# Adaptive Trajectory Synchronization with Time-Delayed Information

Rounak Bhattacharya, Vrithik Raj Guthikonda, Ashwin P. Dani, *Senior Member, IEEE*

***Abstract*—In this paper, an adaptive trajectory synchronization controller is developed that synchronizes the robot joint trajectory to the human joint trajectory in the presence of communication time delay and uncertainty in robot model parameters including nonlinear-in-parameter friction term. The controller synchronizes to the human trajectory by accounting for time delays that arise in human-robot collaboration tasks such as, estimating the human trajectory using image processing, or sensor fusion for trajectory intent estimation, or computational limitations. The developed adaptive time-delayed synchronization controller utilizes a new integral concurrent learning (ICL)-based parameter update law for Neural Network parameter estimation. Uniformly ultimately bounded stability of the synchronization and parameter estimation errors are proved using a Lyapunov-Krasovskii functional analysis. Results of the Monte Carlo simulations are presented to validate the performance of the proposed synchronization controller using a human-robot synchronization example.**

## I. Introduction

Collaboration of human and robot agents has gained vast interest in many applications, specifically in manufacturing robotics and automation [1]. For a joint task which involves coordination of the human and robot, achieving trajectory synchronization can be useful. However, for synchronization tasks, the human trajectory inference is required [2]. Methods for trajectory inference, also known as intent inference, based on image data and fusion with other sensors have been developed in [3]. Image-processing and communication adds a time-delay in the trajectory inference [4]. To handle such delays in trajectory synchronization, this paper develops an adaptive time-delayed trajectory synchronization controller in the presence of general uncertain nonlinear robot model.

Various results are available for control design in the presence of time-delays induced from the state, control input or from the state being communicated [5]–[11]. Time delays are present in many engineered systems such as robot teleoperation [12], systems involving image processing, synchronization of multi-robot systems, networked systems, space robotics [13] and newer applications of human-robot collaboration in cyber-physical human systems. A small communication or information processing delay can lead to unstable behavior and reduced performance in networked autonomous systems [14]. Synchronization of multiple agents is a cooperation of behavior between agents such that the states of agents synchronize with time based information of each other's states for non-equal initial conditions [15], [16]. In [10], [17], [18], a communication-delayed synchronization controllers are developed, where time-delayed information of neighboring agents is used. The methods in [17], [18] develops time-delayed synchronization controller using globally Lipschitz dynamics or dynamics with no drift term. The synchronization controller in [10] considers a general nonlinear dynamics but does not develop an adaptive controller. In our paper, a modified synchronization objective is used, which synchronizes the robot states to the delayed human state, where the delay is present due to information processing. The robot dynamics is a general nonlinear Euler-Lagrange (EL) dynamics including nonlinear-in-parameters (NIP) friction term. The objective is aligned with time-delayed synchronization problem in teleoperation [16], [19].

In teleoperation two robots synchronize based on the state information communicated over a communication channel [12], which adds a time delay to the state information [16], [19]–[25]. Passivity and dissipativity based approaches are used successfully to design adaptive synchronization controllers for bilateral teleoperations in [16] for joint space synchronization with a constant time delay and in [19] for task space synchronization of heterogeneous robot manipulators with varying time delays using gradient update laws. The controllers in [16], [19] guarantee asymptotic stability of the system. Synchronization to human state is achieved in the bilateral teleoperation context in [22] using passivity approach. In [23], the performance and stability of three different configurations of human-in-the-loop telerobotic systems are studied. It is showed how the stability of a human-in-the-loop system is affected by a communication architecture considering independent human reaction and communication time delays. In [24], haptic feedback is provided to improve the performance of the human operator in the human-in-the-loop telerobotic system under the presence of independent human reaction and telecommunication delay. In [25], stability limits of a model reference adaptive control (MRAC) with human-in-the-loop scenario under delay dependent and independent cases are analyzed. However, these studies focus on behavior modeling of human reaction delays and its effects in teleoperation but do not consider the problem of human-robot state synchronization. Also, the robot dynamics is linearly parametrizable.

This work was supported in part by NSF grant no. SMA-2134367 and in part by a Space Technology Research Institutes grant (number 80NSSC19K1076) from NASA Space Technology Research Grants Program. Rounak Bhattacharya, Vrithik Raj Guthikonda and Ashwin P. Dani are with the Department of Electrical and Computer Engineering at University of Connecticut, Storrs, CT 06269. Email: {rounak.bhattacharya;vrithik.guthikonda;ashwin.dani}@uconn.edu.

The contribution of this paper is to develop an adaptive time-delayed trajectory synchronization controller of robot manipulator represented using EL dynamics with the human arm trajectory in joint space. The EL dynamics consists of terms that can be written as linear-in-parameter (LIP) approximated using a single layer neural network (NN) and NIP friction approximated using a 3-layer NN. The synchronization controller uses a delayed state information of the human agent, a newly designed integral concurrent learning (ICL)-based adaptive law for updating single layer NN parameters using finite excitation condition, and gradient parameter update for three-layer NN. It is assumed that the time delay is known, constant and bounded. For the control design using Lyapunov stability analysis, a delay compensating signal is used in the filtered synchronization error. A Lyapunov-Krasovskii (LK) functional is designed such that the delayed terms in the closed-loop dynamics are nullified. This yields an exponential convergence to a bound yielding a uniformly ultimately bounded (UUB) stability of the synchronization and parameter estimation errors, in contrast to the asymptotic stability results found in literature [16], [19], [20], which do not consider NIP uncertainty. To test the performance of the proposed controller for unknown variations in time delays, Monte Carlo (MC) studies are conducted with 100 runs by sampling the delay from a uniform distribution. Root mean squared error (RMSE) and peak error values are computed along with root mean square (RMS) values of the required torque. The case of time-varying delays, similar to [10], [19], will be studied in future. In rest of the paper, Frobenius norm is used for a matrix norm, i.e., $\|A\| = \|A\|_F$ for $A \in \mathbb{R}^{m\times m}$.

