

# Spectroscopy of Twisted Bilayer Graphene Correlated Insulators

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We analytically compute the scanning tunneling microscopy (STM) signatures of integer-filled correlated ground states of the magic angle twisted bilayer graphene (TBG) narrow bands. After experimentally validating the strong-coupling approach at  $\pm 4$  electrons/moiré unit cell, we consider the spatial features of the STM signal for 14 different many-body correlated states and assess the possibility of Kekulé distortion (KD) emerging at the graphene lattice scale. Remarkably, we find that coupling the two opposite graphene valleys in the intervalley-coherent (IVC) TBG insulators does not always result in KD. As an example, we show that the Kramers IVC state and its nonchiral  $U(4)$  rotations do not exhibit any KD, while the time-reversal-symmetric IVC state does. Our results, obtained over a large range of energies and model parameters, show that the STM signal and Chern number of a state can be used to uniquely determine the nature of the TBG ground state.

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**Introduction.**—Near the first magic angle [1–3], both transport [4–19] and spectroscopy [20–29] experiments have uncovered a wealth of superconducting and correlated insulating phases in twisted bilayer graphene (TBG), sparking considerable theoretical effort towards their understanding [30–74]. The physics of TBG near integer fillings with  $\nu$  electrons per moiré unit cell was argued to be in the strong coupling regime, dictated by the interaction-only Hamiltonian projected onto its almost-flat bands [43,44,59,64,69]. The enlarged continuous spin-valley symmetries thereof [43,44,59,75] have rendered a low-energy manifold of its many-body eigenstates [43,44,59,64,69] and few-particle excitations [64,76] exactly solvable at integer fillings. Following numerically validated [56,59,70] analytical arguments [43,59,69,75], the resulting eigenstates of the projected interaction Hamiltonian were shown to be energetically competitive ground-state candidates, if not the actual ground states of the system, for a large range of parameters.

Building on the aforementioned theoretical advances, this Letter identifies spectroscopic signatures of the various competing states. For a given insulator, the differential conductance measured in scanning tunneling microscopy (STM) experiments is proportional to its spectral function [77], which can be computed analytically from the readily available many-body electron and hole excitations [64,76]. We find that the STM features of the proposed correlated states—particularly the presence or absence of a Kekulé distortion (KD) at the graphene lattice scale (i.e., the

modulation of the STM signal at wave vectors connecting the two graphene valleys)—together with the knowledge of their Chern number, can distinguish among the candidate many-body states. Recent experiments [78–80] demonstrating the ability of STM to visualize symmetry-broken states with KD arising from many-body interactions in the zeroth Landau level of monolayer graphene indicate that similar techniques can be employed to discriminate between the correlated insulators of TBG.

The competing correlated states of TBG at an integer filling  $\nu$  can be characterized by their Chern number  $\mathcal{C}$  and valley polarization, being either valley polarized (VP) or intervalley coherent (IVC). Additionally, even for  $\mathcal{C} = 0$ , IVC states may either spontaneously break time-reversal symmetry ( $T$ ), as in the Kramers IVC (K-IVC) state, or preserve it, as in the  $T$ -symmetric IVC (T-IVC) state [43,44,59,75]. Some of these states can be stabilized by magnetic field [12,15,17,26,27,81]. In this Letter, we analyze numerically 14 different TBG correlated insulators, and show analytically that *all* VP states together with the K-IVC states at  $\nu = \pm 2, 0$  exhibit no KD, while generic IVC states do display KD. Furthermore, we show that the strong- versus weak-coupling nature of the system can be uniquely inferred from the spectral function of the  $\nu = \pm 4$  band insulator. The correct asymmetric peak structure obtained in the strong-coupling approach differs significantly from the weak-coupling result and shows dramatic variations as the STM tip moves from the AA to AB moiré regions. While the experimental data display large

sample-to-sample variation in the local density of states (LDOS), some datasets are uniquely compatible with the strong-coupling description.

