

PAPER

## Freeze-in dark matter perturbations are adiabatic

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# Freeze-in dark matter perturbations are adiabatic

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**Abstract.** We show that the large-scale perturbations in the dark matter generated by the freeze-in mechanism are only of adiabatic nature. The freeze-in mechanism is not at odds with the current stringent constraints on isocurvature perturbations.

**Keywords:** physics of the early universe, cosmological perturbation theory, dark matter theory

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## 1 Introduction

It is believed that the cosmic microwave background anisotropies and the large-scale structure of the universe are seeded by the fluctuations generated during a period of inflation during the early evolution of the universe [1]. Such perturbations may be either of adiabatic or of isocurvature nature. Adiabatic, or curvature perturbations, are fluctuations which are generated when there is only one clock in the universe, e.g. the inflaton field during primordial inflation. Isocurvature perturbations require the presence of more than one degree of freedom and require a quite subtle dynamics to be produced. For instance, isocurvature Dark Matter (DM) perturbations may be generated by a curvaton field, but only if the latter decays after the freeze-out temperature of the DM [2].

CMB observations from the Planck collaboration [3] set stringent constraints on the amount of isocurvature fluctuations present on large-scales. A recent study [4] has claimed that too large isocurvature perturbations in the DM component are generated in the so-called freeze-in production mechanism [5–7], when DM particles are generated and never reach chemical equilibrium. If true, the freeze-in mechanism would be ruled out as a production mechanism for the totality of DM. However, this conclusion seems at odd with the generic expectation that no isocurvature perturbations may be generated if perturbations may be ascribed to the presence of only one clock [8, 9]. In this note, we revisit the issue and indeed conclude that DM perturbations are of adiabatic nature. As we discuss in section 3.2, in the freeze-in scenario the energy transferred to DM is only a function of the Standard Model (SM) temperature, and the DM pressure is only a function of the DM energy density (and both are only functions of the SM temperature): these two conditions forbid the generation of DM isocurvature perturbations on large scales, because of the absence of another clock. In conclusion, the freeze-in mechanism does not suffer of constraints on isocurvature perturbations from the CMB anisotropies.

## 2 Freeze-in DM perturbations: heuristic argument

We wish to study the evolution of cosmological perturbations in the freeze-in transition when DM particles are produced by Standard Model (SM) particle interactions and never reach chemical equilibrium.

We first offer the reader a heuristic argument of why DM perturbations generated during the freeze-in are adiabatic. We assume that an adiabatic mode has been produced on superhorizon scales by a period of inflation and communicated to the radiation fluid during the reheating stage [1]. We also assume that the SM particles are relativistic and in thermal

equilibrium, thus composing the radiation field. Finally, we assume, for the moment, that the long mode of the total curvature perturbation  $\zeta_L(\mathbf{x})$  (on length scales  $L$  much larger than the Hubble radius  $H^{-1}$ ) is constant in time. One can redefine the local coordinates to absorb the long mode perturbation. For instance, in the uniform energy density gauge, the perturbed metric can be written as

$$ds^2 = -dt^2 + a^2(t)e^{2\zeta(\mathbf{x})}d\mathbf{x}^2, \quad (2.1)$$

where  $a$  is the scale factor. One can absorb the long mode of the curvature perturbation  $\zeta_L(\mathbf{x})$  by simply redefining the spatial coordinates  $\mathbf{x}' = \exp(\zeta_L(\mathbf{x}))\mathbf{x}$ . In these new coordinates the universe looks homogeneous and isotropic to local observers measuring distances smaller than the Hubble radius. In other words, we adopt the separate universe approach where it is assumed that each comoving region with size much bigger than the Hubble radius looks locally like some unperturbed FRW universe.