## II. Dynamic Model and Synchronization Problem

### A. Joint Space Robot Dynamics

The EL equation of motion for an n-link robot is given by

$$M(q_r)\ddot{q}_r + C(q_r,\dot{q}_r)\dot{q}_r + f(\dot{q}_r) + G(q_r) = \tau_r(t) \tag{1}$$

where $q_r(t) \in \mathbb{R}^n$ represents joint positions, $\dot{q}_r(t) \in \mathbb{R}^n$ represents joint velocities, $\ddot{q}_r(t) \in \mathbb{R}^n$ is joint angular accelerations, $\tau_r \in \mathbb{R}^n$ is the torque, $M(q_r) \in \mathbb{R}^{n\times n}$ represents a positive definite inertia matrix, $C(q_r,\dot{q}_r) \in \mathbb{R}^{n\times n}$ is the centripetal-Coriolis matrix, $f(\dot{q}_r) \in \mathbb{R}^n$ is a nonlinear friction term and $G(q_r) \in \mathbb{R}^n$ is gravity term. The robot dynamics satisfies the following properties which will be subsequently used in the synchronization control design. **Property 1**: The inertia matrix $M(q_r)$ is symmetric, positive definite and satisfies $\underline{m}||y_l||^2 \le y_l^T M(q_r) y_l \le \bar{m}||y_l||^2, \ \forall y_l \in \mathbb{R}^n$, where $\underline{m}, \bar{m} \in \mathbb{R}$ are known positive constants. **Property 2**: The inertia and centripetal-Coriolis matrices satisfy the skew-symmetry property $\bar{\eta}^T(\dot{M} - 2C)\bar{\eta} = 0, \quad \forall \bar{\eta} \in \mathbb{R}^n$,

**Assumption 1.** *It is assumed that $q_r(t)$, $\dot{q}_r(t)$ are measurable.*

**Assumption 2.** *The nonlinear friction terms containing Stribeck effect is upper bounded as $\|f(\dot{q}_r)\| \le 3\bar{\gamma}\sqrt{n}$ for $\bar{\gamma} \in \mathbb{R}^+$ [26].*

### B. Human Arm Agent Model

The human hand motion model is represented by

$$\dot{\bar{q}}_h = f_h(\bar{q}_h) \tag{2}$$

where $\bar{q}_h(t) = \begin{bmatrix} q_h^T & \dot{q}_h^T \end{bmatrix}^T$, $q_h(t) \in \mathbb{R}^n$ are the joint angles of the human hand, $\dot{q}_h(t) \in \mathbb{R}^n$ are the joint angular velocities, $f_h : \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ represented by a NN is given by $f_h(\bar{q}_h) = W_h^T \sigma(V_h^T s) + \epsilon_h(\bar{q}_h)$, where $W_h \in \mathbb{R}^{(n+1)\times 2n}$, $V_h \in \mathbb{R}^{(2n+1)\times n}$ are ideal weight matrices and $s \in \mathbb{R}^{2n+1}$ is the input vector to the NN defined as $s = [q_h^T, \dot{q}_h^T, 1]^T$ and $\sigma(.)$ is the Sigmoid activation function given by $\sigma(s) = [\bar{\sigma}(s_1), \bar{\sigma}(s_2), ..., \bar{\sigma}(s_n), 1]^T$, where $s = [s_1, s_2, ..., s_{2n+1}]^T$ and $\bar{\sigma}(s_i) : \mathbb{R} \to \mathbb{R}, \ \forall i = [1, ..., 2n+1]$ is defined as $\bar{\sigma}(s_i) = 1/(1+\exp(-s_i))$, and $\epsilon_h(\bar{q}_h) \in \mathbb{R}^{2n\times 1}$ is the NN function reconstruction error. More details of the NN training of such models are provided in [27] for estimating $\hat{W}_h$ and $\hat{V}_h$, which will be used to compute $\dot{\bar{q}}_h(t)$.

**Remark 1.** *The human motion trajectories can be fused with other cues such as gaze direction to obtain more accurate trajectories [3]. Computation of $\bar{q}_h$ from image processing may add time delay for computing human states [4].*

### C. Time-Delayed Synchronization Problem

Given the trajectory estimation of the human joints $\bar{q}_h(t - T)$ with a known bounded (constant) delay of $T > 0$ seconds, the robot joint state synchronizes to human state if $\limsup_{t\to\infty} \|q_r(t) - q_h(t-T)\| \le \varepsilon_1$ and $\limsup_{t\to\infty} \|\dot{q}_r(t) - \dot{q}_h(t-T)\| \le \varepsilon_2$ for some small $\varepsilon_1, \varepsilon_2 \in \mathbb{R}^+$. Let $t_T \triangleq t - T$ and let the position synchronization error between the human and the robot be defined as

$$e_r(t) = q_r(t) - q_h(t_T) \tag{3}$$

and an auxiliary error $r_r(t) \in \mathbb{R}^n$ be defined as

$$r_r(t) = \dot{e}_r(t) + \lambda e_r(t) - k_b e_z. \tag{4}$$

where $k_b, \lambda > 0$ are known constant scalar gains, the signal $e_z \in \mathbb{R}^n$ is defined as $e_z \triangleq \int_{t_T}^{t} r_r(l) dl$. Multiplying the time derivative of (4) with $M(q_r)$ and using the double time derivative of (3) for $\ddot{e}_r(t)$, using the definition of $e_z$, and using (1), the open loop error dynamics can be written as

$$\begin{aligned} M(q_r)\dot{r}_r(t) &= \tau_r(t) - C(q_r,\dot{q}_r)\dot{q}_r(t) - f(\dot{q}_r) - G(q_r) \\ &- M(q_r)\ddot{q}_h(t_T) + \lambda M(q_r)\dot{e}_r(t) - k_b M(q_r)[r_r(t) - r_r(t_T)] \end{aligned} \tag{5}$$

Let $f(\dot{q}_r) = W_f^T \sigma_1(V_f^T \dot{q}_{br}) + \epsilon_1(\dot{q}_{br})$, where $\dot{q}_{br} = [\dot{q}_r^T, 1]^T$, $W_f \in \mathbb{R}^{(N_1+1)\times n}$, $V_f \in \mathbb{R}^{(n+1)\times N_1}$ are NN weights, $\sigma_1 : \mathbb{R}^{N_1} \to \mathbb{R}^{N_1+1}$ are the basis functions, and $\epsilon_1(\dot{q}_{br})$ is a function reconstruction error. Let $M(q_r)(\ddot{q}_h(t_T) - \lambda\dot{e}_r) + C(q_r,\dot{q}_r)(\dot{q}_r(t) - r_r(t)) + G(q_r) = \theta^T\sigma_2(\bar{q}) + \epsilon_2(\bar{q})$ where $\bar{q} = [q_r^T, \dot{q}_r^T, r_r^T, \dot{e}_r^T, \ddot{q}_h^T(t_T), 1]^T \in \mathbb{R}^{5n+1}$ and $\sigma_2 : \mathbb{R}^{5n+1} \to \mathbb{R}^p$ is a set of basis functions, $\theta^T \in \mathbb{R}^{n\times p}$ are the ideal NN weights, and $\epsilon_2 \in \mathbb{R}^n$ is the NN function reconstruction error. The open loop error dynamics can then be written as

$$\begin{aligned} M(q_r)\dot{r}_r(t) + C(q_r,\dot{q}_r)r_r(t) &= \tau_r(t) - \theta^T\sigma_2(\bar{q}) - \epsilon_2(\bar{q}) \\ -W_f^T\sigma_1(V_f^T\dot{q}_{br}) - \epsilon_1(q_{br}) &- k_b M(q_r)[r_r(t) - r_r(t_T)] \end{aligned} \tag{6}$$

**Assumption 3.** *The Frobenius norms of ideal weights satisfy the bounds* $\|W_f\|_F^2 \leq \bar{W}_f$, $\|V_f\|_F^2 \leq \bar{V}_f$ *and* $\|\theta\|_F^2 \leq \bar{\theta}$, *where* $\bar{W}_f$, $\bar{V}_f$, $\bar{\theta}_f \in \mathbb{R}^+$, *and the basis functions satisfy* $\|\sigma_2(\bar{q})\| \leq \bar{\sigma}_b$ *for a positive* $\bar{\sigma}_b$.

