Deeply Dissecting Locational Marginal Prices in Look-Ahead Economic Dispatch

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***Abstract*—In the real-time electricity market, look-ahead security constrained economic dispatch (SCED) plays a substantial role in ensuring the sound operation of the electricity spot market with the high penetration of renewable energy. It is crucial to appreciate the effect of unit operation constraints on locational marginal prices (LMPs) in real-time electricity market. To this end, this paper establishes the look-ahead SCED model, derives the LMP formulae under the look-ahead SCED optimization model, and anatomizes the impact of unit operation constraints on the LMP and its components. These formulae and analysis can help market participants to evaluate their own benefits based on their own unit physical parameters. Finally, the correctness of the analysis in this paper is verified via a 3-bus system.**

***Index Terms*—Electricity market, unit operation constraint, look-ahead economic dispatch, locational marginal price.**

NOMENCLATURE

Indices

g	Index of generators.
l	Index of transmission lines.
n	Index of buses.
t	Index of intervals.

Sets

$\mathscr{P}_t$	Set of generator scheduling signals in interval t, $\mathscr{P}_t = \{\mathbf{p}_t, \cdots, \mathbf{p}_{t+W-1}\}$.
$\mathscr{T}_t$	Set of intervals in scheduling window t, $\mathscr{T}_t = \{t, \cdots, t+W-1\}$.
$\mathscr{T}$	Set of intervals in scheduling horizon, $\mathscr{T} = \{1, \cdots, T\}$.
$\mathscr{X}_g$	Operational region of generator g.

Parameters

$\mathbf{a}_{g,t}$	Coefficient vector for generator g in the system-wide constraints in interval time t.
$\mathbf{b}_t$	Limit vector for the system-wide constraints in interval time t.
$F_l^{\min}, F_l^{\max}$	Min/max power flow limits for line l.
G	Total number of generators.
M	Total number of transmission lines.
N	Total number of buses.
$p_g^{\min}, p_g^{\max}$	Min/max generating limits of generator g.
$r_g^{\text{down}}, r_g^{\text{up}}$	Ramp down/up rate limits of generator g.
T	Total number of intervals in look-ahead horizon.
T_{ln}	The (l,n)th entry of the power transfer distribution factor (PTDF) matrix $\mathbf{T}$.
W	Scheduling window size, $W \leq T$.

Variables

$c_{g,t}(\cdot)$	Bid-in cost (energy cost) function of generator g in interval t.
$\hat{\mathbf{d}}_t$	Forecasted net demand vector in interval t.
$\hat{d}_{n,t}$	Forecasted net demand at bus n in interval t.
$\pi_{n,t}^{\text{LMP}}$	LMP at bus n in interval t.
$\mathbf{p}_t$	Output power vector and matrix of generators in interval t.
$\mathbf{P}$	Output power matrix of generators.
$p_{g,t}$	Output power of generator g in interval t.

Dual Variables

λ_t	Dual variable (shadow price) of the system power balance constraint in interval t.
$\mu_{l,t}^{\min}, \mu_{l,t}^{\max}$	Dual variables (shadow prices) of the transmission line l power flow upper and lower limit constraint in interval t.
$\nu_{n,t}^{\min}, \nu_{n,t}^{\max}$	Dual variables (shadow prices) of the minimum and maximum generation output constraints in interval t.
$\xi_{n,t}^{\min}, \xi_{n,t}^{\max}$	Dual variables (shadow prices) of the ramp down and ramp up limit constraints in interval t.

I. INTRODUCTION

A. Research Motivation

COMPARED with the single period market model, the look-ahead optimization model can reduce costs [1], improve social welfare [2], [3], and can also reduce infeasibilities, i.e., not being able to satisfy load or reserve requirements, which can have reliability and pricing implications, such as reduced area control error and avoidance of price spikes [4]. Several regional transmission organizations (RTOs)/independent system operators (ISOs), such as New York ISO (NYISO) and California ISO (CAISO), have already implemented look-ahead SCED; some RTOs/ISOs, e.g., the electric reliability council of Texas (ERCOT), have been contemplating this methodology [1], [5]–[9]. However, there are few references on the LMP in look-ahead security constrained economic dispatch (SCED). In view of this, this paper bridges this gap, which systematically studies the LMP in multi-period optimization and the impact of unit operation constraints on the LMP.

