

Computational Thinking and The Algebra Project: From Voice to Agency

Alan Shaw, Kennesaw State University, alan.shaw@kennesaw.edu

Brian R. Lawler, Kennesaw State University, brian.lawler@kennesaw.edu

William Crombie, The Algebra Project, bill@algebra.org

Tom McKlin, The Findings Group, tom@thefindingsgroup.org

Tamika Richards, Forest Park Middle School, tamika.richards@clayton.k12.ga.us

Through our work to examine mathematical and computational learning in authentic and convivial contexts that requires creativity, imagination, reasoning, and discourse, we have theorized an experiential learning cycle that attends to the development of voice, agency, and identity needed in young people for an earned insurgency—the right to demand change. Our work underscores how the current situation that many students face in classrooms amounts to a type of cognitive segregation that denies these students access to authentic and empowering intellectual agency. By facilitating a process whereby students, using their own creative and imaginative means, intentionally develop a type of ownership over the exploration and application of the mathematical concepts they are being taught, we help students move from simple surface level, syntactic understandings, to deeper semantic learning that is more personally significant and meaningful.

Através do nosso trabalho para examinar a aprendizagem matemática e computacional em contextos autênticos e conviviais que requerem criatividade, imaginação, raciocínio e discurso, teorizamos um ciclo de aprendizagem experiencial que atende ao desenvolvimento da voz, agência e identidade necessários nos jovens para uma insurgência conquistada—o direito de exigir mudanças. Nossa trabalho ressalta como a situação atual que muitos alunos enfrentam nas salas de aula equivale a um tipo de segregação cognitiva que nega a esses alunos o acesso a uma agência intelectual autêntica e capacitadora. Ao facilitar um processo pelo qual os alunos, usando seus próprios meios criativos e imaginativos, desenvolvem intencionalmente um tipo de propriedade sobre a exploração e aplicação dos conceitos matemáticos que estão sendo ensinados, ajudamos os alunos a passar de um nível de superfície simples, entendimentos sintáticos, para um entendimento semântico mais profundo. aprendizagem que é mais pessoalmente significativa.

Introduction

It is perhaps undeniable that one of the key features and requirements of human language systems is a capacity for self-expression. Moreover, this self-expression is, just as undeniably, an important component in the development of our individual identities. No reasonable scholar dealing with these issues would suggest that our acquisition and critical facility with human language does not strongly influence our individual intellectual capacity. Many studies have demonstrated how children deprived of access to a language system at a young age, suffer intellectual deficits because of this that can be somewhat ameliorated with subsequent language exposure (Mayberry, 2007; Morgan, 2014; Zeanah et al., 2011). And yet authors of books like *The Bell Curve* (Herrnstein & Murray, 1996) seem to argue that each individual's intellectual capacity is somehow fixed within some standard deviation measurable by some aptitude test. It does not seem to occur to such authors that when an individual has greater access to, and facility with, a broader range of cognitive-linguistic tools that can enhance their creative self-expression, their intellectual capacity can be enhanced as well. Such authors are firmly stuck in what Carol Dweck (2012) called the *fixed intelligence mindset* that doesn't believe in the power of cognitive growth through intellectual agency, "If success means they're smart, then failure means they're dumb. That's the fixed mindset" (p. 197).

In our research, we have been exploring a paradigm for examining mathematical and computational learning in a context that requires creativity, imagination, reasoning, and discourse. As such, we treat mathematics as an enhancement to the student's language system (Quine, 1981) that provides the student with a particular set of cognitive tools that have the potential to increase their creative and imaginative self-expression. In our view, without the goal of creative and imaginative agency in mathematics, many ultimately see mathematics as primarily a tool to measure a student's aptitude that they believe to be already fixed in place, within some standard deviation. This then influences our society to identify what Herrnstein and Murray (1996) call the *cognitive elite*. To the contrary, we believe this is a viewpoint that is intellectually harmful to our children, and one that is responsible for a sort of *cognitive segregation* in our society, a dystopian vision called for by Herrnstein and Murray but unfortunately is already present—at least in our schools (e.g., Oakes, 1986).