*Model.*—The physics of magic-angle TBG is dominated by the repulsive Coulomb interaction Hamiltonian projected in the almost-flat bands near charge neutrality [43,59,75,82]

$$H_I = \frac{1}{2\Omega_{\text{TBG}}} \sum_{\mathbf{q} \in \text{MBZ}} \sum_{\mathbf{G} \in \mathcal{Q}_0} O_{-\mathbf{q}-\mathbf{G}} O_{\mathbf{q}+\mathbf{G}}, \quad (1)$$

where  $\Omega_{\text{TBG}}$  is the area of the TBG sample, MBZ and  $\mathcal{Q}_0$ , respectively, denote the moiré Brillouin zone (MBZ) and reciprocal lattice, while

$$O_{\mathbf{q}+\mathbf{G}} = \sum_{\substack{\mathbf{k}, \eta, s \\ n, m = \pm 1}} \sqrt{V(\mathbf{q} + \mathbf{G})} M_{mn}^{\eta}(\mathbf{k}, \mathbf{q} + \mathbf{G}) \times \left( \hat{c}_{\mathbf{k}+\mathbf{q}, m, \eta, s}^{\dagger} \hat{c}_{\mathbf{k}, n, \eta, s} - \frac{1}{2} \delta_{\mathbf{q}, 0} \delta_{m, n} \right) \quad (2)$$

are proportional to the flat-band-projected density operators. In particular,  $\hat{c}_{\mathbf{k}, n, \eta, s}^{\dagger}$  is the electron creation operator for the TBG conduction ( $n = +1$ ) and valence ( $n = -1$ ) flat bands from valley  $\eta = \pm$  and spin  $s = \uparrow, \downarrow$ , while  $M_{mn}^{\eta}(\mathbf{k}, \mathbf{q} + \mathbf{G})$  are the TBG form factors. The Fourier-transformed screened Coulomb potential  $V(\mathbf{q}) = 2\pi U_{\xi} \xi^2 (1 - e^{-2\xi q}) / (\xi q)$  (with  $U_{\xi}$  and  $\xi$ , respectively, denoting the interaction energy scale and the screening length) corresponds to the typical single-gate arrangement of the TBG sample in a STM experiment [26].

The TBG single-particle Hamiltonian features a series of discrete symmetries [82]: the  $C_{2z}$ ,  $C_{3z}$ ,  $T$ , and  $C_{2x}$  *commuting* symmetries, as well as an approximate unitary particle-hole  $P$  *anticommuting* symmetry [75,109–112]. The latter enlarges the valley-spin-charge  $U(2) \times U(2)$  rotation symmetry of  $H_I$  to the so-called nonchiral-flat  $U(4)$  symmetry [43,44,59,75], henceforth denoted by  $U_{\text{NC}}(4)$ . Additionally, when the interlayer tunneling amplitude at the AA stacking centers ( $w_0$ ) is neglected compared to the one at AB stacking centers ( $w_1 = 110$  meV)—in the so-called chiral limit ( $w_0/w_1 = 0$ )—the single-particle Hamiltonian enjoys an additional anticommuting chiral  $C$  symmetry [75,113], which further enlarges the symmetry group of  $H_I$  to the chiral-flat  $U(4) \times U(4)$  group [59,75]. Recombining the active TBG bands into Chern-number  $e_Y$  bands with operators  $\hat{d}_{\mathbf{k}, e_Y, \eta, s}^{\dagger} = (1/\sqrt{2})(\hat{c}_{\mathbf{k}, +1, \eta, s}^{\dagger} + ie_Y \hat{c}_{\mathbf{k}, -1, \eta, s}^{\dagger})$ , the 32 generators of the chiral-flat  $U(4) \times U(4)$  group correspond to independent valley-spin rotations within each Chern sector. Away from the chiral limit the  $U(4) \times U(4)$  generators get combined into the 16  $U_{\text{NC}}(4)$  generators such that  $U_{\text{NC}}(4)$  intervalley (intravalley) rotations act on the two Chern sectors in the same (opposite) way [75].