B. Literature Review

Look ahead ED, also known as dynamic ED, originated in the 1980s [10], [11]. Travers *et al.* [12] applied constructive dynamic programming to solving dynamic ED. A multi-period market design for markets with intertemporal constraints was proposed in [13]. In [14], an algorithm for solving dynamic ED was presented. Multi-interval real-time markets in the context of U.S. ISOs and pricing multi-interval dispatch under uncertainty were studied in [5], [15], [16]. References [17], [18] discussed multi-interval pricing models. LMPs in look-ahead ED are analyzed in [2], and [19] researched data perturbation-based sensitivity analysis of real-time look-ahead ED. However, these references did not consider the network loss in the derivation of LMPs.

C. Contributions and the Organization of the Paper

The main differences between our paper and [2] are as follows. Network loss is considered in the derivation of the LMP in this paper, so the formulae obtained are more universal. The principles of the LMP and its components are analyzed in detail based on the Karush-Kuhn-Tucker (KKT) conditions and the envelope theorem from the classical and generator perspectives, respectively. The LMPs calculated from two different perspectives are equal and the influence of unit operation constraints on LMPs are demonstrated via a 3-bus system.

Our main contributions are as follows.

1) *Derivation of the LMP in look-ahead SCED and its relationship to dual multipliers*. The LMP containing network loss is derived in look-ahead SCED, the connotations of the LMP are combed, the inherent relationship between the LMP and the dual multipliers (variables) of constraints is elaborated.
2) *Expressing LMP from the perspective of the generator and anatomizing factors affecting the LMP*. The connotations of the LMP are explained from the perspective of the generator. The influence of unit operation constraints on the LMP is analyzed in depth.
3) *Numerical examples supporting the analysis and derivation in this paper*. We use a 3-bus system to show in detail the calculation of LMP in look-ahead SCED and the impact of unit operation constraints on the LMP.

II. Problem Statement and Formulation of Look-ahead SCED

A. Look-ahead SCED Formulation

1) Generic abstract form: A generic abstract form of look-ahead SCED can be expressed as

$$\min_{\mathbf{P}} \quad f = \sum_{t=1}^{W} \sum_{g=1}^{G} c_{g,t}\left(p_{g,t}\right) \tag{1}$$

$$\text{s. t.} \quad \sum_{g=1}^{G} \mathbf{a}_{g,t} p_{g,t} \leq \mathbf{b}_t, \qquad \forall t, \tag{2}$$

$$\left(c_{g,t}, p_{g,t}\right) \in \mathscr{X}_g, \qquad \forall g, \tag{3}$$

The cost function of generator g in interval t is assumed to be convex and continuously differentiable throughout the paper.

Constraints (2) describe *system-wide* constraints, which are coupled constraints and enforce system-wide requirements, and which usually include power balance, ancillary service, and transmission constraints, and so on. In (2), all constraints are expressed as inequalities. The operational constraints, or *private* inter-temporal constraints, for generator g are aggregated into (3), including the cost function, capacity (output) constraints, ramping constraints, maximum energy constraints, and state-of-charge constraints for energy storage resources.

A typical rolling-window dispatch implementation of a multi-interval real-time market, which is based on look-ahead SCED, is illustrated in Fig. 1. In interval t, the SCED has a look-ahead dispatch window of W intervals, which are represented by $\mathscr{T}_t = \{t, \cdots, t+W-1\}$. In Interval t, look-ahead SCED generates W generator scheduling signals, i.e., $\mathscr{P}_t = \{\mathbf{p}_t, \cdots, \mathbf{p}_{t+W-1}\}$; however, only the scheduling decisions for the binding interval t, which are called the *binding interval* are used; scheduling decisions for the rest of intervals $\mathscr{T}_t = \{t, \cdots, t+W-1\}$, which are called the *advisory intervals* are advisory. Similarly, we refer to prices for the binding intervals as settlement prices; prices for advisory intervals as advisory prices, which are not used for settlement. As interval t goes on, $\mathscr{T}_t$ rolls through the entire dispatch period $\mathscr{T}$.

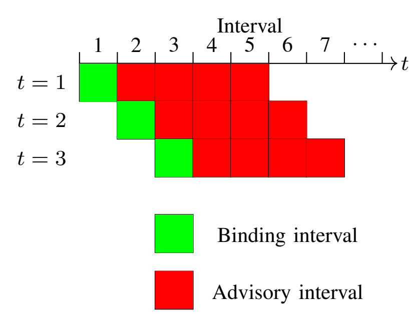

Fig. 1. Look-ahead SCED with the window size $W = 5$.

