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System-level impacts of en-route information sharing considering adaptive routing

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ABSTRACT

Connected and automated vehicles (CAVs) and infrastructure-to-vehicle (I2V) communication have great potential to improve traffic safety and mobility at intersection and road segment levels. However, since transportation is an interconnected network, local transportation state change could lead to broader impacts by rerouting. In this study, we focus on investigating the impacts of en-route locational information updates on the system-level safety and mobility considering adaptive routing of CAVs. We propose a novel two-stage stochastic traffic equilibrium model to characterize the equilibrium traffic patterns considering adaptive routing behavior when en-route traffic information is provided. The proposed methodology is tested using the Orlando transportation network with real traffic and accident data. Different I2V information sharing strategies are analyzed to better understand the impacts of information sharing locations of I2V on transportation network safety and mobility, through which we generate insights on the optimal design of traffic information sharing systems.

1. Introduction

The market penetration of levels 3 and up connected and automated vehicles (CAVs) is projected to reach approximately 8 million by 2025 (ABI, 2018). Although fully automated vehicles may still take years to develop, the improvements in vehicle connectivity, onboard computation, and data sensing in low-level CAVs have great potential to transform transportation systems leveraging increasing availability of information (Elliott et al., 2019).

One of the key information sources for CAVs is infrastructure-to-vehicle (I2V) technology, which collects and processes sensor- or vehicle-generated data and wirelessly shares information with vehicles nearby (Böhm and Frötscher, 2009). With advanced onboard communication and computational capability, CAVs are able to receive and process a large amount of data promptly and suggest optimal travel decisions (e.g., speed control and/or travel paths) based on both the prior knowledge of the transportation system states and real-time information updates received en-route (Malikopoulos et al., 2021). Different information sharing strategies, such as when, where, and what to share with which groups of vehicles in what transportation network conditions will significantly influence the decision making of individual vehicles, which collectively determine the network performance (Unnikrishnan and Waller, 2009; Boyles and Waller, 2011; Lindsey et al., 2014; Acemoglu et al., 2018; Liu and Liu, 2018; Liu and Yang, 2021).

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Existing studies have demonstrated the safety and mobility benefits of CAVs at local levels,[1] such as technology, node, and link levels (e.g., Yue et al., 2018; Rahman et al., 2019; Fagnant and Kockelman, 2015; Zhao et al., 2021; Papadoulis et al., 2018; Ma et al., 2021). However, focusing on local-level impacts may cause biases since a transportation system is an interconnected network. Local node or link transportation state changes, such as the occurrence of weather events, road construction, traffic management activities, traffic accidents, etc, will not only impact the link performance functions and crash risk for the impacted links but also have broader impacts on other links due to vehicles rerouting and traffic redistribution. For instance, if congestion happens on one link, sharing information on this event leads vehicles to use other alternative paths, thus contributing to congestion on those alternative paths at a broader scale. Research addressing the safety and mobility implications of CAVs considering an interconnected transportation network is still limited, which hampers the thorough analysis of CAVs' impacts on transportation systems.

The main motivation for studying system-level safety and mobility impacts of en-route information sharing is to avoid myopic decision making in information system design that seeks to improve only isolated components of the transportation system and ignore the network effects. As we will demonstrate in this study, more information may not always improve the system's safety and mobility. An analytical modeling framework characterizing the equilibrium traffic patterns considering en-route information updates and traffic rerouting is necessary to quantify the system-level safety and mobility impacts. Note that this fundamental motivation is consistent with classic transportation paradoxes, such as Barass's paradox (Rapoport et al., 2009) and Knight–Pigou–Downs paradox (Dechenaux et al., 2014; Morgan et al., 2009), in which improvements in local transportation infrastructure may lead to deterioration in system performance.

In this paper, we aim to investigate the impacts of locational en-route information updates on transportation system safety and mobility considering CAVs' adaptive routing. Note that we do not require autonomous driving features. Instead, we leverage advanced vehicle information sensing and computational technologies that can assist vehicles to make routing decisions, which can be executed by either human-driven vehicles or automated vehicles. While various types of information can be shared, this study focuses on sharing mobility information on specific transportation links with CAVs when they pass by I2V devices. Both the prior knowledge and real-time information on the transportation states will be considered by CAVs to determine the optimal individual (re)routing decisions, which collectively determine the network traffic equilibrium patterns. The contributions of this study are two-fold.

(1) We propose a novel and computationally tractable transportation network model to describe the traffic equilibrium patterns considering the adaptive routing of CAVs with en-route information updates.
(2) We evaluate the impacts of information sharing locations on transportation safety and mobility at a network level with real-world traffic data considering adaptive routing of CAVs.

The rest of the paper is structured as follows. We review the literature about the safety and mobility implications of CAVs and adaptive decision making with information sharing in Section 2. In Section 3, we present the methodology used for this study, which includes traffic network modeling with information updates and data-driven econometric models for safety quantification. Section 4 presents result analyses using the Orlando network. We conclude the study and discuss possible future extensions and policy implications in Section 5.

2. Literature review

I2V and CAV technologies provide tremendous opportunities to improve traffic safety and mobility through information sharing on real-time transportation states. In this section, we review relevant literature on the following two aspects: (1) safety and mobility implications of CAVs; and (2) traffic network modeling with the adaptive decision making of CAVs with information updates.

2.1. Safety and mobility implications of CAVs

Extensive studies have been conducted to demonstrate the effectiveness of safety and mobility improvements leveraging different levels of CAVs. Majority of these existing studies are limited to local safety and mobility improvements, such as motorway (Papadoulis et al., 2019), freeway (Zheng et al., 2019; Xie et al., 2017; Amini et al., 2021; Zhao et al., 2021), and signalized/unsignalized intersection (Morando et al., 2018; Fyfe and Sayed, 2017; Rahman et al., 2019; Du et al., 2018; Lee and Park, 2012; Ma et al., 2021; Wang et al., 2019). For example, Papadoulis et al. (2019) developed a CAV control algorithm and simulated it in VISSIM for motorway segment safety evaluation. Zheng et al. (2019) demonstrated freeway safety and mobility improvement by proposing a cooperative lane-changing strategy with exclusive lanes for CAVs. Du et al. (2018) demonstrated mobility improvement at an unsignalized intersection while proposing a hierarchical coordination strategy for CAVs to traverse through multiple signal-free intersections. Recently, based on V2X communication, Ma et al. (2021) proposed a cooperative adaptive cruise control for CAVs to improve throughput at signalized intersections. However, all the aforementioned studies focus on the safety or mobility implication of CAVs at a local level (i.e., intersections and road segments), without considering the potential network effects of traffic

[1] Local-level (i.e., link/node-level) safety and mobility refers to the crash risk and congestion on neighboring road segments or intersections only, while system-level safety and mobility considers the interconnection of the transportation network and refers to the measures for the whole transportation network by aggregating the local-level safety and mobility for each link/node. In this paper, we use link performance functions and link crash risk functions in different scenarios to represent local transportation mobility and safety.

rerouting. Research on the implication of safety and mobility with CAVs at a network level is still limited. Hasibur Rahman and Abdel-Aty (2021) are among the first to investigate the safety and mobility impact of CAVs at a network level with both V2V and I2V technologies to communicate with surrounding vehicles and traffic signals. However, this study relies on microscopic simulation, which may not be easily generalized.

2.2. Adaptive decision making with information updates

Studies have investigated the transportation system dynamics in the framework of simulation-based Dynamic Traffic Assignment (DTA) (Mahmassani, 2001; Gao and Chabini, 2006; Lin et al., 2008; Sundaram et al., 2011; Antoniou et al., 2011; Gao and Huang, 2012; Gao, 2012; Ma et al., 2016; Hasibur Rahman and Abdel-Aty, 2021). Some studies have explicitly considered the impacts of real-time information updates using iterative approaches to account for drivers' experience and learning processes, which may not lead to convergence. For example, Gao (2012) proposed a fixed-point formulation for the user equilibrium with real-time information, which was solved by a heuristic method of successive averages. Based on the space–time network, Ma et al. (2016) proposed an agent-based optimization modeling framework to determine information provision, which can be solved by integrating Lagrangian relaxation-based heuristics and a mesoscopic DTA simulator. Although simulation-based DTA approaches can characterize system dynamics in detail, it may be challenging to calibrate the simulation parameters for a large network. In addition, rigorous mathematical properties (e.g., existence, uniqueness, and convergence) and structural analyses of system interaction may be challenging to achieve (Rambha et al., 2018).

To mitigate the drawbacks of simulation-based DTA approaches, another stream of literature adapts the classic traffic equilibrium notions (e.g., Wardrop user equilibrium Wardrop, 1952; Beckmann et al., 1955, stochastic user equilibrium (SUE) Daganzo and Sheffi, 1977, and DTA Friesz and Han, 2019) to capture the impacts of information updates. For example, Acemoglu et al. (2018) generalized classic Wardrop user equilibrium and proposed an information-constrained Wardrop equilibrium model to study the impact of information on traffic congestion. Studies also investigated the route choice behavior and traffic equilibrium patterns in the context of variable message signs (VMSs) (Li et al., 2016; Ban et al., 2009; Lam and Chan, 1996) and advanced travelers information system (ATIS) (Levinson, 2003; Yang, 1998; Yang et al., 1993; Hall, 1996; Henn and Ottomanelli, 2006) using network modeling approaches. For instance, Li et al. (2016) proposed a stochastic network equilibrium model to determine the optimal location of VMS to share travel time information. Yang (1998) proposed a mixed equilibrium assignment model, where informed drivers and uninformed drivers choose routes in user-optimal and stochastic-equilibrium manners, respectively. However, these studies do not consider the adaptive routing of vehicles.

Du et al. (2014, 2015) proposed equilibrium routing decision (ERD), an extension of SUE at an individual level and in a shorter time frame. However, ERD treats decision making at each time step as independent of each other and does not consider the network impacts of rerouting. Unnikrishnan and Waller (2009), Boyles and Waller (2010, 2011), and Rambha et al. (2018) studied user equilibrium with recourse (UER) in an uncertain transportation network where travelers adjust their travel routes depending on en-route information updates. Calderone and Sastry (2017) leveraged the potential function to represent the Markov decision process routing games in the context of ride-sourcing services, where drivers consider the routing over an entire time horizon with real-time information updates. Most of the above-mentioned research on adaptive decision making depicts information implicitly in routing policies, which is challenging for further information systems design and analyses. In addition, the number of routing policies grows exponentially with both uncertain scenarios and paths, which brings significant challenges for real-world applications. To mitigate these issues, we propose a novel and scalable equilibrium-based network model based on the concept of non-anticipativity in stochastic programming (e.g., Rockafellar and Wets (1976)) to characterize the equilibrium traffic patterns considering adaptive decision making with information updates.