**Assumption 4.** *Using the universal function approximation property of the NN, the function reconstruction errors can be bounded as* $\|\epsilon_1\| \leq \bar{\epsilon}_1$, *and* $\|\epsilon_2\| \leq \bar{\epsilon}_2$.

## III. Time Delayed Synchronization Control

### A. Control and Parameter Update Law

The robot torque control input is designed as

$$\tau_r(t) = -K_1 r_r(t) - K_2 e_r(t) + \hat{\theta}^T \sigma_2(\bar{q}) + \hat{W}_f^T \sigma_1(\hat{V}_f^T \dot{q}_{br}) \quad (7)$$

where $\hat{\theta}(t)$,$\hat{W}_f(t)$, $\hat{V}_f(t)$ are the NN weight parameter estimates and $K_1, K_2 > 0$ are scalar constants.

*ICL-based Adaptive Law:* For ICL adaptive control [28] the parameter update law is defined as follows

$$\dot{\hat{\theta}} = -\Lambda \sigma_2(\bar{q}) r_r^T(t) + k\Lambda \sum_{i=1}^{N} [\mathcal{Y}_i (\mathcal{U}_i - \hat{\theta}^T \mathcal{Y}_i)^T] - \alpha_{s1} \Lambda \hat{\theta} \quad (8)$$

where $\Lambda \in \mathbb{R}^{p \times p}$ a positive-definite matrix and $k \in \mathbb{R}^+$ are adaptation gains, $\alpha_{s1}$ is a scalar gain, $N \in \mathbb{Z}^+$. Let $q_a = [q_r^T, \dot{q}_r^T, \ddot{q}_r^T, 1]^T \in \mathbb{R}^{3n+1}$, $\mathcal{Y}_i \in \mathbb{R}^p$ and $\mathcal{U}_i \in \mathbb{R}^n$ are defined as

$$\mathcal{U}_i = \int_{max(t-\Delta t,0)}^{t} \tau_r(\xi) d\xi, \; \mathcal{Y}_i = \int_{max(t-\Delta t,0)}^{t} \sigma_2(V_a^T q_a(\xi)) d\xi \quad (9)$$

Integrating both sides of (1) yields

$$\int_{t-\Delta t}^{t} \tau_r(\psi) d\psi = \theta^T \int_{t-\Delta t}^{t} \sigma_2(V_a^T q_a(\psi)) d\psi + Z_i \quad (10)$$

where $M(q_r)\ddot{q}_r + C(q_r, \dot{q}_r)\dot{q}_r + G(q_r) = \theta^T \sigma_2(V_a^T q_a) + \epsilon_1(q_a)$, $Z_i \triangleq \int_{t-\Delta t}^{t} \epsilon_1(\psi) + f(\psi) d\psi$, $V_a \in \mathbb{R}^{3n+1 \times p}$ is a constant computed according to the batch intrinsic plasticity (BIP) algorithm of extreme learning machine NN [29]. Using (9), (10) is written as $\mathcal{U}_i = \theta^T \mathcal{Y}_i + Z_i$. Therefore (8) reduces to

$$\dot{\hat{\theta}} = -\Lambda \sigma_2(\bar{q}) r_r^T(t) + k\Lambda \sum_{i=1}^{N} [\mathcal{Y}_i (\theta^T \mathcal{Y}_i + Z_i - \hat{\theta}^T \mathcal{Y}_i)^T] - \alpha_{s1} \Lambda \hat{\theta} \quad (11)$$

The NN parameter update laws are designed as

$$\dot{\hat{W}}_f = -\Gamma_1 \hat{\sigma}_1 r_r^T - \Gamma_1 \hat{\sigma}_1' \hat{V}_f^T \dot{q}_{br} r_r^T - \alpha_{s2} \Gamma_1 \hat{W}_f \quad (12)$$

$$\dot{\hat{V}}_f = \Gamma_2 \dot{q}_{br} (\hat{\sigma}_1'^T \hat{W}_f r_r)^T - \alpha_{s3} \Gamma_2 \hat{V}_f \quad (13)$$

where $\Gamma_1 \in \mathbb{R}^{(N_1+1)\times(N_1+1)}$, and $\Gamma_2 \in \mathbb{R}^{(n+1)\times(n+1)}$, $\hat{\sigma}_1$ is the basis function computed at $\hat{V}_f$ and $\hat{\sigma}_1' : \mathbb{R}^{N_1} \to \mathbb{R}^{N_1+1 \times N_1}$ is the Jacobian of the basis functions.

**Remark 2.** *Although regressor* $\sigma_2(q_a)$ *is a function* $\ddot{q}_r$, $\mathcal{Y}_i$ *is computed by integrating* $\sigma_2(q_a)$, *which reduces the effects of noise in computation of* $\ddot{q}_r$.

