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TEACHERS' KNOWLEDGE AND USE OF VISUAL FRACTION REPRESENTATIONS

Karl W. Kosko	Christine K. Austin	Maryam Zolfaghari
Kent State University	Kent State University	Kent State University
kkosko1@kent.edu	causti17@kent.edu	mzolfagh@kent.edu

The purpose of this study was to examine the relationship between in-service teachers (ISTs') reported use of manipulatives and their pedagogical content knowledge for teaching fractions (PCK-Fractions). The study's results indicated no significant relationship between ISTs' reported use of visual representations and their PCK fractions. However, trends were observed across ISTs' education, taught grade levels, PCK, and use of visual representation. The implications and future needs for the study are discussed in the paper.

Keywords: Mathematical representations; instructional activities and practices; teacher knowledge.

Mathematical Knowledge for Teaching (MKT) involves the content and pedagogical content knowledge used by teachers to engage in pedagogy in its various forms (Ball et al., 2008; Izsák, 2008). Over the past two decades, research on MKT has shown various factors are associated with lower and higher MKT. For example, while years of experience alone are positively associated with higher MKT scores (Copur-Gencturk & Li, 2023; Hill, 2010), a stronger association is typically found when focusing on experience in contexts where specific concepts are taught (Herbst & Kosko, 2014; Hill, 2010; Zolfaghari et al., 2022). Various other factors have been explored including those associated with indicators of content knowledge (Copur-Gencturk & Li, 2023; Hill, 2010; Ko & Herbst, 2020) and pedagogy used (Hill et al., 2008; Jacobson et al., 2021; Morin, 2013). This paper focuses on one particular pedagogical approach often posed as associated with higher MKT: use of visual representations and manipulatives.

Manipulative use has long been tied with definitions of MKT. For example, Hill et al. (2005) describe developing items of specialized content knowledge so that teachers could "show or represent numbers or operations using pictures or manipulatives…" (p. 388). Later validating the Mathematical Quality of Instruction rubric, Hill et al. (2008) included use of representations, including manipulatives, as a category that aligned with MKT scores. Examining teachers' MKT for fraction multiplication, Izsák (2008) noted a relationship between teachers' skill and frequency in using visual representations for fraction arithmetic and their demonstrated MKT. Morin (2013) found that the relationship was particularly evident in one teacher's incorporation of a concrete to figurative to abstract progression for students' meaning-making. This included knowledge of varying manipulatives that allowed for adjusting activities should one representation not facilitate the connections to underlying concepts the teacher sought. Examining the topic more explicitly, Jacobson et al. (2021) found that teachers' evaluation of visual representations corresponded with their demonstrated pedagogical content knowledge. Despite the common assumption of manipulative use being tied to higher MKT scores, the bulk of such scholarship is qualitative. Thus, the purpose of this exploratory study is to examine whether and to what degree Inservice teachers' pedagogical content knowledge for teaching fractions (PCK-Fractions) is associated with their reported use of manipulatives for teaching fractions.

Pedagogical Content Knowledge for Teaching Fractions

Research on MKT for fractions is common, with studies providing evidence for a lack of content knowledge amongst preservice and inservice teachers (Erdam, 2016; Huang et al., 2009; Izsák et al., 2019). For example, Izsák et al. (2019) reported many teachers were unable to coordinate three levels of units – an important conceptual level for understanding fraction multiplication and division. Such reasoning is also important as Izsák (2008) observed teachers' ability to coordinate such units corresponded with how they used visual representations. Similarly, Thurtell (2019) found that preservice teachers who had less robust content knowledge of fractions were limited in their use of visual representations. Thurtell (2019) suggested that an overreliance on symbolic representation as an ideal for representing mathematics was at fault. Rather, "the undercurrent of calculational views evident in the preservice teachers' learning approaches were reflected clearly in their teaching approaches" (p. 305) despite dispositions generally supportive of using visual representations. More recently, Zolfaghari et al. (2022) found that preservice teachers who have field experiences in upper elementary grades (3-5) demonstrated higher PCK for fractions. Key in this finding is that Zolfaghari et al. (2022) defined higher PCK as an ability to assess more sophisticated unit coordination in students' reasoning. Taken altogether, scholarship on teachers' MKT for fractions suggest content knowledge is generally weaker than it should be. However, preservice teachers would benefit from explicit experience in upper elementary classrooms (Zolfaghari et al., 2022) and by interrogating an overreliance on symbolic representations for teaching fractions (Thurtell, 2019).