2) Concrete form: Based on the reduced-form DC power flow, the generic abstract form shown in (1)–(3) can be converted into the following concrete form parameterized by the time-varying demand:

$$
\begin{aligned}
\min_{\mathbf{P}} \quad & f(\hat{\mathbf{d}}) = \sum_{t=1}^{W}\sum_{n=1}^{N} c_{n,t}\left(p_{n,t}\right) && (4)\\
\text{s.t.} \quad & \sum_{n=1}^{N}\left(p_{n,t}-\hat{d}_{n,t}\right) = \mathrm{loss}(\mathbf{p}_t,\hat{\mathbf{d}}_t) : \lambda_t, \forall t, && (5)\\
& \sum_{n=1}^{N-1} T_{ln}\left(p_{n,t}-\hat{d}_{n,t}\right) \le F_l^{\max} \;\; : \mu_{l,t}^{\max},\ \forall l,\ \forall t, && (6)\\
& \sum_{n=1}^{N-1} T_{ln}\left(p_{n,t}-\hat{d}_{n,t}\right) \ge F_l^{\min} \;\; : \mu_{l,t}^{\min},\ \forall l,\ \forall t, && (7)\\
& p_{n,t} \le p_n^{\max} \;\; : \nu_{n,t}^{\max},\ \forall n,\ \forall t, && (8)\\
& p_{n,t} \ge p_n^{\min} \;\; : \nu_{n,t}^{\min},\ \forall n,\ \forall t, && (9)\\
& p_{n,t}-p_{n,t-1} \le r_n^{\text{up}} \;\; : \xi_{n,t}^{\max},\ \forall n,\ 2\le t\le T, && (10)\\
& p_{n,t}-p_{n,t-1} \ge -r_n^{\text{down}} \;\; : \xi_{n,t}^{\min},\ \forall n,\ 2\le t\le T. && (11)
\end{aligned}
$$

System power balance constraints including linearized loss are described by (5). Transmission line constraints are represented by (6) and (7). Generating limits of generators and ramp rate limits of generators are denoted by (8)–(9) and (10)–(11), respectively.

III. Derivation of the LMP and Factors Affecting the LMP in Look-Ahead SCED

A. Derivation of the LMP

The associated Lagrangian function of the look-ahead SCED formulated in the model (4)–(11) is written as

$$
\begin{aligned}
&\mathscr{L}\left(\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\mu}^{\max},\boldsymbol{\mu}^{\min},\boldsymbol{\nu}^{\max},\boldsymbol{\nu}^{\min},\boldsymbol{\xi}^{\max},\boldsymbol{\xi}^{\min},\hat{\mathbf{d}}\right)\\
&= \sum_{t=1}^{W}\sum_{n=1}^{N} c_{n,t}\left(p_{n,t}\right)\\
&- \sum_{t=1}^{W}\lambda_t\left[\sum_{n=1}^{N}\left(p_{n,t}-\hat{d}_{n,t}\right)-\mathrm{loss}(\mathbf{p}_t,\hat{\mathbf{d}}_t)\right]\\
&- \sum_{t=1}^{W}\sum_{l=1}^{M}\mu_{l,t}^{\max}\left[F_l^{\max}-\sum_{n=1}^{N-1}T_{ln}\left(p_{n,t}-\hat{d}_{n,t}\right)\right]\\
&- \sum_{t=1}^{W}\sum_{l=1}^{M}\mu_{l,t}^{\min}\left[\sum_{n=1}^{N-1}T_{ln}\left(p_{n,t}-\hat{d}_{n,t}\right)+F_l^{\min}\right]\\
&- \sum_{t=1}^{W}\sum_{n=1}^{N}\nu_{n,t}^{\max}\left(p_n^{\max}-p_{n,t}\right)\\
&- \sum_{t=1}^{W}\sum_{n=1}^{N}\nu_{n,t}^{\min}\left(p_{n,t}-p_n^{\min}\right)\\
&- \sum_{t=2}^{W}\sum_{n=1}^{N}\xi_{n,t}^{\max}\left(r_n^{\text{up}}-p_{n,t}+p_{n,t-1}\right)\\
&- \sum_{t=2}^{W}\sum_{n=1}^{N}\xi_{n,t}^{\min}\left(p_{n,t}-p_{n,t-1}+r_n^{\text{down}}\right).
\end{aligned}
\tag{12}
$$