The presence of enlarged symmetries renders some of the eigenstates of  $H_I$  exactly solvable at integer fillings. Up to rotations  $\hat{U}$  belonging to the symmetry group of  $H_I$ , the TBG ground states have been shown to be Slater determinants obtained by populating the active TBG bands one Chern-valley-spin sector  $(e_Y, \eta_j, s_j)$  at a time [43,56,59,69,70,75,76]

$$|\varphi\rangle = \hat{U} \prod_{\mathbf{k}} \prod_{j=1}^{4+\nu} \hat{d}_{\mathbf{k}, e_Y, \eta_j, s_j}^{\dagger} |0\rangle. \quad (3)$$

In the chiral limit,  $|\varphi\rangle$  is an *exact* eigenstate of  $H_I$  for any choice of the filled Chern-valley-spin sectors and  $\hat{U} \in U(4) \times U(4)$  [69]. Away from the chiral limit, only the insulators from Eq. (3) with fully filled or fully empty valley-spin flavors and  $\hat{U} \in U_{\text{NC}}(4)$  are *exact*, with the rest being *perturbative* eigenstates of  $H_I$  [69].

*Spectral function.*—For a given state  $|\varphi\rangle$  from Eq. (3), the differential conductance as a function of bias voltage measured in STM experiments is proportional to its spectral function [77]

$$A(\mathbf{r}, \omega) = \sum_{\xi, s} [|\langle \xi | \hat{\psi}_s^{\dagger}(\mathbf{r}) | \varphi \rangle|^2 \delta(\omega - E_{\xi} + E_{\varphi}) + |\langle \xi | \hat{\psi}_s(\mathbf{r}) | \varphi \rangle|^2 \delta(\omega + E_{\xi} - E_{\varphi})], \quad (4)$$

where  $\psi_s(\mathbf{r})$  denotes the electron field annihilation operator corresponding to spin  $s = \uparrow, \downarrow$ , and a summation is performed over all the *many-body* eigenstates  $|\xi\rangle$  of  $H_I$  with energy  $E_{\xi}$  [82]. Expressing the field operators in the TBG energy-band basis  $\hat{\psi}_s^{\dagger}(\mathbf{r}) = \sum_{\eta, n} V_{\mathbf{r}, \mathbf{k}n\eta} \hat{c}_{\mathbf{k}, n, \eta, s}^{\dagger}$  (where the factors  $V_{\mathbf{r}, \mathbf{k}n\eta}$  depend on the carbon  $p_z$  orbitals and the TBG flat band wave functions and include contributions from both graphene layers), we find that

$$A(\mathbf{r}, \omega) = \sum_{\substack{\mathbf{k} \in \text{MBZ}, \\ n, \eta, n', \eta' \\ c = \pm}} [\mathcal{M}_{\varphi}^c(\omega)]_{\mathbf{k}n\eta, \mathbf{k}n'\eta'} [\mathcal{B}(\mathbf{r})]_{\mathbf{k}n\eta, \mathbf{k}n'\eta'}. \quad (5)$$

In Eq. (5), we have introduced the spatial factor matrix  $[\mathcal{B}(\mathbf{r})]_{\mathbf{k}n\eta, \mathbf{k}n'\eta'} = V_{\mathbf{r}, \mathbf{k}n\eta} V_{\mathbf{r}, \mathbf{k}n'\eta'}^*$  (which depends only on the TBG single-particle Hamiltonian) and the spectral function matrices (which depend on the state  $|\varphi\rangle$ )

