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ORIGINAL PAPER

Enhancing prospective secondary teachers' potential competence for enacting core teaching practices—through experiences in university mathematics and statistics courses

Yvonne Lai[1] · Jeremy F. Strayer[2] · Andrew Ross[3] · Kingsley Adamoah[2] · Cynthia O. Anhalt[4] · Chris Bonnesen[2] · Stephanie Casey[3] · Brynja Kohler[5] · Alyson E. Lischka[2]

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Abstract

In 1908, Felix Klein suggested that to mend the discontinuity that prospective secondary teachers face, university instruction must account for teachers' needs. More than a century later, problems of discontinuity remain. Our project addresses the dilemma of discontinuity in university mathematics courses through simulating core teaching practices in mathematically intensive ways. In other words, we interpret teachers' needs to include integrating content and pedagogy. We argue that doing so has the potential to impact teachers' competence. To make this argument, we report findings from the Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S2)) project. The results are based on data from 324 prospective secondary mathematics teachers (PSMTs) enrolled in courses using curricular materials developed by the project in four content areas (algebra, geometry, modeling, and statistics). We operationalized competence in terms of PSMTs' content knowledge for teaching and their motivation for enacting core teaching practices. We examined pre- and post-term data addressing these constructs. We found mean increases in PSMTs' outcomes in content knowledge for teaching and aspects of motivation.

Keywords University mathematics · Secondary mathematics teacher education · Mathematical knowledge for teaching · Expectancy-value theory · Core teaching practices

1 Introduction

Secondary teacher education faces a disconnection problem: a perceived incongruity between tertiary mathematics experiences and secondary mathematics teachers' needs (e.g., Gueudet et al., 2016; Winsløw & Grønbæk, 2014). Winsløw and Grønbæk (2014) identified various dimensions of disconnection, including the contrasting positions of future secondary *teacher* and current university *student*. In line with this observation, we along with others advocate that tertiary mathematics courses should provide explicit bridges from their content to secondary mathematics teaching (see Lai et al. (in press) for a review). Consistent with Baumert et al.'s (2010) conception of content knowledge for secondary mathematics teaching, we explore university mathematics courses that develop teachers' "profound mathematical understanding" of secondary mathematics (p. 142). Such courses address secondary level topics with a sophistication commensurate with tertiary level coursework.

We hypothesize that for teachers to experience the greatest connection between university mathematics courses and secondary teaching, bridging must take place in two ways. First, the prospective secondary teachers themselves must have opportunities to simulate core teaching practices that draw on the content taught in the university mathematics course. We operationalize core teaching practices to be those that promote discussion and elicit student thinking about content in ways rooted in disciplinary norms. Second, the university instructor must showcase these teaching practices

Yvonne Lai and Jeremy F. Strayer contributed equivalently.

✉ Jeremy F. Strayer
Jeremy.Strayer@mtsu.edu

[1] University of Nebraska-Lincoln, Lincoln, USA

[2] Middle State Tennessee University, Murfreesboro, USA

[3] Eastern Michigan University, Ypsilanti, USA

[4] University of Arizona, Tucson, USA

[5] Utah State University, Logan, USA

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in their own instruction. We argue that such pedagogical coordination supports the development of teachers' competence in secondary mathematics teaching.

In our research, we operationalized competence as cognitive and affective, and examined content knowledge, expectancy, and value. We examined data in four content areas—algebra, geometry, mathematical modeling, and statistics. These data were collected by the Mathematics of Doing, Learning, and Educating for Secondary Schools (MODULE(S2)) project, which we lead. We address research questions:

RQ1. How did prospective secondary mathematics teachers (PSMTs)' content knowledge for teaching change over the duration of a term-long experience with coordinated instruction and applications to teaching?
RQ2. How did PSMTs' expectancy and value for carrying out core teaching practices change?
RQ3. What aspects of this experience influenced changes in PSMTs' potential competence for enacting core teaching practices?

1.1 Terminology

We use *PSMT* to refer to a prospective secondary mathematics teacher, *student* to refer to a secondary student, and *instructor* or *faculty* to refer to university faculty. *Content* refers to areas of mathematics or statistics. *Secondary* refers to grades 6–12. Over 90% of participants in the study prepared to teach grades 6–12. Our project data is collected primarily from the United States, with some participants from Canada. *Term* refers to an academic term in the US, typically 10- to 15-weeks in duration depending on institution.

2 Background and perspective

Overall, PSMTs have opportunities to learn many areas of mathematics–and yet, there is scant evidence that university mathematics course taking impacts future teaching or its underlying cognitive or motivational aspects (see Tatto et al., 2018 for a review). Among results that indicate a positive influence of tertiary coursework on prospective and practicing secondary teachers (e.g., Buchholtz & Kaiser, 2013; Burroughs et al., 2023; Wasserman & McGuffey, 2021), there is a commonality. One can explain positive results in terms of intentional course design that meets secondary teachers' needs. These needs may involve how tertiary faculty teach (Buchholtz & Kaiser, 2013), or whether there are explicit links from course content to secondary teaching practice (Burroughs et al., 2023; Wasserman & McGuffey, 2021).

To our knowledge, there has been no systematic evaluation of university mathematics coursework that does both. That is, evaluation of coursework offering tasks that explicitly connect content to secondary teaching practice, and whose materials support university instructors in modeling exemplary teaching practices. Our project incorporates such tasks, and coordinates the teaching practices showcased in these tasks with guidance for instructors' teaching of the course as a whole. We see our work as an answer at last to Tatto et al.'s (2018) lament that "studies exploring the distinct impact of coherent course offerings are rare" (p. 416); the latest citation in their statement dates to 1999.

2.1 Applications of content to teaching

In examining the nature of discontinuity, Winsløw and Grønbæk (2014) described three dimensions: institution (university vs. school), positioning (student vs teacher), and content (elementary vs advanced). Historically, mending discontinuity has focused on the last dimension (e.g., Klein, 1908/1932; Murray & Star, 2013). Yet the discontinuity remains (e.g., Zazkis & Leikin, 2010). One potential explanation is that other dimensions need attention as well. To support PSMTs experiencing their tertiary mathematics content as coherent with secondary teaching, university courses may need embedded and explicit connections to secondary teaching practice. In particular, teachers should be asked to simulate the work of secondary teaching in ways that draw on the tertiary course experiences. In this way, course materials attend to the dimensions of institution and positioning, by engaging teachers' images of the role of teachers in the institution of secondary school.