The Lagrange dual function of the look-ahead SCED formulated in the model (4)–(11) is defined as

$$
\begin{aligned}
&\mathscr{D}\left(\boldsymbol{\lambda},\boldsymbol{\mu}^{\max},\boldsymbol{\mu}^{\min},\boldsymbol{\nu}^{\max},\boldsymbol{\nu}^{\min},\boldsymbol{\xi}^{\max},\boldsymbol{\xi}^{\min},\hat{\mathbf{d}}\right)\\
&:= \inf_{\mathbf{P}\in\mathcal{D}}\mathscr{L}\left(\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\mu}^{\max},\boldsymbol{\mu}^{\min},\boldsymbol{\nu}^{\max},\boldsymbol{\nu}^{\min},\boldsymbol{\xi}^{\max},\boldsymbol{\xi}^{\min},\hat{\mathbf{d}}\right)
\end{aligned}
\tag{13}
$$

The Lagrange dual problem of the look-ahead SCED formulated in the model (4)–(11) is defined to be

$$
\begin{aligned}
\max \quad & \mathscr{D}\left(\boldsymbol{\lambda},\boldsymbol{\mu}^{\max},\boldsymbol{\mu}^{\min},\boldsymbol{\nu}^{\max},\boldsymbol{\nu}^{\min},\boldsymbol{\xi}^{\max},\boldsymbol{\xi}^{\min},\hat{\mathbf{d}}\right) && (14)\\
\text{s.t.} \quad & \boldsymbol{\mu}^{\max}\ge\mathbf{0},\boldsymbol{\mu}^{\min}\ge\mathbf{0},\boldsymbol{\nu}^{\max}\ge\mathbf{0},\boldsymbol{\nu}^{\min}\ge\mathbf{0}, && (15)\\
& \boldsymbol{\xi}^{\max}\ge\mathbf{0},\boldsymbol{\xi}^{\min}\ge\mathbf{0}. && (16)
\end{aligned}
$$

Definition 1 (LMP). *$\pi_{n,t}^{\text{LMP}}$ is defined as the least cost to the system of serving the next increment of the load at that bus in that interval.*

Mathematically, $\pi_{n,t}^{\text{LMP}}$ measures the sensitivity of the optimal value of the DC optimal power flow (OPF) problem with respect to the load at bus n in interval t, i.e.,

$$
\pi_{n,t}^{\text{LMP}} := \frac{\partial}{\partial d_{n,t}}\sum_{t=1}^{W}\sum_{n=1}^{N} c_{n,t}(p_{n,t}(\hat{d}_{n,t})). \tag{17}
$$

According to the *envelope theorem*, (17) can be transformed into*

$$
\begin{aligned}
\pi_{n,t}^{\text{LMP}} &:= \frac{\partial}{\partial d_{n,t}}\mathscr{L}\Big(\mathbf{P}^*(\hat{\mathbf{d}}),\boldsymbol{\lambda}^*(\hat{\mathbf{d}}),\boldsymbol{\mu}^{\max*}(\hat{\mathbf{d}}),\boldsymbol{\mu}^{\min*}(\hat{\mathbf{d}}),\\
&\qquad \boldsymbol{\nu}^{\max*}(\hat{\mathbf{d}}),\boldsymbol{\nu}^{\min*}(\hat{\mathbf{d}}),\boldsymbol{\xi}^{\max*}(\hat{\mathbf{d}}),\boldsymbol{\xi}^{\min*}(\hat{\mathbf{d}}),\hat{\mathbf{d}}\Big)\\
&= \frac{\partial}{\partial d_{n,t}}\mathscr{L}\Big(\mathbf{P}^*,\boldsymbol{\lambda}^*,\boldsymbol{\mu}^{\max*},\boldsymbol{\mu}^{\min*},\boldsymbol{\nu}^{\max*},\boldsymbol{\nu}^{\min*},\\
&\qquad \boldsymbol{\xi}^{\max*},\boldsymbol{\xi}^{\min*},\hat{\mathbf{d}}\Big)\\
&= \underbrace{\lambda_t^*}_{\text{energy component}} + \underbrace{\left[-\sum_{l=1}^{M}\left(\mu_{l,t}^{\max*}-\mu_{l,t}^{\min*}\right)T_{ln}\right]}_{\text{congestion component}}\\
&\quad + \underbrace{\left[\lambda_t^*\frac{\partial\,\mathrm{loss}(\mathbf{p}_t,\hat{\mathbf{d}}_t)}{\partial d_{n,t}}\right]}_{\text{loss component}}, \quad n=1,\cdots,N-1
\end{aligned}
\tag{18}
$$