In the freeze-in mechanism, the local DM energy density on scales  $\ell \ll H^{-1}$  changes according to the equation (we assume for the moment and for the sake of the argument a non-relativistic DM)

$$\dot{\rho}_{\text{DM}}(\mathbf{x}', t) + 3H\rho_{\text{DM}}(\mathbf{x}', t) = \Gamma, \quad (2.2)$$

where the dot denotes differentiation with respect to the coordinate time  $t$ ,  $\Gamma$  is the production rate (which depends on the number density of the SM particles) and  $H$  is the Hubble rate. For instance, if DM particles are generated by the annihilations of SM particles with relative velocity  $v_{\text{rel}}$  and cross section  $\sigma_{\text{ann}}$ , the production rate reads ( $m_{\text{DM}}$  being the DM massss)

$$\Gamma = m_{\text{DM}}\langle\sigma_{\text{ann}}v_{\text{rel}}\rangle n_{\text{SM}}^2. \quad (2.3)$$

To be more concrete, in the freeze-in scenario where the DM is a millicharged fermion  $\chi$  with electrical charge  $Q_\chi$  produced from pair annihilation of  $e^+e^-$ ,

$$\Gamma = m_{\text{DM}}\langle\sigma_{\text{ann}}v_{\text{rel}}\rangle n_e^2 \simeq m_{\text{DM}} \left( \frac{9\zeta^2(3)\alpha_{\text{EM}}Q_\chi^2}{2\pi^4} \right) T^4, \quad (2.4)$$

where  $\alpha_{\text{EM}} = e^2/(4\pi)$  is the electromagnetic coupling constant and  $\zeta(3) \approx 1.20$  is the Riemann zeta function. As the SM bath is in thermal equilibrium,  $\Gamma$  is a function only of the local temperature (besides the masses of the SM particles, the DM mass, and coupling constants),

$$\Gamma = \Gamma(T(\mathbf{x}', t)), \quad (2.5)$$

and therefore the local DM number density after the freeze-in is only a function of the local temperature  $T(\mathbf{x}', t)$ . The latter inherits the large-scale superhorizon fluctuations seeded by inflation [10]

$$T(\mathbf{x}', t) = T_{\text{bg}}(t)e^{\zeta_L(\mathbf{x})/5}, \quad (2.6)$$

where  $T_{\text{bg}}$  is the background temperature. This already suggests that there is only one clock governing the fluctuations of the freeze-in DM, the one generated by the curvature perturbation  $\zeta_L(\mathbf{x})$ , and that no isocurvature perturbation can be generated in the absence of an additional separate source of perturbations [9]. As we will see, the gauge-invariant expression for the DM-radiation isocurvature perturbation depends only on the difference

$$\frac{\delta\rho_{\text{DM}}(\mathbf{x}, t)}{\dot{\rho}_{\text{DM}}} - \frac{\delta\rho_\gamma(\mathbf{x}, t)}{\dot{\rho}_\gamma}. \quad (2.7)$$

Since

$$\frac{\delta\rho_{\text{DM}}(\mathbf{x}, t)}{\dot{\rho}_{\text{DM}}} = \frac{\delta T(\mathbf{x}, t)}{\dot{T}} = \frac{\delta\rho_\gamma(\mathbf{x}, t)}{\dot{\rho}_\gamma}, \quad (2.8)$$

we expect that no isocurvature perturbations are generated through the freeze-in mechanism in the DM component, in agreement as well with our previous assumption that the total curvature perturbation is constant in time on superhorizon scales. In the following we will arrive at the same conclusion through a rigorous gauge-invariant treatment.

### 3 Freeze-in DM perturbations: the gauge-invariant treatment

To study the freeze-in DM perturbations in a gauge-invariant manner, we follow the gauge-invariant approach developed in ref. [11] for the general case of an arbitrary number of interacting fluids in general relativity.

#### 3.1 The background equations

The evolution of the background FRW universe during the freeze-in stage is governed by the Friedmann constraint

$$H^2 = \frac{8\pi G}{3}\rho, \quad (3.1)$$

$$\dot{H} = -4\pi G(\rho + P), \quad (3.2)$$

and the continuity equation

$$\dot{\rho} = -3H(\rho + P), \quad (3.3)$$

where  $\rho$  and  $P$  are the total energy density and the total pressure of the system. The total energy density and the total pressure are related to the energy density and pressure of the DM field and radiation by

$$\begin{aligned} \rho &= \rho_{\text{DM}} + \rho_\gamma, \\ P &= P_{\text{DM}} + P_\gamma, \end{aligned} \quad (3.4)$$

where  $P_\gamma$  is the radiation pressure. The DM field and the radiation component have energy-momentum tensor  $T_{\text{DM}}^{\mu\nu}$  and  $T_\gamma^{\mu\nu}$ , respectively. The total energy momentum tensor