Bass (2005) and Stylianides and Stylianides (2010) argued that connections to school teaching practice, where teachers simulate content intensive work of teaching, enact a form of applied mathematics. Álvarez et al. (2020) used their arguments to advocate for these connections in university mathematics courses where PSMTs may enroll. In parallel to Álvarez et al., we refer to these connections as *applications of content to teaching*.

Such applications have only recently been systematically embedded in university mathematics curricula (Lai et al., in press). There are few reports of the effects of incorporating applications to secondary mathematics teaching. Wasserman and McGuffey (2021) documented secondary classroom teachers attributing teaching decisions to prior experience with real analysis materials designed with such applications. This study is unique and promising—and it only examined six teachers. Burroughs et al. (2023) studied calculus, discrete mathematics, abstract algebra, and statistics courses featuring applications to teaching. They reported that undergraduates in these courses valued understanding secondary students' thinking. Their study demonstrates the possibility of incorporating applications to teaching in a wide range of courses. It is limited in that it did not systematically examine change in cognitive or affective aspects.

There is a need to further examine the impact of embedding applications to teaching into university mathematics courses. Our project does so in a way that moves beyond a 'proof of concept' study. We examine data across multiple content areas and multiple institutions, and we report on changes in PSMTs' content knowledge, expectancy, and value.

2.2 Operationalizing competence and socialization

In their professional work, teachers simultaneously call on knowledge, affect, and beliefs. Competence for teaching, then, must reflect both the situations to which teachers respond and underlying cognitive and motivational dispositions (Blömeke et al., 2015). As Blömeke et al. (2015) theorized, situation-specific skills—which depend on intentions, knowledge, and the particular circumstances—may mediate between underlying dispositions and ultimate performance. We now discuss our project's perspective on cognitive aspects, situation-specific skills, and then motivation.

Cognitively, we focus on content knowledge for teaching. We refer here to the disciplinary knowledge for teaching entailed in recurrent work of teaching mathematics or statistics (cf. Baumert et al., 2010). Content knowledge for teaching depends on the domain; for instance, content knowledge for teaching geometry differs from that for teaching other areas of secondary mathematics (Herbst & Kosko, 2014). Moreover, we are interested in the influence of tertiary experiences on teachers' knowledge in specific areas (namely, algebra, geometry, mathematical modeling, and statistics). Hence we designed our study to have separate assessments for each area.

We see assessments of content knowledge for teaching as proxies for situation-specific skills. As early work in this area indicates, assessment items with teaching context can simulate situation-specific skills (Hill et al., 2004). Scholars who have worked on assessing content knowledge for teaching at the secondary level broadly agree that instruments should contain at least some items of this sort, in that they describe context such as lesson goals or student talk (e.g., Baumert et al., 2010; Tatto, 2013).

Blömeke et al. (2015) observed that approaches to competence stemming from educational research tend to focus on "identifying a person's [underlying characteristics] and how these best can be developed" (p. 5). In line with this aim, our project operationalized motivation in terms of expectancy and value (Eccles & Wigfield, 2020) for carrying out core teaching practices. Teachers' expectancy and value, and motivation more broadly, have been shown to predict instructional quality (e.g., Holzberger et al., 2014; Zee & Koomen, 2016).

A person's *expectancy* is the expectation of success at enacting a task in a particular situation (Wigfield & Eccles, 2000). *Value* is the importance of carrying out a task well, and can encompass utility, enjoyment, and personal fulfillment (see Eccles & Wigfield, 2020, for a review). We use *core teaching practices* to refer to teaching practices that promote discussion and elicit student thinking about content in ways rooted in disciplinary norms. Such teaching practices are associated with instructional quality and student learning outcomes at the secondary levels (e.g., Baumert et al., 2010). Our notion of core practices is also consistent with various unpackings of mathematically intensive teaching practices (e.g., Baumert et al., 2010; Tatto et al., 2018).

A person's cognitive and motivational dispositions are influenced by socialization. The form and orientation of teacher education influences what teachers' learn (Werler & Tahirsylaj, 2020). We operationalize PSMTs' socialization as their opportunities to learn and use content (including its applications to teaching), the tasks an instructor uses to enact these opportunities, and the instructor's own teaching practices (cf. Schmidt et al., 2008).

To examine the role of socialization, we distinguish two perspectives: PSMTs-as-*university-students* and PSMTs-as-*future-secondary-teachers*. We investigate whether PSMTs-as-*university-students* experienced and observed core practices in their tertiary instruction. We explore the relationship between this perception and PSMTs-as-*future-secondary-teachers'* expectancy and value.

Figure 1 provides a visual outline of our conceptual perspective. In our diagram, we specialize the schematic shown in Blömeke et al. (2015) to our constructs of interest, and we expand their schematic to account for socialization (Eccles & Wigfield, 2020).

To summarize, the scope of our study allows us to address two main gaps in the literature. First, there is a need for more recent studies on mathematics courses that intentionally provide coherence across mathematics and pedagogy (Tatto et al., 2018), where pedagogy means both what PSMTs learn about teaching and what PSMTs experience as university students. This coherence is provided in MODULE(S2) through applications that connect course content to core teaching practices and support for instructors in enacting these practices. Second, although there are indications that courses with applications of content to teaching may promote teachers' dispositions and shape future teaching decisions, systematic investigations that also measure change in teachers' cognitive or motivational aspects is scant. Our study explores relationships between tertiary instruction and PSMTs' dispositions in the context of instructional coherence. We examine PSMTs' change in content knowledge, expectancy, and value. The results of