$$
\pi_{N,t}^{\text{LMP}} := \lambda_t^* \tag{19}
$$

According to the stationarity of the KKT conditions, i.e.,

$$
\begin{aligned}
&\nabla_{\mathbf{P}}\mathscr{L}\Big(\mathbf{P}^*,\boldsymbol{\lambda}^*,\boldsymbol{\mu}^{\max*},\boldsymbol{\mu}^{\min*},\boldsymbol{\nu}^{\max*},\boldsymbol{\nu}^{\min*},\boldsymbol{\xi}^{\max*},\\
&\qquad \boldsymbol{\xi}^{\min*},\hat{\mathbf{d}}\Big)=\mathbf{0},
\end{aligned}
\tag{20}
$$

*That $\mathscr{L}$ is continuously differentiable is assumed.

we have

$$\frac{\partial \mathscr{L}}{\partial p_{n,t}} = \frac{dc_{n,t}\left(p_{n,t}^{*}\right)}{dp_{n,t}} - \lambda_t^{*} + \left[\lambda_t^{*}\frac{\partial\, \mathrm{loss}(\mathbf{p}_t, \hat{\mathbf{d}}_t)}{\partial p_{n,t}}\right] + \sum_{l=1}^{M}\left(\mu_{l,t}^{\max *} - \mu_{l,t}^{\min *}\right)T_{ln} + \left(\nu_{n,t}^{\max *} - \nu_{n,t}^{\min *}\right) + \left(\xi_{n,t}^{\max *} - \xi_{n,t+1}^{\min *}\right) + \left(\xi_{n,t+1}^{\min *} - \xi_{n,t}^{\min *}\right) = 0, \quad n = 1, \cdots, N-1. \tag{21}$$

$$\frac{\partial \mathscr{L}}{\partial p_{N,t}} = \frac{dc_{N,t}\left(p_{N,t}^{*}\right)}{dp_{N,t}} - \lambda_t^{*} + \left[\lambda_t^{*}\frac{\partial\, \mathrm{loss}(\mathbf{p}_t, \hat{\mathbf{d}}_t)}{\partial p_{n,t}}\right] + \left(\nu_{N,t}^{\max *} - \nu_{N,t}^{\min *}\right) + \left(\xi_{N,t}^{\max *} - \xi_{N,t+1}^{\min *}\right) + \left(\xi_{N,t+1}^{\min *} - \xi_{N,t}^{\min *}\right) = 0. \tag{22}$$

By combining (18)–(19) and (21)–(22), we can get the formula of $\pi_{n,t}^{\mathrm{LMP}}$ derived from the perspective of the generator as follows:

$$\pi_{n,t}^{\mathrm{LMP}} = \underbrace{\frac{dc_{n,t}\left(p_{n,t}^{*}\right)}{dp_{n,t}}}_{\text{marginal cost component}} + \underbrace{\left[\lambda_t^{*}\frac{\partial\, \mathrm{loss}(\mathbf{p}_t, \hat{\mathbf{d}}_t)}{\partial d_{n,t}}\right]}_{\text{loss component}} + \underbrace{\left[\lambda_t^{*}\frac{\partial\, \mathrm{loss}(\mathbf{p}_t, \hat{\mathbf{d}}_t)}{\partial p_{n,t}}\right]}_{\text{second loss component}} + \underbrace{\left(\nu_{n,t}^{\max *} - \nu_{n,t}^{\min *}\right)}_{\text{generating limit component}} + \underbrace{\left(\xi_{n,t}^{\max *} - \xi_{n,t+1}^{\max *}\right) + \left(\xi_{n,t+1}^{\min *} - \xi_{n,t}^{\min *}\right)}_{\text{ramping component}}, \quad n = 1, \cdots, N-1. \tag{23}$$

$$\pi_{N,t}^{\mathrm{LMP}} = \underbrace{\frac{dc_{N,t}\left(p_{N,t}^{*}\right)}{dp_{N,t}}}_{\text{marginal cost component}} + \underbrace{\left[\lambda_t^{*}\frac{\partial\, \mathrm{loss}(\mathbf{p}_t, \hat{\mathbf{d}}_t)}{\partial p_{N,t}}\right]}_{\text{second loss component}} + \underbrace{\left(\nu_{N,t}^{\max *} - \nu_{N,t}^{\min *}\right)}_{\text{generating limit component}} + \underbrace{\left(\xi_{N,t}^{\max *} - \xi_{N,t+1}^{\max *}\right) + \left(\xi_{N,t+1}^{\min *} - \xi_{N,t}^{\min *}\right)}_{\text{ramping component}}, \quad n = N. \tag{24}$$

B. Scrutinizing the LMP From Different Perspectives

In this paper, the LMP and its components are discussed in detail from the classical and generator perspectives, respectively, which is seldom done in the existing literature.