$$T^{\mu\nu} = T_{\text{DM}}^{\mu\nu} + T_\gamma^{\mu\nu} \quad (3.5)$$

is covariantly conserved, but we allow for energy transfer between the fluids,

$$\begin{aligned} \nabla_\mu T_{\text{DM}}^{\mu\nu} &= Q_{\text{DM}}^\nu, \\ \nabla_\mu T_\gamma^{\mu\nu} &= Q_\gamma^\nu, \end{aligned} \quad (3.6)$$

where  $Q_{\text{DM}}^\nu$  and  $Q_\gamma^\nu$  are the generic energy-momentum transfer to the inflaton and radiation sector respectively and are subject to the constraint

$$Q_{\text{DM}}^\nu + Q_\gamma^\nu = 0. \quad (3.7)$$

The continuity equations for the energy density of the DM field and radiation in the background are thus ( $Q_{\text{DM}} = Q_{\text{DM}}^0$ ,  $Q_\gamma = Q_\gamma^0$ )

$$\begin{aligned} \dot{\rho}_{\text{DM}} &= -3H(\rho_{\text{DM}} + P_{\text{DM}}) + Q_{\text{DM}}, \\ \dot{\rho}_\gamma &= -3H(\rho_\gamma + P_\gamma) + Q_\gamma. \end{aligned} \quad (3.8)$$

In the following we parametrise the energy transfer between radiation and the DM by

$$\begin{aligned} Q_{\text{DM}} &= \Gamma(\rho_\gamma), \\ Q_\gamma &= -\Gamma(\rho_\gamma). \end{aligned} \quad (3.9)$$

This assumption is motivated again by the fact that in the freeze-in mechanism DM particles are generated out of chemical equilibrium and by SM degrees of freedom which are relativistic and therefore a function of only the temperature or radiation energy density (besides the masses and coupling constants). The background energy conservation equations therefore read

$$\dot{\rho}_{\text{DM}} = -3H(\rho_{\text{DM}} + P_{\text{DM}}) + \Gamma, \quad (3.10)$$

$$\dot{\rho}_\gamma = -4H\rho_\gamma - \Gamma, \quad (3.11)$$

$$\dot{\rho} = -H[3(\rho_{\text{DM}} + P_{\text{DM}}) + 4\rho_\gamma]. \quad (3.12)$$

### 3.2 Gauge-invariant linear perturbations

Linear scalar perturbations about a spatially-flat FRW background model are defined by the line element [1]

$$ds^2 = -(1 + 2\varphi)dt^2 + 2aB_{,i}dtdx^i + a^2[(1 - 2\psi)\delta_{ij} + 2E_{,ij}]dx^i dx^j, \quad (3.13)$$

where we have introduced the gauge-dependent curvature perturbation,  $\psi$ , the lapse function,  $\varphi$ , and scalar shear,  $\chi \equiv a^2\dot{E} - aB$ . The perturbed energy transfer rates including terms up to first order, are written as

$$-Q_{\text{DM}}(1 + \varphi) - \delta Q_{\text{DM}} \text{ and } -Q_\gamma(1 + \varphi) - \delta Q_\gamma, \quad (3.14)$$

where the gravitational redshift (time-dilation) factor  $(1 + \varphi)$  has been made manifest. Both the density perturbations  $\delta\rho_{\text{DM}}$  and  $\delta\rho_\gamma$  and the gravitational potential  $\psi$  are in general gauge-dependent. However gauge-invariant combinations can be constructed which describe the density perturbations on uniform curvature slices or, equivalently the curvatures of uniform density slices. The total curvature perturbation  $\zeta$  on uniform total density hypersurfaces is given by

$$\zeta = -\psi - H\frac{\delta\rho}{\dot{\rho}}, \quad (3.15)$$

while the curvature perturbation on uniform DM energy density and radiation energy density hypersurfaces are respectively defined as

$$\begin{aligned} \zeta_{\text{DM}} &= -\psi - H\frac{\delta\rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}}, \\ \zeta_\gamma &= -\psi - H\frac{\delta\rho_\gamma}{\dot{\rho}_\gamma}. \end{aligned} \quad (3.16)$$