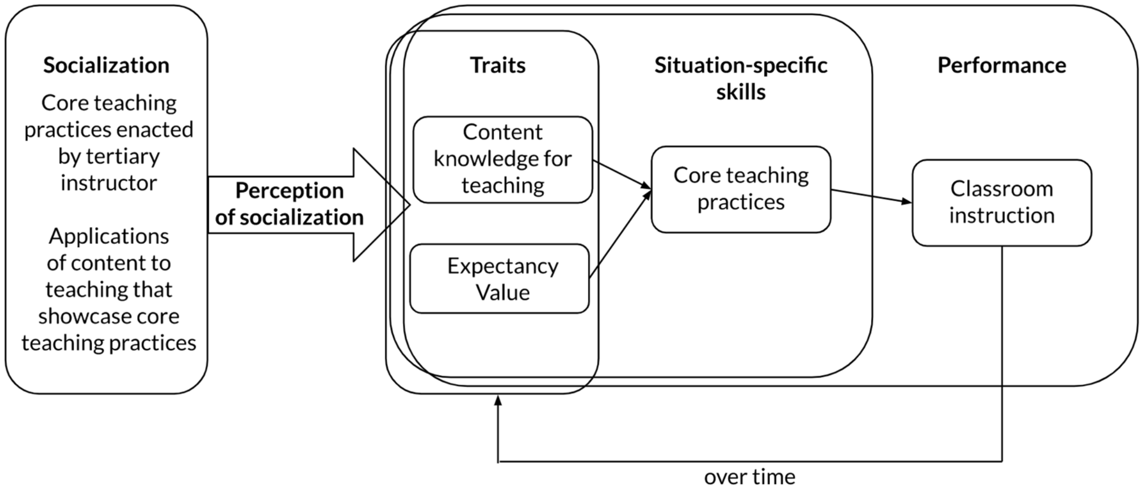

Fig. 1 Conceptual perspective

this study are important to the field to begin filling these gaps, particularly as teachers' motivation-related beliefs tend to be malleable early in their career and resistant to change later (Holzberger et al., 2014). If we find positive change in teacher traits, this suggests that instructional coherence may be an effective approach to designing mathematics courses for PSMTs. If we are not able to find positive change, we may need to reconsider how exactly to address the dilemma of discontinuity.

3 Study context

In our context, many PSMTs take separate "methods" and "content" courses. Education faculty typically teach methods, which address pedagogy, and mathematics faculty typically teach content, which addresses mathematics and statistics (Tatto et al., 2010). In this respect, MODULE(S2) integrates what has typically been bifurcated: we offer opportunities to apply content to teaching in content courses.

For each of the areas of algebra, geometry, mathematical modeling, and statistics, the MODULE(S2) team designed curriculum modules intended to span approximately 3 months of instruction. These modules were intended for use in content courses for PSMTs. We designed modules to coordinate content with a selection of core teaching practices related to generating discussion and understanding learners' thinking. All modules featured routine opportunities to use content learned to address teaching situations such as using student work to seed a class discussion. Further, each area featured multiple extended opportunities to apply recently learned content to teaching. Here, PSMTs were asked to depict teaching moves in writing or in video in response to a given context. Figures 2 and 3 show example prompts. Finally, all materials came in PSMT-facing and instructor-facing versions. Instructor-facing versions described how instructors might enact core practices in their own instruction, and discussed features of core practices in applications of content to teaching. We delivered in-person and virtual professional development sessions for instructors using the modules.

Course content treated secondary mathematics from an advanced perspective. In some assignments, PSMTs in algebra and geometry were asked to generate proofs of a complexity commensurate with tertiary level coursework. Similarly, PSMTs in statistics and mathematical modeling were asked to write mathematical analyses of situations that were of a depth and rigor expected in tertiary education. Assignments offered opportunities for PSMTs to connect mathematical practice and content across secondary and tertiary levels based on teaching scenarios. Figure 2 depicts an Algebra teaching scenario. Based on the scenario PSMTs were asked to describe how they would lead a whole class discussion of the approaches in Group 1 and Group 2 to advance students' understanding of the connections between procedures for finding intercepts and the definition of a graph of an equation. Figure 3 depicts a modeling teaching scenario. Based on the scenario PSMTs were asked to describe how they would lead a whole class discussion where students practice articulating the benefits, drawbacks, and similarities of multiple approaches to a mathematical modeling problem.

4 Data and method

We investigated changes in participating PSMTs' content knowledge for teaching (RQ1). We then examined changes in their expectancy and value for enacting core

Students in your 11th grade class have been working with graphs of equations. They have been given the following definition.

> The **graph of an equation** in x and y is defined as the set of points (a, b) such that evaluating the equation at $x = a$ and $y = b$ results in a true statement.

You have given students the following task:

> Where are the y-intercepts of the graph of the equation $(x-2)^2 + y^2 = 5$?

As students are working in groups on this problem, you have these two conversations:

Group 1	Group 2
You: How did you start?	**You:** Tell me what you're thinking about.
Student A: I put in a 0 for the x-value.	**Student C:** I don't think we can find the intercepts. This is a circle with a radius of $\sqrt{5}$ and that's a messy number. So the intersection with the y-axis will also be a messy number.
You: Let's talk about that. If we're finding a y-intercept, why do we start by putting in a 0 for the x?	**Student D:** I think we might be able to find it either by plugging in 0 for the x-value or 0 for the y-value, but I can't remember which one.
Student A: You want things to cancel, and 0 makes things cancel?	**Student C:** Oh yeah that sounds right. Which one should we use?
Student B: Because it's like finding the intersection of the graph and the line $x = 0$?	

Fig. 2 Teaching scenario provided in algebra materials

teaching practices (RQ2). Finally, we analyzed PSMTs' perceptions of the extent to which they experienced and observed core practices in the course and their reports of influential course features (RQ3).

4.1 Participants

Participants for RQ1 were 132 PSMTs enrolled in tertiary mathematics courses using MODULE(S2) materials at 22 different institutions across the US and Canada. These courses were intended for PSMTs. Participants for RQ2 were 192 PSMTs at 31 different institutions across the United States and Canada. Participants for RQ3 were 70 of the 192 PSMTs for RQ2. All participants consented to participate. For all forms, we defined "completion" as completing a majority of questions.

We recruited PSMTs through their instructors. Institutions ranged from large public research universities to small private colleges to regional public universities, and from those that served predominantly white to predominantly minoritized populations. Hence a variety of classroom sizes and teacher characteristics are present in our sample. (Note: "college" here refers to 4-year bachelors granting institutions without graduate programs.)

4.2 Instruments

4.2.1 Content knowledge for teaching

We measured content knowledge for teaching in each area at the beginning and end of the term. All content knowledge for teaching assessments included applications of content to teaching.

For algebra, we used items from the Exponential, Quadratics, and Linear assessment (Howell et al., 2016). Analysis of cognitive interviews from 186 responses from 23 practicing and prospective secondary teachers were used to validate that its items represented teachers' reasoning and that the contexts provided were authentic to secondary mathematics teaching (Howell et al., 2016). The mathematical topics aligned with the algebra modules for the project. The items were refinements of those developed for the Measures of Effective Teaching Study for Algebra I, and follow the item design theory reported in Hill et al. (2004). Some items asked PSMTs to write mathematical proof. Some items used notation typically introduced in tertiary mathematics.

For geometry, we used Geometry Assessments for Secondary Teaching (GAST; Mohr-Schroeder et al., 2017), in consultation with author Mohr-Schroeder on selecting items aligned with our geometry materials. Through a study

The modeling task below asks students to design a method for figuring out the area of the reservation land from the 1851 and 1877 maps of the Sioux (a North-American Indigenous Nation) Reservation to calculate the percentage in area reduction of the land.