Visual Representations and Manipulatives

Visual representations are commonly advocated for the teaching and learning mathematics (Bolden et al., 2015). Visual representations are defined as both concrete and pictorial, with common mathematical visuals including groups of and array models for multiplication (Kosko, 2018) and fraction strips, number lines, and pie charts for fractions (Cramer et al., 2008; Tunç-Pekkan, 2015). Different studies have reported teachers' usage of visual representations at various grade levels. For instance, surveying 603 primary and 336 secondary teachers, Howard et al. (1997) found that most teachers felt confident in using visual representations. However, the use of visual representations was significantly lower in secondary mathematics classrooms compared to elementary classrooms. Similarly, Gilbert and Bush (1988) surveyed 220 elementary teachers and found that as teachers' grade level increased, their use of visual representations decreased. Examining 820 teachers from K–10, Swan and Marshall (2010) found teachers' reduced use of visual representations was due to the complexity of topics taught. Rather, "teachers associate the use of mathematics manipulatives with concept formation and hence to be abandoned when the mathematics becomes more complex." (p.17). These findings were corroborated by similar surveys (O'Meara et al., 2020; Uribe-Flórez & Wilkins, 2010). Additionally, O'Meara et al. (2020) found that teachers' use of manipulatives was negatively associated with a lack of training available in how to use them.

Method

Participants & Measures (1 paragraph)

The sample consisted of 47 in-service teachers (ISTs) who taught in Midwestern schools districts. Much of this sample identified as white (n = 45), female (n = 36), and had an average of 18.23 years of teaching experience. Our sample consisted of 12 third grade, 18 fourth grade, 6 fifth grade, and 11 sixth grade teachers. Of the 47 participants, 37 reported to have a master's degree which includes a master's in general education/curriculum (n = 10), elementary education

(n = 3), secondary mathematics (n = 2), literacy/reading (n = 10), administration/leadership (n = 6), education psychology (n = 2), and education technology (n = 1).

Participants were recruited via email from nearby school districts that taught grades third through sixth and were asked to participate in a survey to be completed on Qualtrics. The survey consisted of 14 pictures of fraction manipulatives that the ISTS had to indicate if they were aware of (see Table 1). If they indicated that they knew the manipulative they were then asked how often they used the manipulative (physical and virtual) on a Likert scale (0 = Never, 1 = Rarely, 2 = Sometimes, 3 = Often, 4 = Always). Then, the ISTs were given a Pedagogical Content Knowledge of Fractions instrument (PCK-Fractions) that was designed to measure a teacher's level of PCK for the teaching and learning of fractions of upper elementary mathematics (Zolfaghari et al., 2021; 2022). The instrument consisted of 17 multiple choice items that have the teachers look at a student's mathematical work and assess their reasoning.

Analysis & Results

ISTs demonstrated higher PCK-Fraction scores (M = 0.35, SD = 0.96), which is to be expected when considering the measure assesses teachers from preservice to inservice (see Zolfaghari et al., 2022). On average, most of the sampled ISTs are aware of various fraction manipulatives (Table 1). However, fewer participants knew of using Geoboards (n = 24) or playdoh (n = 5) for teaching fractions. Spearman Rho correlation coefficients were calculated to examine the relationship between ISTs' PCK-Fractions, their knowledge of various fraction manipulatives and their reported use. To facilitate this, we created composite variables for all visual representations (excluding symbolic numeral and symbolic number lines) for knowledge of (α=.82, M=.72, SD=.23) and use of (α=.83, M=1.33, SD=.61) various visual representations of fractions. The correlation analysis indicates a strong significant relationship between the number of manipulatives known and the average reported use (ρ = 0.580, p < .001). However, no statistically significant relationship was observed between PCK-Fractions with either knowledge (ρ = -0.230, p = .879) or reported use (ρ = -0.097, p = .516) of fractions visual representations.