3. Methodology

The proposed research aims to investigate the safety and mobility implications of CAVs and I2V technologies considering en-route information updates and rerouting. We denote the locations where information is provided to travelers passing by as information nodes. Travelers can update their routing decisions based on the information they have received. Note that en-route rerouting decision does not necessarily imply a dynamic traffic state. En-route rerouting decisions could also be because a (static) transportation state is not known to travelers until they pass by information nodes. In this paper, we focus on static traffic patterns.

3.1. Network modeling

We use network modeling techniques to capture the interdependence between traffic flow at different links and formulate the adaptive decision making of CAVs when they pass by the I2V infrastructure and receive information updates. Depending on the locations of I2V infrastructure and the travel paths of CAVs, different CAVs may receive different information. Similar to the classic traffic assignment (Wardrop, 1952), the link travel time and the route choices of all CAVs are coupled. To describe the traffic flow pattern given information updates, we propose a novel computationally tractable static-equilibrium-based formulation of traffic assignment with adaptive routing decision making. In this subsection, we will start with a brief summary of the classic Beckmann's user equilibrium (UE) models (Beckmann et al., 1955). Then, we discuss how to generalize the classic user equilibrium models to reflect adaptive decision making with information updates.

3.1.1. Beckmann's models for UE

We denote a transportation network as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the node set (indexed by n) and $\mathcal{A}$ is the set of links (indexed by a). Furthermore, we define the OD set $\mathcal{C} \doteq \mathcal{R} \times \mathcal{S}$ (indexed by rs), where $\mathcal{R}, \mathcal{S} \subseteq \mathcal{N}$ are sets of origin and destination nodes. The route (path) set for connecting an OD pair rs ($\in \mathcal{C}$) is denoted as $\mathcal{P}_{rs}$. By Wardrop first principle (Wardrop, 1952), $\forall p \in \mathcal{P}_{rs}$ with positive path flow (i.e., $x_p > 0$),

$$c_p(\boldsymbol{x}) = \min_{q \in \mathcal{P}_{rs}} c_q(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x}$ is the path flow vector and $c_p(\boldsymbol{x}) = \sum_{a \in p} t_a v_a$, where $c_p(\cdot)$ and $t_a(\cdot)$ is the path and link travel costs functions, respectively.

Based on the definition of Wardrop user equilibrium, Beckmann et al. (1955) proposed an equivalent convex mathematical formalization as (2) to facilitate the calculation of link traffic flow $\boldsymbol{v}$ for large-scale transportation networks. Wardrop UE models have been widely used to characterize vehicle traffic in different contexts, such as charging infrastructure planning (Guo et al., 2016), EV power grid services (Baghali et al., 2022), and spatial pricing of ride-sourcing (Afifah and Guo, 2022).

$$\underset{\boldsymbol{v}}{\text{minimize}} \quad \sum_{a \in \mathcal{A}} \int_0^{v_a} t_a(z)\, dz \tag{2a}$$

$$\text{subject to} \quad \boldsymbol{v} \in \mathcal{X}_v, \tag{2b}$$

where $\mathcal{X}_v$ is the projection into the arc flows space of feasible flow set $\mathcal{X} =: \{(\boldsymbol{v}, \boldsymbol{x})\}$.

However, the definition of Wardrop equilibrium and the corresponding formulation and properties are limited to the deterministic case. Stochastic user equilibrium, pioneered by Daganzo and Sheffi (1977), was proposed to model one-stage routing decision making considering perceived uncertainties of travel time. Travelers navigate through the network based on their perceived travel time and there are no en-route information updates. Therefore, the routing decision making of travelers is made at the beginning of the trip, i.e., single-stage decision making. However, in our proposed model, travelers know the probability distribution of travel times a priori and could receive information updates on the actual realization if they pass information nodes.

3.1.2. Adaptive traffic assignment models

To distinguish from the classic (one-stage) stochastic user equilibrium notion (Daganzo and Sheffi, 1977), we denote our traffic equilibrium as stochastic user equilibrium with recourse (SUER) in this study, as formally defined in Definition 1. We assume that drivers are rational and risk-neutral, and have a common prior belief about the link travel time. The prior belief is measured by the probability distribution of system uncertain parameters ξ, which can be estimated from historical traffic data. Note that the probability distribution of parameters ξ is exogenously given while the equilibrium travel time endogenously determined by all the travelers' routing decisions. I2V devices will provide information updates on ξ when CAVs pass through I2V nodes, denoted as $\mathcal{N}_I$. We further assume that information nodes will provide true and consistent information on system uncertain parameters ξ, so that for the first time a driver passing through node $n \in \mathcal{N}_I$, he/she will receive a specific realization of ξ, which will be taken into account for their future routing decisions. This setting indicates that there are at most two possible stages in which a traveler could be in, namely before and after receiving information. In the first stage (i.e., before they receive information updates), travelers make routing decisions based on the expectation of the path travel time given prior knowledge of the probability distributions of link performance functions. In the second stage (i.e., after they receive information updates), travelers adjust their routes based on the new information. Travelers will receive information when they pass an information node and we do not assume travelers need to receive all information at the same time. Note that our traffic equilibrium aligns with the user equilibrium with recourse (UER) notion originally proposed in Unnikrishnan and Waller (2009). However, instead of using a policy-based modeling approach, we adopt a completely different modeling strategy inspired by two-stage stochastic programming with recourse. We note that since the proposed modeling framework is based on two-stage stochastic programming concepts, the proposed modeling strategies can be naturally extended to a multi-stage SUER case, where information nodes reveal partial information about the systems. To keep the focus clear, we left the treatment of multi-stage SUER for the future.

Definition 1 (*Two-Stage SUER*)**.** In two-stage SUER, the following two conditions hold for each origin and destination:

- *before receiving information, the expected travel times/costs on all paths used from the origin to the destination in the first stage are equal, and less than those which would be experienced by a single vehicle on average on any unused path;*
- *after receiving information, the travel times/costs on all paths used from the (first) information node to the destination in the second stage are equal, and less than those which would be experienced by a single vehicle on any unused path.*

Notice that the drivers may adjust their travel routes after receiving information updates. Therefore, "paths selected in the first stage" should bundle all the possible paths that a driver can still choose after they receive information. More formally, we define the "bundled paths" as *hyperpath*, as shown in Definition 2.

Definition 2 (*Hyperpath*)**.** A hyperpath, denoted as $\mathcal{P}_k^{rs}$, is the kth set of paths that connect the same OD pair rs and share the same sequence of links from origin r to the first information node i ($i = s$ if there are no information nodes on a path).

Inspired by the mathematical formulation of Wardrop equilibrium proposed by Beckmann et al. (1955), we construct a convex optimization problem, whose optimal solutions are consistent with our definition of two-stage SUER. The convex optimization model that can generate the equilibrium outcome according to Definition 1 is presented in model (3).

$$\min_{x_p(\xi)\geq 0,\ \forall p,\xi} \quad \mathbb{E}_\xi \sum_{a\in\mathcal{A}} \int_0^{v_a(\xi)} t_a(u,\xi)\mathrm{d}u \tag{3a}$$

$$\text{s.t.} \quad v_a(\xi) = \sum_{rs\in RS}\sum_{p\in\mathcal{P}^{rs}} \delta_{ap} x_p(\xi),\ \forall a\in\mathcal{A},\xi\in\Xi \tag{3b}$$

$$(\gamma^{rs}(\xi)) \quad \sum_{p\in\mathcal{P}^{rs}} x_p(\xi) = q^{rs},\ \forall rs\in RS,\xi\in\Xi \tag{3c}$$

$$(\lambda^{rs}_{a,k}(\xi)) \quad \sum_{p\in\mathcal{P}^{rs}_k} \delta^+_{ap} x_p(\xi) = x^{rs}_{a,k},\ \forall rs\in RS, a\in\mathcal{A},\xi\in\Xi \tag{3d}$$

where:

v_a : traffic flow on link a;

$t_a(\cdot,\cdot)$: travel time function of link a;

q^{rs} : travel demand from r to s (model input);

$x^{rs}_p(\xi)$: traffic flow on path p that connects r,s at scenario ξ;

$x^{rs}_{a,k}$: traffic flow on link a from r to s belong to kth hyperpath that have not received information on ξ;

δ_{ap} : link-path incidence scaler, which equals to 1 if link a belongs to path p, and 0 otherwise;

δ^+_{ap} : link-path incidence scaler, which equals to 1 if link a belongs to path p and has not passed through any node n $\in \mathcal{N}_I$, and 0 otherwise;

$\mathcal{P}^{rs}_k$: The path set for those paths share the same sequence of links before they receive uncertainties information, with k denote the kth set for rs;

$\boldsymbol{\gamma},\boldsymbol{\lambda}$: dual variables of corresponding constraints.

The basic idea of the model (3) is that we first relax path flow x_p and links flow v_a to be scenario dependent, and then we enforce non-anticipitivity constraint (3d) to guarantee that the paths flow is measurable according to the uncertainty set, i.e. the flow on the path segments before receiving information (i.e., $\delta^+_{ap}=1$) should not be measurable by ξ. In other words, the traffic flow before receiving information should be the same regardless of the scenario realization. Note that all scenarios are coupled in constraint (3d) and we cannot solve the traffic flow for each scenario separately. Constraint (3b) aggregates the path flow to each link for each scenario ξ; constraint (3c) restricts the summation of path flow connecting each OD pair should equal to OD demand. The objective function (3a) is constructed in this way so that the optimal solutions of the optimization problem (3) satisfies the two conditions of two-stage SUER, as stated in Definition 1. This result is stated more formally in Theorem 1.

Theorem 1 (*Two-Stage SUER*)**.** *The traffic flow pattern is following two-stage SUER principles (i.e., Definition 1) if and only if it is the optimal solution to the optimization problem (3).*

Proof. See Appendix. □

The proposed formulation is intuitive in the sense that the final formulation, i.e., model (3), closely relates to the classic UE formulation. However, the rigorous proof of the equivalence between the solutions of the proposed model (3) and the traffic patterns following the definition of two-stage SUER (i.e., Definition 1) is non-trivial and novel. As far as the authors are aware, no other work has utilized a stochastic programming concept to model a UER problem, which could lead to significant computational advantages since the proposed models belong to multi-stage stochastic programming problems, for which a large body of literature has been dedicated to developing efficient computational algorithms.

In addition, we note that there are two other well-known modeling frameworks, DTA and Markovian traffic equilibrium (MTE), that could be relevant for the studied problem, but are not adopted in this study. The reasons are explained as follows.