During class, a group of students chose to segment the area of the land into familiar polygons, to calculate the area of each polygon, and then find the sum of the areas, while another group is overlaying a grid over a map and estimating the squares that cover the reservation land to calculate the area.

Mathematical Modeling Task: Shrinking Area of the Sioux Reservation
Based on the two maps of the Great Sioux Reservation in 1851 and 1877, develop a procedure that can be used to approximate the area of the Great Sioux Reservation and use it to calculate the percentage in area reduction between 1851 and 1877.

- Describe your method for estimating the area based on the map images.
- Estimate the accuracy of your solution and describe changes you would make to improve the accuracy.
- Develop an improved procedure based on your initial solution.

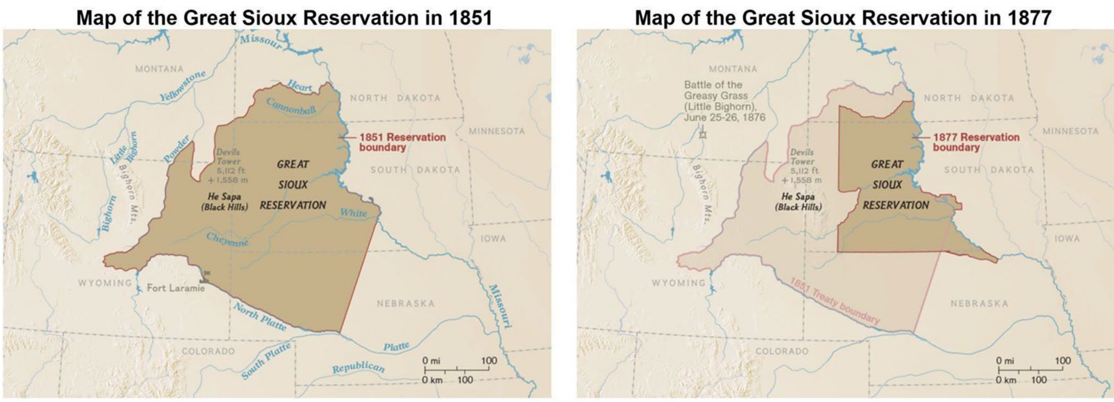

Fig. 3 Teaching scenario provided in mathematical modeling materials

involving 157 practicing and prospective secondary teachers and 3,698 students, Mohr-Schroeder et al. (2017) reported that the GAST has predictive validity for student outcomes. Some items required PSMTs to analyze mathematical proof and geometric transformations using knowledge typically taught at the tertiary level.

For mathematical modeling, we used Anhalt and Cortez's (2016) questionnaire on conceptions of mathematical modeling. These authors triangulated 11 PSMTs' responses to the questionnaire with mathematical modeling work done over the duration of a semester. There is no other instrument that we are aware of for assessing teachers' proficiency with modeling. For this study, we developed a rubric for scoring questionnaire responses. To test and refine the completeness of this rubric, we used data from PSMTs enrolled in courses using MODULE(S2) mathematical modeling materials collected prior to the data reported in the present study.

For statistics, we were not aware of any instrument that measured teachers' statistical knowledge for teaching grades 6–12 topics, so we developed a 7-item Statistical Knowledge for Teaching (Groth, 2013) test and an accompanying scoring rubric (Casey et al., 2022). We designed the instrument to assess secondary teachers' knowledge for teaching the statistics standards in the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Multiple items used released student work on tasks from the LOCUS project (n.d.) to represent student conceptions of statistics.

4.2.2 Expectancy and value for enacting core teaching practices

We measured expectancy and value for enacting core teaching practices at the beginning and end of the term. To measure expectancy, we asked PSMTs to evaluate their comfort carrying out a selection of core teaching practices when teaching middle or high school students. The phrasing was drawn from instruments validated to measure subject-area expectancy (e.g., Wigfield & Eccles, 2000) The selection of core teaching practices were:

- (Conjecture) *Ask students questions so that they make conjectures*;
- (Explain) *Ask students questions that help them come up with explanations*;

- (Connect representations) *Ask students questions that help them make connections between different representations of the same idea;*
- (Build upon) *Ask questions so that students understand how to build on their thinking and what to revise*; and
- (Analyze thinking) *Analyze students' responses to understand their reasoning*.

PSMTs responded using a Likert scale from 0 (not at all) to 5 (very much) for each of these teaching practices across a set of key concepts in each area, shown in Table 1.

A typical item read: "Suppose you are teaching middle or high school **algebra students** how to think about functions in terms of how changes in one variable may impact the value of the other variable. How well does this statement describe how you feel? I would be comfortable regularly *asking questions so that middle or high school students make conjectures*." Here, **boldface** represents content area; in actual items, this was "algebra", "geometry". "mathematical modeling", or "statistics". Underlining represents key concepts. For each key concept, PSMTs were asked to respond to 5 statements corresponding to the listed teaching practices (*italicized*). For each content area, we averaged responses over all key concepts to produce one expectancy rating per respondent per core practice.

To measure value for enacting core teaching practices, we used items parallel to the expectancy items, e.g., "How much do you personally agree with these ideas about teaching middle and/or high school students about **algebra**? I think it is important to *regularly ask questions so that middle or high school students make conjectures*." One item was posed for each of the five core teaching practices. We used a scale of 0 (not at all) to 5 (very much).

In a prior round of piloting, PSMTs remarked how they learned things that they did not know *could* be learned. This phenomenon has been observed elsewhere as *response shift bias* and identified as problematic for the internal validity of pre-/post-difference analysis (Howard, 1980). To account for the possibility that this bias was at work in our study, the end of term surveys included two ratings for each expectancy and value item—a post-rating and a *retrospective* pre-rating. We used the stem, "Looking back, how well did these statements describe you at the beginning of the course, AND now at the end of the course?" Likert scales were identical to those above. In total, we asked PSMTs to provide three ratings: an *actual* pre-rating at the beginning of term, and a *retrospective* pre-rating and actual post-rating at the end of term.

Internal reliability (Cronbach's α) was α = 0.91 for the expectancy assessment, and α = 0.79 for the value assessment. A common guideline for Cronbach's α is to consider values over 0.7 as acceptable and over 0.9 as excellent (Nunnally, 1978).

To assess validity, we used Kane's (2004) argument-based approach. See Sect. 4.3.2.