### B. Closed-loop Error Dynamics

By substituting $\tau_r$ from (7) into (6), the closed-loop dynamics can be written as

$$M(q_r)\dot{r}_r(t) + C(q_r, \dot{q}_r) r_r(t) = -K_1 r_r(t) - K_2 e_r(t) - \theta^T \sigma_2(\bar{q}) + \hat{\theta}^T \sigma_2(\bar{q}) - \epsilon_2 - k_b M(q_r)[r_r(t) - r_r(t_T)] - (W_f^T \sigma_1(V_f^T \dot{q}_{br}) + \epsilon_1 - \hat{W}_f^T \sigma_1(\hat{V}_f^T \dot{q}_{br})) \quad (14)$$

Taylor series expansion of friction NN terms yields

$$M(q_r)\dot{r}_r(t) + C(q_r, \dot{q}_r) r_r(t) = -K_1 r_r(t) - K_2 e_r(t) - \tilde{\theta}^T \sigma_2(\bar{q}) - k_b M(q_r)[r_r(t) - r_r(t_T)] - (\tilde{W}_f^T \hat{\sigma}_1 - \tilde{W}_f^T \hat{\sigma}_1' \hat{V}_f^T \dot{q}_{br} + \hat{W}_f^T \hat{\sigma}_1' \tilde{V}_f^T \dot{q}_{br} + \epsilon_f) \quad (15)$$

where $\epsilon_f = -\tilde{W}_f^T \hat{\sigma}_1' V_f^T \dot{q}_{br} - W_f^T O(\tilde{V}_f^T \dot{q}_{br}) + \epsilon_1(\dot{q}_{br}) + \epsilon_2(\bar{q})$ is such that $\|\epsilon_f\| \leq c_1 + c_2\|r_r\|$, for positive constants $c_1$ and $c_2$, $O(\cdot)$ are the higher order terms. Let $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$, $\tilde{W}_f(t) = W_f - \hat{W}_f(t)$ and $\tilde{V}_f(t) = V_f - \hat{V}_f(t)$ be the NN and weight matrix estimation errors for $\theta$, $W_f$ and $V_f$, respectively. Using (11), the parameter estimation error dynamics is written as

$$\dot{\tilde{\theta}} = \Lambda \sigma_2 r_r^T - k\Lambda \sum_{i=1}^{N} [\mathcal{Y}_i \mathcal{Y}_i^T] \tilde{\theta} - k\Lambda \sum_{i=1}^{N} \mathcal{Y}_i Z_i^T + \alpha_{s1} \Lambda \hat{\theta} \quad (16)$$

and the NN weight matrix estimation errors are given by

$$\dot{\tilde{W}}_f = \Gamma_1 \hat{\sigma}_1 r_r^T + \Gamma_1 \hat{\sigma}_1' \hat{V}_f^T \dot{q}_{br} r_r^T + \alpha_{s2} \Gamma_1 \hat{W}_f$$

$$\dot{\tilde{V}}_f = -\Gamma_2 \dot{q}_{br} (\hat{\sigma}_1'^T \hat{W}_f r_r)^T + \alpha_{s3} \Gamma_2 \hat{V}_f \quad (17)$$

**Assumption 5.** *The system is sufficiently excited over a finite time period* $T_{fe}$, *this implies* $\exists \lambda_m > 0$ *and* $\exists T_{fe} > \Delta t$ : $\forall \Delta t \geq T_{fe}$, $\lambda_{\min} \left\{ \sum_{i=1}^{N} \mathcal{Y}_i^T \mathcal{Y}_i \right\} \geq \lambda_m$.

### C. Stability Analysis

The stability of the adaptive synchronization controller is analyzed in two phases. The first theorem is derived when Assumption 5 is not satisfied and second theorem is derived when Assumption 5 is satisfied.

**Theorem 1.** *For the system defined in (1), the synchronization controller in (7) and adaptive law in (8)-(13) ensure bounded synchronization and parameter estimation errors if Assumptions 1-4 and the gain conditions* $k_b \in \left( \frac{\alpha_r + c_2 - K}{m - \frac{\bar{m}}{2}}, \min\left\{ \frac{4\lambda}{\gamma_r^2}, \frac{2}{K_2} \right\} \right)$ *and* $K > 0$ *are satisfied, where* $\gamma_r, \lambda_r$ *are constants defined subsequently and* $\lambda > 0$.

*Proof.* Let $y(t) = [r_r^T \; e_r^T \; \text{vec}(\tilde{\theta})^T \; \text{vec}(\tilde{W}_f)^T \; \text{vec}(\tilde{V}_f)^T]^T \in \mathbb{R}^{3n+np+N_1(2n+1)}$ be a vector of closed-loop signals, vec$(\cdot)$ is vectorization operator. For the stability analysis, LK functional $P$ and $Q$ are defined as

$$P = \omega_r \int_{t_T}^{t} \int_{s}^{t} r_r^T(l) r_r(l) dl ds, \;\; Q = K \int_{t_T}^{t} r_r^T(l) r_r(l) dl. \quad (18)$$

where $\omega_r \in \mathbb{R}^+$ and $K \in \mathbb{R}^+$ are constants. Let $z(t) = [y^T \sqrt{Q} \sqrt{P}\,]^T \in \mathcal{D} \subset \mathbb{R}^{3n+np+2+N_1(2n+1)}$ be an auxiliary vector. A positive definite Lyapunov functional candidate $V(z,t) : \mathcal{D} \times [0,\infty) \to \mathbb{R}^+$ is then defined as

$$V(z,t) = \frac{1}{2}(r_r^T M(q_r) r_r + K_2 e_r^T e_r + \mathrm{tr}(\tilde{\theta}^T \Lambda^{-1} \tilde{\theta}) + \mathrm{tr}(\tilde{W}_f^T \Gamma_1^{-1} \tilde{W}_f) + \mathrm{tr}(\tilde{V}_f^T \Gamma_2^{-1} \tilde{V}_f)) + P + Q \quad (19)$$

The bounds on $V(z,t)$ can be derived as $\beta_1 \|z\|^2 \leq V(z,t) \leq \beta_2 \|z\|^2$, where $\beta_1 \triangleq \frac{1}{2}\min\{2, K_2, \underline{m}, \lambda_{\min}(\Lambda^{-1}), \lambda_{\min}(\Gamma_1^{-1}), \lambda_{\min}(\Gamma_2^{-1})\}$ and $\beta_2 \triangleq \frac{1}{2}\max\{2, K_2, \overline{m}, \lambda_{\max}(\Lambda^{-1}), \lambda_{\max}(\Gamma_1^{-1}), \lambda_{\max}(\Gamma_2^{-1})\}$. Taking the time derivative of $V(z,t)$ and substituting (4), (15), (16) and (17) yields

$$\begin{aligned}
\dot{V}(z,t) &= r_r^T(t)(-C_r(q_r,\dot{q}_r) r_r(t) - K_1 r_r(t) - K_2 e_r(t) \\
&- \tilde{\theta}^T \sigma_2 - k_b M(q_r)[r_r(t) - r_r(t_T)] \\
&- \tilde{W}_f^T \hat{\sigma}_1 - \tilde{W}_f^T \hat{\sigma}_1' \hat{V}_f^T \dot{q}_{br} + \hat{W}_f^T \hat{\sigma}_1' \tilde{V}_f^T \dot{q}_{br} - \epsilon_f) \\
&+ K_2 e_r^T(t)(r_r(t) - \lambda e_r(t) + k_b e_z) + \frac{1}{2} r_r^T(t) \dot{M} r_r(t) \\
&+ \mathrm{tr}\Big(\tilde{\theta}^T [\sigma_2 r_r(t)^T - k \sum_{i=1}^N [\mathcal{Y}_i \mathcal{Y}_i^T] \tilde{\theta} + \alpha_{s1} \Lambda \hat{\theta} - k \sum_{i=1}^N \mathcal{Y}_i Z_i^T]\Big) \\
&+ \mathrm{tr}(\tilde{W}_f^T [\hat{\sigma}_1 r_r^T + \hat{\sigma}_1' \hat{V}_f^T \dot{q}_{br} r_r^T + \alpha_{s2} \hat{W}_f]) \\
&+ \mathrm{tr}(\tilde{V}_f^T [-\dot{q}_{br} (\hat{\sigma}_1'^T \hat{W}_f r_r)^T + \alpha_{s3} \hat{V}_f]) + \dot{P} + \dot{Q} \qquad (20)
\end{aligned}$$