On the one hand, we adopt static-equilibrium-based models instead of DTA for three main reasons. First, we focus on the stochastic nature of the transportation system instead of its dynamic nature. In our setting, we assume link performance functions are uncertain, whose realizations can only be known when travelers pass by certain information nodes. In other words, the realizations of link performance functions are static (do not depend on time), although the link performance functions themselves are uncertain (unknown before reaching information nodes). Second, while DTA could be used to model en-route rerouting with uncertainties, it is computationally costly, while our proposed method is a convex optimization, as shown in Section 3.1.2. Third, static approaches have been demonstrated to be effective to represent the adaptive routing decision of travelers in transportation networks (Unnikrishnan and Waller, 2009).

On the other hand, although the fundamental concept of MTE appears relevant, the current mathematical framework of MTE (e.g., Baillon and Cominetti (2008)) may not be suitable for studying the proposed problem. This is because the MTE formulation focuses on the variation of travelers' perception of link travel time (Baillon and Cominetti, 2008). In other words, the stochasticity in

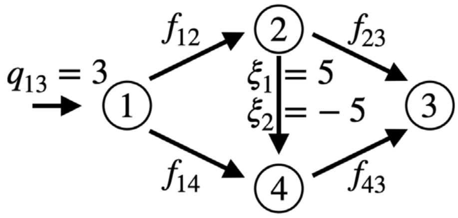


Fig. 1. Four-node network with stochastic link cost.

MTE refers to the stochasticity in travelers' perception of the link travel time, rather than to the stochasticity of the link performance function itself. This is an important distinction because following the MTE setting, one can adopt discrete choice models to model the traffic distribution at any given node. However, in the case when the link performance function is uncertain, travelers make decisions considering expected travel time based on prior knowledge and information updates on the probability distributions and the discrete choice model is not applicable.

In this paper, we consider information provided to all travelers passing by information nodes. But we note that the proposed model (3) can be extended to consider OD-dependent information sharing. First, when information is only shared with travelers departing from or arriving at certain nodes, those nodes can be treated information nodes and all we need to do is to update information node set $\mathcal{N}_I$ accordingly. Second, when en-route information is not provided to all travelers passing by the information nodes but only to a subset of travelers belonging to certain origins and/or destinations, we need to specify the non-anticipativity constraint (3d) to reflect that travelers from each OD can receive different information. The key parameter needed to be specified here is δ^+_{ap}, which indicates whether travelers choosing path p have received information or not when they arrive at link a. For an OD pair rs that information is not shared with, $\delta^+_{ap} = 0$ for all $a \in \mathcal{A}, p \in \mathcal{P}^{rs}$.

3.1.3. Illustrative example for two-stage SUER

To illustrate two-stage SUER, we use a simple four-node network, as shown in Fig. 1.

In Fig. 1, there are 3 units of travel demand from node 1 to node 3. The link cost, as a function of link flow f, is shown on each link. The travel cost of link 2-4, denoted as a random variable ξ, is uncertain, which could be influenced by random events, such as traffic accident/incidents, weather, tolls, recommended speed, etc. We assume ξ has two realizations, 5 and -5,[2] with equal probability in this toy example. Node 2 is the only information node that reveals the realization on ξ to the drivers passing by node 2. From node 1 to node 3, there are three paths: $p_1 = \{1,2,3\}, p_2 = \{1,2,4,3\}, p_3 = \{1,4,3\}$. There are two possible hyperpaths in the first stage: $\mathcal{P}^{rs}_1 = \{p_1, p_2\}$ and $\mathcal{P}^{rs}_2 = \{p_3\}$. In the first stage, drivers will decide whether to take link 1-2 or 1-4, considering the historic probability distribution of ξ. If drivers choose link 1-2 in the first stage, they will have an option later to decide whether to go on route 2-4-3 or route 2-3 based on the new information received at node 2; while drivers who choose link 1-4 will not receive information updates during their travel and will follow path 1-4-3.

The optimal solutions of model (3) are shown in Table 1. We can see that the traffic flow on the hyperpaths $\mathcal{P}^{rs}_1$ and $\mathcal{P}^{rs}_2$ are independent of scenario parameter ξ. This is because of non-anticipitivity constraints so that drivers are not able to make scenario-dependent routing decisions before vehicles receive information. But after vehicles receive information updates, the traffic will redistribute between different paths in each hyperpath depending on scenario ξ. For example, we can see that all traffic (7/3 units) in hyperpath $\mathcal{P}^{rs}_1$ select path p_1 in scenario ξ_1 and they select path p_2 in scenario ξ_2. From the (expected) costs, as presented in Table 1, we can verify that: (1) Hyperpath $\mathcal{P}^{rs}_1 = \{p_1, p_2\}$ and $\mathcal{P}^{rs}_2 = \{p_3\}$ have the same expected cost[3]; and (2) p_1 has lower second-stage travel cost than p_2 when $\xi = 5$ so that p_1 is selected while p_2 is unused in the second stage for this scenario. Opposite observations hold when $\xi = -5$. Therefore, the solutions of model (3) indeed satisfy Definition 1.

The traffic patterns reported in Table 1 are dramatically different from the classic user equilibrium solutions with expected link costs on link $2-4$, where $x_{p_1} = x_{p_3} = 1.5$, and $x_{p_2} = 0$. The interpretation for this difference is that because I2V is available at node 2, CAVs expect to receive information updates if they travel through $\{p_1, p_2\}$ in the first stage. Therefore, more CAVs are attracted to choose $\{p_1, p_2\}$ compared to $\{p_3\}$ in a two-stage SUER in contrast to the user equilibrium solutions under Wardrop first principal.

We also note that equilibrium traffic patterns may also be sensitive to where information is shared. For example, if information on ξ is shared at node 1, all drivers will have perfect information about link cost of $2-4$ at the beginning of the trips. Therefore, the traffic equilibrium solutions are scenario dependent with $x_{p_1} = x_{p_3} = 1.5$, $x_{p_2} = 0$ at scenario ξ_1 and $x_{p_1} = x_{p_3} = 0$, $x_{p_2} = 3$ at scenario ξ_2.

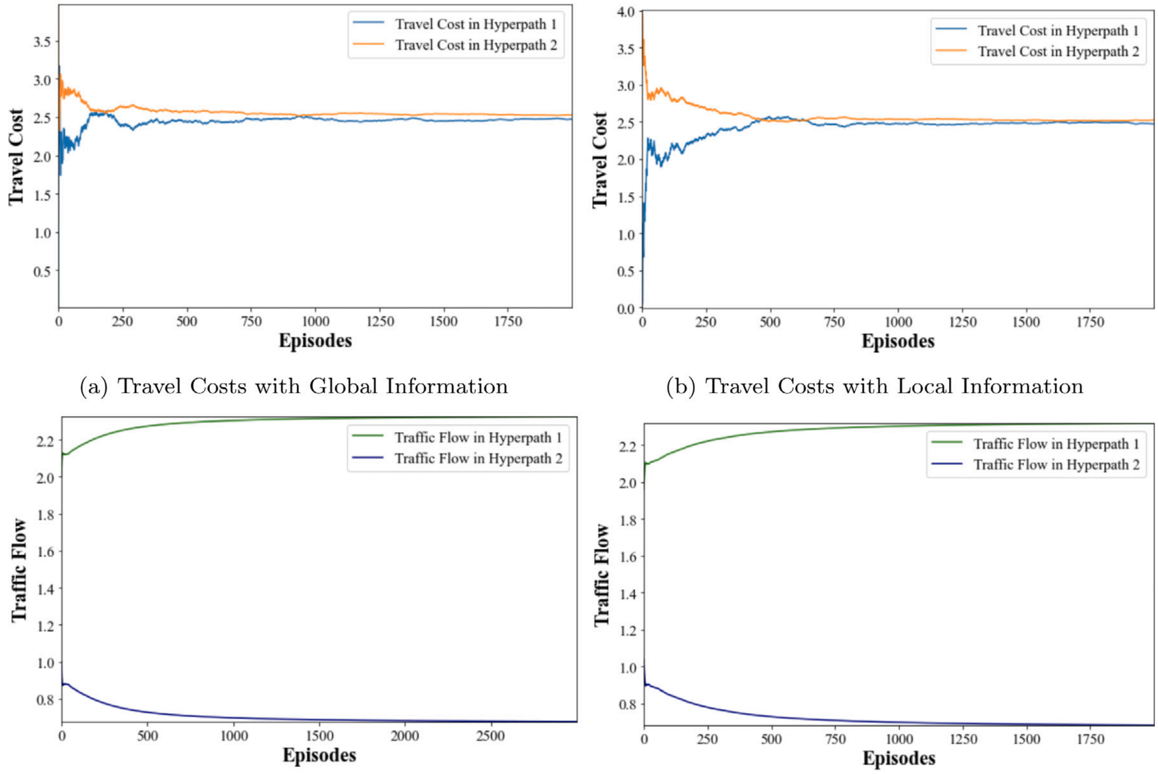
To demonstrate the traffic patterns will be indeed stabilized given that travelers will update their prior knowledge on the probability distribution of link performance functions and receive en-route information updates, we have simulated the travelers' en-route rerouting behaviors in response to uncertainties and information updates using the same example shown in Fig. 1. We consider

[2] We select 5 and −5 to simplify the verification of solutions without losing generality. Negative travel costs are possible given the possibility that drivers may have incentives (e.g., scenery) to use certain routes.

[3] Note that although the expectation of travel costs of both $\mathcal{P}^{rs}_1$ and $\mathcal{P}^{rs}_2$ are the same, $\mathcal{P}^{rs}_1$ has higher variance of travel costs compared to $\mathcal{P}^{rs}_2$. Therefore, for those travelers value reliability of travel costs, more traffic will be shifted from $\mathcal{P}^{rs}_1$ to $\mathcal{P}^{rs}_2$. This is beyond the scope of this research and will be left for future investigation.

Table 1
Equilibrium solutions with risk-neutral adaptive behaviors.