1) Classical Perspective: It can be seen that from (18)–(19), the LMP is decomposed into three parts, i.e., energy component, congestion component, and loss component. The energy component is the marginal cost of energy at the slack bus N (same for all buses). The congestion component is the marginal cost of congestion at bus n relative to bus N. The loss component is the marginal cost of loss at bus n relative to bus N. Each component can be positive or negative. Due to losses, LMPs may depend on the selection of the slack bus. However, under the condition that the network loss is ignored, LMPs do not depend on the selection of the slack bus, but the components do.

2) Perspective of the generator: From (23), it can be seen that the LMP is composed of marginal cost component, loss component, second loss component, generating limit component, and ramping component. To be concrete, $\pi_{n,t}^{\mathrm{LMP}}$ is related to the marginal cost of generator n in interval t, the shadow prices of the capacity constraints for generator n in interval t, and the shadow prices of the ramp constraints for generator n in intervals t and $t+1$. Equation (23) explains the structure of the LMP from the perspective of the generator, which is helpful to analyze and understand the influence of unit operation constraints on the LMP. The formulae of the LMP derived from these two different perspectives are not contradictory; they are essentially the same instead.

C. Factors Affecting the LMP

From (23)–(24), we can see that $\pi_{n,t}^{\mathrm{LMP}}$ is positively correlated with $\frac{dc_{n,t}(p_{n,t})}{dp_{n,t}}$, $\nu_{n,t}^{\max *}$ and $\xi_{n,t}^{\max *}$ $\xi_{n,t+1}^{\min *}$, and negatively correlated with $\nu_{n,t}^{\min *}$ $\xi_{n,t+1}^{\max *}$ and $\xi_{n,t}^{\min *}$.

Applying the complementary slackness of the KKT to the model (4)–(11), we have

$$\mu_{l,t}^{\max}\left[F_l^{\max} - \sum_{n=1}^{N-1} T_{ln}\left(p_{n,t} - \hat{d}_{n,t}\right)\right] = 0, \ \forall l, \ \forall t, \tag{25}$$

$$\mu_{l,t}^{\min}\left[\sum_{n=1}^{N-1} T_{ln}\left(p_{n,t} - \hat{d}_{n,t}\right) + F_l^{\min}\right] = 0, \ \forall l, \ \forall t, \tag{26}$$

$$\nu_{n,t}^{\max}\left(p_n^{\max} - p_{n,t}\right) = 0, \ \forall l, \ \forall t, \tag{27}$$

$$\nu_{n,t}^{\min}\left(p_{n,t} - p_n^{\min}\right) = 0, \ \forall l, \ \forall t, \tag{28}$$

$$\xi_{n,t}^{\max}\left(r_n^{\mathrm{up}} - p_{n,t} + p_{n,t-1}\right) = 0, \ \forall l, \ , 2 \le t \le T, \tag{29}$$

$$\xi_{n,t}^{\min}\left(p_{n,t} - p_{n,t-1} + r_n^{\mathrm{down}}\right) = 0, \ \forall l, \ 2 \le t \le T. \tag{30}$$

When there is no transmission congestion, that is, the constraints (6)–(7) are non-binding, from (25)–(26) we know that $\mu_{l,t}^{\max} = 0$ and $\mu_{l,t}^{\min} = 0$. Therefore, the LMPs of all buses except for the slack bus only have the energy component and the loss component; if the network loss is further ignored, then the LMPs of all buses in the whole network are the same. The energy component directly reflects the basic price of the balanced system power, which is the marginal power generation cost of the marginal unit of the system. When the branches of the power grid are congested, that is, the constraints (6)–(7) are binding, the LMPs of different buses are coupled with each other through the power transfer distribution factor (PTDF) matrix of the system, and the congestion price of the branch will be proportional to the PTDF, resulting in the difference in the LMP of all buses of the network, thus generating the price signal to guide users' electricity consumption behavior.

When the output of unit n is between the maximum and minimum outputs of the unit and the ramp up and ramp down rates of the unit are sufficient, i.e., constraints (8)–(11) are non-binding, according to (27)–(30), we can have $\nu_{n,t}^{\max}=0$, $\nu_{n,t}^{\min}=0$, $\xi_{n,t}^{\max}=0$, $\xi_{n,t}^{\min}=0$. Therefore, the LMP of bus n equals to the marginal cost of unit n in the absence of binding ramping and generating limit constraints.