Canceling the terms in trace with the corresponding NN terms and using Property 3 of the EL system, (20) reduces to

$$\begin{aligned}
\dot{V}(z,t) &= r_r^T(t)(-K_1 r_r(t) - K_2 e_r(t) - k_b M(q_r)[r_r(t) \\
&- r_r(t_T)] - \epsilon_f + K_2 e_r^T(t)(r_r(t) - \lambda e_r(t) + k_b e_z) \\
&+ \mathrm{tr}\Big(\tilde{\theta}^T (-k \sum_{i=1}^N [\mathcal{Y}_i \mathcal{Y}_i^T] \tilde{\theta} - k \sum_{i=1}^N \mathcal{Y}_i Z_i^T + \alpha_{s1} \Lambda \hat{\theta})\Big) \\
&+ \dot{P} + \dot{Q} + \alpha_{s2} \mathrm{tr}(\tilde{W}_f^T \hat{W}_f) + \alpha_{s3} \mathrm{tr}(\tilde{V}_f^T \hat{V}_f) \qquad (21)
\end{aligned}$$

The time-derivatives of $P$ and $Q$ from (18) using the Leibniz integral rule for differentiation under the integral are

$$\dot{P} = -\omega_r \int_{t_T}^t r_r^T(l) r_r(l) dl \qquad (22)$$

$$\dot{Q} = K(r_r^T(t) r_r(t) - r_r^T(t_T) r_r(t_T)) \qquad (23)$$

Substituting (22) and (23) in (21), and using $K_1 = 2K$ yields

$$\begin{aligned}
\dot{V}(z,t) &= -\Big(K r_r^T(t) r_r(t) + K r_r^T(t_T) r_r(t_T) \\
&+ k_b r_r^T(t) M(q_r) r_r(t) + K_2 \lambda e_r^T(t) e_r(t) \\
&+ \mathrm{tr}(k \tilde{\theta}^T \sum_{i=1}^N \mathcal{Y}_i \mathcal{Y}_i^T \tilde{\theta} + k \tilde{\theta}^T \sum_{i=1}^N \mathcal{Y}_i Z_i^T - \alpha_{s1} \tilde{\theta}^T \hat{\theta}\Big) \\
&+ \omega_r \int_{t_T}^t r_r^T(l) r_r(l) dl + \big(k_b r_r^T(t) M(q_r) r_r(t_T) \\
&+ K_2 k_b e_r^T(t) e_z\big) - r_r^T \epsilon_f - \alpha_{s2} \|\tilde{W}_f\|^2 - \alpha_{s3} \|\tilde{V}\|^2 \\
&+ \alpha_{s2} \mathrm{tr}(\tilde{W}_f^T W_f) + \alpha_{s3} \mathrm{tr}(\tilde{V}_f^T V_f) \qquad (24)
\end{aligned}$$

Substituting $\hat{\theta}(t) = \theta - \tilde{\theta}(t)$, and using the fact that $\sum_{i=1}^N \mathcal{Y}_i^T \mathcal{Y}_i$ is positive semi-definite when Assumption 5 is not satisfied, $\dot{V}(z,t)$ can be upper bounded as

$$\begin{aligned}
&\dot{V}(z,t) \leq -K\|r_r(t)\|^2 - K\|r_r(t_T)\|^2 - k_b \underline{m} \|r_r(t)\|^2 \\
&- K_2 \lambda \|e_r(t)\|^2 + k_b \bar{m} \|r_r(t)\| \|r_r(t_T))\| + K_2 k_b \|e_r(t)\| \\
&. \|e_z(t))\| + \|r_r\| \|\epsilon_f\| - \omega_r \int_{t_T}^t r_r^T(l) r_r(l) dl - \alpha_{s2} \|\tilde{W}_f\|^2 \\
&- \alpha_{s3} \|\tilde{V}_f\|^2 + \alpha_{s2} \|\tilde{W}_f\| \bar{w}_f + \alpha_{s3} \|\tilde{V}_f\| \bar{v}_f - \alpha_{s1} \|\tilde{\theta}\|^2 + \alpha_1 \|\tilde{\theta}\|
\end{aligned} \qquad (25)$$

where the bound on terms $\mathcal{Y}_i$ and the friction $f$ (and therefore $Z_i$) developed using Assumptions 2 and 3 can be used to define $\alpha_1 \triangleq (\alpha_{s1} \bar{\theta} + k N \Delta t^2 \bar{\sigma}_b [3\bar{\gamma}\sqrt{n} + \bar{\epsilon}])$. The following terms in (25) can be upper bounded by using Young's inequality as

$$\begin{aligned}
k_b \bar{m} \|r_r(t)\| \|r_r(t_T)\| &\leq \frac{k_b \bar{m}}{2} \|r_r(t)\|^2 + \frac{k_b \bar{m}}{2} \|r_r(t_T)\|^2 \\
K_2 k_b \|e_r(t)\| \|e_z(t)\| &\leq \frac{K_2 k_b \gamma_r^2}{4} \|e_r(t)\|^2 + \frac{K_2 k_b}{\gamma_r^2} \|e_z(t)\|^2
\end{aligned} \qquad (26)$$

where $\gamma_r \in \mathbb{R}^+$ such that $\gamma_r > \sqrt{\frac{2T}{\omega_r}}$. Hence, $-\omega_r < -\frac{2T}{\gamma_r^2}$.