$$[\mathcal{M}_{\varphi}^c(\omega)]_{\mathbf{k}n\eta, \mathbf{k}n'\eta'} = \sum_{\lambda_{-c}, s} \langle \lambda_{+} | \hat{c}_{\mathbf{k}, n', \eta', s} | \lambda_{-} \rangle \langle \lambda_{-} | \hat{c}_{\mathbf{k}, n, \eta, s}^{\dagger} | \lambda_{+} \rangle \times \delta(\omega - E_{\lambda_{-}} + E_{\lambda_{+}}), \quad (6)$$

where  $|\lambda_c\rangle = |\varphi\rangle$  and we assumed no breaking of the moiré translation symmetry. Since a  $\hat{c}_{\mathbf{k}, n, \eta, s}^{\dagger}$  operator acts in *one* single-layer graphene (SLG) valley,  $[\mathcal{B}(\mathbf{r})]_{\mathbf{k}n\eta, \mathbf{k}n'\eta'}$  is only modulated at the level of the SLG and TBG lattices.

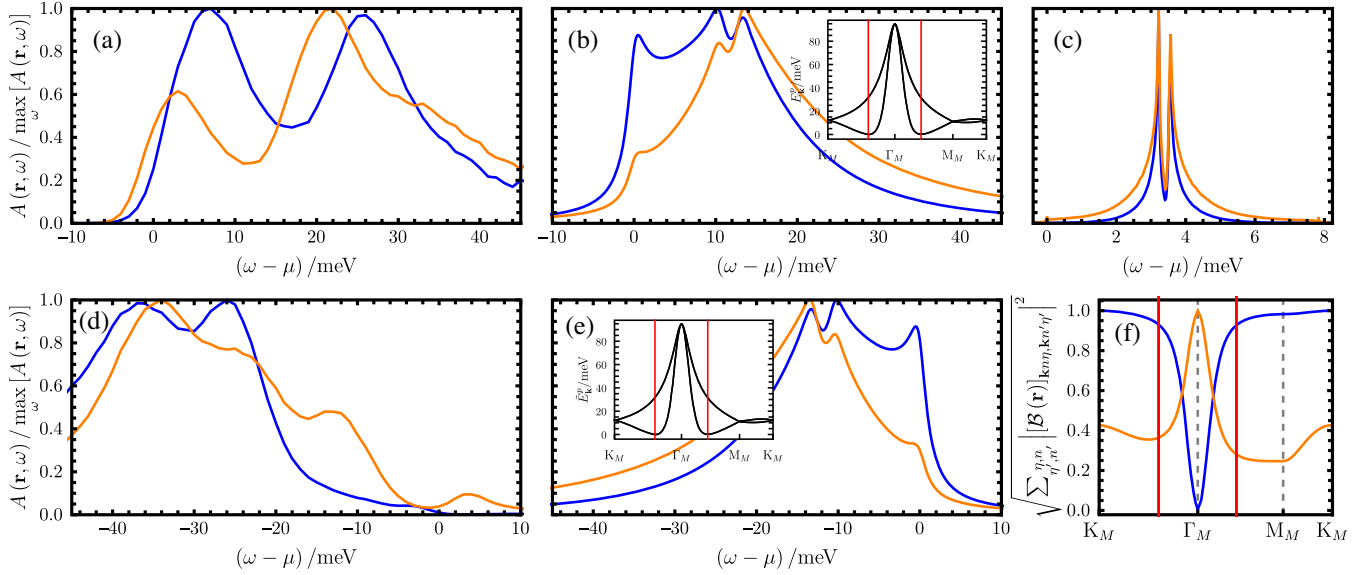


FIG. 1. The TBG spectral function for the  $\nu = \pm 4$  insulator. For  $\nu = -4$  ( $\nu = +4$ ), we compare the experimentally measured STM signal in (a) [(d)] with the spectral function computed from the charge-one excitation of  $H_I$  in (b) [(e)]. We use  $U_\xi = 24$  meV,  $\xi = 300$  nm, and  $w_0/w_1 = 0.8$ . For reference, the TBG spectral function at  $\nu = -4$  derived from the single-particle TBG Hamiltonian is given in (c). The signal at the center of the AA (AB) site is shown in blue (orange) and normalized by its maxima in the energy range at that particular location. The theoretically computed spectral function is averaged over three SLG unit cells. The inset in (b) [(e)] shows the electron [hole] excitation dispersion  $E_{\mathbf{k}}^p$  ( $\tilde{E}_{\mathbf{k}}^p$ ) computed from Eq. (7) (the red vertical lines are a guide for the eye pointing the minima of the dispersion). (f) provides the spatial factor at the AA and AB sites along the high symmetry line of the MBZ (the red lines indicate the position of minima).