The total curvature perturbation (3.15) is thus a weighted sum of the individual perturbations

$$\zeta = \frac{\dot{\rho}_{\text{DM}}}{\dot{\rho}}\zeta_{\text{DM}} + \frac{\dot{\rho}_\gamma}{\dot{\rho}}\zeta_\gamma, \quad (3.17)$$

while the difference between the two curvature perturbations describes a relative gauge-invariant entropy (or isocurvature) perturbation

$$\mathcal{S}_{\text{DM}\gamma} = 3(\zeta_{\text{DM}} - \zeta_{\gamma}) = -3H \left( \frac{\delta\rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}} - \frac{\delta\rho_{\gamma}}{\dot{\rho}_{\gamma}} \right). \quad (3.18)$$

From the definitions of the total curvature perturbation (3.17) and the entropy perturbation (3.18), we get for instance that

$$\begin{aligned} \zeta_{\text{DM}} &= \zeta + \frac{1}{3} \frac{\dot{\rho}_{\gamma}}{\dot{\rho}} \mathcal{S}_{\text{DM}\gamma} \\ \zeta_{\gamma} &= \zeta - \frac{1}{3} \frac{\dot{\rho}_{\text{DM}}}{\dot{\rho}} \mathcal{S}_{\text{DM}\gamma}. \end{aligned} \quad (3.19)$$

On wavelengths larger than the horizon scale, the perturbed energy conservation equations for the DM energy density and the radiation energy density can be written, including energy transfer, as

$$\begin{aligned} \dot{\delta\rho}_{\text{DM}} + 3H(\delta\rho_{\text{DM}} + \delta P_{\text{DM}}) - (\rho_{\text{DM}} + P_{\text{DM}}) 3\dot{\psi} &= Q_{\text{DM}}\varphi + \delta Q_{\text{DM}}, \\ \dot{\delta\rho}_{\gamma} + 3H(\delta\rho_{\gamma} + \delta P_{\gamma}) - (\rho_{\gamma} + P_{\gamma}) 3\dot{\psi} &= Q_{\gamma}\varphi + \delta Q_{\gamma}. \end{aligned} \quad (3.20)$$

Using the perturbed  $(0i)$ -component of Einstein's equations for super-horizon wavelengths

$$\dot{\psi} + H\varphi = -\frac{H}{2} \frac{\delta\rho}{\rho}, \quad (3.21)$$

we can re-write eq. (3.20) in terms of the gauge-invariant curvature perturbations  $\zeta_{\text{DM}}$  and  $\zeta_{\gamma}$  [11]

$$\begin{aligned} \dot{\zeta}_{\text{DM}} &= -\frac{H}{\dot{\rho}_{\text{DM}}} (\delta Q_{\text{intr,DM}} + \delta Q_{\text{rel,DM}}) + \frac{3H^2}{\dot{\rho}_{\text{DM}}} \delta P_{\text{intr,DM}}, \\ \dot{\zeta}_{\gamma} &= -\frac{H}{\dot{\rho}_{\gamma}} (\delta Q_{\text{intr},\gamma} + \delta Q_{\text{rel},\gamma}) + \frac{3H^2}{\dot{\rho}_{\gamma}} \delta P_{\text{intr},\gamma}, \end{aligned} \quad (3.22)$$

where

$$\begin{aligned} \delta Q_{\text{intr,DM}} &\equiv \delta Q_{\text{DM}} - \frac{\dot{Q}_{\text{DM}}}{\dot{\rho}_{\text{DM}}} \delta\rho_{\text{DM}}, & \delta P_{\text{intr,DM}} &\equiv \delta P_{\text{DM}} - \frac{\dot{P}_{\text{DM}}}{\dot{\rho}_{\text{DM}}} \delta\rho_{\text{DM}}, \\ \delta Q_{\text{intr},\gamma} &\equiv \delta Q_{\gamma} - \frac{\dot{Q}_{\gamma}}{\dot{\rho}_{\gamma}} \delta\rho_{\gamma}, & \delta P_{\text{intr},\gamma} &\equiv \delta P_{\gamma} - \frac{\dot{P}_{\gamma}}{\dot{\rho}_{\gamma}} \delta\rho_{\gamma}, \end{aligned} \quad (3.23)$$

are the gauge-invariant perturbations for the intrinsic non-adiabatic energy transfer and the pressure, and