Utilizing (26), the inequality for $\omega_r$, and $\|e_z(t)\|^2 \leq T \int_{t_T}^t r_r^T(l) r_r(l) dl$ computed using the Cauchy-Schwarz inequality in (25) results in

$$\begin{aligned}
\dot{V}(z,t) &\leq -\Big(K + k_b \underline{m} - \frac{k_b \bar{m}}{2}\Big) \|r_r(t)\|^2 \\
&- \Big(K - \frac{k_b \bar{m}}{2}\Big) \|r_r(t_T)\|^2 - K_2 \Big(\lambda - \frac{k_b \gamma_r^2}{4}\Big) \|e_r(t)\|^2 \\
&- \frac{(2 - K_2 k_b)}{\gamma_r^2} T \int_{t_T}^t r_r^T(l) r_r(l) dl + \|r_r\| c_1 + c_2 \|r_r\|^2 \\
&- \alpha_{s1} \|\tilde{\theta}\|^2 - \alpha_{s2} \|\tilde{W}_f\|^2 - \alpha_{s3} \|\tilde{V}_f\|^2 \\
&+ \alpha_{s2} \|\tilde{W}_f\| \bar{w}_f + \alpha_{s3} \|\tilde{V}_f\| \bar{v}_f + \alpha_1 \|\tilde{\theta}\| \qquad (27)
\end{aligned}$$

Let $\alpha_2, \alpha_3, \alpha_4, \alpha_r \in \mathbb{R}^+$. Completing the squares on terms $\|\tilde{W}_f\|^2$, $\|\tilde{V}_f\|^2$, $\|\tilde{\theta}\|^2$, $\|r\|^2$, and using $\alpha_i$ with further simplification yields

$$\begin{aligned}
\dot{V}(z,t) &\leq -\Big(K + k_b \underline{m} - \frac{k_b \bar{m}}{2} - \alpha_r - c_2\Big) \|r_r(t)\|^2 \\
&- \Big(K - \frac{k_b \bar{m}}{2}\Big) \|r_r(t_T)\|^2 - K_2 \Big(\lambda - \frac{k_b \gamma_r^2}{4}\Big) \|e_r(t)\|^2 \\
&- \frac{(2 - K_2 k_b)}{\gamma_r^2} T \int_{t_T}^t r_r^T(l) r_r(l) dl - (1 - \alpha_2) \alpha_{s2} \|\tilde{W}_f\|^2 \\
&- (1 - \alpha_3) \alpha_{s3} \|\tilde{V}_f\|^2 - (1 - \alpha_4) \alpha_{s1} \|\tilde{\theta}\|^2 + c_3 \qquad (28)
\end{aligned}$$

where $c_3 = \frac{1}{4\alpha_r} c_1^2 + \frac{\alpha_{s2}}{4\alpha_2} \bar{w}^2 + \frac{\alpha_{s3}}{4\alpha_3} \bar{v}^2 + \frac{\alpha_1^2}{4\alpha_4 \alpha_{s1}}$, $1 - \alpha_2 > 0$, $1 - \alpha_3 > 0$, and $1 - \alpha_4 > 0$. Splitting the integral term of (28) and utilizing the following inequality

$$-\frac{1}{\gamma_r^2} \int_{t_T}^t \int_s^t r_r^T(l) r_r(l) dl ds \geq -\frac{T}{\gamma_r^2} \int_{t_T}^t r_r^T(l) r_r(l) dl \qquad (29)$$

and using (18) yields the following result

$$\dot{V}(z,t) \leq -\Big(K + k_b\underline{m} - \frac{k_b\bar{m}}{2} - \alpha_r - c_2\Big)\|r_r(t)\|^2 - K_2\Big(\lambda - \frac{k_b\gamma_r^2}{4}\Big)\|e_r(t)\|^2 - \frac{(2-K_2k_b)}{2K\gamma_r^2}TQ - \frac{(2-K_2k_b)}{2\gamma_r^2\omega_r}P - (1-\alpha_2)\alpha_{s2}\|\tilde{W}_f\|^2 - (1-\alpha_3)\alpha_{s3}\|\tilde{V}_f\|^2 - (1-\alpha_1)\alpha_{s1}\|\tilde{\theta}\|^2 + c_3. \quad (30)$$

Writing (30) as $\dot{V}(z,t) \leq -\bar{\eta}V(z,t) + c_3$, where $\eta_1 = \min\Big(K + k_b\underline{m} - \frac{k_b\bar{m}}{2} - \alpha_r - c_2,\ K_2(\lambda - \frac{k_b\gamma_r^2}{4}), (1-\alpha_4)\alpha_{s1}, \frac{(2-K_2k_b)}{2K\gamma_r^2}T, \frac{(2-K_2k_b)}{2\gamma_r^2\omega_r}\Big)$. Using the bounds on $V(z,t)$ and solving the linear differential inequality in $V(z,t)$, the bound on the system states is computed to be $\|z(T_{fe})\| \leq \sqrt{\frac{\beta_2}{\beta_1}}\|z(0)\|e^{-\frac{\bar{\eta}T_{fe}}{2}} + \sqrt{\frac{c_3}{\bar{\eta}\beta_1}},\ t \in [0, T_{fe})$, where $\bar{\eta} = \frac{\eta_1}{\beta_2}$. ☐

**Theorem 2.** *For the system defined in (1), if Assumptions 1-5 and gain conditions in Theorem 1 are satisfied then the synchronization controller in (7) and adaptive laws in (8)-(13) ensure uniformly ultimately bounded stability of all signals of the closed-loop system in (15)-(17). The robot state synchronizes to the delayed human state $\bar{q}_h(t_T)$ with an ultimate bound*

$$\|z(t)\| \leq \sqrt{\frac{\beta_2}{\beta_1}}\|z(0)\|e^{-\frac{\eta}{2}(t-T_{fe})} + \sqrt{\frac{c_3}{\eta\beta_1}},\ \ t \in [T_{fe}, \infty) \quad (31)$$

*where $\eta \triangleq \frac{\eta_2}{\beta_2}$, where $\eta_1 \in \mathbb{R}^+$ is subsequently defined.*

*Proof.* Utilizing the Lyapunov analysis presented in Theorem 1 until (24), from the finite excitation condition of Assumption 5, $\sum_{i=1}^{N}\mathcal{Y}_i^T\mathcal{Y}_i$ is positive definite. So $\dot{V}(z,t)$ can be upper bounded as

$$\dot{V}(z,t) \leq -K\|r_r(t)\|^2 - K\|r_r(t_T)\|^2 - k_b\underline{m}\|r_r(t)\|^2 - K_2\lambda\|e_r(t)\|^2 + k_b\bar{m}\|r_r(t)\|\|r_r(t_T)\| - k\lambda_m\|\tilde{\theta}(t)\|^2 + K_2k_b\|e_r(t)\|\|e_z\| - \omega_r\int_{t_T}^{t} r_r^T(l)r_r(l)dl + \|r_r\|\|\epsilon_f\| - \alpha_{2s}\|\tilde{W}_f\|^2 - \alpha_{3s}\|\tilde{V}_f\|^2 + \alpha_{2s}\|\tilde{W}_f\|\bar{w}_f + \alpha_{3s}\|\tilde{V}_f\|\bar{v}_f - \alpha_{s1}\|\tilde{\theta}\|^2 + \alpha_1\|\tilde{\theta}\| \quad (32)$$