Path	Flow		Travel costs			
	ξ_1	ξ_2	ξ_1	ξ_2	Exp.	Var.
p_1	7/3	0	14/3	–	2.5	9.4
p_2	0	7/3	–	1/3		
p_3	2/3	2/3	4/3	11/3	2.5	2.7



(a) Travel Costs with Global Information (b) Travel Costs with Local Information

(c) Traffic Flow with Global Information (d) Traffic Flow with Local Information

Fig. 2. Convergence of traffic patterns and expected travel time.

two cases. The first case (denoted as the global information case) assumes that travelers will learn the probability distributions for all links regardless of whether they travel on the links or not. This case is plausible considering travelers can get access to the realization of link performance through news or social media afterward. In the second case (denoted as the local information case), we assume that travelers will only learn the probability distributions on the links they travel on. In the simulation, travelers will update their knowledge of probability distributions and select the routes to minimize the expected travel time based on their up-to-date information set. Figs. 2(a) and 2(b) demonstrate that the expected travel costs of both hyperpaths will converge to an equilibrium travel costs identical to our model solution in Table 1 for both cases. Similarly, the traffic patterns converge to those in Table 1, as shown in Figs. 2(c) and 2(d).

3.2. Parametric estimation

Network modeling and system safety assessment require link-level information (i.e., traffic link performance function and link crash risk). In this subsection, we will discuss how to estimate the key parameters using measurable data.

3.2.1. Link performance function

Researchers have used different link performance functions in the past (Branston, 1976). By far, the Bureau of Public Roads (BPR) function is one of the most widely adopted functions because of its simplicity and convenience to represent congestion behavior and free-flow travel time. We adopt the BPR function in this paper. The BPR functions are expressed as Eq. (4).

$$t = t_0[1 + \alpha(\frac{v}{c})^{\beta}] \tag{4}$$

where t_0, α, β, and c are parameters that define the shape of the BPR function and needed to be estimated for different types of roadways. The travel time will increase with the ratio of the flow v and the capacity c to represent congestion effects.

In a stochastic environment, parameters t_0, α, c, and β in link performance function (4) could be random variables. In this paper, we assume that with an increasing amount of traffic data, parameters in link performance functions can be categorized as probability distributions. The uncertainties could be due to recurring events that will affect the link travel time, such as weather events, construction/maintenance activities, traffic accidents, etc. Travelers will learn the probability distribution of link performance functions through day-to-day learning. While the link performance functions are uncertain, the realization of uncertainties in a given scenario does not change during the study period, thus warranting the stability of the traffic. The link performance function at a specific scenario $\xi \in \Xi$ can be expressed in Eq. (5).

$$t_{\xi} = t_{0,\xi}[1 + \alpha_{\xi}(\frac{v_{\xi}}{c_{\xi}})^{\beta_{\xi}}] \tag{5}$$

where $\boldsymbol{\xi} = \{t_{0,\xi}, \alpha_{\xi}, c_{\xi}, \beta_{\xi}\}$ are random parameters. At the beginning of the trips, drivers have a prior belief about the probability distribution of the random parameters, which could be updated along the CAVs' travel paths if I2V information sharing is available. Note that, without loss of generality, the information that CAVs receive from I2V infrastructure can be an updated probability distribution of these random parameters instead of a specific realization.

Link travel time t will depend on the uncertain scenario $\boldsymbol{\xi}$ in addition to link flow v. To estimate the uncertain parameters for a specific scenario, one can use non-linear least squares regression. The input data for the regression model is (t, v), which could be simulated in a microsimulation platform when they are not able to be directly measured from real-world traffic sensors.

3.2.2. Collision risk function

Collision risks can be defined in multiple ways. When crash data is available, collision risk can be measured as the probability of collision happening under specific traffic conditions. In this section, we will present the methodology to calculate collision risk. The calculated collision risk will be further used to estimate the relationship between collision risk and link traffic flow.

When real-world collision data is available, we encode the collision risk using a binary variable Y, with 1 indicating collision happens and 0 indicating no collision. Since Y is a binary variable, we propose a discrete choice-based crash risk model to formulate the probability of a collision p. The discrete choice-based crash risk model is also adopted by Shi and Abdel-Aty (2015) and Hasibur Rahman and Abdel-Aty (2021).

The target variable is a binary indicator Y indicating whether collision happens or not with probability p. In other words, Y is a Bernoulli trial and can be expressed as a probability distribution (6).

$$Y \sim \text{Bernoulli}(p) \tag{6}$$

Discrete choice model is able to estimate the probability p, indicating that with a probability of p collision may happen (i.e., $Y = 1$) and with a probability of $1 - p$ otherwise. The formulation of the logit model is shown in Eq. (7) for any given link.

$$logit\,(p) = \log\left(\frac{p}{1-p}\right) = \beta_0 + X\beta_1 \tag{7}$$

where:

p : the likelihood of collision happens

β_0 : the constant

X : the explanatory variables, such as traffic flow q

β_1 : the coefficients of the explanatory variables

While explanatory variables X can include different variables, in this study, we focus on traffic flow q and treat other factors as exogenous variables, whose impacts will be included in link-specific constant coefficient β_0. Therefore, the collision risk for each link is shown in Eq. (8).

$$CR^{\text{Link}} = p = \frac{e^{\beta_0+\beta_1 v}}{1+e^{\beta_0+\beta_1 v}} \tag{8}$$

Remark 1. Since traffic flow is one of the most important factors influencing traffic safety (Abdel-Aty and Radwan, 2000), we characterize the collision risk of a link (Eq. (8)) as a function of the traffic flow of that link. However, we note that our proposed model can be extended to incorporate the potential correlation of link safety risk functions among neighboring links for different scenarios. For example, in a given scenario ξ, Eq. (8) can be updated as $CR^{\text{Link},\xi} = p = \frac{e^{\beta_{0,\xi}+\beta_{1,\xi}q}}{1+e^{\beta_{0,\xi}+\beta_{1,\xi}q}}$. This can offer the flexibility to model the link collision risk considering the effect of neighbor links because the parameters for neighbor links can be specified for each scenario to consider their correlation. Since the focus of this paper is to propose a new network model to study the traffic equilibrium with en-route information updates, we will leave the estimation and implementation of the neighboring effects for the future.

Remark 2. Scenario probability and crash risk are interdependent. But there are three main reasons that we did not consider the endogeneity of scenario probability in this modeling framework. First, the scenario we consider is not solely based on whether accidents occurred or not. The link performance functions in each scenario are determined by multiple factors, including weather events, traffic management, and/or accidents. The probability distribution of the link performance function is learned from the travel experience of travelers. Therefore, the probability of collision does not equal the probability of a certain scenario. Since accidents are only part of the factors that define the scenarios, in this manuscript, we treat the scenario probability as exogenous. Second, accidents are rare events and the probability of accidents is usually low and difficult to measure directly using a probability distribution function. In such cases, only focusing on accidents to represent each scenario may not be practical in a real-world setting. Third, if scenario only represents accidents and we can measure accident probability, we can use iterative approaches to find the fixed point of the following function $x = f(x)$, where x is the probability vector of scenarios (i.e., $pr(\xi)$) and $f(\cdot)$ is the mapping from $pr(\xi)$ to crash risk, which combines model (3) and Eq. (8). However, we note that the convergence of such an iterative approach may not be guaranteed for high-dimensional scenarios.

3.3. Network safety analyses

In order to conduct an analysis on the impact of information sharing locations on network safety, we need to mathematically formulate different information sharing strategies and embed them in the two-stage SUER model (3), and quantify the network safety metrics. This subsection focuses on these two aspects.

3.3.1. Representation of information sharing locations

Denote the set of I2V information sharing locations as $\mathcal{I} \subset \mathcal{N}$. When CAVs passed by these locations, CAVs will receive information updates for their future routing decision making.

Denote information node indicator variable $y_n, \forall n \in \mathcal{N}$. $y_n = 1$ if I2V infrastructure is placed at node n; and $y_n = 0$ otherwise. Denote a path that consists of K links, as $p = \{a_1, a_2, \dots, a_K\}$. δ^+_{ap} in Model (3) can be calculated using Eq. (9).

$$\delta^+_{ap} = \prod_{n \in \mathcal{N}_{ap}} (1 - y_n) \tag{9}$$

where $\mathcal{N}_{ap}$ is the node set on path p before reaching to link a.

Using Fig. 1 as an example, if I2V infrastructure is placed at node 2,

$$y_n = \begin{cases} 1, & \text{if } n = 2 \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

For path $p_2 = \{1, 2, 4, 3\}$ and link $a = \{(4, 3)\}$, we have $\mathcal{N}_{ap} = \{1, 2, 4\}$. Therefore, $\delta^+_{ap} = \prod_{n \in \mathcal{N}_{ap}} (1 - y_n) = 0$. In other words, CAVs traveling on path p_2 have received the information before they travel on link $\{(4, 3)\}$.

3.3.2. Network safety assessment

Given crash risk CR at link level as a function of link traffic flow q (i.e., Eq. (8)), we can derive the collision risk (CR) for the network as the total risk experienced by all the vehicles in the network. In a stochastic environment, the parameters β_0/β_1 in Eq. (8) for any link a will be random variables. We denoted the collision risk for link a at scenario ξ as $CR^{\text{Link}}_{a,\xi}$. Collision risk for the network at a specific scenario $\xi \in \Xi$ can be expressed in Eq. (11).

$$CR^{\text{Network}}_{\xi} = \sum_{a \in A} CR^{\text{Link}}_{a,\xi} v_{a,\xi} \qquad \forall \xi \in \Xi \tag{11}$$

where $v_{a,\xi}$ is the link flow at link a in scenario ξ.

