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Numerical simulations of musical instruments

Nicholas Giordano

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JASA REFLECTIONS

The *Reflections* series takes a look back on historical articles from *The Journal of the Acoustical Society of America* that have had a significant impact on the science and practice of acoustics.

Numerical simulations of musical instruments

Article: Numerical simulations of piano strings. I. A physical model for a struck string using finite difference methods
Author: Antoine Chaigne and Anders Askenfelt
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ARTICLE OVERVIEW

The article by Chaigne and Askenfelt[1] described a systematic approach for modeling the excitation and subsequent vibrations of a piano string. A companion paper that appeared in the *Journal of the Acoustical Society of America* (*JASA*) a month later[2] applied this approach to model the behavior of real piano strings and study how that behavior depends on a number of specific properties of piano strings and hammers. Their modeling predictions agreed well with the experimentally measured behavior for quantities such as the transverse string velocity as a function of position and time (shown in the figure reproduced here) and the way the force between the hammer and string varies with time.

The basic modeling framework of Chaigne and Askenfelt has several important features. (1) It is based on an equation of motion derived from the first principles of mechanics. For the piano string, this is the wave equation including string stiffness, damping, and an external force from the hammer. (2) It has an algorithm that can be used to obtain a numerical solution of the equation of motion, with attention to numerical stability and accuracy. (3) It has a formalism that can be readily extended to treat other components of an instrument and accommodate improvements in the equation of motion (such as the inclusion of both transverse string modes). (4) It has the goal of basing all parameters in the model on the properties of real instruments, enabling quantitative comparisons with experiment and an understanding of how those properties affect the behavior.

Individual pieces of this modeling framework were discussed and implemented before Ref. 1, but bringing them all together was crucial for realizing the full power of numerical modeling as applied to musical instruments.

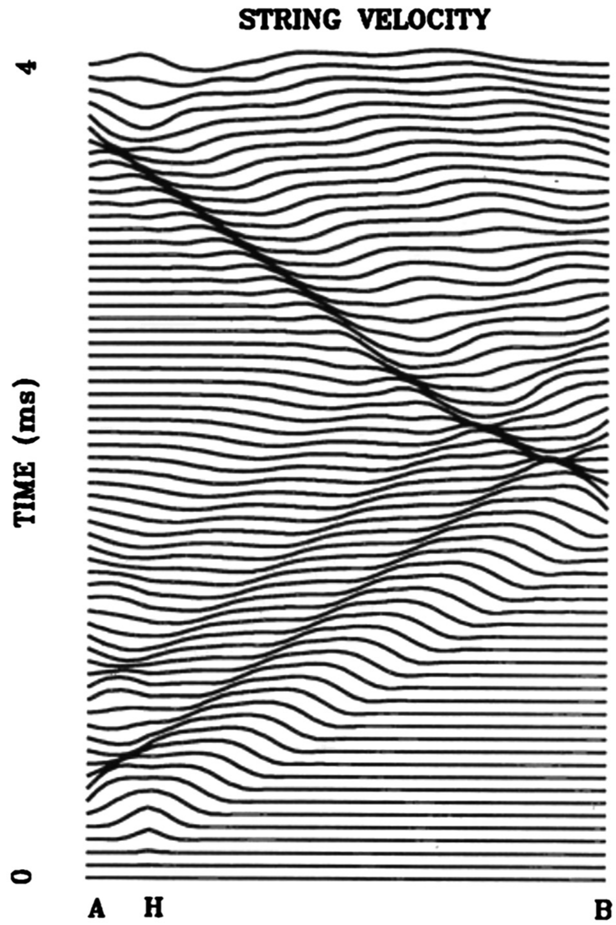

Figure 4 from Chaigne and Askenfelt (Ref. 1). Simulated velocity profile of a piano string during the first 4 ms after a blow from the hammer. (A) and (B) indicate the ends of the string at the agraffe and bridge, and (H) indicates the hammer strike point. First contact is shown in the profile at the bottom. The string excitation then travels away from the strike point and reflects from the ends. Reprinted with permission from A. Chaigne and A. Askenfelt, J. Acoust. Soc. Am. 95, 1112–1118 (1994). Copyright 1994 Acoustical Society of America (Ref. 1).

WHY AND HOW THIS WORK HAPPENED

The problem of a vibrating string has a long history in physics, with discussions by d'Alembert, Bernoulli, Fourier, von Helmholtz, Rayleigh, Raman, and many others. Thus, much was known about the vibrational properties of strings and plates and other components found in musical instruments. However, the nontrivial geometries of most real instruments together with complications due to factors such as string stiffness make closed form analytic solutions impractical. As computers became available, a number of researchers began applying them to the problem of piano strings and hammers. Important examples were the work of Hiller and Ruiz[3] on applying finite difference algorithms to plucked, struck, and bowed strings and the careful study of stability and accuracy of algorithms for dealing with stiff (dispersive) strings by Chaigne.[4] At the same time, careful studies of piano strings, hammers, and their interaction (e.g., Ref. 5) were yielding new insights, qualitative and quantitative. (More references to this prior work are given in Ref. 1.) To their credit, Chaigne and Askenfelt realized that a "marriage" of the numerical methods and the quantitative instrument parameters was needed; this led to a collaboration that benefited from an extended stay (by Chaigne) in Stockholm, leading to Refs. 1 and 2.

IMPACT

The basic modeling approach demonstrated in Ref. 1 is not limited to piano strings but has since been applied to many other problems in musical acoustics, by Chaigne and Askenfelt and their collaborators and by many other researchers. For example, it has been used to model other components of the piano[6] and (with increasing realism) to model the complete instrument.[7] It is not limited to the piano, but has been used in studies of other stringed instruments, such as the guitar, percussion instruments (including drums and xylophones), and wind instruments (including recorders, flutes, and clarinets). Work on complete instruments requires that one model how a vibrating soundboard or air column excites sound in the surrounding air. It is worth noting that this problem was discussed in a seminal paper published in the same volume of *JASA*,[8] which was itself the subject of a recent Reflections paper.[9]

By NICHOLAS GIORDANO
Department of Physics, Auburn University, Auburn, Alabama 36849, USA

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