In this paper we will discuss how a student's *math identity* (Aguirre et al., 2013) can inform whether or not they view mathematics as an avenue for creative, imaginative and discursive self-expression. This relates to what Dweck (2012) describes as the *growth mindset* versus the *fixed mindset*, and the fixed mindset is all too often the more likely point of view of the student because of an entrenched societal cognitive segregation that we believe needs to be challenged. In order for a student to have an agency-based and personally empowering math identity, the student needs to see within the mathematics that they are being taught a type of *thick authenticity* (Shaffer & Resnick, 1999), and a type of intellectual *earned insurgency* (Moses et al., 2009).

We believe that mathematics education can be a genuine game changer in the mind of both students and teachers (Moses & Cobb, 2001). By this we mean that it has the ability to produce a paradigm shift away from the elitist viewpoint, and toward a viewpoint of intellectual agency and empowerment. In a previous paper (Shaw et al., 2021), we argued that the authentic social and cultural voice expressed through young people's orality must be embraced in order for students to actively agree to expand those voices into the more formalized regimented language system of mathematics. However, the challenge presented by doing that requires expanding the potential for creative and imaginative self-expression, reasoning, and discourse within mathematics. We argue here that this expansion can be done in a natural and authentic way by overlapping traditional mathematics with computer science (CS) and computational thinking (CT).

Beginning with Student Voice

We have been working with a model of student growth and efficacy which developed from the Algebra Project's 5-Step Curricular Process (Bucci & McEwan, 2015). We see the first application of the model at the level of the individual student in a classroom. But it also provides a dynamical model for teams of students or even for the class as a whole. The model is built upon three dynamic variables: voice, agency, and identity. Voice refers to both the talking that a student does to him or herself and the talking that students do with each other. The curricular provocation to engage student voice is a shared concrete experience, the first of the five steps. The experience necessarily needs to be both accessible and engaging in order to capture the attention of students. Voice is the first level of engagement in the Algebra Project's 5-Step Curricular Process.

After the shared concrete experience, students create a model or picture of what they found most interesting in the experience. They write about it. They talk about it from a perspective that they own (Shaw et al., 2021). These first steps in the curricular process create a space where students bring their voice to what will ultimately be the mathematical table, where their opinions matter in the process of mathematizing a shared concrete experience. Students create a space where they can express their imagination and creativity in first creating a picture or model of their shared experience and then discussing and writing about it. Their reflections on the shared experience and their considerations of the features at play in the experience are a developing expression of student agency.

In the last two steps of the curricular process, students conceptualize and capture in symbolic representations the mathematical relationships that were originally only implicit in the initial shared experience. This action of casting ideas in symbolic form is both an expression of student *agency* (they are creating a little piece of mathematics) and a means to develop that agency. Thus, we view agency through the perspective of increasing levels of competence within the domains of discourse, reasoning, imagination and creativity. What students say (internally and externally as *voice*), and

what students do (individually and collectively as agency), in the long run, contribute to the building of their mathematical *identities*. The realization on the part of students that they can do the required mathematics through their own voice and agency is how they recognize their right to make a demand on the educational system for a quality education, what Bob Moses (2009) called their *earned insurgency*.

Mathematics in a Discursive Context

When mathematical learning occurs in a discursive context, the actors experience and thus view mathematical knowledge creation as a cooperative and communal activity, not as something only accessible by a cognitive elite activity. In our efforts to build both computational thinking and mathematics fluency, we find the development of a discursive learning context in the mathematics classroom to be a paradigm shift for both students and teachers.