In this study, the network safety indicator is measured as the expected network safety risks. We seek to evaluate the impacts of providing information at different locations on the expected network safety risks, CR^{Network}, which can be calculated in Eq. (12).

$$CR^{\text{Network}} = \mathbb{E}_{\xi}\{CR^{\text{Network}}_{\xi}\} \tag{12}$$

4. Numerical results

We tested our proposed methodologies on the Orlando transportation network, as shown in Fig. 3, which serves the traffic between three major attractions in Orlando metropolitan area, which are Orlando International Airport (Node 17), Universal Studio (Node 11), and Disney World (Node 3). The network covers an area of 78.87 mile^2. This network has a total of 27 two-way links, which comprise both freeways and multi-lane highways. The archived traffic data, collected by the Regional Integrated Transportation Information System (RITIS) from February 1st, 2020 to April 30th, 2020, contains speed, traffic volume, and traffic occupancy. The crash records during the studied time period were collected from Florida Highway Safety and Motor Vehicles. Based on these raw data, we are able to estimate the link flow, link travel time, and link crash risks at each hour, which will be used to estimate the link performance functions and link crash risk functions, as discussed in Section 3.2. In this section, we demonstrate the impacts of different information sharing strategies on network traffic distribution, mobility, and safety. For the base case, incident links include 14-17, 17-14, 3-5, and 5-3, which are four links with relatively higher collision risks. Each incident link could be in two scenarios: *normal scenario* and *incident scenario*. We differentiate the *incident scenario* from *normal scenario* by reducing the incident

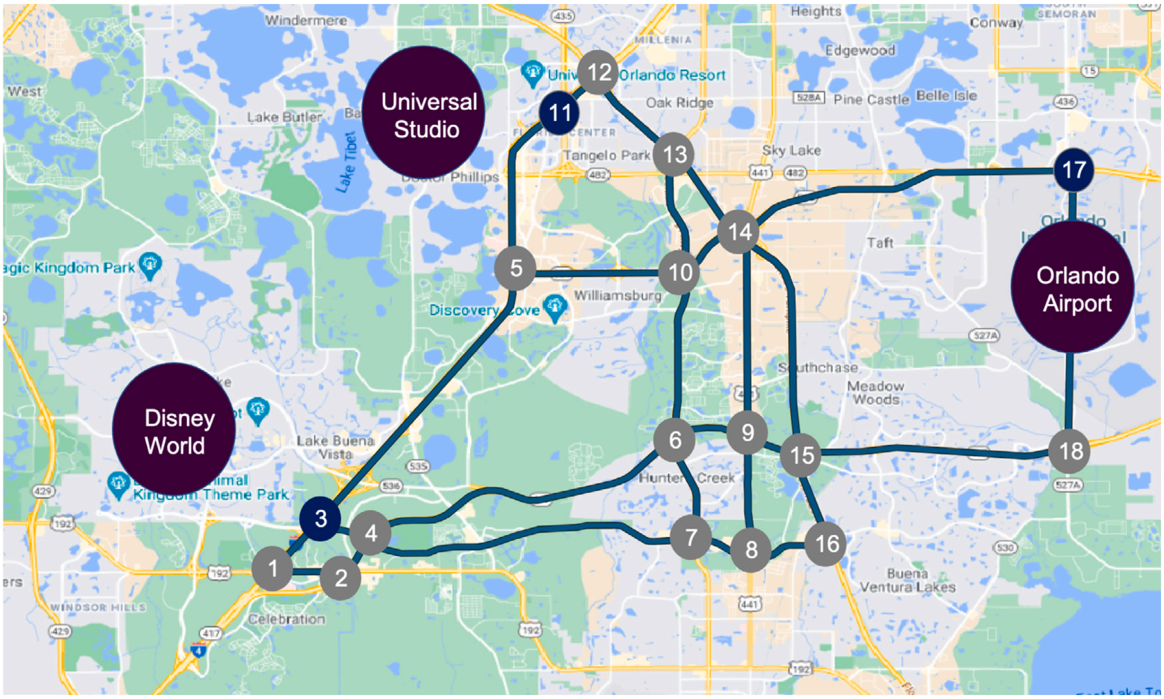


Fig. 3. Study area in Orlando.

links' capacity by 50% in Eq. (5). The capacity reduction could be due to incidents such as traffic accidents, weather events, and road condition deterioration. We consider information sharing at node 10 (a central node in the network) as the base case if not further specified.

In the remainder of this section, we first analyze the network mobility and safety for the base case; then we conduct sensitivity analyses on key parameter settings (including information sharing locations, incident severity, and OD demand).

4.1. Base case results

Both traffic congestion level and network collision risk could be different under different information sharing strategies due to rerouting. Travelers' routing decisions can be changed as en-route information is received. To demonstrate this, we show the traffic flow changes for O-D from node 17 (Orlando Airport) to node 3 (Disney World) in Fig. 4, which depicts the change in link traffic between *normal scenario* and *incident scenario* based on the information they receive at node 10. When travelers receive information at node 10 indicating that no incident happens in link 5-3 (i.e., *normal scenario*), more traffic will use the path 10-5-3. In contrast, when they receive information at node 10 indicating incident happens (i,e, *incident scenario*), travelers originally take the path 10-5-3 in *normal scenario* will re-route to path 10-6-4-3. The traffic on all the other links does not change because they either do not receive information updates along their routes or do not have better routing options despite knowing the incident occurred.

To quantify the impacts of information sharing on network mobility, we measure congestion using the "v/C" ratio (i.e., link flow/link capacity) of a link. Fig. 5 shows the relative difference in traffic congestion levels between two cases: (1) information is shared at node 10 and (2) no information is shared. Fig. 5(a) shows that in the *normal scenario*, when information is shared, links 10-5-3 significantly increase the congestion levels, while links 10-6-4-3 reduce the congestion levels, compared with the case when information is not shared. Opposite observation holds for the *incident scenario*. The reason is that for traffic going from node 10 to node 3, their routing will depend on the information they received about the state of link 5-3. On the other hand, for the links before information is received, congestion levels slightly increase in links 17-14-10 while slightly decrease in links 17-18-15-9-6 regardless of the actual realization on the incident links. This is because when node 10 is an information node, more traffic will prefer to pass by node 10 from node 17 so that they can receive information updates to inform their future rerouting decisions. From Fig. 5, we also observe that information sharing at node 10 mainly affects the traffic routing to node 3. The reason why information sharing does not affect the routing of travelers with destination to other nodes is that those travelers may not have better routing options to avoid incident links despite knowing incidents occurred (e.g., travelers to node 17 prefers link 14-17 regardless of incidents) or they simply do not utilize incident links regardless of which scenario the system is in (e.g., travelers to node 11).

Traffic rerouting caused by information sharing may further lead to changes in link collision risk over the network. Fig. 6 illustrates the relative difference of collision risk in each link between two cases: (1) information is shared at node 10 and (2) no information is shared. Fig. 6(a) shows that in the *normal scenario*, link 10-5 has higher collision risk, while links 10-6-4-3 have lower

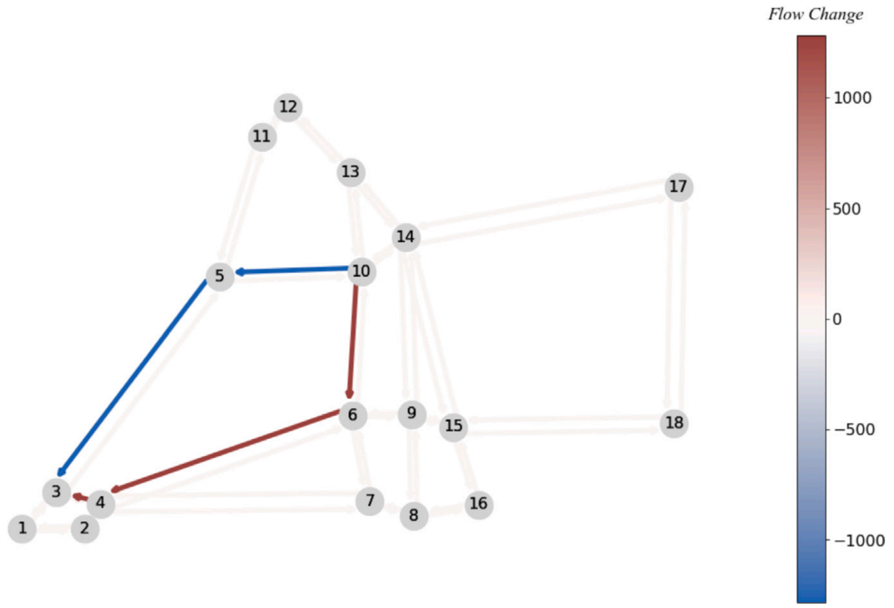


Fig. 4. Link flow changes between normal scenario and incident scenario for the base case.

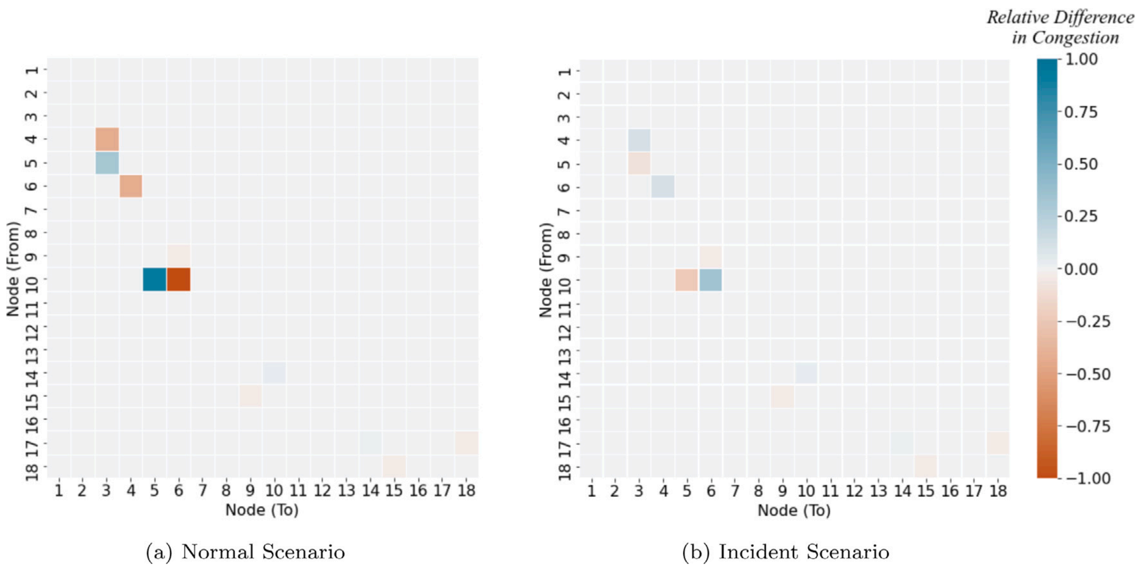


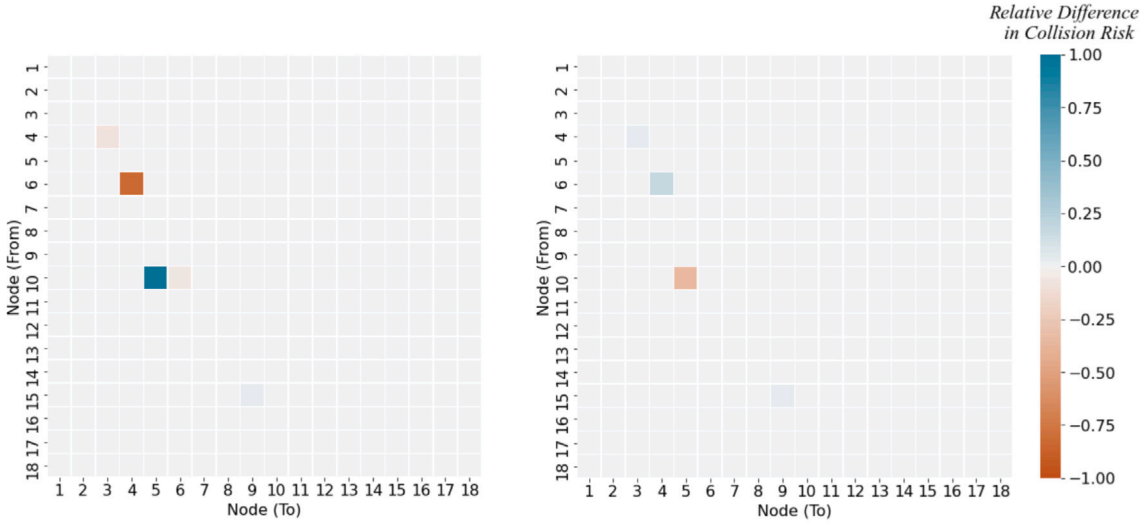
Fig. 5. Relative difference of congestion level between information sharing and no information sharing.

collision risk when information is shared compared with when no information is shared. Opposite results for these links are observed in Fig. 6(b) for the *incident scenario*. The increase/decrease of collision risk in these links is consistent with the increase/decrease of congestion levels as shown in Fig. 5(b). However, we note that a lower congestion level does not always imply a lower collision risk. For example, link 15-9 has a lower congestion level when information is shared in both the *normal and incident scenarios*, but this link has a slightly higher collision risk in both scenarios. These results indicate the value of using a network perspective to model traffic redistribution and to gain a better understanding of the impact of information sharing on network traffic safety.