$$\begin{aligned} \delta Q_{\text{rel,DM}} &= \frac{Q_{\text{DM}}\dot{\rho}}{2\rho} \left( \frac{\delta\rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}} - \frac{\delta\rho}{\dot{\rho}} \right) = -\frac{Q_{\text{DM}}}{6H\rho} \dot{\rho}_{\gamma} \mathcal{S}_{\text{DM}\gamma}, \\ \delta Q_{\text{rel},\gamma} &= \frac{Q_{\gamma}\dot{\rho}}{2\rho} \left( \frac{\delta\rho_{\gamma}}{\dot{\rho}_{\gamma}} - \frac{\delta\rho}{\dot{\rho}} \right) = +\frac{Q_{\gamma}}{6H\rho} \dot{\rho}_{\text{DM}} \mathcal{S}_{\text{DM}\gamma} \end{aligned} \quad (3.24)$$

are the relative gauge-invariant non-adiabatic perturbed energy transfer due to the presence of relative entropy perturbations [11]. The intrinsic pressure perturbations  $\delta P_{\text{intr}}$  for radiation

and DM both vanish. Indeed, for the radiation one has  $P_\gamma = \rho_\gamma/3$  and for the DM, even for a non-thermal DM phase space density, one has that both  $P_{\text{DM}}$  and  $\rho_{\text{DM}}$  are functions of the thermal temperature and therefore one can always write the pressure  $P_{\text{DM}}$  as a function of the energy density  $\rho_{\text{DM}}$  (in the relativistic limit one would have  $P_{\text{DM}} = \rho_{\text{DM}}/3$ , which is valid even for a non-thermal DM distribution in phase space; in the non-relativistic limit  $P_{\text{DM}} = 0$ ).

The evolution equations (3.22) for the curvature perturbation on uniform DM density hypersurfaces,  $\zeta_{\text{DM}}$ , and uniform radiation density hypersurfaces,  $\zeta_\gamma$ , are given by

$$\dot{\zeta}_{\text{DM}} = \frac{\Gamma}{6\rho} \frac{\dot{\rho}_\gamma}{\dot{\rho}_{\text{DM}}} \mathcal{S}_{\text{DM}\gamma} - \frac{H}{\dot{\rho}_{\text{DM}}} \delta\Gamma_{\text{DM}}, \quad (3.25)$$

$$\dot{\zeta}_\gamma = \frac{\Gamma}{6\rho} \frac{\dot{\rho}_{\text{DM}}}{\dot{\rho}_\gamma} \mathcal{S}_{\text{DM}\gamma} - \frac{H}{\dot{\rho}_\gamma} \delta\Gamma_\gamma, \quad (3.26)$$

where [12]

$$\begin{aligned} \delta\Gamma_{\text{DM}} &= \delta\Gamma - \dot{\Gamma} \frac{\delta\rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}}, \\ \delta\Gamma_\gamma &= -\delta\Gamma + \dot{\Gamma} \frac{\delta\rho_\gamma}{\dot{\rho}_\gamma} \end{aligned} \quad (3.27)$$

are the gauge-invariant perturbations of the freeze-in production rate. Taking  $\Gamma = \Gamma(\rho_\gamma)$ , we find

$$\begin{aligned} \delta\Gamma_{\text{DM}} &= \delta\Gamma - \dot{\Gamma} \frac{\delta\rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}} = \dot{\Gamma} \left( \frac{\delta\rho_\gamma}{\dot{\rho}_\gamma} - \frac{\delta\rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}} \right) = \frac{\dot{\Gamma}}{3H} \mathcal{S}_{\text{DM}\gamma}, \\ \delta\Gamma_\gamma &= 0. \end{aligned} \quad (3.28)$$

