B_\parallel$ (Fig. 4 inset). We elaborate on the above crossing in the SM [28] based on our measured strengths of the different FQHSs near $\nu = 3/2$ and show that it occurs in a very narrow range of $B_\parallel$ near 8 T (see Fig. S5 of SM [28]).

Next, we discuss how HH-LH coupling in 2DHSs combines $N = 0$ and $N > 0$ components in the wave function of one LL, which can explain the unusual behavior observed in our experiments. In the EFA, LLs in 2D systems can be described using two sets of dimensionless ladder operators $a, a^\dagger$ and $b, b^\dagger$, corresponding to two harmonic oscillators [70,75,76], see also SM [28]. The single-particle Hamiltonian H only depends on the a oscillators; thus, the a oscillators define the energy spectrum of H as a function of $B_\perp$. The b operators commute with H, and represent an oscillator with frequency zero that describes the degeneracy of the LLs.

In the EFA applied to the s-like electrons in the conduction band of GaAs, the wave functions have two spinor components corresponding to a spin $s = \pm 1/2$. However, ignoring the weak spin-orbit coupling in electron systems, the spin is decoupled from the orbital motion, and eigenstates of H have only one nonzero spinor component s. Therefore, the quantum number $N = 0, 1, 2, \ldots$ that labels the eigenstates $|N\rangle$ of the a oscillator can also label the eigenstates of H with energies $E_N = \hbar\omega_c(N + \frac{1}{2}) \pm \Delta_Z$ and Zeeman energy Δ_Z.

The p-like holes in the valence band require four spinor components representing $s = \pm 3/2$ (HH) and $\pm 1/2$ (LH) [77]. The hole eigenstates of H are generally a linear combination of HH and LH states. However, to a good approximation, the eigenstates of H remain eigenstates of total angular momentum $j = l + s$ (axial approximation [70,75]), where $l = xp_y - yp_x = \hbar(a^\dagger a - b^\dagger b)$ is the orbital angular momentum. The eigenstates of H become

$$\psi_{\mathcal{N}} = \sum_s \left| N = \mathcal{N} - s - \frac{3}{2} \right\rangle \xi_s^{\mathcal{N}} u_s, \qquad (1)$$

with $|N\rangle = 0$ when $N < 0$. Here, u_s are the base spinors with spin s, and $\xi_s^{\mathcal{N}}$ gives the relative weights of the HH and LH spinors. We ignore, in Eq. (1), the b oscillators that are not relevant for the present discussion. $\mathcal{N} = 0, 1, 2, \ldots$ labels the energies of the LLs. In contrast to 2DESs, N is not a good quantum number for hole LLs because of the HH-LH mixing that is parametrized via the coefficient $\xi_s^{\mathcal{N}}$ [78].

Note that, in Fig. 4, the $\boldsymbol{\alpha}$ level is a pure HH LL ($s = -3/2, \mathcal{N} = 0$) containing only $N = 0$, consistent with Eq. (1). The $\boldsymbol{\gamma}$ level with $\mathcal{N} = 3$, on the other hand, is strongly affected by HH-LH coupling, which makes this level a mixture of different oscillator states $|N\rangle$ associated with the different spinor components representing the HH and LH parts of its wave function. The decomposition of the $\boldsymbol{\gamma}$ level yields approximately 49% $s = +3/2$ with $N = 0$, 45% $s = +1/2$ with $N = 1$, and 6% $s = -3/2$ with $N = 3$. This can explain why the traces we observe at and near $\nu = 3/2$ at large angles, e.g., at $\theta \simeq 70°$, are intriguingly unique as they exhibit features normally seen in an electron system in LLs with different N: $N = 0$ (numerous odd-denominator FQHSs at 6/5, 11/9, etc.), $N = 1$ (an even-denominator FQHS), and $N > 1$ LLs (states with extremely anisotropic transport coefficients). It is also worth emphasizing that, while the anisotropic $\nu = 3/2$ FQHS in our sample abruptly appears after the LL crossing at $B_\parallel \simeq 8$ T, it persists well past the crossing, even when $B_\parallel \simeq 15$ T.

We emphasize that the above calculations are performed in purely perpendicular magnetic fields and should be interpreted cautiously when tilted fields are involved. In large $B_\parallel$, the HH-LH coupling may be different. It is unlikely that the reduced symmetry in tilted fields would reduce the amount of HH-LH coupling experienced by the $\boldsymbol{\gamma}$ level. Indeed, it is more likely that the eigenstates in a tilted field would become yet more complicated linear combinations of different oscillator states $|N\rangle$. Note that such combinations are unique in GaAs 2DHSs and cannot be achieved in GaAs 2DESs. Our results presented here demonstrate the power of applying $B_\parallel$ as a knob to engineer the properties of 2D hole LLs and create exotic, new ground states. They also highlight the need for calculations of 2DHS LLs in titled magnetic fields; such calculations, while computationally very challenging, would be very helpful in understanding the physics of the observed ground states.

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