We view mathematics as the product of human activity (Quine, 1981), and mathematical knowing/knowledge as constructed (Glaserfeld, 1995; Papert, 1990). Such a view disrupts the onto-epistemological hierarchy of the Platonic view of knowledge very commonly applied to mathematical knowledge (Bowers & Lawler, 2021). Further, rather than viewing the teacher as arbiter of mathematical truth, students are positioned as authority, and knowledge emerges communally, democratically. The teacher may be invited to interject as someone knowledgeable of the discipline (or curriculum per Dewey, 1902). When mathematics is a product of human activity, each young person's mathematical ideas are (equally) valued, in line with organizing principles (Moses et al., 1989) of the Civil Rights Era in the U.S.

Only with these shifts in orientation to what and whose knowledge counts can a truly discursive mathematical learning community exist. The Algebra Project pedagogy is geared toward ensuring this shift of authority. Throughout the many curricular experiences of the 5-Step Process, there are mathematical problems to be resolved. This discourse structure follows a pattern of individual thinking (production), small group work (publication), and finally whole group discussion (peer-review). The production phase ensures every student has some idea or question or concern to contribute to a small group discussion. Through that small group discussion phase—free of expert oversight—students have the opportunity to rehearse sharing of ideas in a communal space. One member of every group then must report to the whole class on their work with the problem. Here, students are developing voice. Collectively, the young people in the class consider one another's approaches, and together refine a strategy that all understand and agree to.

This discourse structure was enacted by the sharecroppers of the Mississippi Delta as they fought for the right to vote. By struggling with a problem and shaping solutions, the sharecroppers as do the students in the Algebra Project classroom, find an agency to change oppressive forces in their lives.

Mathematics in a Thick Authenticity context

Shaffer and Resnick (1999), computational thinking and computer science education researchers in MIT's Media Lab, define *thick authenticity* as having four tenets:

Activities that are personally meaningful, connected to important and interesting aspects of the world beyond the classroom, grounded in a systematic approach to thinking about problems and issues, and which provide for evaluation that is meaningfully related to the topics and methods being studied. (p. 203)

While the original work was a response to a debate in education around authentic instruction and assessment, the primary tenets of thick authenticity are timely and relevant to this work. Placing our work within the guard rails of authenticity requires building instructional interventions that are personally relevant to students, are infused with real-world tools and tasks, are discipline-focused, and allow students to metacognitively assess their own learning and what comes next in their learning. In the Algebra Project curricular and pedagogical designs, we ask students to engage in mathematics as mathematicians and scientists do; to use the vocabulary and affordances of mathematics to reason about real-world problems familiar to students; to use approaches, methods, and vocabulary of mathematicians; and to check the quality of their solutions as mathematicians would.

An inauthentic activity is one that does not adhere to the four tenets. One example in introductory computing is asking high school students to program a checkbook registry, a common activity in introductory CS courses in the 1980s. This activity is inauthentic in that it may have little personal meaningfulness to students since they may have little to no experience writing checks and therefore provides little opportunity for students to describe whether they are learning CS concepts since much of their cognitive energy is focused on learning how checkbook registries work. For the fourth tenet, assessment, Shaffer and Resnick (1999) explain that portfolio assessment, common in the arts, may simply become a collection of old homework assignments in a different context. Therefore, authentic assessment must take the learning content and context into consideration. Ideally, students should be able to describe what comes next in their learning (Davies, 2020).

Mathematics in an Earned Insurgency Context

In short, mathematics that is not personally meaningful lacks a sense of authenticity, which in turn leads many students to a feeling of being alienated from the subject. This sense of alienation can lead to a sense of failure, causing students to question their own capacity to do mathematics. As Dweck (2012) points out, and as we referenced above, this all leads to that fixed mindset in which the students who are struggling are tempted to believe that their difficulties are because of a lack of ability, not because of a lack of an authentic opportunity for personally meaningful and relevant constructivist(-ionist) learning. Constructivist learning would lead to an experience of cognitive integration, rather than the cognitive segregation model argued for by Herrnstein and Murray

(1996). When students are stuck in an alienating, discordant context, they do not experience a feeling of ownership over their mathematics work, which can have the unfortunate outcome of validating the sense of cognitive segregation with which the students are left.