4.2. Sensitivity analyses

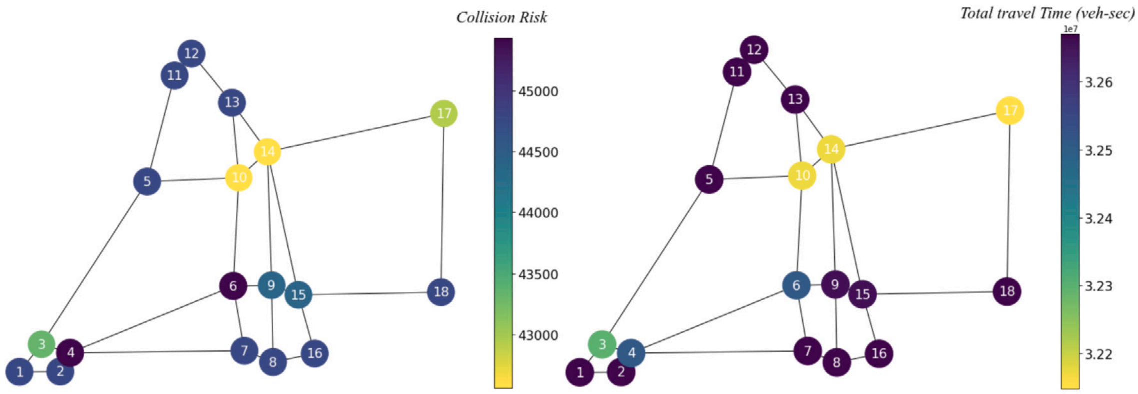
4.2.1. Sensitivity of information sharing strategies

First, we investigate the network effects of information sharing at different individual nodes on network safety and mobility, as shown in Fig. 7. In Fig. 7(a), we can see that information sharing at nodes 10 or 14 provides the most network benefits in



(a) Normal Scenario (b) Incident Scenario

Fig. 6. Comparison of collision risk at each link between information sharing vs. no information sharing.



(a) Network Safety (b) Network Mobility

Fig. 7. Effect of information sharing at individual nodes separately.

terms of reducing network collision risk. The reason could be that nodes 10 and 14 are two central nodes in the network and information sharing at these two locations allow traffic passing by (traffic volume 11,240 veh/h and 13,549 veh/h at node 10 and 14, respectively) to update their routing decisions, which are beneficial to the whole system. However, one should not conclude that sharing information at nodes with high traffic volume is always better. For example, information sharing at node 4 and node 6, which also have high traffic volume at around 11,255 veh/h, leads to the highest network collision risks. This again indicates the necessity of using a network modeling strategy to capture the complex interaction among the system.

In Fig. 7(b), we see that information sharing at node 17 leads to the least total travel time in the network. Notice that information sharing at nodes 10 or 14 also benefits travelers in terms of network mobility. This is because incidents may happen at the links 3-5/5-3 and 14-17/17-14, which are on the shortest path of OD demand 3-17 and 17-3. Information sharing at nodes 10 and 14 allows a large amount of traffic from node 3 (47% of the total incoming traffic at nodes 10/14) and node 17 (50% of the total incoming traffic at nodes 10/14) to choose alternative paths depending on if an incident happens or not. In contrast, information sharing at node 5 is not a good option because, on one hand, traffic from node 3 will be encouraged to utilize link 3-5 to access information, which may experience a delay if an incident happens on that link. On the other hand, traffic from link 10-5 and link 11-5, which is 52% of the total incoming traffic at node 5, do not benefit from receiving information due to a lack of alternative travel routes.

Second, we compare between four information sharing strategies, including (1) *best single node*: information sharing at the best single node (i.e., nodes 10 or 14 for safety and node 17 for mobility), (2) *worst single node*: information sharing at the worst single node (i.e., nodes 4 or 6 for safety and nodes 1, 2, 5, 7, 8, 9, 11, 12, 13, 15, 16, or 18 for mobility), (3) *perfect information sharing*:

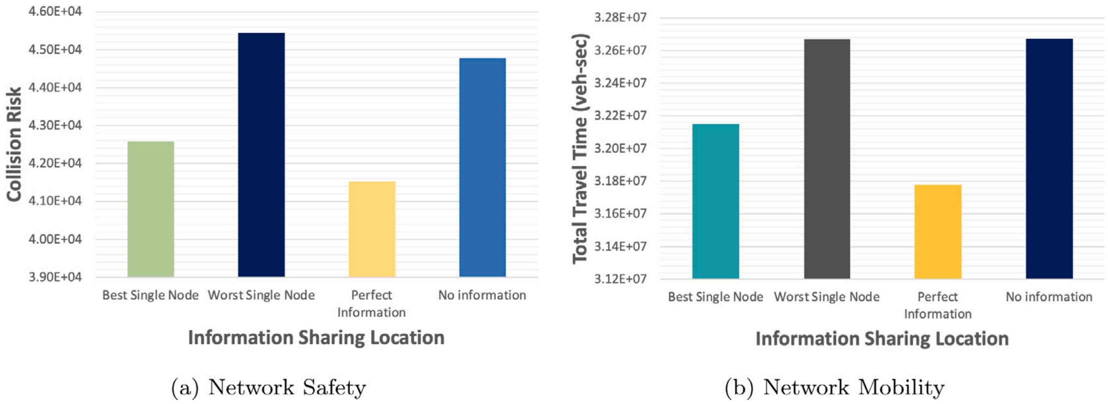


Fig. 8. Impact of perfect and no information sharing.

information sharing at all the nodes, and (4) *no information sharing*: no information sharing in any nodes, with network safety and mobility results shown in Figs. 8(a) and 8(b), respectively. We can see that *perfect information sharing* is the best strategy for both network safety and mobility. However, we note that no information sharing is not always the worst strategy. For example, in Fig. 8(a), sharing information at node 4 or 6 is the worse strategy compared with no information sharing in terms of network safety. In Fig. 8(b), no information sharing and information sharing at the worst single nodes will lead to the similar total travel time in the network. The reason for this is that information sharing allows for additional travelers rerouting but travelers make rerouting decisions to optimize their own utilities without considering the potential negative externalities for other travelers.

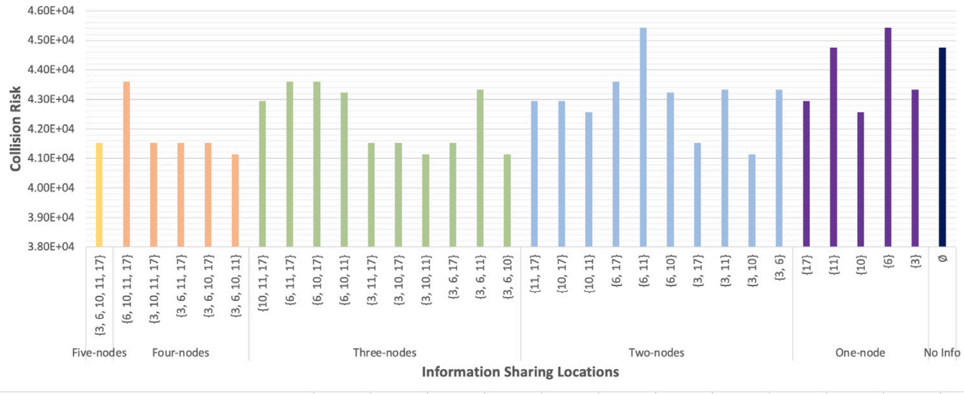
Third, we investigate the Pareto optimal information sharing locations for both network safety and mobility. We assume nodes 3, 6, 10, 11, and 17 are candidate information sharing nodes. Figs. 9(a) and 9(b) show the impacts of information sharing for all the subsets of the candidate information sharing nodes for network safety and mobility, respectively. From Fig. 9(a), it can be seen that the network collision risk is the lowest when information is shared at nodes 3 and 10.[4] Notice that this is even better than the perfect information sharing strategy as shown in Fig. 8(a). This observation indicates that more information is not necessarily better for the whole system, which is consistent with observations in existing studies on "information paradox" (Unnikrishnan and Waller, 2009; Lindsey et al., 2014; Acemoglu et al., 2018; Liu and Liu, 2018). In terms of mobility, Fig. 8(b) shows that if the information is shared at nodes 3 and 17, it provides the best system mobility outcome. To determine the Pareto optimal information sharing strategies, Fig. 9(c) shows the Pareto frontier when both safety and mobility are considered. Information sharing at nodes {3,17} and nodes {3,10} are two Pareto optimal information sharing strategies with the least amount of information sharing nodes.

4.2.2. Sensitivity of O-D demand

In this section, we investigate the sensitivity of different levels of O-D demand on the average collision risk and the average travel time[5] of the network for both information shared and no information shared cases. We consider a range of O-D demand from 40% decrease to 40% increase from the base case O-D demand, with the results shown in Fig. 10. From Fig. 10(b), we can see that a higher level of O-D demand results in a monotone increase in the system average travel time because of the non-linear congestion effects. The average travel time in the network is slightly higher when information is not shared than when information is shared for all O-D demand levels (Fig. 10(b)). However, from the perspective of network safety, we observed a more complex pattern. As shown in Fig. 10(a), the average collision risk in the network decreases slightly when O-D demand increases from 60% to 80% and increases significantly from 80% to 120% when information is shared. The increase rate slows down when OD-demand increases from 120% to 140%. When information is not shared, the average collision risk is much higher than when information is shared, except when O-D demand is at 60% of the base case value. This is because information sharing affects system collision risk in two major ways. On one hand, the travelers receive information can make route adjustments to avoid experiencing high collision risks on the incident links. On the other hand, information sharing may encourage travelers to travel to the information nodes, which may have negative impacts on system collision risk. When the transportation network is not congested, collision risks may not be high on the incident links. But when information is shared at a node, travelers may still try to select the routes to access information updates and reroute afterward to maximize their own expected utility, which creates more negative externalities to the system compared with the no-information-sharing case.