4.2.3 Perceptions of learning

At the end of the term, we measured PSMTs' perceptions of the extent to which their instructors enacted core teaching practices. We examined whether each PSMT: (1) individually contributed to conjectures, explanations, and

Table 1 Key concepts for each area featured in expectancy measures

Area	Key concepts
Algebra	1. Using the definitions of graph and coordinate plane to help explain the concepts of functions and relations 2. Correspondence and co-variational views of functions support and complement each other 3. The parallels between exponential properties and the number and operation properties that students learn in elementary and middle grades 4. How the process of solving equations relies on structural properties of the real numbers that we often take for granted
Geometry	1. Axiomatic systems and how axioms are used logically to prove theorems 2. Definitions of rigid transformations (translations, rotations, reflections, glide reflections) 3. Similarity in terms of transformations
Mathematical Modeling	1. Aspects of the mathematical modeling process 2. The process of fitting modeling parameters to data 3. How to validate models by comparing them with real data
Statistics	1. Designing statistical studies to anticipate variability and variables not controlled by the study 2. Gaining insight through visualization and analysis 3. How inferential statistics enable us to infer, though with uncertainty, beyond the data we have to a broader set of individuals or circumstances 4. How association means that information about one variable changes our idea about what happened with the other variable, but does not necessarily establish a causation relationship

representations; (2) felt that the instructor led discussions where conjectures, explanations, and representations took place; and (3) felt that the instructor made efforts to build on PSMTs' thinking. The Perceptions of Learning survey included items such as "I made mathematical explanations throughout the course", "My class participated in many discussions where we made conjectures", and "My instructor regularly asked us questions that helped us understand each other's ideas." Items used a scale from 0 (not at all) to 5 (very much). Instrument reliability (Cronbach's α) was $\alpha = 0.93$ for this measure, which is considered excellent ($\alpha > 0.9$; Nunnally, 1978).

The survey concluded with open-response items asking PSMTs what they learned about content, teaching the content, and what was most helpful for this learning.

4.3 Analysis

4.3.1 Change in PSMTs' content knowledge for teaching, expectancy, and value

We quantified change in PSMT traits using differences in *actual* pre-, *retrospective* pre-, and post-term ratings. We computed paired pre/post differences and analyzed how meaningfully different from zero they were with a focus on *effect size*, as measured by Cohen's d (Cohen, 1988). We noted where p-values are less than 0.05 (statistical significance), but we focused on effect size (practical significance) rather than p-values. Effect size provides a standardized measure of the *magnitude* of the difference between pre and post scores, whereas small p-values provide evidence for *whether* a difference exists (no matter how small) in the theoretical population (here, all PSMTs who learn with MODULE(S2) materials). Recent quantitative analyses often favor a focus on effect size because it is easier to interpret (e.g., a smaller p-value does not mean evidence of a larger difference, but larger effect size *does*), and p-value is sample size dependent while effect size is not (Sullivan & Feinn, 2012). Common benchmarks for interpreting Cohen's d are 0.2 for a small but non-negligible effect, 0.5 for a medium effect, 0.8 for a large effect, and 1.3 for a very large effect (Sullivan & Feinn, 2012). We also used comparative dotplots of percentage of maximum score to analyze changes in content knowledge for teaching.

4.3.2 Validating measures of expectancy and value

We used Kane's (2004) argument-based approach to validating the measures of expectancy and value. To test the assumption that our measures capture change in PSMTs' expectancy and value, we examined PSMTs' open responses to the Perceptions of Learning survey. We determined the percentage of PSMTs who made statements of expectancy, value, and attributions as operationalized in Table 2. Attribution here means that PSMTs attribute a change in their expectancy or value to a course feature. We coded each statement as positive or negative.

Our logic is as follows. If PSMTs made more positive than negative statements of expectancy and value while attributing learning to course features, and the measures indicated increases in these constructs, then we have evidence that the measures capture change. However, if we do not generally find positive statements from PSMTs, and the measures indicate increases in the constructs, then we have evidence against the validity of the measures for capturing change.

4.3.3 Examining factors in changes in expectancy and value

To examine instructional factors in changes in PSMTs' expectancy and value, we used Likert and open responses to the Perceptions of Learning survey. Using Likert responses, we analyzed relationships between PSMTs' expectancy and value and instructors' use of core teaching practices. To do so, we used Pearson's correlation coefficient r to measure effect size of correlations of perception of instruction, expectancy, and value. We report p-values to determine if there is evidence of a non-zero correlation in the theoretical population, but focus on practical significance. Common benchmarks for practical significance with r are roughly the same as those for Cohen's d. We triangulated results with PSMTs' open response statements.

Table 2 Open response codes

Code	Description
Expectancy	Statement of confidence or facility, or change in confidence or facility, in aspects of doing mathematics, learning mathematics, or teaching mathematics
Value	Statement of importance, benefit, worth, or enjoyment ascribed to aspects of doing mathematics, learning mathematics, or teaching mathematics
Attribution	Statement that attributes change in expectancy or value to instruction, where instruction includes course activities, norms, or interaction (e.g., Cohen et al., 2003)

5 Results

5.1 Change in PSMTs' content knowledge for teaching (RQ1)

PSMTs in all areas exhibited a mean increase in content knowledge for teaching as shown in Table 3. As Fig. 4 shows, 75% of content scores increased from pre to post. All differences are statistically significant ($p < 0.05$). More importantly, three effect sizes are above the threshold for large practical significance, and the fourth is above the threshold for a medium effect. Because the maximum possible score differed across content areas, we expressed mean scores as a percent of the maximum possible score on the applicable assessment.

5.2 PSMTs' change in expectancy and value for carrying out core teaching practices (RQ2)

We analyzed differences in PSMTs' expectancy and value for carrying out core teaching practices. We report results for pre/post differences using *actual* pre-ratings and then *retrospective* pre-ratings. All ratings were self-reported on a Likert scale from 0 (not at all) to 5 (very much). (See Appendices 1 and 2 in the electronic supplementary material for mean, standard deviation, sample size, and Cohen's *d* values. Measures referenced here are shaded.)