Students who fall victim to a sense of cognitive segregation can become discouraged and disengaged in their intellectual pursuits. To overcome this, they would need to value their own learning potential and thereby increase their academic engagement. Students need to believe in their own agency as active participants in the educational processes they are involved in. Bob Moses (2009), in his article *An Earned Insurgency: Quality Education as a Constitutional Right* shows how this situation is analogous to the plight of disempowered Black southerners during the civil rights movement.

Moses argues that political disengagement was the result of an extremely oppressive and life-threatening sociopolitical environment for Black southerners, and although this does not completely correspond to what underserved children experience in the classroom, both circumstances require an earned insurgency to overcome systemic issues involved. Moses explains this by addressing what happened to a group of young Freedom Riders who took a bus from Washington, DC (USA) bound for New Orleans during the civil rights movement to protest Jim Crow policies. The bus was attacked and firebombed by a mob on May 14, 1961, and the riders were severely beaten. The U.S. president at the time, John F. Kennedy, not understanding the empowering symbolism connected to this journey, ordered an end to the trip. But John Lewis and others continued on despite that, and Moses (2009) describes the outcome in the following way:

They rode an *earned insurgency*, watched by the nation and the world, and forced Kennedy's new administration to confront the boundaries of state and national citizenship and jurisdiction. (p. 372)

Confronting an immobilizing impediment whether social, political, or academic, provides an opportunity for a deeper sense of significance when and if that barrier is overcome, and this idea is at the heart of the concept of the earned insurgency. This type of insurgency shines an attitude-changing light on the potential for future successes in this same area, and it highlights the agency of the individual in their own empowerment. And the reason that such an insurgency is necessary is because the previous lack of empowerment is steeped in deeply held elitist justifications that both the elites and their victims have bought into.

It took knocking against the hard heads of Jim Crow Nation, but it also took knocking on the minds of sharecroppers for them to create the demand for change. Earning the insurgency in the Delta took more than facing down the terror of Jim Crow. It took facing down the logic of Jim Crow, too. (p. 377)

From the top to the bottom, or from the bottom to the top, the mindset and the legitimacy of the status quo needed to change. And the civil rights movement changed

it at the bottom, while legal challenges and political action sought to change it at the top.

Judge Wisdom of the Fifth Circuit Court of Appeals in Baton Rouge, Louisiana, handed down the court's opinion in *United States v. the State of Louisiana* (1963). He said, in effect, that this nation, having refused to educate freed slaves and their descendants because they didn't intend for them to vote, cannot now deny them the vote because they are not so educated. (p. 376)

In the classroom, we believe students can be confronted with the idea that their own creative, imaginative, and discursive agency can dismantle the elitist view of mathematics education and the cognitive segregation under which they have been suffering. Through such agency they can experience an earned insurgency that can serve as a bottom-up paradigm shift, which we argue needs to complement the top-down aspect of any curricular intervention. Moses (2009) called this *working the demand side* of the movement for educational reform.

Mathematics in a Convivial Context

Understanding this perspective has led us to work on curricular interventions that we believe are an empowering set of cognitive tools for mathematics that have creative, imaginative, and discursive agency at their core. Another term for these types of tools is what Ivan Illich (1973) calls *convivial*. Convivial tools involve agency and interagency in ways that are both internally and externally directed.

I consider conviviality to be individual freedom realized in personal interdependence and, as such, an intrinsic ethical value. (p. 24)

To Illich, a convivial approach is in opposition to the purely top-down institutional approach. Although, sadly, he believes most interventions in urban populations are not convivial.