We then obtain

$$\dot{\zeta}_{\text{DM}} = \left( \frac{\Gamma}{6\rho} \frac{\dot{\rho}_\gamma}{\dot{\rho}_{\text{DM}}} - \frac{\dot{\Gamma}}{3\dot{\rho}_{\text{DM}}} \right) \mathcal{S}_{\text{DM}\gamma} = \frac{\dot{\rho}}{\dot{\rho}_{\text{DM}}} \left( \frac{\Gamma}{2\rho} \frac{\dot{\rho}_\gamma}{\dot{\rho}_{\text{DM}}} - \frac{\dot{\Gamma}}{\dot{\rho}_{\text{DM}}} \right) (\zeta - \zeta_\gamma), \quad (3.29)$$

$$\dot{\zeta}_\gamma = \frac{\Gamma}{6\rho} \frac{\dot{\rho}_{\text{DM}}}{\dot{\rho}_\gamma} \mathcal{S}_{\text{DM}\gamma} = \frac{\dot{\rho}}{\dot{\rho}_\gamma} \frac{\Gamma}{2\rho} \frac{\dot{\rho}_{\text{DM}}}{\dot{\rho}_\gamma} (\zeta_{\text{DM}} - \zeta) = \frac{\dot{\rho}}{\dot{\rho}_\gamma} \frac{\Gamma}{\rho} (\zeta - \zeta_\gamma). \quad (3.30)$$

It follows that

$$\dot{\mathcal{S}}_{\text{DM},\gamma} = \frac{3\dot{\rho}}{\dot{\rho}_{\text{DM}}^2} \left( \frac{\dot{\rho}_\gamma^2 - \dot{\rho}_{\text{DM}}^2}{\dot{\rho}_\gamma} \frac{\Gamma}{2\rho} - \dot{\Gamma} \right) (\zeta - \zeta_\gamma). \quad (3.31)$$

Now, the initial condition of such evolution equations, before the freeze-in transition, is

$$\zeta(\mathbf{x}, t \ll t_{\text{fi}}) = \zeta_\gamma(\mathbf{x}, t \ll t_{\text{fi}}), \quad (3.32)$$

from which we conclude that

$$\zeta(\mathbf{x}, t \gg t_{\text{fi}}) = \zeta_\gamma(\mathbf{x}, t \gg t_{\text{fi}}) \text{ and } \mathcal{S}_{\text{DM}\gamma}(\mathbf{x}, t \gg t_{\text{fi}}) = 0 \quad (3.33)$$

are fixed points of the evolution.

DM perturbations inherit the same adiabatic perturbations generated during inflation. This rigorous result confirms the argument that no isocurvature perturbations may be generated in the presence of only one clock.

Let us also add the following comment. Our conclusion is valid also for the DM produced out of chemical equilibrium through a decay of a SM degree of freedom whose perturbation is adiabatic, that is of the curvature type. In such a case, one will have again simply  $\zeta_{\text{DM}} = \zeta$ . In general, the situation with the production of the DM by the freeze-in mechanism through interactions of the SM particles which dominate the energy density of the universe and whose perturbations are curvature perturbations is completely analogous to what happens in the case in which DM particles are produced by the curvaton decay if the latter happens after the freeze-out epoch. Such DM particles do not reach chemical equilibrium and if the curvaton at the time of its decay dominates the energy density, it simply transfers its curvature perturbation to the DM, with no residual isocurvature perturbation [2].

Notice also that one can extend our result to any order in perturbation theory for perturbations on large scales. As a consequence, the freeze-in DM perturbation receives the same non-Gaussianity of the curvature perturbation, therefore negligible in single-field models of inflation [13].

As a final remark, we notice that the definition of the isocurvature perturbation in eq. (3.18) is gauge-invariant also if there is an energy transfer between radiation and DM (as it is the case for freeze-in production). This is not the case for the alternative definition of Kodama and Sasaki [14, eq. (5.38)], in presence of energy transfer: this can be easily seen by writing that expression in a gauge where  $\delta T^{0i} = 0$ , so that

$$\mathcal{S}_{\text{DM}\gamma}^{[\text{KS}]} = \frac{\delta\rho_{\text{DM}}/\rho_{\text{DM}}}{1+w_{\text{DM}}} - \frac{\delta\rho_{\gamma}/\rho_{\gamma}}{1+w_{\gamma}} = -3H \left( \frac{\delta\rho_{\text{DM}}}{\dot{\rho}_{\text{DM}} - Q_{\text{DM}}} - \frac{\delta\rho_{\gamma}}{\dot{\rho}_{\gamma} - Q_{\gamma}} \right) \quad (3.34)$$

whose evolution equations [14, 15] are more involved than those in ref. [11], that we discuss here. Maybe this could be the source of the disagreement with ref. [4].

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