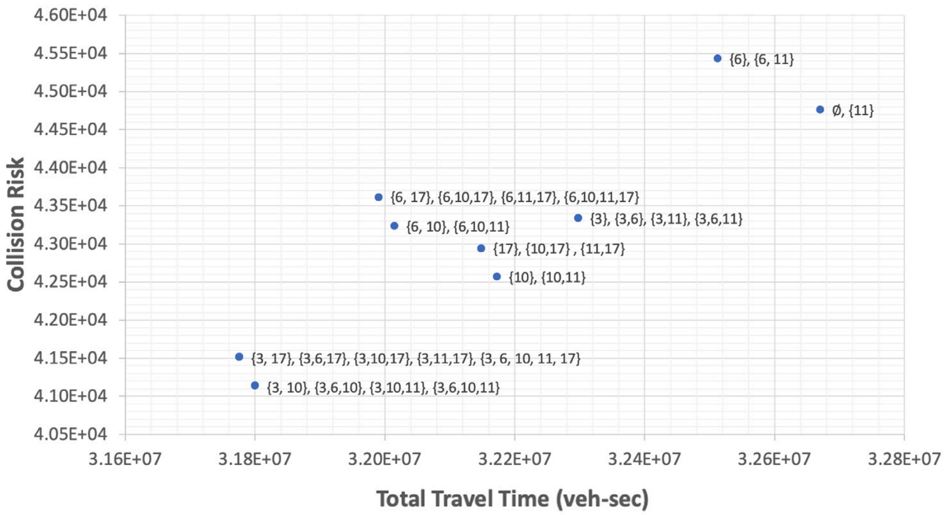
[4] When different combinations have the same impacts on the system, we prefer the combination with the least information sharing nodes to save installation costs.

[5] The reason we measure mobility and safety using average risk/time instead of total risk/time is to normalize the impacts by O-D demand level.



(a) Network Safety

(b) Network Mobility

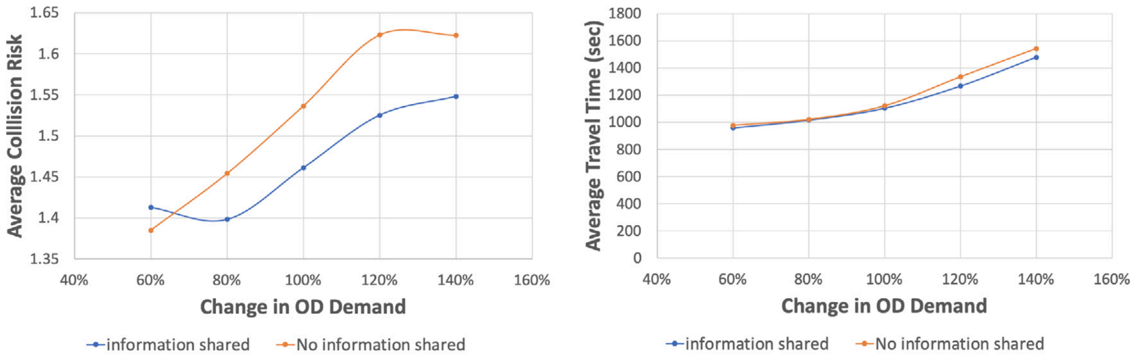


(c) Pareto Graph

Fig. 9. Impact of at information sharing at selected nodes.

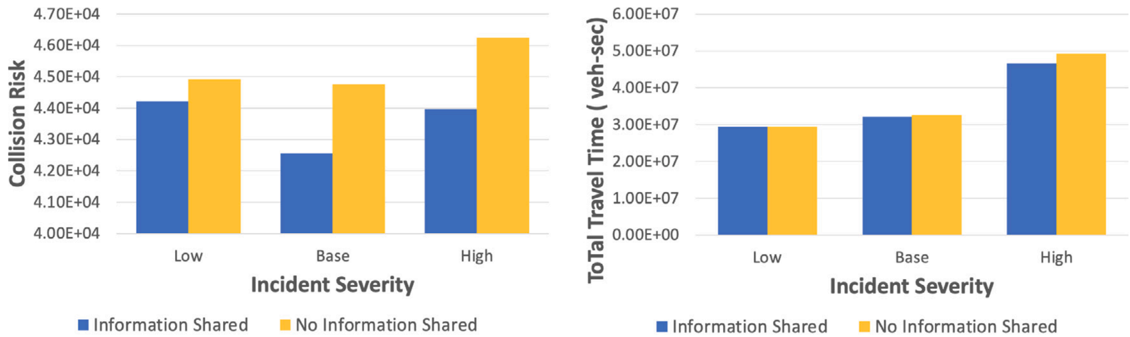
4.2.3. *Sensitivity analysis on incident severity*

Fig. 11 depicts the impact of various incident severity levels on the safety and mobility of the network for both information shared and no information shared cases. Low severity (20% capacity reduction) and high severity (80% capacity reduction) levels



(a) Network Safety (b) Network Mobility

Fig. 10. Effect of change in O-D demand.



(a) Network Safety (b) Network Mobility

Fig. 11. Effect of incident severity on network mobility and safety.

have been used and compared with the base case (50% capacity reduction). Fig. 11 shows that both the network collision risks and the total travel time are slightly higher when no information is shared compared with when information is shared, which means information sharing is beneficial regardless of different incident severity levels. Fig. 11(b) shows that the total travel time of the network increases as the incident severity level increases, which is intuitive since the overall network capacity decreases with a higher level of incident severity. This observation is consistent regardless of whether the information is shared or not. However, for network safety, the relationship between the network collision risk and the severity of the incident may not be monotone especially when information is shared. For example, a more severe incident from the "Low" case to the "Base" case leads to a much lower level of network collision risks. This is because different levels of incident severity will increase the travel time differently on those impacted links, which leads to different rerouting strategies of individual travelers when information is shared, resulting in different safety externalities on the system.

5. Discussion

In this study, we investigate the impact of information sharing locations on transportation network safety and mobility. We propose a transportation network modeling framework to model the adaptive routing behavior of CAVs given information updates at different information sharing locations. We estimate the traffic mobility parameters using traffic data for the Orlando network and the crash risk of each link from real crash data. The proposed methodology allows us to identify the Pareto optimal locations to share information with CAVs that helps to promote the safety and mobility of the whole network. Through numerical experiments, we found that: (1) the optimal information sharing strategies depend on specific network configurations and more/less information is not always better/worse for the network mobility and safety; (2) locational information sharing will encourage the traffic to travel through information nodes so that they can make informed rerouting decision; (3) while mobility is monotone decreasing with the levels of OD demand and incident severity, network collision risks may have a more complex relationship, which can only be quantified with a network modeling perspective and real historical crash data.

This research can be extended in several directions. First, we assume there is only one universal piece of information in the network. How to extend the proposed methodology to consider heterogeneous information sharing remains to be solved. But extending the proposed SUER from two-stage to multi-stage will not change the overall modeling strategies. Second, in addition to information sharing locations, other aspects of information sharing strategies can be investigated, such as what information to share, to which group of CAVs to share information. Third, the study can be extended for mixed traffic that forms of CAVs and regular vehicles. Fourth, given increasing concerns of cybersecurity, investigating how transportation network safety and mobility will be influenced by erroneous information is also a valuable next step.

CRediT authorship contribution statement

Fatima Afifah: Formal analysis, Investigation, Writing – original draft, Writing – review & editing. **Zhaomiao Guo:** Conceptualization, Methodology, Writing – original draft, Supervision, Writing – reviewing and editing. **Mohamed Abdel-Aty:** Data collection, Data preprocessing, Advising, Writing – reviewing and editing.

Data availability

Data will be made available on request.

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Appendix. Proof of Theorem 1

Firstly, the objective function (3a) is convex because t_a is monotone increasing with respected to traffic flow. In addition, the constraints for problem (3) are all linear. Therefore, the optimization problem (3) is convex. The Lagrangian function of problem (3) is as follows:

$$
\begin{aligned}
&\mathcal{L}(x_{a,k}^{rs}, x_p(\xi), \lambda_{a,k}^{rs}(\xi), \gamma^{rs}(\xi)) = \mathbb{E}_\xi \sum_{a\in\mathcal{A}} \int_0^{v_a(\xi)} t_a(u,\xi)\mathrm{d}u \\
&- \sum_{rs\in RS}\sum_{\xi\in\Xi} \gamma^{rs}(\xi)[\sum_{p\in\mathcal{P}^{rs}} x_p(\xi) - q^{rs}] \\
&- \sum_{rs\in RS}\sum_{a\in\mathcal{A}}\sum_{\xi\in\Xi} \lambda_{a,k}^{rs}(\xi)[\sum_{p\in\mathcal{P}_k^{rs}} \delta_{ap}^{+} x_p(\xi) - x_{a,k}^{rs}]
\end{aligned}
$$

The first-order derivatives of $\mathcal{L}$ with respect to $x_{a,k}^{rs}$ and $x_p(\xi)$ are as follows:

$$
\begin{aligned}
&\frac{\partial\mathcal{L}}{\partial x_{a,k}^{rs}} = \sum_{\xi\in\Xi} \lambda_{a,k}^{rs}(\xi), \quad \forall a,k,rs \\
&\frac{\partial\mathcal{L}}{\partial x_p(\xi)} = Pr_\xi \sum_{a\in\mathcal{A}} \delta_{ap} t_a(v_a(\xi),\xi) - \sum_{a\in\mathcal{A}} \delta_{ap}^{+} \lambda_{a,k}^{rs}(\xi) - \gamma^{rs}(\xi), \quad \forall p,\xi
\end{aligned}
$$

where Pr_ξ is the probability measurement of scenario ξ.