The city child is born into an environment made up of systems that have a different meaning for their designers than for their clients. The inhabitant of the city is in touch with thousands of systems, but only peripherally with each... Learning by primary experience is restricted to self-adjustment in the midst of packaged commodities... People know what they have been taught, but learn little from their own doing. (p. 73)

The alternative to this is the convivial society, which is the balance between top-down and bottom-up approaches:

What is fundamental to a convivial society is not the total absence of manipulative institutions and addictive goods and services, but the balance between those tools which create the specific demands they are specialized to satisfy and those complementary, enabling tools which foster self-realization. The first set of tools produces according to abstract plans for men in general; the other set enhances the ability of people to pursue their own goals in their unique way. (p. 37)

In our work, our goal is a similar effort to provide students with creative and imaginative agency inside of an educational setting filled with discourse and collaborative activities. We facilitate this using computer science tools and computational thinking within a microworld that we are calling an epistemic playground (Figure 1).

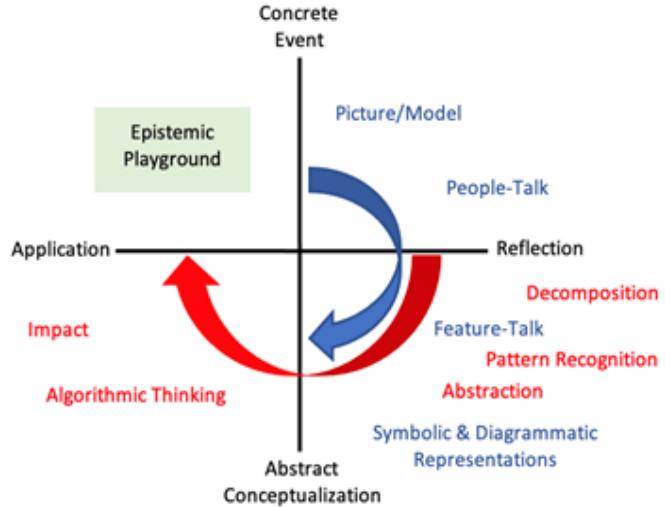


Figure 1: The epistemic playground in our experiential learning cycle.

We argue that learning a challenging new idea is an active process that can be characterized by a developmental learning cycle. When learning occurs that involves a challenging concept, a person must make an intellectual accommodation for the concept by integrating it into their own broader intellectual understanding of the domain in question. This active accommodation from the learner will involve the student doing the work of exploring, analysing, and probing a new idea until it becomes familiar enough to be abstracted or generalized so the concept can be applied appropriately in whatever task is subsequently given to the student. The student's exploration, analysis, and examination that makes up the student's intellectual work is, by in large, an internalized process that is aided by different types of educational resources. Figure 1 gives a picture of how we see those resources playing their part when the 5-Step developmental cycle of the AP (blue) along with CT interventions (red) are fully in place.

In the experiential learning cycle that we have adapted from the Algebra Project's model, the introduction to the conceptual material starts with the active experience of a concrete event, which is then modelled in a physical way, such as through a picture, chart, or graph. This opens the door to informal and formal discourse about the event that directs the student into progressively deepening reflections. CT activities are introduced during these reflections that assist the student in representing the mathematical features abstractly and symbolically. In this way, these steps offer students a bridge from a concrete external event to something that involves internalized conceptual understandings. When students are not provided with such a bridge, we believe its absence makes it more difficult for students to build the appropriate intellectual scaffolding a mathematical concept may require.

When students are successful at producing appropriate intellectual structures, it allows them to generalize a concept and apply it across different scenarios. What this means is that the student is able to take what they have internalized, and then externalize it in multiple ways that are relevant, giving the student the ability to explore how what they have learned can produce various types of impacts. This is displayed in the 3rd quadrant of the learning cycle diagram (Figure 1). From a constructivist viewpoint, this stage of the learning process is no less important than the earlier stage, because it is here that the student can actually explore being creative and imaginative when determining new ways to apply the newly acquired concept. Because of this, we argue that entering straight into a testing (playground) phase after learning some new abstract idea is not the best way to help students get a firm grip on concepts that may be difficult to thoroughly digest. Students need opportunities to chew on an idea before being tested on how well they have digested it.