Table 3 Pre-term and post-term means and effect size, reported in terms of percentage of maximum possible score on each area content assessment

Area	Mean pre	Mean post	Mean pre–post difference	SD_d	Cohen's *d*
Algebra ($n = 9$)	20.8%	39.2%	18.3%*	13.5%	1.36
Geometry ($n = 63$)	25.6%	35.9%	10.3%*	10.1%	1.02
Mathematical modeling ($n = 20$)	31.3%	44.4%	13.1%*	17.9%	0.73
Statistics ($n = 40$)	26.6%	42.2%	15.6%*	16.4%	0.95

*$p < 0.05$

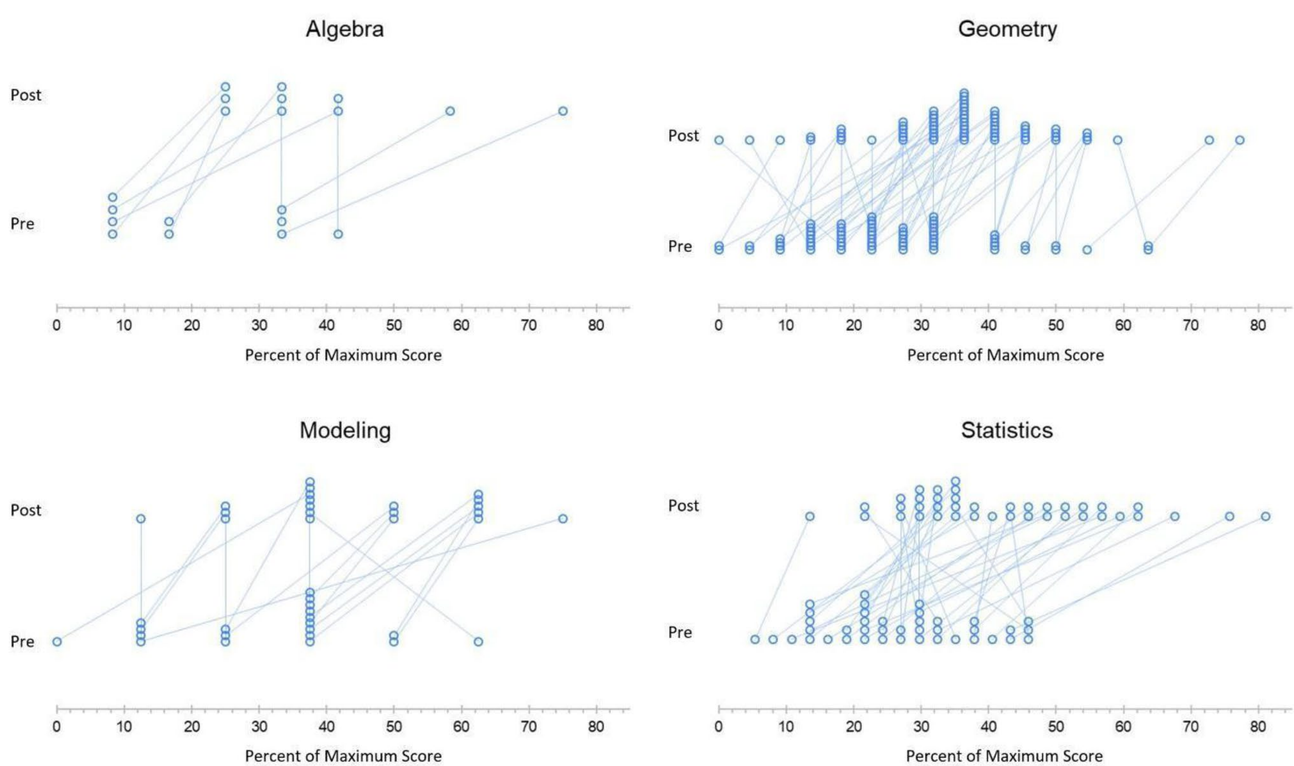

Fig. 4 Content knowledge for teaching measures: pre-term and post-term paired dotplots

Across all content areas, PSMTs' expectancy means in actual pre-ratings for teaching practices ranged from 3.42 to 3.77, and post-rating means ranged from 4.08 to 4.54. Across all 4 content areas and all 5 core teaching practices, the 20 ($=4\times5$) mean differences for expectancy (paired by PSMT) were statistically significantly greater than zero—providing evidence that the larger post-term means for each teaching practice are likely not due to chance alone. Differences were also practically significant; Cohen's d values ranged from 0.42–0.5 (non-negligible to medium significance) for the statistics content area, and 0.71–0.99 (medium to strong significance) for other areas. Across all content areas, PSMTs' value means in *actual* pre-ratings ranged from 4.39–4.75, and post-rating means ranged from 4.56–4.89. Only 5 of the 20 differences were statistically significantly above zero, and they indicated only non-negligible to medium practical significance. Thus, as a result of our analysis of *actual* pre- to post-term expectancy means, we have evidence that a difference exists. We also have evidence that a meaningful difference exists for the scale we used. However, we have less evidence that these differences exist for value means using these *actual* pre-ratings.

We now turn to differences in post- and *retrospective* pre-ratings. All PSMTs' mean expectancy and value differences, across all core practices and all content areas, were statistically significant. All differences in PSMTs' expectancy means, across all areas, indicated strong practical significance. Differences in PSMTs' value means indicated strong practical significance in algebra and mathematical modeling, and medium to strong practical significance in geometry and statistics. For the scale we used, our analysis thus provides evidence that a meaningful difference exists in *retrospective* pre- to post-term expectancy and value means for enacting core practices.

5.3 Validity of expectancy and value surveys using an argument-based approach

Analysis of 277 statements in open responses to the Perception of Learning survey provided evidence for the validity of our expectancy and value measures for capturing change in these constructs. Table 4 reports the percentage of PSMTs who made at least one statement regarding expectancy, value, or course attribution to their learning, and the number of those statements that were positive or negative in nature.

Negative statements were rare (15 statements) compared to positive statements (262 statements) across all areas. In positive statements, PSMTs described increased facility in content knowledge and working with students. In algebra and geometry, multiple PSMTs cited increased knowledge of "why things work" and benefit to their future teaching (e.g., "I learned that algebra could be understood in a more general way. This was shown by demonstrating how to use the general proof structure in order to prove certain mathematical ideas", "Being challenged to dig deeper into these ideas will be helpful in my future career"). In mathematical modeling and statistics, multiple PSMTs described little prior knowledge of these topics and more confidence that they could teach the topic as a result of the course (e.g., "I think viewing and practicing modeling problems ourselves made it easier to see what modeling is and does … I think class discussions about other students' modeling approaches … helped me better understand what to look at when teaching").

As for negative statements, two PSMTs stated that content in algebra was overly theoretical for their context; five characterized experiences with mathematical modeling as redundant; three discussed "unnecessary" inclusion of social justice issues; three stated that though they had learned statistics, they still felt uncomfortable with some statistical concepts; and 1 stated they had insufficient opportunity to apply statistics to teaching. In other words, the PSMTs who made negative statements may have learned content, but they perhaps did not learn it as well as they wanted to, or in the manner that they wanted to.