Table 1: Percent of Participants who Know of Manipulatives and Average Reported Use

	Know	Use		Know	Use
Symbolic Numerals w/ Pictures	95.75%	M=2.93	Counters	80.85%	M=1.36
Symbolic Numerals	100%	M=2.60	Fraction Tiles	74.47%	M=1.30
Symbolic Number Line	97.87%	M=2.50	Fraction Squares	74.47%	M=1.11
Fraction Strips	89.36%	M=2.11	Linking Cubes	70.21%	M=1.02
Fraction Circles	93.62%	M=1.89	Cuisenaire Rods	68.10%	M=0.77
Tactile Number Line	80.85%	M=1.51	Geoboards	51.06%	M=0.53
Pattern Blocks	74.47%	M=1.41	Play-Doh	10.64%	M=0.06

Likert Use Scale: 0 = Never, 1 = Rarely, 2 = Sometimes, 3 = Often, 4 = Always

To better understand these results, we examined the potential effect of various demographic variables that may influence teachers' use of manipulatives. Of particular interest, we found that the highest degree earned (bachelors or masters) had a statistically significant and negative relationship with the use of manipulatives (ρ = -0.397, p = .006). Table 2 reports participants by their highest degree earned and the grade level taught along with the average years of experience and average reported use (0 = Never, 1 = Rarely, 2 = Sometimes, 3 = Often, 4 = Always). As seen in Table 2, other than fifth grade, ISTs who have a master's degree, on average, reported using manipulatives less frequently. Another interesting trend evident in Table 2 is that the specific grade level taught appears to influence reported manipulative use, with grade 6 teachers

reporting the lowest use of visual representations and grade 5 the highest. Unfortunately, the relatively small sample size per subgroup in Table 2 prevented further statistical analysis.

Table 2: Average Reported Years' Experience by Highest Degree Earned and Grade Level Taught

Highest Degree	Grade	PCK	Experience	Use
Bachelors (n = 10)	Third (n = 3)	M = -0.03 SD = 0.66	M = 7.00 SD = 6.56	M = 1.73 SD = 1.00
	Fourth (n = 5)	M = 0.22 SD = 1.19	M = 15.60 SD = 11.87	M = 2.08 SD = 0.90
	Fifth (n = 2)	M = -1.23 SD = 1.82	M = 6.50 SD = 2.12	M = 1.45 SD = 1.06
Masters (n = 37)	Third (n = 9)	M = 0.43 SD = 0.72	M = 13.11 SD = 8.45	M = 1.07 SD = 0.46
	Fourth (n = 13)	M = 0.49 SD = 1.00	M = 20.46 SD = 7.68	M = 1.28 SD = 0.72
	Fifth (n = 4)	M = 1.58 SD = 0.39	M = 25.00 SD = 4.69	M = 1.68 SD = 0.19
	Sixth (n = 11)	M = 0.15 SD = 0.58	M = 23.73 SD = 6.97	M = 0.84 SD = 0.49

Discussion

Results presented here are from an exploratory study examining the relationship between PCK for fractions and ISTs' reported knowledge and use of visual representations for fractions. Despite significant scholarship positing a relationship between use of visual representations and MKT (Hill et al., 2008; Izsák, 2008; Jacobson et al., 2021), we found no statistically significant relationships. However, trends in the descriptive data suggest that grade level and graduate coursework may have varying effects on visual representation use and the role of PCK. Our study did not examine the role of content knowledge, and future work should consider this given prior research suggesting its influence on how teachers use visual representations (Izsák, 2008; Thurtell, 2019). Additional research is also needed to better understand how graduate work may detract from use of manipulatives. More focus may be drawn to the concentration of such degrees (i.e., the majority in this study were in literacy due partly to a state mandate), the quality of degrees themselves, and so forth. One final implication from this study is that despite knowledge of various visual representations, most teachers use such representations less than "often" when teaching fractions. This finding is alarming, given that more effective use of manipulatives occurs at least on a weekly basis (Uribe-Flórez & Wilkins, 2010).

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