Utilizing development similar to Theorem 1 from (26) to (28) with $-k\lambda_m\|\tilde{\theta}(t)\|^2$ term and using (18) the bound on $\dot{V}(z,t)$ can be written as

$$\dot{V}(z,t) \leq -\Big(K + k_b\underline{m} - \frac{k_b\bar{m}}{2} - \alpha_r - c_2\Big)\|r_r(t)\|^2 - K_2\Big(\lambda - \frac{k_b\gamma_r^2}{4}\Big)\|e_r(t)\|^2 - (k\lambda_m + (1-\alpha_4)\alpha_{s1})\|\tilde{\theta}\|^2 - \frac{(2-K_2k_b)}{2K\gamma_r^2}TQ - \frac{(2-K_2k_b)}{2\gamma_r^2\omega_r}P - (1-\alpha_2)\alpha_{s2}\|\tilde{W}_f\|^2 - (1-\alpha_3)\alpha_{s3}\|\tilde{V}_f\|^2 + c_3 \quad (33)$$

Thus, (33) can be written as

$$\dot{V}(z,t) \leq -\eta_2\|z\|^2 + c_3, \quad \forall t \in [T_{fe}, \infty) \quad (34)$$

where $\eta_2$ is same as $\eta_1$ except the third term inside the min operator is $k\lambda_m + (1-\alpha_4)\alpha_{s1}$. Using the bounds on $V(z,t)$, (34) can be written as $\dot{V}(z,t) \leq -\eta V(z,t) + c_3,\ \forall t \in [T_{fe}, \infty)$. The bound in (31) is obtained by solving the linear differential inequality similar to the proof of Theorem 1. Using standard signal chasing argument, all the closed loop signals and torque input are bounded. ☐

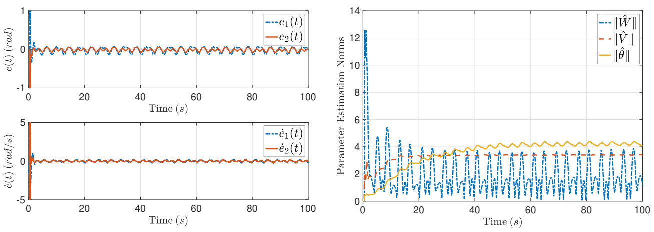
**Remark 3.** *A data selection algorithm to collect $\{q_r(t), \dot{q}_r(t), \ddot{q}_r(t), \tau_r(t)\}_{t=t_i-\Delta t}^{t=t_i}, \forall i = [1, ..., N]$ is used such that the minimum singular value in Assumption 5 is maximized, [30], $\lambda_{\min}$ is proportional to $\eta$ of Theorem 2.*

**Remark 4.** *The replacement or addition of data points from history stack does not affect LK functional, thus (19) can be used as a common LK functional similar to [30].*

## IV. Simulation Studies

The adaptive time-delayed synchronization controller is tested in simulation using a 2DoF robot which synchronizes its joint angle trajectories and velocities with the 2DoF model of human joint states. Since the human motion is not externally controlled, it is in effect one-way synchronization, where the robot synchronizes its motion with human states. In simulation, the human joint trajectories are generated from a predefined motion intent model presented in Section II-B. The developed adaptive synchronization controller is coded in Matlab R2022b. The robot is simulated with following parameters: masses and the lengths of each link are $m_1 = 0.85$kg, $m_2 = 2.3$kg, $l_1 = 1.1$m, $l_2 = 0.9$m. The acceleration due to gravity $g = 9.81\text{m/sec}^2$. The sampling time is chosen to be 0.01sec and the total simulation time is 100sec. To study the performance of the controller to the variations in time delay, the delay is sampled from a uniform distribution with mean delay of $T = 0.45$sec sampled from $[0.4-0.5]$sec interval. Monte Carlo runs of the simulation are performed by running the simulation 100 times. The steady state RMSE and the RMS values for the robot torque are calculated for each of the 100 runs. The human trajectory, for its motion in a horizontal plane for moving an object horizontally, is obtained from the model and is described as $\bar{q}_h(t) = [2\sin(\frac{\pi t}{50}), \sin(\frac{\pi t}{50}), \frac{2\pi}{50}\cos(\frac{\pi t}{50}), \frac{\pi}{50}\cos(\frac{\pi t}{50})]^T$.

To incorporate time-delayed adaptive synchronization with ICL parameter update law, the following control gains are selected: $\lambda = 5I_2$, $k_b = 0.1$, $k = 0.05I_{11}$, $K_1 = 38$ and $K_2 = 80$, where $I_n$ is an identity matrix of size $n$. Parameters of the ICL term are selected as $N = 20$ and $\Delta t = 1.5 \times 10^{-1}$ sec, where $N$ is selected using $N \geq \lceil \frac{n.p}{n} \rceil$, $\lceil\cdot\rceil$ denotes the ceiling function, which yields $N \geq 11$. The robot state is initialized to $[q_r^T(0), \dot{q}_r^T(0)]^T = [1\ 0\ 1\ 0]^T$. The NN parameter estimates $\hat{\theta}(0)$, $\hat{V}(0)$ and $\hat{W}(0)$ are initialized using a normal distribution. The basis functions are selected to be Sigmoid function. The parameter update law gains are selected to be $\Gamma_1 = 8I_9$, $\Gamma_2 = 3I_3$, $\Lambda = 0.25I_{11}$, $\alpha_{s1} = 0.16$, $\alpha_{s2} = 0.09$, $\alpha_{s3} = 0.001$. In Fig. 1(a), synchronization errors for joint angles and joint angular velocities of the 2-DOF robot are shown and the norm of the parameter estimates




(a) (b)

Figure 1. Results of simulation: (a) position and velocity synchronization errors between human and the robot, (b) parameter estimate norms.

are shown in Fig. 1(b). From Fig. 1(a), it is observed that the maximum peak error (out of two errors) in the steady state is of the order of $0.14$ rad. The mean and the standard deviations of the steady state RMSE computed over all the MC runs are $0.1342$rad and $0.00194$rad, respectively. RMS of $\tau$ computed over the MC runs is $26.378$N-m.

## V. Conclusion and Future Work

An adaptive time-delayed trajectory synchronization controller is developed in joint space for synchronizing robot and human motion. Considering the time delays in human state estimation in the synchronizing control for EL dynamics, the Lyapunov stability analysis using LK functional guarantees UUB stability of synchronization and parameter estimation errors. MC simulation studies show that the developed controller synchronizes to the human state with a small bound as seen from RMSE. Time varying time delay case will be studied as future work and the performance of the developed controller will be tested using a robot platform.