Based on (18) and (23), we have

$$
\begin{aligned}
\underbrace{\lambda_t}_{\text{energy component}} &= \underbrace{\frac{dc_{n,t}\left(p_{n,t}^*\right)}{dp_{n,t}}}_{\text{marginal cost component}} \\
&+ \underbrace{\left(\nu_{n,t}^{\max *}-\nu_{n,t}^{\min *}\right)}_{\text{generating limit component}} \\
&+ \underbrace{\left(\xi_{n,t}^{\max *}-\xi_{n,t+1}^{\min *}\right)+\left(\xi_{n,t+1}^{\min *}-\xi_{n,t}^{\min *}\right)}_{\text{ramping component}} - \\
&\underbrace{\left[-\sum_{l=1}^{M}\left(\mu_{l,t}^{\max}-\mu_{l,t}^{\min}\right)T_{ln}\right]}_{\text{congestion component}} + \underbrace{\left[\lambda_t^*\frac{\partial\,\text{loss}(\mathbf{p}_t,\hat{\mathbf{d}}_t)}{\partial p_{N,t}}\right]}_{\text{second loss component}}, \\
&n=1,\cdots,N-1,
\end{aligned} \tag{31}
$$

From (31), it can be seen that the energy component, the congestion component and the loss component are related to the marginal cost of the generator and the operation constraints of generators, which can deepen our understanding of the LMP.

IV. Simulation Results

As shown in Fig. 2, we consider a 3-bus system with two generating units and one load, and the network loss is ignored in these case studies. Bus 3 is chosen as the slack bus. $W=5$. The physical parameters of the network and units and the forecasted net demand at Bus 3 are shown in Tables I–III, respectively.

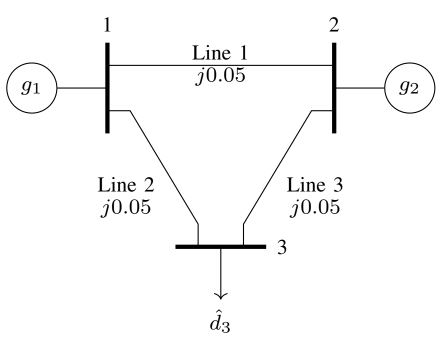

Fig. 2. A 3-bus system.

TABLE I
NETWORK PARAMETERS OF THE 3-BUS SYSTEM

Line	X_l (p.u.)	$F_l^{\max}$ (MW)	$F_l^{\min}$ (MW)
1	0.05	40	-40
2	0.05	160	-160
3	0.05	160	-160

TABLE II
PHYSICAL PARAMETERS OF UNITS

Unit	$p_n^{\min}$ (MW)	$p_n^{\max}$ (MW)	r_n^{up}, r_n^{down} (MW/5min)	MC[1] ($/MWh)
1	0	150	4, 4	25
2	0	60	40, 40	50

[1] MC stands for the marginal cost. In this example, $c_{g,t}(\cdot)$ is assumed to be a linear function.

TABLE III
FORECASTED NET DEMAND

	1	2	3	4	5
$\hat{d}_{3,t}$ (MW)	160	165	150	155	156

The following cases are researched.

Case 1: All all parameters are listed in Tables I–III.

Case 2: Except for $p_1^{\max}=110\text{MW}$, other parameters are the same as those in Tables I–III.

Case 3: Except for $r_1^{\text{up}}=0.4\,\text{MW}/5\text{min}$ and $r_1^{\text{down}}=0.4\,\text{MW}/5\text{min}$, other parameters are the same as those in Tables I–III.

The outputs of generators and the power flow of branches in Interval 1 for Cases 1–3 are shown in Tables IV, VII, and X, respectively. The results of $\pi_{n,1}^{\text{LMP}}$ calculation for Cases 1–3 from the classical perspective and the perspective of the generator are shown in Tables V–VI, Tables VIII–IX, and Tables XI–XII, respectively.