In contrast,  $[\mathcal{B}(\mathbf{r})]_{\mathbf{k}\eta\eta',\mathbf{k}'\eta'(-\eta)}$  contains an additional modulation corresponding to wave vectors linking the two Dirac points of the same graphene layer, which manifests in real space as a KD of the SLG.

As shown in Eqs. (5) and (6), computing the TBG spectral function requires the eigenstates of  $H_I$  containing an extra electron or hole compared to  $|\varphi\rangle$  (i.e., the charge-one excitations). Despite  $H_I$  being a quartic Hamiltonian, the exact charge-( $\pm$ )one excitations on top of  $|\varphi\rangle$  can be computed as a zero-body problem using the *charge-one commutation relations* [64,76]. For example, the electron commutation relation reads as

$$[H_I - \mu\hat{N}, \hat{c}_{\mathbf{k},n,\eta,s}^\dagger]|\varphi\rangle = \sum_m R_{mn}^\eta(\mathbf{k})\hat{c}_{\mathbf{k},m,\eta,s}^\dagger|\varphi\rangle, \quad (7)$$

where  $\mu$  denotes the chemical potential,  $\hat{N}$  is the total fermion number operator, and the matrix  $R(\mathbf{k})$  depends on  $\nu$ , the active TBG wave functions and the Coulomb repulsion potential [82]. As such, the  $\hat{c}_{\mathbf{k},n,\eta,s}^\dagger$  and  $\hat{c}_{\mathbf{k},n,\eta,s}$  operators can be recombined into exact electron and hole excitations, allowing for the analytical calculation of the spectral function of  $|\varphi\rangle$  [82].

**Signatures of strong correlation.**—We first analyze the spectral function of the  $\nu = -4$  ( $\nu = +4$ ) TBG insulator from Eq. (3), for which the active TBG bands are fully empty (fully filled) and no ambiguity in the choice of ground state arises. As shown in Fig. 1, the strong-coupling

and weak-coupling spectral functions at  $\nu = \pm 4$  are markedly different as a result of the large interaction-induced dispersion of single-particle excitations in the strong-coupling regime [insets in Figs. 1(b) and 1(e)] compared to the almost-flat dispersion in the weak-coupling (i.e., noninteracting) regime, as well as from different van Hove singularities and Dirac points [64,76].

We will discuss the  $\nu = -4$  insulator from Figs. 1(a)–1(c), with the  $\nu = +4$  insulator [Figs. 1(d) and 1(e)] following analogously from the many-body charge-conjugation symmetry of TBG [75]. Details about the experimental measurements and signal normalization are provided in Ref. [82]. For  $\nu = -4$ , we focus on positive-energy biases ( $\omega - \mu \geq 0$ ), such that the electrons tunnel into the fermion states of the active TBG bands, recombined into the electron excitations according to Eq. (7). The electron excitation energies  $E_{\mathbf{k}}^p$  [inset of Fig. 1(b)], obtained by diagonalizing the charge-one commutation matrices from Eq. (7) [82], are comprised of four sets of twofold [spin SU(2)] degenerate bands, which are further paired by the approximate  $C_{2z}P$  symmetry of  $H_I$  into two sets of almost fourfold degenerate bands [64,76,109,111]. For small biases, the electrons start tunneling into the regions at the bottom of the excitation bands away from any high-symmetry points (e.g., halfway between the  $\Gamma_M$  and  $M_M$  points of the MBZ), giving rise to the peak near  $\omega - \mu \approx 0$  meV in the spectral function at both the AA and AB centers. Upon increasing the bias to  $\omega - \mu \approx 20$  meV, the electrons tunnel into the almost-flat