PSMTs' descriptions of learning were readily characterized through expectancy and value as they attributed their learning to aspects of the course. Overall, the positive statements overshadowed the negative statements in substance and quantity. There were more than 17 times as many

Table 4 Expectancy and value statements in open responses to perceptions of learning survey

	% Participants mentioning expectancy	% Participants mentioning value	% Participants mentioning attributions	Expectancy statements (Pos + Neg)*	Value statements (Pos + Neg)*	Attribution statements (Pos + Neg)*
Algebra ($n=28$)	82.1%	60.7%	67.9%	37 + 0	25 + 2	29 + 0
Geometry ($n=6$)	100.0%	83.3%	50.0%	12 + 0	7 + 0	5 + 0
Mathematical modeling ($n=23$)	78.3%	95.7%	73.9%	30 + 0	48 + 2	25 + 5
Statistics ($n=13$)	92.3%	76.9%	61.5%	21 + 1	14 + 2	9 + 3

Pos positive statement; *Neg* negative statement positive statements as negative statements. Although only 50% of PSMTs in geometry mentioned attributions, there were only 6 participants. These participants made positive statements about expectancy and value, but did not make an explicit attribution as to why. The lack of attribution does not contradict an apparent mean increase in expectancy and value. Altogether, the analysis of open responses provides evidence that our measures captured change in teachers' expectancy and value in alignment with Kane's (2004) argument-based approach to validity.

5.4 Potential factors influencing PSMTs' competence (RQ3)

To explore potential explanations for changes in PSMTs' competence, we analyzed quantitative and qualitative data collected with the Perceptions of Learning survey (described in Sect. 4.2.3).

Quantitatively, we observed statistically significant positive correlations with non-negligible effects ($p<0.05$, $r>0.2$) between PSMTs' perception of experiencing each core teaching practice during the semester studied and the increase in their expectancy *actual* pre- and *retrospective* pre-ratings differences (see Table 5). Correlations are similar between differences with *actual* pre-ratings ($0.25<r<0.32$) and *retrospective* pre-ratings ($0.21<r<0.28$). All correlations between PSMTs' perception of experiencing each core teaching practice and the increase in their value *actual* pre- and *retrospective* pre-ratings differences are positive, but not statistically significant. In summary, perceptions of core teaching practices are weakly correlated with changes in expectancy, and we see little to no correlation with changes in value.

Table 5 Correlations of PSMTs' perceptions of learning and their change in expectancy and value ($n=192$)

Correlation to change in expectancy or value	Core practice perceived	Pearson's correlation coefficient r	
		Using actual pre-ratings	Using retrospective pre-ratings
Expectancy	Conjecture	0.25*	0.25*
	Explain	0.32*	0.28*
	Connect	0.27*	0.28*
	Build	0.25*	0.21*
	Analyze	0.27*	0.24*
Value	Conjecture	0.07	0.14
	Explain	0.08	0.08
	Connect	0.06	0.12
	Build	0.01	0.05
	Analyze	0.03	0.00

*$p<0.05$

Qualitatively, we identified emergent themes for factors in PSMTs' learning using open responses to the Perception of Learning survey. We now discuss the most frequently occurring themes in the 277 statements across the content areas.

Quality of instruction was mentioned in all content areas (35 mentions; e.g., "Having conversations with peers and being given time to absorb and reflect on ideas was really helpful", "In this class, [instructor] was probably the most helpful. She did a great job pushing us to talk and discuss each problem. Then looking back, you can see the results of those discussions. Being able to do that myself will be a massive help").

Structure and content of materials were mentioned in all areas (23 mentions; e.g., "When I heard the phrase 'in the future your students will ask you' I never really thought about it, but after witnessing it firsthand [in my field experience] and with the exact same topics from class […] I was shook and thankful that I have this class"; "I learned about the modeling cycle. The most helpful part of this course was the practice with modeling"; "I was able to learn some statistics throughout this course and a lot of that was through visual aids, CODAP, StatKey, etc.").

Finally, *applications of content to teaching* were mentioned in all areas (22 mentions; e.g., "The videos we had to create where we looked at a student's answer… get them to think where they might come up with the answer on their own without me giving them the answer I found very beneficial and helpful!"; "Understanding the principles of statistics and applying them to student answers was most helpful.").

Across these statements, PSMTs described the positive effect of discussion, instructional practices, or instructor demeanor. Some PSMTs expressed a desire to mirror the instructional practices they experienced in their future teaching. PSMTs also praised the topics and mathematical practices addressed. In discussing applications to teaching, PSMTs alluded to aspects of core teaching practices. They valued their encounters with student ideas and opportunities to anticipate teaching moves in response to these ideas.

Altogether, qualitative results suggest that PSMTs benefited from instructors' enactment of core teaching practices, and the quantitative results do not contradict this. The qualitative results indicate that PSMTs found the applications of content to teaching useful, that they drew on course content in applications to teaching, and that they attended to core practices within these applications.

6 Discussion

According to Baumert et al. (2010), ''One of the major findings of qualitative studies on mathematics instruction is that the repertoire of teaching strategies and the pool of alternative mathematical representations and explanations available to teachers in the classroom are largely dependent on the breadth and depth of their conceptual understanding of the subject'' (p. 138). This "repertoire" includes core teaching practices, which promote students' explanations and representations. In our study, we investigated the impact of university mathematics courses on PSMTs' competence, operationalized as their content knowledge for teaching, and their expectancy and value for carrying out core teaching practices. We further examined course factors that PSMTs attributed to their learning.

Course materials in four areas were intended to develop teachers' "profound mathematical understanding" of secondary mathematics; such an understanding is deeper than what is addressed at the secondary level, and is akin to secondary mathematics from an advanced perspective (Baumert et al., 2010, p. 142). A principal design feature of the materials is instructional coherence; the materials feature applications to apply course content to core teaching practices, while supporting instructors themselves in enacting these practices.

6.1 Enhancing teachers' potential competence through experiences in university mathematics courses

Our results address two gaps in the literature. First, there are preliminary indications that courses with applications of content to teaching may benefit PSMTs. However, there is to date no systematic measurement of PSMTs' changes as a result of such courses. Our results for RQ1 (content knowledge for teaching) and RQ2 (expectancy and value) address this gap. Our results showed increases in teachers' content knowledge for teaching. Across all content areas, these increases were statistically significant, with moderate to very strong effects. We also found retrospective increases in expectancy and value for carrying out core teaching practices. These increases were statistically significant with moderate to very strong effects. With actual pre-ratings, we found statistically significant increases in expectancy, but not in value. Our qualitative analysis for RQ3 (course factors) indicate that PSMTs benefit from integrating course content and core practices through applications of content to teaching.