TABLE IV
OUTPUTS OF GENERATORS AND THE POWER FLOW OF BRANCHES IN INTERVAL 1 FOR CASE 1

$p_{1,1}$ (MW)	$p_{2,1}$ (MW)	$F_{11,1}$ (MW)	$F_{13,1}$ (MW)	$F_{23,1}$ (MW)
140	20	40	100	60

TABLE V
$\pi_{n,1}^{\text{LMP}}$ CALCULATION FOR CASE 1 FROM THE CLASSICAL PERSPECTIVE

Bus	Energy Component ($/MWh)	Congestion Component ($/MWh)	$\pi_{n,1}^{\text{LMP}}$ ($/MWh)
1	37.5	-12.5	25
2	37.5	12.5	50
3	37.5	/	37.5

V. Conclusion

In this paper, the abstract and concrete forms of look-ahead SCED including network loss are formulated, and the expressions of the LMP and their relations with the corresponding dual variables are derived from the classical and generator perspectives, respectively. Furthermore, the principles of the LMP and its components are analyzed in detail based on the the KKT conditions and the envelope theorem from the classical and generator perspectives, respectively. Moreover,

TABLE VI
$\pi_{n,1}^{\text{LMP}}$ CALCULATION FOR CASE 1 FROM THE PERSPECTIVE OF THE GENERATOR ($/MWh)

Bus/Unit	MC	$\nu_{n,1}^{\max*}$	$\nu_{n,1}^{\min*}$	$\xi_{n,1}^{\max*}$	$\xi_{n,2}^{\max*}$	$\xi_{n,2}^{\min*}$	$\xi_{n,1}^{\min*}$	$\pi_{n,1}^{\text{LMP}}$
1	25	0	0	0	0	0	0	25
2	50	0	0	0	0	0	0	50

TABLE VII
OUTPUTS OF GENERATORS AND THE POWER FLOW OF BRANCHES IN INTERVAL 1 FOR CASE 2

$p_{1,1}$ (MW)	$p_{2,1}$ (MW)	$F_{11,1}$ (MW)	$F_{13,1}$ (MW)	$F_{23,1}$ (MW)
110	50	20	90	70

TABLE VIII
$\pi_{n,1}^{\text{LMP}}$ CALCULATION FOR CASE 2 FROM THE CLASSICAL PERSPECTIVE

Bus	Energy Component ($/MWh)	Congestion Component ($/MWh)	$\pi_{n,1}^{\text{LMP}}$ ($/MWh)
1	50	0	50
2	50	0	50
3	50	/	50

TABLE IX
$\pi_{n,1}^{\text{LMP}}$ CALCULATION FOR CASE 2 FROM THE PERSPECTIVE OF THE GENERATOR ($/MWh)

Bus/Unit	MC	$\nu_{n,1}^{\max*}$	$\nu_{n,1}^{\min*}$	$\xi_{n,1}^{\max*}$	$\xi_{n,2}^{\max*}$	$\xi_{n,2}^{\min*}$	$\xi_{n,1}^{\min*}$	$\pi_{n,1}^{\text{LMP}}$
1	25	25	0	0	0	0	0	50
2	50	0	0	0	0	0	0	50

TABLE X
OUTPUTS OF GENERATORS AND THE POWER FLOW OF BRANCHES IN INTERVAL $t = 1$ FOR CASE 3

$p_{1,1}$ (MW)	$p_{2,1}$ (MW)	$F_{11,1}$ (MW)	$F_{13,1}$ (MW)	$F_{23,1}$ (MW)
135.8	24.2	37.2	98.6	61.4

TABLE XI
$\pi_{n,1}^{\text{LMP}}$ CALCULATION FOR CASE 3 FROM THE CLASSICAL PERSPECTIVE

Bus	Energy Component ($/MWh)	Congestion Component ($/MWh)	$\pi_{n,1}^{\text{LMP}}$ ($/MWh)
1	50	0	50
2	50	0	50
3	50	/	50

TABLE XII
$\pi_{n,1}^{\text{LMP}}$ CALCULATION FOR CASE 3 FROM THE PERSPECTIVE OF THE GENERATOR ($/MWh)

Bus/Unit	MC	$\nu_{n,1}^{\max*}$	$\nu_{n,1}^{\min*}$	$\xi_{n,1}^{\max*}$	$\xi_{n,2}^{\max*}$	$\xi_{n,2}^{\min*}$	$\xi_{n,1}^{\min*}$	$\pi_{n,1}^{\text{LMP}}$
1	25	0	0	0	0	25	0	50
2	50	0	0	0	0	0	0	50

the influence factors of the LMP in look-ahead SCED are anatomized. These derivations and analyses help to analyze and deeply understand the principles of the LMP from different perspectives, especially from the perspective of the generator, and provide certain references for market participants on the impact of unit physical parameters on revenue.

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