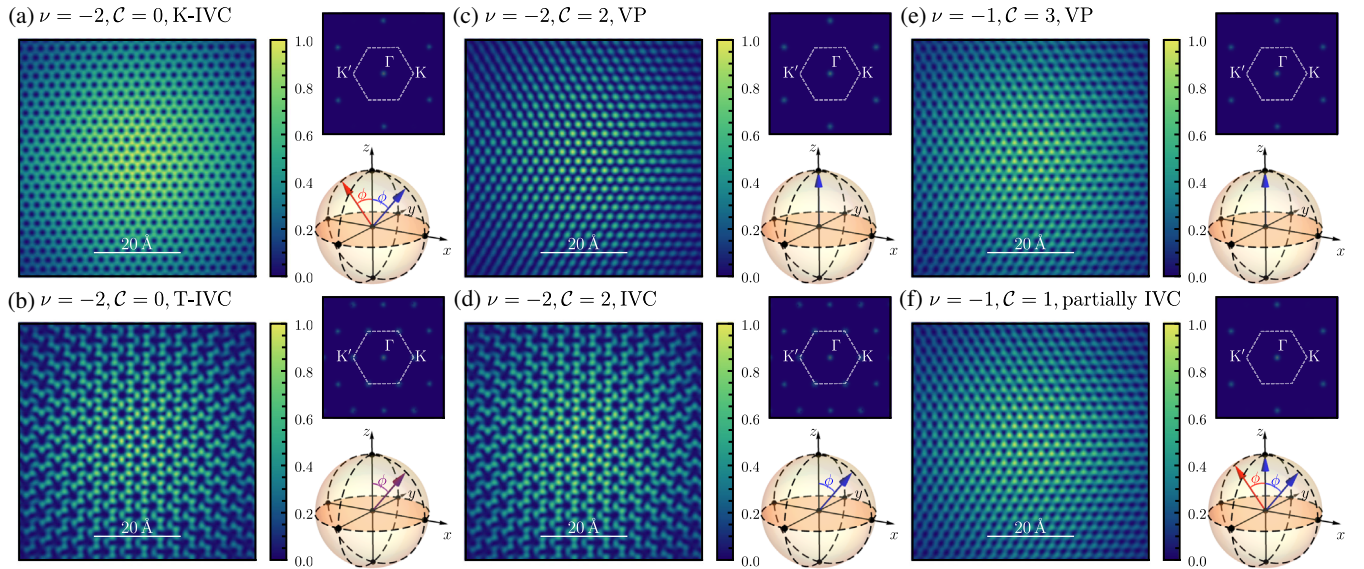


FIG. 2. Kekulé distortion and intervalley coherence. For each insulator [(a)–(f)], we show the real-space spectral function centered at the AA site (left panel), its Fourier transformation (top-right panel), as well as the valley polarizations of the occupied Chern bands as blue ( $e_Y = +1$ ) or red ( $e_Y = -1$ ) unit vectors in the valley Bloch sphere. The valley polarization can be oriented parallel to the  $\hat{z}$  axis or at an angle  $\phi = \pi/2$  to it. We consider the  $\nu = -2$ ,  $C = 0$ , K-IVC [(a)], and T-IVC [(b)] states, the  $\nu = -2$ ,  $C = 2$ , VP [(c)], and IVC [(d)] Chern insulators, as well as the  $\nu = -1$ ,  $C = 3$  fully VP [(e)] and  $\nu = -1$ ,  $C = 1$  partially IVC [(f)] Chern insulators. The presence of KD in (b) and (d) appears as a threefold enlargement of the SLG unit cell and as a signal at the K and K' points of the SLG Brillouin zone. We use  $U_\xi = 8$  meV,  $\xi = 300$  nm,  $w_0/w_1 = 0.8$ .

regions near the boundary of the MBZ, giving rise to two close peaks in the theoretical spectral function, merging into one in the experimental STM signal. For larger biases, the spectral function decreases as the electron tunnel in the strongly dispersive bands near  $\Gamma_M$ .