## References

[1] Y. Wang and F. Zhang, *Trends in control and decision-making for human-robot collaboration systems*. Springer, 2017.

[2] A. P. Dani, I. Salehi, G. Rotithor, D. Trombetta, and H. Ravichandar, "Human-in-the-loop robot control for human-robot collaboration: Human intention estimation and safe trajectory tracking control for collaborative tasks," *IEEE Control Systems Magazine*, vol. 40, no. 6, pp. 29–56, 2020.

[3] D. Trombetta, G. Rotithor, I. Salehi, and A. P. Dani, "Variable structure human intention estimator with mobility and vision constraints as model selection criteria," *Mechatronics*, vol. 76, p. 102570, 2021.

[4] G. Macesanu, V. Comnac, F. Moldoveanu, and S. M. Grigorescu, "A time-delay control approach for a stereo vision based human-machine interaction system," *Journal of Intelligent & Robotic Systems*, vol. 76, pp. 297–313, 2014.

[5] M. Krstic, *Delay compensation for nonlinear, adaptive, and PDE systems*. Springer, 2009.

[6] F. Mazenc, M. Malisoff, and T. N. Dinh, "Robustness of nonlinear systems with respect to delay and sampling of the controls," *Automatica*, vol. 49, no. 6, pp. 1925–1931, 2013.

[7] N. Olgac and R. Sipahi, "An exact method for the stability analysis of time-delayed linear time-invariant (LTI) systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 5, pp. 793–797, 2002.

[8] N. Sharma, C. M. Gregory, and W. E. Dixon, "Predictor-based compensation for electromechanical delay during neuromuscular electrical stimulation," *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 19, no. 6, pp. 601–611, 2011.

[9] N. Fischer, A. P. Dani, N. Sharma, and W. E. Dixon, "Saturated control of an uncertain nonlinear system with input delay," *Automatica*, vol. 49, no. 6, pp. 1741–1747, 2013.

[10] J. R. Klotz, S. Obuz, Z. Kan, and W. E. Dixon, "Synchronization of uncertain Euler–Lagrange systems with uncertain time-varying communication delays," *IEEE Transactions on Cybernetics*, vol. 48, no. 2, pp. 807–817, 2017.

[11] M. Jankovic, "Control Lyapunov-Razumikhin functions and robust stabilization of time delay systems," *IEEE Transactions on Automatic Control*, vol. 46, no. 7, pp. 1048–1060, 2001.

[12] G. Niemeyer and J.-J. E. Slotine, "Telemanipulation with time delays," *The International Journal of Robotics Research*, vol. 23, no. 9, pp. 873–890, 2004.

[13] T. Fong, J. Rochlis Zumbado, N. Currie, A. Mishkin, and D. L. Akin, "Space telerobotics: unique challenges to human–robot collaboration in space," *Reviews of Human Factors and Ergonomics*, vol. 9, no. 1, pp. 6–56, 2013.

[14] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[15] S.-J. Chung and J.-J. E. Slotine, "Cooperative robot control and concurrent synchronization of Lagrangian systems," *IEEE Trans. Robot.*, vol. 25, no. 3, pp. 686–700, 2009.

[16] N. Chopra, M. W. Spong, and R. Lozano, "Synchronization of bilateral teleoperators with time delay," *Automatica*, vol. 44, no. 8, pp. 2142–2148, 2008.

[17] S. Liu, L. Xie, and F. L. Lewis, "Synchronization of multi-agent systems with delayed control input information from neighbors," *Automatica*, vol. 47, no. 10, pp. 2152–2164, 2011.

[18] L. Ding and W. X. Zheng, "Network-based practical consensus of heterogeneous nonlinear multiagent systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 8, pp. 1841–1851, 2016.

[19] Y.-C. Liu and N. Chopra, "Controlled synchronization of heterogeneous robotic manipulators in the task space," *IEEE Transactions on Robotics*, vol. 28, no. 1, pp. 268–275, 2011.

[20] E. Nuno, R. Ortega, L. Basanez, and D. Hill, "Synchronization of networks of nonidentical Euler-Lagrange systems with uncertain parameters and communication delays," *IEEE Transactions on Automatic Control*, vol. 56, no. 4, pp. 935–941, 2011.

[21] V. Chawda and M. K. O'Malley, "Position synchronization in bilateral teleoperation under time-varying communication delays," *IEEE/ASME Transactions on Mechatronics*, vol. 20, no. 1, pp. 245–253, 2014.

[22] T. Hatanaka, N. Chopra, and M. Fujita, "Passivity-based bilateral human-swarm-interactions for cooperative robotic networks and human passivity analysis," in *IEEE Conference on Decision and Control*, 2015, pp. 1033–1039.

[23] E. Yousefi, Y. Yildiz, R. Sipahi, and T. Yucelen, "The effect of remote-signal feedback architectures on the stability of human-in-the-loop telerobotics in the presence of time-delays," in *IEEE Conference on Control Technology and Applications*, 2017, pp. 1288–1293.

[24] ——, "Stability analysis of a human-in-the-loop telerobotics system with two independent time-delays," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 6519–6524, 2017.

[25] T. Yucelen, Y. Yildiz, R. Sipahi, E. Yousefi, and N. Nguyen, "Stability limit of human-in-the-loop model reference adaptive control architectures," *International Journal of Control*, vol. 91, no. 10, pp. 2314–2331, 2018.

[26] C. Makkar, G. Hu, W. G. Sawyer, and W. E. Dixon, "Lyapunov-based tracking control in the presence of uncertain nonlinear parameterizable friction," *IEEE Trans. Automat. Control*, vol. 52, pp. 1988–1994, 2007.

[27] H. C. Ravichandar, A. Kumar, and A. Dani, "Gaze and motion information fusion for human intention inference," *International Journal of Intelligent Robotics and Applications*, vol. 2, no. 2, pp. 136–148, 2018.

[28] A. Parikh, R. Kamalapurkar, and W. E. Dixon, "Integral concurrent learning: Adaptive control with parameter convergence using finite excitation," *International Journal of Adaptive Control and Signal Processing*, vol. 33, no. 12, pp. 1775–1787, 2019.

[29] I. Salehi, G. Rotithor, G. Yao, and A. P. Dani, "Dynamical system learning using extreme learning machines with safety and stability guarantees," *International Journal of Adaptive Control and Signal Processing*, vol. 35, no. 6, pp. 894–914, 2021.

[30] G. V. Chowdhary, "Concurrent learning for convergence in adaptive control without persistency of excitation," Ph.D. dissertation, Georgia Institute of Technology, 2010.