It is worth considering possible factors for these increases outside of the mathematics courses. For instance, increases in content knowledge and expectancy may be influenced by field experiences or methods course taking. Pedagogical experience could enhance responses to content knowledge items. We cannot discount these effects.

Nonetheless, we believe that PSMTs' opportunities to apply content to teaching can explain these results. Applying content to teaching offers a way for PSMTs to develop stronger understanding of the content and see how it is relevant to their future. These opportunities may also set up an expectation of enacting core practices in teaching, and at the same time scaffold these practices. However, they may not necessarily change PSMTs' valuing of core practices. For instance, PSMTs may not have been asked to reflect on the utility of core practices; they were only asked to simulate them.

Our results overall contribute empirical evidence toward the benefits of applications to teaching embedded into university content courses. These opportunities may enhance teachers' potential competence by developing PSMTs' content knowledge for teaching and their expectancy for carrying out core practices.

The second gap we address pertains to the promise of instructional coherence in the preparation of teachers. Despite its potential, there is a lack of recent studies on the effects of content courses for teachers adhering to this characteristic (Tatto et al., 2018). In view of the conceptual and empirical advances made in the past decades on secondary content knowledge for teaching (see Baumert et al., 2010 for a review), it is time to revisit this notion.

Our results for RQ3 (course factors) indicate that PSMTs noticed that their instructors used core practices. Moreover, PSMTs referred to features of core practices in their statements about applications of content to teaching. We observe that these statements came from end-of-semester surveys associated with the course, and so the PSMTs may have felt an obligation to make positive statements. However, PSMTs made overwhelmingly more positive statements than negative statements across different content areas, instructors, and institutions. Thus we find it plausible that the materials' instructional coherence may explain our results. When PSMTs' instructors model core practices, the PSMTs may then see these core practices as part of the course context. Thus the PSMTs may feel more disposed to activities featuring descriptions of these core practices, such as the applications of content to teaching. We conclude that instructional coherence is indeed a desirable design feature for university mathematics courses for teachers.

6.2 Response shift bias

In our study, we analyzed changes in expectancy and value, which are aspects of motivation. As motivation is typically measured by self-report, we wanted to account for response shift bias (Howard, 1980). We therefore collected and analyzed actual and retrospective differences in PSMTs' expectancy and value for enacting core practices. When comparing actual pre-ratings to retrospective pre-ratings, we found evidence consistent with response shift bias.

Based on our findings, we recommend that when evaluating programmatic impact on teachers' affective dispositions—when using measures that rely on self-report—researchers measure both actual and retrospective differences. At the beginning of their programs, PSMTs may not be aware how much there is to grow, or how valuable it is to enact core practices skillfully. Although retrospective differences may be more useful for capturing PSMTs' perception of growth, actual pre-ratings are useful for capturing PSMTs' actual dispositions when beginning their program.

6.3 Putting PSMTs' content knowledge for teaching into perspective

Despite the PSMTs' mean gains in content knowledge having a medium/large effect size and being statistically significant, the mean post-scores still fell below 50% of maximum possible scores. The mean post-score percentages appear comparable across areas, ranging from 39.2% to 44.4% across areas. Our findings are consistent with Milewski et al.'s (2019) report of PSMT performance compared to practicing mathematics teacher performance on an instrument validated to measure mathematical knowledge for teaching geometry. In their results, PSMTs' mean score, even after taking a university mathematics course in geometry, was comparable to scores in the bottom quartile of practicing teachers' scores.

In concert with Milewski et al.'s (2019) results, our findings appear to suggest the hypothesis that there is a ceiling on how much content knowledge for teaching can be learned by PSMTs. Yet what is unknown is the cumulative effect of the mathematical teacher preparation as a whole; our results, and Milewski et al.'s, only address changes from a semester's worth of instruction. If we were to compare PSMTs' performance on these knowledge measures in their first year of university to their end-of-program performance, would we see more of an effect? In other words, part of putting these findings into perspective may involve understanding PSMTs' knowledge trajectory before mathematics courses likely taken in their final years of university.

6.4 Limitations

Although many factors influence teacher competence, we focused our study on enhancements to individual PSMTs' content knowledge for teaching, and their expectancy and value for enacting core practices. As a result, our study only addresses competence as an individual trait, and it does not address competence as a social trait. We did not examine PSMTs' work with actual students, nor did we examine the impacts of their larger social context.

We are limited in conclusions that we can draw about socialization. We did not directly analyze enactments of instruction; we focused on PSMTs' perceptions of instruction. We provided professional development to instructors, and we relied on the design of the course materials to provide guidance and structure for instructors' enactment of core practices, but instruction will vary in response to specific contexts. We also cannot ignore the fact that PSMTs have experiences outside of their mathematics courses.

We acknowledge limitations in instrumentation. The sample sizes were not large enough within each content area to quantify reliability for measures of content knowledge for teaching beyond the work that had already been done on those instruments that were the basis for our measures. As is common practice, we used identical pre-/post-instruments and surveys, and so test familiarity may have impacted results. Additional reliability and validity studies for our measures would strengthen our design, particularly for affective aspects.

7 Conclusion

It has been more than a century since Klein (1908/1932) named a problem of discontinuity in secondary teacher education. Our strategy for tackling discontinuity was to simulate core teaching practices in mathematically intensive ways. We mean this in two ways. First, PSMTs are given the opportunity to do so, through applications of content to teaching. Second, the university instructors are supported in modeling these core practices while teaching secondary mathematics from an advanced perspective.

Winsløw and Grønbæk (2014) proposed that resolving any discontinuities required distinct attention to three interdependent dimensions: position (student or teacher), content (secondary or advanced), and institution (university or school). Our project confirms the relevance of these three dimensions. Applications of content to teaching formed links from PSMT-as-university-student to PSMT-as-future-secondary-teacher. We treated secondary level content from an advanced perspective. In doing so, we connected secondary mathematics to tertiary complexity and depth. We have only so far referred to assignments and activities. The final, but perhaps most important design feature is instructional coherence. We designed the materials to support instructors' enactment of the core practices featured in applications to teaching. In this way, university instruction in these courses can reinforce the bridge formed through the applications of content to teaching.

Our results indicate that PSMTs noticed the instructors' enactment of core teaching practices and attended to core practices through applications to teaching. Furthermore, their content knowledge for teaching and expectancy for carrying out core teaching practices increased. These results held across four content areas and multiple types of tertiary institutions. Our findings provide evidence that instructional coherence may enhance teachers' potential competence. At the same time, we observe that enacting instructional coherence means more than changing written curriculum. Resolving the dilemma of discontinuity may call for changing participation structures within and across tertiary classrooms.

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