The variation of the magnitude of  $[\mathcal{B}(\mathbf{r})]_{\mathbf{k}\eta\eta',\mathbf{k}'\eta'\eta'}$  with  $\mathbf{k} \in \text{MBZ}$  at the AA and AB sites [Fig. 1(f)] *qualitatively* explains the change in the STM signal between the two stacking centers: at the AA site, the spatial factor has roughly the same magnitude in the MBZ for the two almost-flat regions of the excitation bands, resulting in similar magnitudes for the LDOS peaks at  $\omega - \mu \approx 0$  meV, 20 meV. At the AB site, the spatial factor has a larger amplitude on the boundary of the MBZ, diminishing the peak at  $\omega - \mu \approx 0$  meV compared to the one at  $\omega - \mu \approx 20$  meV. A similar decrease is also present in the experimental data [Fig. 1(a)], while clearly absent in the noninteracting LDOS [Fig. 1(c)]. Moreover, the half-maximum width of the spectral function ( $\Delta\omega$ ) is much smaller in the noninteracting case ( $\Delta\omega < 1$  meV, comparable to the active TBG bandwidth) than in the experiment and the strong-coupling prediction ( $\Delta\omega = 20$ –30 meV, comparable to  $U_\xi = 24$  meV and much larger than the resolution of the experiment  $\delta\omega \approx 3$  meV [82]). While the STM signal is sample dependent and may vary from different AA or AB sites, this dataset indicates evidence of strong correlations governing the physics of TBG near charge neutrality.

*Discriminating correlated insulating phases.*—We now investigate the effects of intervalley coherence on the spatial variation of  $A(\mathbf{r}, \omega)$  for various  $|\nu| < 4$  insulating states. Naively, coupling the two graphene valleys in an IVC insulator results in IVC charge-one excitations and should lead to the emergence of KD in the corresponding STM signal. However, due to the discrete symmetries of TBG, breaking the valley U(1) symmetry does *not* guarantee the emergence of KD in  $A(\mathbf{r}, \omega)$  [82]. For instance, Figs. 2(a) and 2(b) show the simulated STM patterns for two *fully* IVC TBG insulators at  $\nu = -2$  [59,69]:

$$\begin{aligned}
 |\text{K-IVC}\rangle &= \prod_{\mathbf{k}} \prod_{e_Y=\pm 1} \frac{\hat{d}_{\mathbf{k},e_Y,+, \uparrow}^\dagger + e_Y \hat{d}_{\mathbf{k},e_Y,-, \uparrow}^\dagger}{\sqrt{2}} |0\rangle, \\
 |\text{T-IVC}\rangle &= \prod_{\mathbf{k}} \prod_{e_Y=\pm 1} \frac{\hat{d}_{\mathbf{k},e_Y,+, \uparrow}^\dagger + \hat{d}_{\mathbf{k},e_Y,-, \uparrow}^\dagger}{\sqrt{2}} |0\rangle.
 \end{aligned} \quad (8)$$

The K-IVC (T-IVC) state is obtained from a fully filled valley-spin flavor by rotating the two Chern bands in the  $xz$  valley plane in opposite (identical) directions, as shown in Figs. 2(a) and 2(b). Remarkably, while the STM patterns of the T-IVC state show clear signs of KD, no KD emerges for the K-IVC state.