Based on the Karush–Kuhn–Tucker (KKT) theorem, the optimality conditions of problem (3) is equivalent to the following complementarity conditions in additions to constraints ((3b) ∼ (3c)):

$$
0 \le x_{a,k}^{rs} \perp \sum_{\xi\in\Xi} \lambda_{a,k}^{rs}(\xi) \ge 0, \quad \forall a,k,rs \tag{A.1a}
$$

$$
0 \le x_p(\xi) \perp Pr_\xi \sum_{a\in\mathcal{A}} \delta_{ap} t_a(v_a(\xi),\xi) - \sum_{a\in\mathcal{A}} \delta_{ap}^{+} \lambda_{a,k}^{rs}(\xi) - \gamma^{rs}(\xi) \ge 0, \quad \forall p,\xi \tag{A.1b}
$$

A.1. Sufficient condition of Theorem 1

We first prove the sufficient condition by showing that every optimal solution of optimization problem (3) is a Stochastic User Equilibrium through the following two steps.

Step 1: We first show that all the used subpaths in the second stage have the same cost, which is no more than the travel cost of the unused subpaths.

For any given path p, we separate the links before and after the first information node into two groups: $\mathcal{A}_p^{+}$ and $\mathcal{A}_p^{-}$. Notice that for the paths do not pass through any information node, $\mathcal{A}_p^{-}$ will be empty set and $\mathcal{A}_p^{+}$ will contain all the links of path p. Furthermore, for those paths p that share the same set of $\mathcal{A}_p^{+}$ and same rs, we group them into hyperpath set P_k^{rs}, with k denoting the kth hyperpath set for rs.

For a given 3-tuple (k, rs, ξ) and $\forall p \in P_k^{rs}$, based on (A.1b) we have:

$$\{Pr_{\xi}[\sum_{a\in\mathcal{A}_p^+} t_a(v_a(\xi),\xi) + \sum_{a\in\mathcal{A}_p^-} t_a(v_a(\xi),\xi)] - \sum_{a\in\mathcal{A}_p^+} \lambda_{a,k}^{rs}(\xi) - \gamma^{rs}(\xi)\}(x_p(\xi) - x_p^*(\xi)) \geq 0 \tag{A.2}$$

Because we consider $p \in P_k^{rs}$, by definition, these paths have the same $\mathcal{A}_p^+$. We denote $Pr_{\xi} \sum_{a\in\mathcal{A}_p^+} t_a(v_a(\xi),\xi) - \sum_{a\in\mathcal{A}_p^+} \lambda_{a,k}^{rs}(\xi) \doteq t_{k,rs}^+(\xi)$. On one hand, for any $p \in P_k^{rs}$ that $x_p(\xi) > 0$, (A.2) can be simplified into:

$$\sum_{a\in\mathcal{A}_p^-} t_a(v_a(\xi),\xi) = \frac{1}{Pr_{\xi}}(\gamma^{rs}(\xi) - t_{k,rs}^+(\xi)) \tag{A.3}$$

where the right hand side is independent of p. On the other hand, for those $p \in P_k^{rs}$ that $x_p(\xi) = 0$, (A.2) can be simplified into:

$$\sum_{a\in\mathcal{A}_p^-} t_a(v_a(\xi),\xi) \geq \frac{1}{Pr_{\xi}}(\gamma^{rs}(\xi) - t_{k,rs}^+(\xi)) \tag{A.4}$$

Combining (A.3) and (A.4), we have shown that all the used subpaths in the second stage have the same cost, which is no more than the travel cost for the unused subpaths.

Step 2: Then we show that the expected travel cost for all the first stage used paths connecting a pair of origin r and destination s are the same, which is no more than the travel cost for the unused paths connecting rs.

For a given 2-tuple (k, rs), if there exist $a \in \mathcal{A}$ such that $x_{a,k}^{rs} > 0$, we have $\sum_{\xi\in\Xi} \lambda_{a,k}^{rs}(\xi) = 0$ based on (A.1a). Because of (3d), for any scenario $\xi \in \Xi$, there exists at least one path, denoted as $p(\xi)(\in P_k^{rs})$, such that $x_{p(\xi)}(\xi) > 0$. Therefore, based on (A.1b), we have the following:

$$Pr_{\xi} \sum_{a\in\mathcal{A}} \delta_{ap(\xi)} t_a(v_a(\xi),\xi) - \sum_{a\in\mathcal{A}} \delta_{ap(\xi)}^+ \lambda_{a,k}^{rs}(\xi) - \gamma^{rs}(\xi) = 0 \tag{A.5}$$

Take summation of (A.5) over all $\xi \in \Xi$, we have:

$$\sum_{\xi\in\Xi} [Pr_{\xi} \sum_{a\in\mathcal{A}} \delta_{ap(\xi)} t_a(v_a(\xi),\xi) - \sum_{a\in\mathcal{A}} \delta_{ap(\xi)}^+ \lambda_{a,k}^{rs}(\xi) - \gamma^{rs}(\xi)] = 0 \tag{A.6}$$

Because of the following two facts:

(1) $\sum_{\xi\in\Xi} \lambda_{a,k}^{rs}(\xi) = 0$ if $x_{a,k}^{rs} > 0$;

(2) $\delta_{ap(\xi)}^+$ is identical for all ξ as long as $p(\xi) \in P_k^{rs}$;

we have $\sum_{\xi\in\Xi} \sum_{a\in\mathcal{A}} \delta_{ap(\xi)}^+ \lambda_{a,k}^{rs}(\xi) = \sum_{a\in\mathcal{A}} \delta_{ap(\xi)}^+ \sum_{\xi\in\Xi} \lambda_{a,k}^{rs}(\xi) = 0$. Therefore, (A.6) becomes:

$$\sum_{\xi\in\Xi} Pr_{\xi} \sum_{a\in\mathcal{A}} \delta_{ap(\xi)} t_a(v_a(\xi),\xi) = \sum_{\xi\in\Xi} \gamma^{rs}(\xi) \tag{A.7}$$

Notice that the left hand side of (A.7) represent the expected travel cost of first stage used path k and the right hand side of (A.7) is independent of k, which means the expected travel cost of all the first stage used paths connecting rs are identical.

For a given 2-tuple (k, rs), if $\forall a \in \mathcal{A}$, $x_{a,k}^{rs} = 0$, which means kth first stage path is unused for connecting rs, we have $\sum_{\xi\in\Xi} \lambda_{a,k}^{rs}(\xi) \geq 0$. Same processes as above, we will have:

$$\sum_{\xi\in\Xi} Pr_{\xi} \sum_{a\in\mathcal{A}} \delta_{ap(\xi)} t_a(v_a(\xi),\xi) = \sum_{a\in\mathcal{A}} \delta_{ap(\xi)}^+ \sum_{\xi\in\Xi} \lambda_{a,k}^{rs}(\xi) + \sum_{\xi\in\Xi} \gamma^{rs}(\xi) \geq \sum_{\xi\in\Xi} \gamma^{rs}(\xi) \tag{A.8}$$

Till now, we have finished our proof on the sufficient condition of Theorem 1.

A.2. Necessary condition of Theorem 1

Secondly, we prove the necessary condition by showing that every Stochastic User Equilibrium is one of the optimal solutions of optimization problem (3).

Based on Definition 1, we have the following two variational inequalities:

$$\{\sum_{\xi\in\Xi} Pr_{\xi}[\sum_{a'\in\mathcal{A}_p^+} t_{a'}(v_{a'}^*(\xi),\xi) + t_k^-(\xi)] - t^{rs}\}(x_{a,k}^{rs} - x_{a,k}^{rs*}) \geq 0,\ \forall a \in \mathcal{A}_p^+, p \in P_k^{rs}, k \in K^{rs}, rs \tag{A.9}$$

$$[\sum_{a\in\mathcal{A}_p^-} t_a(v_a^*(\xi),\xi) - t_k^-(\xi)](x_p(\xi) - x_p^*(\xi)) \geq 0,\ \forall p \in P_k^{rs}, k \in K^{rs}, rs, \xi \tag{A.10}$$

where $t_k^-(\xi)$ is the travel cost after receiving information and t^{rs} is the expected travel cost.

From VI (A.10), we have the following:

$$[\sum_{a\in\mathcal{A}_p} t_a(v_a^*(\xi),\xi) - \sum_{a\in\mathcal{A}_p^+} t_a(v_a^*(\xi),\xi) - t_k^-(\xi)](x_p(\xi) - x_p^*(\xi)) \geq 0,\ \forall p \in P_k^{rs}, k \in K^{rs}, rs, \xi \tag{A.11}$$

Construct $\lambda_{a,k}^{rs}(\xi)$ and $\gamma^{rs}(\xi)$ in the following way:

$$\gamma^{rs}(\xi) \doteq Pr_{\xi} t^{rs} \tag{A.12}$$

$$\lambda_{a,k}^{rs}(\xi) \doteq \frac{1}{|\mathcal{A}_p^+|}\{Pr_\xi[\sum_{a\in\mathcal{A}_p^+} t_a(v_a^*(\xi),\xi) + t_k^-(\xi)] - \gamma^{rs}(\xi)\} \tag{A.13}$$

so that:

$$\sum_{\xi\in\Xi}\gamma^{rs}(\xi) = t^{rs} \quad (Because \sum_{\xi} Pr_\xi = 1) \tag{A.14}$$

$$\sum_{a\in\mathcal{A}_p^+}\lambda_{a,k}^{rs}(\xi) + \gamma^{rs}(\xi) \doteq Pr_\xi[\sum_{a\in\mathcal{A}_p^+} t_a(v_a^*(\xi),\xi) + t_k^-(\xi)] \tag{A.15}$$

Plug Eq. (A.15) in VI (A.11), we will have condition (A.1b).
Plug Eq. (A.15) in VI (A.9), we will have:

$$\{\sum_{\xi\in\Xi}[\sum_{a'\in\mathcal{A}_p^+}\lambda_{a',k}^{rs}(\xi) + \gamma^{rs}(\xi)] - t^{rs}\}(x_{a,k}^{rs} - x_{a,k}^{rs*}) \geq 0,\ \forall a\in\mathcal{A}_p^+, p\in P_k^{rs}, k\in K^{rs}, rs \tag{A.16}$$

Because of (A.14), VI (A.16) can be simplified to:

$$\{\sum_{\xi\in\Xi}\sum_{a'\in\mathcal{A}_p^+}\lambda_{a',k}^{rs}(\xi)\}(x_{a,k}^{rs} - x_{a,k}^{rs*}) \geq 0,\ \forall a\in\mathcal{A}_p^+, p\in P_k^{rs}, k\in K^{rs}, rs \tag{A.17}$$

Based on our construction (A.13), $\lambda_{a',k}^{rs}$ is independent of a'. So VI (A.17) is equivalent to condition (A.1a).
Till now, we have finished our proof on the necessary condition of Theorem 1. □

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