Then, in the 4th and final quadrant of the experiential learning cycle, the student can engage in convivial explorations inside what we call an epistemic playground. This playground is the place where a student is in a safe place to explore and experiment with ideas, similar to how in a physical playground a student can enter a sandbox and make and break constructions in a playful and creative context. We have found that this type of safe conceptual space can be realized inside of a microworld that is designed to provide mathematical constructs that students can make use of inside of a virtual epistemic sandbox.

We, like Papert (1980), define a microworld to be a digital environment where students have tools that they can use in creative ways to explore concepts related to a specific conceptual domain. A microworld might involve programming, and it might not, however, in our research we are focused on creating a programming microworld that is focused on exploring mathematics and CT concepts and activities. Papert built his microworld using the Logo programming language. Ours is built using Python. In both cases the environments support open-ended explorations of the student.

We believe that by adding activities that involve CT concepts and programming activities, students can engage in the full developmental cycle shown above. The cycle involves internalization and externalization, reflection and application, discourse and reasoning, rigorous analysis and abstraction, as well as imaginative and creative play. An interesting feature about the developmental cycle as we have outlined it, is that it starts with a shared concrete event, and when it progresses all the way to the epistemic playground students are able to engage in explorations and experiments that can also be shared as concrete events with other students. The developmental cycle begins with a community of learners sharing ideas and explorations after experiencing as a group a concrete math-rich activity, and it ends in a shared communal context as well, but this time by engaging in an epistemic playground.

Using the microworld model as a context, CT concepts and programming activities can be constructive, but they are not culturally neutral. Students will have been exposed to both positive and negative examples of CT and programming artifacts, and technology

in general, and this brings with it a challenge to understand what Papert described as the criterion for appropriable activities. If an activity is not appropriable because of negative affective connotations, then that activity will not provide opportunities for creative constructions and affirming internalization or externalizations. Papert lists three principles that determine appropriable activities: the continuity principle, the power principle, the principle of cultural resonance (1980, p. 54). Understanding these principles is an important part in introducing new technologies in a constructive way.

The continuity principle argues that appropriable activities will connect with some “well-established personal knowledge” that comes from those involved with the activity. The power principle establishes that one must be involved in work that is personally meaningful and that could not be done as well in other available activities. The principle of cultural resonance states that the activity must “make sense in terms of a larger social context.” With these principles in mind, the challenge is always to think critically about how new technologies are introduced, and to find a holistic, well designed, socially informed and culturally sensitive approach. Finding a way to integrate material and practices that affirm the unique identities and cultures of the students involved is always a good first step.

Conclusion

Mathematics should be viewed as an activity over which everyone can feel some amount of ownership. It is a tool of a type of regimented language as put forth by Quine (1981), that anyone can claim as their own. But for this to be realizable in each student’s experience with mathematics, there must be an opportunity for the type of authentic conceptualization with the tool that involves creative, imaginative, discursive, and convivial activities. This in turn, we argue, can be supported effectively using a specific type of developmental learning cycle that we have described in this paper. This learning cycle involves both opportunities for constructive internalization and externalization through shared events and discourse that involve computational thinking activities and a programming microworld. What this type of engagement leads to is opportunities for individual and collective agency as well as the beginning of the development of a constructive mathematics identity. A mathematics identity that is based on the idea of cognitive integration and a growth mindset, instead of the fixed mindset and cognitive segregation. With this different mindset, students can rightfully and authentically challenge the educational systems and assumptions that undermine them, and to embrace an intellectual earned insurgency that seeks to empower them.

It is important to note that this work was focused on middle school mathematics and early Algebra. We intend to continue this work in high school mathematics, focusing on Algebra II and early Calculus.

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