The counterintuitive absence of KD in the  $\nu = -2$  K-IVC state is part of a more general *exact* result, relying on the  $C_{2z}$ ,  $T$ , and  $P$  symmetries of TBG [82]: a VP even- $\nu$  insulator with only fully filled and fully empty valley-spin flavors and all its  $U_{\text{NC}}(4)$  rotations have *identical* spectral

functions, without exhibiting KD. Note that these are precisely the theoretically proposed exact ground states of  $H_I$  at even filling and away from the chiral limit [43,59,69,70]. Moreover, even when the  $P$  symmetry is broken, we find that  $C_{2z}$  and  $T$  are enough to guarantee the *exact* absence of KD in the K-IVC state, although not necessarily in its general  $U_{NC}(4)$  rotations [82].

When  $|\varphi\rangle$  is *not* a  $U_{NC}(4)$  rotation of an insulator with only fully filled or fully empty valley-spin flavors, inter-valley coherence *can* lead to KD, but fine-tuned counter-examples do exist. For maximally spin polarized states, we have derived simple rules governing the presence of KD [82]: (1) Filling a single IVC Chern band gives rise to KD; (2) An exact cancellation of the KD signal occurs upon filling a pair of Chern bands with *opposite* Chern numbers whose valley polarization projections in the valley  $xy$  plane of the Bloch sphere are nonzero and cancel out, e.g., the K-IVC from Fig. 2(a). In Figs. 2(c)–2(f), we illustrate these rules for  $C \neq 0$  states, which at odd filling are the theoretical ground states, while at even filling are the ground states in-field [12,15,17,26,27,81]. The  $\nu = -2$ ,  $C = 2$  and  $\nu = -1$ ,  $C = 3$  VP Chern insulators trivially harbor no KD. The  $\nu = -2$ ,  $C = 2$  IVC insulator does exhibit KD, since the valley polarizations of the two bands projected in the  $xy$  plane of the valley Bloch sphere do not cancel out. Finally, the  $\nu = -1$ ,  $C = 1$  partially IVC insulator has one VP filled Chern band and a pair of filled IVC Chern bands, satisfying rule 2 and therefore not displaying any KD. Further examples are presented in Ref. [82]. Between states showing no KD, further LDOS differences exist because the Chern band operators  $\hat{a}_{\mathbf{k},e_Y,\eta,s}^\dagger$  are primarily located on a single SLG sublattice, depending on  $e_Y\eta = \pm 1$ . In the  $\nu = -2$ ,  $C = 2$  state, two Chern bands with  $e_Y = +1$  are occupied in the same valley  $\eta = +$  (and two spin sectors), leading to the appearance of a triangular lattice in the STM signal from Fig. 2(c). To obtain the VP  $\nu = -1$ ,  $C = 3$  (IVC  $\nu = -1$ ,  $C = 1$ ) state, a Chern band with  $\eta = -$ ,  $e_Y = +1$  ( $\eta = +$ ,  $e_Y = -1$ ) is added, which is polarized primarily on the other graphene sublattice. Hence two interpenetrating triangular lattices of weight 2:1 appear in the LDOS patterns from Figs. 2(e) and 2(f).

**Conclusions.**—We analyzed the STM signal of a multitude of the predicted candidates for the correlated insulators in TBG and showed that it can be used to differentiate between the ground states. Both numerically and analytically, we found that the  $C = 0$  celebrated K-IVC states at  $\nu = \pm 2$ , 0 do not exhibit a KD, while other IVC states, including the  $\nu = \pm 2$  T-IVC or  $C = 2$  states do display KD. We experimentally measured the STM signal for the  $\nu = \pm 4$  band insulator, and used it to validate the strong-coupling regime. The broadening of the signal and the specific variations of the signal from the AA to the AB sites are unique signatures of the strong-coupling regime.

Our Letter paves the road toward unambiguous identification of the TBG correlated insulators.

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**Note added.**—Recently, we became aware of Ref. [114], which also computes the STM signal of various TBG states using Hartree-Fock methods. Where they overlap, our conclusions (i.e., the vanishing of KD in the K-IVC state) agree with the ones presented in Ref. [114].

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